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$Sp_L(6) \times U_Y(1)$ extension of the electroweak theory

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The generation structure of quarks and leptons is incorporated with the electroweak theory in an $Sp_L(6)\times U_Y(1)$ model. In this model, the first-generation fermions are naturally light because their masses are generated only as two-loop corrections. Also the inequality $m_u < m_d$ is related to $m_t > m_b$.

In two previous reports,[1,2] we proposed an extended electroweak model based on the gauge group $SU_L(2)\times SU_R(3)$. Here $SU_R(3)$ not only is a part of the electroweak group, but also plays the role of a horizontal symmetry group.[3] It was shown that the model results in small masses of the first-generation fermions, as we will summarize briefly in the following.[4] There, right-handed quarks and leptons are assigned to triplets of $SU_R(3)$, and left-handed ones to singlets. For instance, (u_R,c_R,b_R) and (d_R,s_R,t_R) form $\underline{3}_R$ and $\underline{\bar{3}}_R$, respectively. If we further assume that there is one Higgs multiplet ϕ, transforming as $(\underline{2}_L,\underline{3}_R)$, then the Higgs-Yukawa couplings of quarks, for instance, become

$$\sum_I (f_I\bar{u}_R q_L^I\phi + g_I\bar{d}_R q_L^I\phi^\dagger)+\text{H.c.} \quad , \tag{1}$$

where q_L^I are left-handed doublets with the generation index $I=1$, 2, 3, and where u_R (d_R) represents the right-handed $\underline{3}$ ($\underline{\bar{3}}$), generically. In (1) we can always make $f_1=g_1=0$ by redefining q_L^I, and hence q_L^1, or the first generation, stays massless naturally after the $SU_L(2)$ breakdown. It was then noticed[2] that this family can subsequently become massive via two-loop corrections when a set of scalar fields is introduced. As a bonus, the inequality $m_u < m_d$ is tied in naturally with $m_t > m_b$. It was also realized that the symmetry group including $SU_c(3)$ can be extended even further to

$$SU(4)\times Sp_L(6)\times Sp_R(6) \quad , \tag{2}$$

under which the left-handed and the right-handed fermions transform as $[\underline{4},\underline{6},\underline{1}]$ and $[\underline{4},\underline{1},\underline{6}]$, respectively.[5] The symmetry group (2) contains the $SU_L(2)\times SU_R(3)$, the $SU_L(2)\times SU_R(2)$, and thus the standard $SU_L(2)\times U_Y(1)$ groups as its subgroups, with the feature that, except for the right-handed neutrinos (N_R), the model contains no new, unobserved fermions. It is in this sense that (2) is the maximal symmetry group unifying the generation structure with the electroweak symmetry.

In spite of these features, however, the $SU_L(2)\times SU_R(3)$ model gives one unacceptable result. Since q_L^1, i.e., both u_L and d_L, cannot mix with the other generation at the tree level, the resulting Cabibbo angle θ_C, induced simultaneously with m_1, can be no larger than m_1/m_2, where m_i is a typical value of the ith-family mass. This difficulty made us investigate other subgroups of (2), among which the group

$$SU_c(3)\times Sp_L(6)\times U_Y(1) \tag{3}$$

turned out to resolve the θ_C problem while reproducing the previous features.

We now wish to describe the model [Eq. (3)] in detail. The fermion assignments under (3) are obtained, by decomposition, from those under the group (2). They are therefore $q_L^i[\underline{3},\underline{6};+\frac{1}{6}]$, $l_L^i[\underline{1},\underline{6};-\frac{1}{2}]$, $u_R^I[\underline{3},\underline{1};+\frac{2}{3}]$, $d_R^I[\underline{3},\underline{1};-\frac{1}{3}]$, and $e_R^I[\underline{1},\underline{1};-1]$, where $i=1,\ldots,6$ is the spinor index of $Sp_L(6)$ (Refs. 4,5) and $I=1,2,3$ is the generation index. A summary of particle assignments is given in Table I. Notice that N_R need not be introduced. It is now evident that our model requires merely a minimal extension of the standard model. Upon the breakdown $Sp_L(6)\to SU_L(2)$, q_L^i (l_L^i) decomposes to three doublets $q_L^{\rho a}$ ($l_L^{\rho a}$), where $\rho=1,2$ is the $SU_L(2)$ spinor index, $a=1,2,3$ corresponds to the generation, and $i=a+3(\rho-1)$ (Refs. 4,6).

The breakdowns $Sp_L(6)\to[SU_L(2)]^3\to SU_L(2)$ can be induced by two $[\underline{1},\underline{14};0]$ Higgs fields $H^{(\alpha)}$, $\alpha=1,2$ (Ref. 7). Since $Sp_L(6)$ contains direct flavor-changing neutral-current (FCNC) interactions, the scale of the first breaking is restricted to be $\gtrsim 100$ TeV. The second scale may be as low as ~ 1 TeV,[8] although we will assume, for simplicity, that the two breaking scales are identical. The standard Higgs multiplet is embedded into $[\underline{1},\underline{6};+\frac{1}{2}]$. We actually introduce two of them $\phi^{(\alpha)i}$, $\alpha=1,2$, so that the Higgs-Yukawa couplings are

$$-\sum_I\sum_\alpha [f_I^{(\alpha)}\phi_i^{(\alpha)\dagger}\bar{d}_R^I q_L^i + f_L'^{(\alpha)}\eta_{ij}\phi^{(\alpha)i}\bar{u}_R^I q_L^j + g_I^{(\alpha)}\phi_i^{(\alpha)\dagger}\bar{e}_R^I l_L^i]+\text{H.c.} \quad , \tag{4}$$

TABLE I. The particle assignments under the gauge group $SU_c(3)\times Sp_L(6)\times U_Y(1)$: here q_L^i (l_L^i) are left-handed quarks (leptons), and $(u_R^I)=(u_R,c_R,t_R)$, for example, are right-handed quarks. The scalar fields $H^{(\alpha)}$, $\phi^{(\alpha)}$, and χ (χ') carry $Sp_L(6)$ spinor indices $i,j=1,\ldots,6$. The generation index $I=1,2,3$ is not a group index, nor is $\alpha=1,2$.

	q_L^i	l_L^i	u_R^I	d_R^I	e_R^I	$H^{(\alpha)ij}$	$\phi^{(\alpha)i}$	χ (χ')
$SU_c(3)$	3	1	3	3	1	1	1	3
$Sp_L(6)$	6	6	1	1	1	14	6	1
$U_Y(1)$	$+\frac{1}{6}$	$-\frac{1}{2}$	$+\frac{2}{3}$	$-\frac{1}{3}$	-1	0	$+\frac{1}{2}$	$-\frac{1}{3}$

where η_{IJ} is the metric of the $Sp_L(6)$ spinor.[4,5] Since we can always redefine the right-handed fermions d_R^I, etc., such that

$$f_1^{(\alpha)}=f_1'^{(\alpha)}=g_1^{(\alpha)}=0 \ , \qquad (5)$$

(4) naturally results in one massless generation of fermions. Under the breakdown $Sp_L(6)\rightarrow SU_L(2)$, $\phi^{(\alpha)I}$ decompose similarly to $\phi^{(\alpha)\rho a}$. When $SU_L(2)\times U_Y(1)$ is subsequently broken by $\langle\phi^{(\alpha)2a}\rangle$ $(\equiv v_a^{(\alpha)})$, the fermion mass terms are induced to be

$$-\sum_I\sum_a[\bar{\lambda}_R^I M_{Ia}^{(\lambda)}\lambda_L^a+\text{H.c.}],\ \ \lambda=u,d,e \ , \qquad (6)$$

where $M^{(\lambda)}$ is of the form

$$M^{(\lambda)}=\begin{pmatrix}0&0&0\\ *&*&*\\ *&*&*\end{pmatrix} \ . \qquad (7)$$

This structure shows clearly that there exists one generation of massless fermions. Notice that λ_R^1 ($\lambda=u$, d, or e), instead of λ_L^1 as in (1), are decoupled from $M^{(\lambda)}$. The Cabibbo angle θ_C is consequently nonvanishing even at the tree level, and thus free from the difficulty of the previous model.[1,2]

We will make one technical remark concerning (7).[4] At first sight $v_a^{(\alpha)}$, $\alpha=1,2$, seem to have the most general structure except that $v_1^{(1)}$, for example, can be chosen to be real. Nevertheless, since they behave as $\underline{3}$ of the $SU_L(3)$ subgroup of $Sp_L(6)$, we can apply $SU_L(3)$ rotations to $v_a^{(\alpha)}$ so that they may have simpler forms. Such transformations, however, are restricted by the condition

$$\vec{T}_L\langle H^{(\alpha)}\rangle=0 \ , \qquad (8)$$

where $\vec{T}_L$ are the $SU_L(2)$ generators. To study this restriction, let us write down $\vec{T}_L$ and the $SU_L(3)$ generators explicitly,

$$\vec{T}_L=\vec{\sigma}\otimes 1_3,\ \ \Lambda^1=\begin{pmatrix}\lambda^1 & \\ & -\lambda^{1*}\end{pmatrix},\ \text{etc.} \ , \qquad (9)$$

where 1_3 is the 3×3 unit matrix. Among the Λ's, only Λ^2, Λ^5, and Λ^7 commute with $\vec{T}_L$, and hence can be used to reduce $v_a^{(\alpha)}$ without conflicting with (8). Notice that rotations generated by them are all real rotations. We may first make $v_2^{(1)}=$real by a Λ^7 rotation, and then $v_1^{(1)}=0$ by a Λ^2 rotation. No subsequent rotations by Λ^2, Λ^5, or Λ^7 can reduce $v_a^{(\alpha)}$ any further. Hence, $v_a^{(\alpha)}$ can be reduced at best to

$$(v^{(1)})=(0,*,*),\ \ (v^{(2)})=(*,*,*) \ , \qquad (10)$$

which does not change (7). Were it not for the restriction (8), we could reduce $v_a^{(\alpha)}$ further. It is well known that an SU(2) doublet (v_1,v_2) can be transformed to $(v,0)$, $v=$real, by an SU(2) rotation. Since this applies to each SU(2) subgroup of $SU_L(3)$, $v_a^{(\alpha)}$ could be reduced, by an $SU_L(3)$ rotation R, from (10) to

$$(v_0^{(1)})=(0,0,*),\ \ (v_0^{(2)})=(0,*,*) \ , \qquad (11)$$

yielding

$$M_0^{(\lambda)}=\begin{pmatrix}0&0&0\\ 0&*&*\\ 0&*&*\end{pmatrix} \qquad (7')$$

instead of (7). It is (8), therefore, that made θ_C nonvanishing, because without (8) we would have concluded from (7′) that $\theta_C=0$. The transformation R, though not applicable, is nevertheless useful for studying the diagonalization of (7). It actually shows the finiteness of radiatively induced masses, as will be explained later.

To induce the first-generation masses radiatively, it is necessary to introduce more fields which couple with fermions, since otherwise an unbroken discrete chiral symmetry, under which $u_R\rightarrow -u_R$, etc., would emerge and prohibit the mass terms. Here we follow the simplest procedure proposed in Ref. 2, by introducing scalar fields. The quantum numbers of them can be determined from those of the q_L^2 and $(u_R,d_R)^2$ channels to be $[\underline{3},\underline{1};-\frac{1}{3}]$. We can then write the Yukawa couplings,

$$\left[\frac{G_L}{2}\eta_{IJ}q_L^{IT}(-C^{-1})q_L^J+\sum_{I,J}G_R^{IJ}d_R^{IT}(-C^{-1})u_R^J\right]\chi+\text{H.c.} \ , \qquad (12)$$

where χ is the scalar field in question and C is the charge-conjugation matrix. In (12) the Levi-Civita tensor of $SU_c(3)$ is not written down explicitly. The same quantum number allows actually another set of lepton-quark Yukawa couplings,

$$\left[F_L\eta_{IJ}l_L^{IT}(-C^{-1})q_L^J+\sum_{I,J}F_R^{IJ}e_R^{IT}(-C^{-1})u_R^J\right]\chi'^{\dagger}+\text{H.c.} \ , \qquad (13)$$

where χ' is also $[\underline{3},\underline{1};-\frac{1}{3}]$. Here we will avoid the proton-decay problem by demanding that the baryon number be a good global symmetry and that χ and χ' carry $-\frac{2}{3}$ and $+\frac{1}{3}$, respectively.[9] It is straightforward to rewrite (12) and (13) as[4]

$$\sum_{I,J}[\tilde{G}_L^{IJ}\tilde{d}_L^{IT}(-C^{-1})\tilde{u}_L^J+\tilde{G}_R^{IJ}\tilde{d}_R^{IT}(-C^{-1})\tilde{u}_R^J]\chi+\left[\sum_{I,J}\tilde{F}_L^{IJ}\tilde{e}_L^{IT}(-C^{-1})\tilde{u}_L^J-F_L\tilde{\nu}_L^I(-C^{-1})\tilde{d}_L^I+\sum_{I,J}\tilde{F}_R^{IJ}\tilde{e}_R^I(-C^{-1})\tilde{u}_R^J\right]\chi'^{\dagger}+\text{H.c.} \qquad (14)$$

in terms of the tree mass eigenstates $\tilde{d}_L^I$, etc. In (14) $\tilde{G}_L^{IJ}$, etc., are products of G_L, etc., with the unitary matrices diagonalizing (7). A less trivial fact is that

$$\tilde{G}_L^{1I}=\tilde{G}_L^{I1}=\tilde{F}_L^{1I}=\tilde{F}_L^{I1}=0\ \ \text{for } I=2,3 \ . \qquad (15)$$

This is a direct consequence of the fact that χ (χ') is an $Sp_L(6)$ singlet, and can be proved explicitly by the use of the rotation R and (7′), mentioned previously.[4] The finiteness of induced masses also guarantees (15) because the one-loop correction shown in Fig. 1(a) would otherwise yield divergent first-generation masses in conflict with the theorem.[10] Actually the one-loop correction vanishes identically due to (15). We thus have to study two-loop corrections given in Fig. 1(b). The relation (15) proves their finiteness again, while the included $Sp_L(6)$ gauge interactions

$$g_L V_\mu^A\sum_{\lambda,\lambda'}\bar{\lambda}_L^I\gamma^\mu(\tilde{T}^A)^{\lambda(I)\lambda'(J)}\lambda_L'^J \ , \qquad (16)$$

where $\{\lambda,\lambda'\}$ runs over all pairs of $\{\tilde{u},\tilde{d}\}$ or $\{\tilde{\nu},\tilde{e}\}$, make them nonvanishing. Explicit evaluation yields finally that m_d, m_u, and m_e are the absolute values of

$$\frac{g_L{}^2}{128\pi^4}\sum 2\tilde{G}_R^{1I*}m_I^u[-(\tilde{T}^A)^{u(J)u(I)}\tilde{G}_L^{KJ}(\tilde{T}^A)^{d(K)d(1)}+(\tilde{T}^A)^{d(J)u(I)}\tilde{G}^{JK}(\tilde{T}^A)^{u(K)d(1)}]\Psi\left(\frac{M_A{}^2}{m_\chi{}^2}\right) , \tag{17a}$$

$$\frac{g_L{}^2}{128\pi^4}\sum\left[2\tilde{G}_R^{I1*}m_I^d[-(\tilde{T}^A)^{d(J)d(I)}\tilde{G}_L^{JK}(\tilde{T}^A)^{u(K)u(1)}+(\tilde{T}^A)^{u(J)u(I)}\tilde{G}_L^{KJ}(\tilde{T}^A)^{d(K)u(1)}]\Psi\left(\frac{M_A{}^2}{m_\chi{}^2}\right)\right.$$
$$\left.+F_R^{I1*}m_I^e[-(\tilde{T}^A)^{e(J)e(I)}\tilde{F}_L^{JK}(\tilde{T}^A)^{u(K)u(1)}+(\tilde{T}^A)^{\nu(J)e(I)}F_L(\tilde{T}^A)^{d(J)u(1)}]\Psi\left(\frac{M_A{}^2}{m_{\chi'}{}^2}\right)\right] , \tag{17b}$$

$$\frac{g_L{}^2}{128\pi^4}\sum 3\tilde{F}_R^{1I*}m_I^u[-(\tilde{T}^A)^{u(J)u(I)}\tilde{F}_L^{KJ}(\tilde{T}^A)^{e(K)e(1)}+(\tilde{T}^A)^{d(J)u(I)}F_L(\tilde{T}^A)^{\nu(J)e(1)}]\Psi\left(\frac{M_A{}^2}{m_{\chi'}{}^2}\right) , \tag{17c}$$

respectively. In (17), m_I^u, etc., for $I=2,3$, denote the tree masses of the Ith family, while M_A, m_χ, and $m_{\chi'}$ are the masses of $\tilde{V}_\mu^A$, χ, and χ'. The approximation $m_I^u, m_I^d, m_I^e \ll M_A, m_\chi, m_{\chi'}$ is used to reach (17), and the function $\Psi(z)$ is defined by[11]

$$(1-z)\,l(1-z^{-1})-(1-z^{-1})\,l(1-z)+\tfrac{1}{2}\ln^2 z-\ln z+(\pi^2/6)(z-z^{-1}) , \tag{18}$$

where $l(x)=x+x^2/4+\cdots$ is the Spence function.[12] The relative factors 1, 2, and 3 in (17) are due to the color structures $\delta_\alpha^\gamma\delta_\gamma^\beta=\delta_\alpha^\beta$, $\epsilon_{\alpha\gamma\delta}\epsilon^{\beta\gamma\delta}=2\delta_\alpha^\beta$, and $\delta_\alpha^\beta\delta_\beta^\alpha=3$.

In conclusion, the present model reproduces the qualitative features of Refs. 1 and 2: The masses of the first generation (m_1) vanish at the tree level naturally. Then the two-loop corrections induce m_1 from those of the third (m_3) as in (17), and hence m_1/m_3 contains at least $O(g_L{}^2/128\pi^4)\sim 10^{-4}$. Moreover, the inequality $m_u < m_d$ is related naturally with $m_t > m_b$ when the sums of (17) are saturated by t and b quarks. The advantage of the present model over the previous one[2] is that it yields these preferable consequences without entailing a small Cabibbo angle ($\lesssim m_1/m_2$). On the other hand, the use of two Higgs multiplets $\phi^{(\alpha)I}$ is somewhat arbitrary.

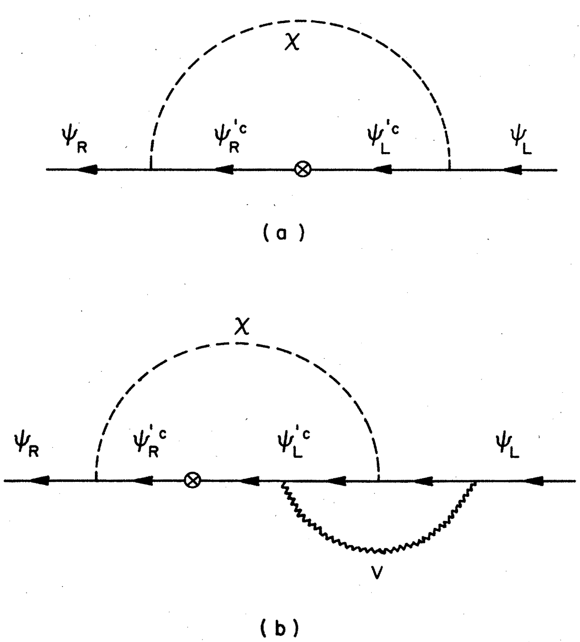

FIG. 1. Diagrams to induce the first-generation masses radiatively. The broken and wavy lines correspond, respectively, to the χ (χ') field and to the $Sp_L(6)$ gauge field V. The ψ (ψ') fields are the tree mass eigenstates of either quarks or leptons with appropriate chiralities, and ψ^c, for instance, denotes $C\bar{\psi}^T$. The symbol ⊗ on the fermion lines shows that, in the numerator of the propagator, only the mass term contributes. The one-loop correction (a) vanishes identically, while the two-loop correction (b) remains finite, for the first generation.

Several comments are in order:[13]

First of all, the exhaustive study of (2) and its subgroups is now finished. Among them the present model is the least extension of the standard model, and yet yields the specific results on the first generation without any apparent difficulties. Among the new $Sp_L(6)$ gauge fields, certain combinations of $[SU(2)]_L{}^3$ ones may be observed in the supercollider experiments because their masses can possibly be $\gtrsim 1$ TeV.

Secondly, to the best of our knowledge, there is no simple gauge group which can unify this model without destroying the results on the masses. Fortunately, however, because of the embeddings $SU_L(2)\subset[SU_L(2)]^3\subset Sp_L(6)$, the $Sp_L(6)$ gauge coupling ($=3\alpha_2$) becomes as large as α_s, and hence does not necessarily require a unification. We can therefore unify the remainder through $SU_c(3)\times U_Y(1)\subset SU(4)\times SU_R(2)\subset SU(6)$ to explain the ratio α/α_s, while reproducing the present model as a low-energy effective theory. The group theory and the renormalization-group analysis will be given in Ref. 4, showing that the SU(6) scale is $\sim 4\times 10^{14}$ GeV. This unification also sets restrictions on the Yukawa couplings (4), (12), and (13), but their implications on masses, as well as on proton decays, are yet to be studied.

Thirdly, a single ϕ Higgs field would keep even the second-generation massless as was noted. However, no mechanism is found yet to produce their reasonably large masses through radiative corrections.

To generalize the present models for an arbitrary number of generations N_G, all one has to do are to replace Sp(6) by $Sp(2N_G)$ and to make corresponding changes elsewhere.

Finally we would like to point out that the symplectic group Sp(2N) is a very interesting candidate for the horizontal symmetry. Since $Sp(2N)$ has no complex representations, its gauge theories are always anomaly free. The group $Sp_L(2N)$ can contain N generations naturally according to the breakdown $Sp_L(2N)\to[SU_L(2)]^N\to SU_L(2)$. Last, but not least, unlike the $SU(N)$ or $SO(4N+10)$

groups, no superfluous fermions ever need to be introduced, in models such as (2) or its subgroups. These features, in addition to the occurrence of naturally light families, suggest that the symplectic group deserves more attention than it is usually given.[14] It is our belief that the symplectic group plays an important role in the attempt to understand the generation structure.

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[1]T. K. Kuo and N. Nakagawa, Phys. Lett. **138B**, 135 (1984).

[2]T. K. Kuo and N. Nakagawa, Phys. Lett. **140B**, 63 (1984).

[3]See, for example, T. K. Kuo and N. Nakagawa, Nucl. Phys. **B232**, 263 (1984), for a list of earlier works on the horizontal symmetry.

[4]For a more complete discussion, see T. K. Kuo and N. Nakagawa, Purdue University Report No. PURD-TH-84-14 (unpublished).

[5]For a review of the symplectic group Sp(6) see, for example, Ref. 4 or H. Georgi, *Lie Algebras in Particle Physics: From Isospin to Unified Theories* (Benjamin, New York, 1982).

[6]For later convenience, we note that, under the $SU_L(3)$ subgroup, $q_L^{1a} \equiv u_{La}$ transforms as $\underline{3}$, while $q_L^{2a} \equiv d_L^a$ transforms as $\underline{\bar{3}}$. (See Ref. 4.)

[7]Here $\underline{14}$ is the antisymmetric second-rank spinor.

[8]This restriction is imposed by the "absence" of the second W and/or Z^0.

[9]Certain extended models, such as $SU(6) \times Sp_L(6)$ which will be mentioned later and in Ref. 4, contain a large mass scale ($\sim 10^{15}$ GeV). There χ' can be identified safely with χ provided that it is superheavy.

[10]S. Weinberg, Phys. Rev. D **5**, 1962 (1972).

[11]The definition of $\Psi(z)$ given in Ref. 2 [denoted by $2G(z)$ there] is erroneous, and is corrected in (18).

[12]J. Schwinger, *Particles, Sources and Fields* (Addison-Wesley, Reading, Mass., 1973), Vol. 2.

[13]More complete accounts of each item are given in Ref. 4.

[14]Some previous applications may be found in G. Racah, Phys. Rev. **63**, 367 (1943); **76**, 1352 (1949); A. Goshen and H. J. Lipkin, in *Spectroscopic and Group Theoretical Methods in Physics*, edited by F. Bloch *et al.* (North-Holland, Amsterdam, 1968), and references cited therein; P. Kramer, M. Moshinsky, and T. H. Seligman, in *Group Theory and Its Application*, edited by E. M. Loebl (Academic, New York, 1975); Dzh. L. Chkareuli, Pis'ma Zh. Eksp. Teor. Fiz. **29**, 166 (1979) [JETP Lett. **29**, 148 (1979)].