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Bounds on mixing between light and heavy gauge bosons

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Upper bounds are derived for the mixing between light and heavy gauge bosons in terms of the closeness of the observed W or Z mass to the SU(2)×U(1) prediction m_0. The mixing angles between light and heavy bosons with masses m_1 and m_2, respectively, are less than $[(m_0^2-m_1^2)/(m_2^2-m_0^2)]^{1/2}$. Existing data imply a limit of $\lesssim 0.04$ for the mixing angle between the W and a 1-TeV heavy boson. A similar result holds for the Z if there are no Higgs multiplets with $I > \frac{1}{2}$ with significant vacuum expectation values.

The standard SU(2)×U(1) electroweak model[1] has been extremely successful in its predictions for charged- and neutral-current phenomena.[2] In addition, the predictions[3]

$$M_{W_0}=83.0^{+2.9}_{-2.7}\text{ GeV},\quad M_{Z_0}=93.8^{+2.4}_{-2.2}\text{ GeV} \tag{1}$$

of the standard model (with $\sin^2\theta_W=0.217\pm0.014$ and only Higgs doublets and singlets) are in excellent agreement with the results

$$M_{W_1}=\begin{cases}80.9\pm1.5\pm2.4\text{ GeV}, & \text{UA1 (Ref. 4)}\ ,\\ 81.0\pm2.5\pm1.3\text{ GeV}, & \text{UA2 (Ref. 5)}\ ,\end{cases}$$
$$M_{Z_1}=\begin{cases}95.6\pm1.4\pm2.9\text{ GeV}, & \text{UA1}\ ,\\ 91.9\pm1.3\pm1.4\text{ GeV}, & \text{UA2}\ ,\end{cases} \tag{2}$$

found by the UA1 and UA2 collaborations at CERN.

Despite these successes, it is possible that SU(2)×U(1) is embedded in a larger electroweak group. The closeness of the observed masses of the W and Z to the SU(2)×U(1) predictions does not, by itself, place any constraints on the masses of additional gauge bosons. It seems intuitively likely, however, that this closeness does limit the possible mixing of the W and Z with new bosons, especially if the latter are very heavy (e.g., $M \gtrsim 1$ TeV). Such results have in fact been found in a number of specific models,[6] but the analyses have generally depended on the number of new gauge bosons, the group, the Higgs representations, or the couplings of the new gauge bosons to fermions. In this paper I will derive upper bounds on the mixings between light and heavy gauge bosons that depend only on the deviations $m_0^2-m_1^2$ of the observed boson masses (m_1^2) from their SU(2)×U(1) predictions (m_0^2), and on $m_i^2-m_0^2$, where m_i is the physical mass of the ith heavy boson. The only assumptions needed are that M_{W_0} and M_{Z_0} take their canonical values (i.e., I am assuming that there are no important contributions from Higgs triplets, etc.) and that in the absence of mixing the W and Z would be the lightest bosons. After stating and proving the inequalities I will apply them to the W and Z bosons and compare the results with other phenomenological bounds on mixing.

For orientation, first consider the case of two neutral[7] or charged gauge bosons, B_1^0 (which is the ordinary W_0 or Z_0) and B_2^0. Assume that in the absence of mixing between B_1^0 and B_2^0, the masses would be m_0 and $m_H \geq m_0$, respectively. Including mixing, the Hermitian[8] mass-squared matrix becomes

$$M^2=\begin{pmatrix} m_0^2 & b\\ b^* & m_H^2\end{pmatrix}\ , \tag{3}$$

where $b\equiv|b|e^{i\alpha}$ is an arbitrary mixing mass squared. For $b\neq0$, the physical bosons are

$$\begin{aligned} B_1&=\cos\theta\, B_1^0+e^{i\alpha}\sin\theta\, B_2^0\\ B_2&=-e^{-i\alpha}\sin\theta\, B_1^0+\cos\theta\, B_2^0\end{aligned} \tag{4}$$

and the physical squared masses are m_1^2 and m_2^2. It is easily seen by a direct computation of θ, m_1^2, and m_2^2 that

$$m_1^2 < m_0^2 \leq m_H^2 < m_2^2 \tag{5}$$

and that the mixing angle is given by

$$\tan^2\theta=\frac{m_0^2-m_1^2}{m_2^2-m_0^2}\ . \tag{6}$$

Therefore, θ must be small if the observed masses satisfy $m_1 \lesssim m_0$ and $m_2 >> m_0$.

Now consider the case of n charged or neutral[7] bosons B_i^0, with an $n\times n$ Hermitian[8] mass-squared matrix

$$M^2=\begin{pmatrix} m_0^2 & b_2 & b_3 & \cdots & b_n\\ b_2^* & & & & \\ \vdots & & M_H^2 & & \\ b_n^* & & & & \end{pmatrix}\ , \tag{7}$$

where m_0 is M_{W_0} or M_{Z_0}, M_H^2 is the $(n-1)\times(n-1)$ mass-squared matrix for the $n-1$ heavy bosons, and b_i, $i=2,\ldots,n$ are arbitrary parameters with lead to mixing between the light and heavy bosons. Without loss of generality, we may work in a basis for which M_H^2 is diagonal with elements $a_2,\ldots,a_n$. Furthermore, we may assume $b_i\neq0$, $i=2,\ldots,n$ [if any b_i vanishes, then B_i^0 decouples and the problem reduces to an $(n-1)$-dimensional system] and $a_2<a_3<\cdots<a_n$ (if two or more a's are degenerate all but one of the B_i^0 in the degenerate subspace decouple and may be ignored). Finally, I will make the physical assumption that $m_0^2 \leq a_2$. Then the physical squared masses

m_i^2, $i=1,\ldots,n$, can be shown to satisfy

$$m_1^2 < m_0^2 \leq a_2 < m_2^2 < a_3 < m_3^2 \cdots < a_n < m_n^2 \quad (8)$$

(i.e., the lowest mass is decreased by the mixing while the other masses are increased). Furthermore, the physical gauge bosons are

$$B_i = \sum_{j=1}^{n} (u_i)_j^* B_j^0 \ , \quad (9)$$

where u_i is the ith (normalized) eigenvector of M^2:

$$\sum_k M^2{}_{jk}(u_i)_k = m_i^2 (u_i)_j \ . \quad (10)$$

My major results are then

$$\frac{|(u_1)_i|}{|(u_1)_1|} \leq \left[\frac{m_0^2 - m_1^2}{m_i^2 - m_0^2}\right]^{1/2}, \quad i=2,\ldots,n \quad (11)$$

and[9]

$$|(u_i)_1| < \left[\frac{m_0^2 - m_1^2}{m_i^2 - m_0^2}\right]^{1/2}, \quad i=2,\ldots,n \ . \quad (12)$$

These inequalities imply that for $m_1 \lesssim m_0$, $m_2 >> m_0$, the lightest physical boson is mainly W_0 or Z_0 with very little admixture of other bosons, and conversely the physical heavy bosons have little admixture of the W_0 and Z_0. For $n=2$, (11) becomes an equality, reproducing (6), while for $n>2$, (11) becomes a strict inequality.

I will now sketch the proofs of (8), (11), and (12). It is straightforward to derive an expression for $\det(M^2 - m_i^2)$, from which it is easily seen that m_i^2 can equal neither m_0^2 nor any a_i as long as all of the b_i are nonzero. The signs of the inequalities (e.g., $a_2 < m_2^2 < a_3$) then follow from consideration of the special case in which the b_i's are small.

Equation (11) is derived from the eigenvector equations for u_1, viz.,

$$\begin{aligned} &(m_0^2 - m_1^2)(u_1)_1 + \sum_{j=2}^{n} b_j (u_1)_j = 0 \ , \\ &b_i^*(u_1)_1 + (a_i - m_1^2)(u_1)_i = 0, \quad i=2,\ldots,n \ . \end{aligned} \quad (13)$$

From (13) one finds $(u_1)_1 \neq 0$ and hence

$$\begin{aligned} \frac{|(u_1)_i|^2}{|(u_1)_1|^2} &= \frac{|b_i|^2}{(a_i - m_1^2)^2} \\ &\leq \frac{1}{a_i - m_1^2} \sum_{j=2}^{n} \frac{|b_j|^2}{a_j - m_1^2} = \frac{m_0^2 - m_1^2}{a_i - m_1^2} \\ &\leq \frac{m_0^2 - m_1^2}{m_i^2 - m_0^2} \ , \end{aligned} \quad (14)$$

where the last step follows from (8) and

$$\mathrm{tr} M^2 = \sum_{i=1}^{n} m_i^2 \ .$$

Both inequalities in (14) become equalities for $n=2$ and strict inequalities for $n>2$.

Equation (12) may be derived by inverting the similarity transformation that diagonalizes M^2, which leads to

$$|(u_1)_1|^2 m_1^2 + \sum_{j=2}^{n} |(u_j)_1|^2 m_j^2 = m_0^2 \ . \quad (15)$$

But

$$\sum_{j=1}^{n} |(u_j)_1|^2 = 1 \quad (16)$$

by completeness, so

$$\begin{aligned} |(u_i)_1|^2 &\leq \frac{1}{m_i^2 - m_1^2} \sum_{j=2}^{n} |(u_j)_1|^2 (m_j^2 - m_1^2) = \frac{m_0^2 - m_1^2}{m_i^2 - m_1^2} \\ &< \frac{m_0^2 - m_1^2}{m_i^2 - m_0^2} \ . \end{aligned} \quad (17)$$

The bonds in (11) and (12) are rigorous within the stated assumptions. However, there is a basic difficulty in their application; one needs the value of m_0^2. Even assuming that the only Higgs fields with important vacuum expectation values are SU(2) doublets and singlets, one still needs[10] the value of $\sin^2\theta_W$ in order to predict M_{W_0} and M_{Z_0}. The value of $\sin^2\theta_W$ determined from the neutral-current data[2,3] may, in principle, be affected by the existence of heavier Z bosons. It is difficult to limit such effects in a completely model-independent way, but in view of the excellent agreement between the $SU(2)\times U(1)$ model and a great variety of neutral-current experiments, it seems unlikely that additional bosons could do more than slightly perturb the value of $\sin^2\theta_W$ and therefore those of M_{W_0} and M_{Z_0}. I will therefore assume that the present values and uncertainties in M_{W_0} and M_{Z_0} are correctly given by (1).

From (2) it is apparent that there is no evidence for any deviation from the $SU(2)\times U(1)$ model. To illustrate the results I will take $M_{W_1} > 77.00$ GeV and $M_{Z_1} > 89.2$ GeV from (2), where I have very conservatively added the statistical and systematic errors linearly. I will also use the upper limits $M_{W_0} < 85.9$ GeV and $M_{Z_0} < 96.2$ GeV from (1). Then one finds upper limits on $|(u_1)_i|/|(u_1)_1|$ and $|(u_i)_1|$ of 0.21, 0.077, and 0.038 for $M_{W_i} = 200$, 500, and 1000 GeV, respectively, with similar results for Z bosons. These constraints are already strong enough to imply that one can ignore mixings with the W_0 and Z_0 when considering the production and decay of $m_i \gtrsim 1$-TeV bosons at future colliders.[11] In the future one expects significant improvements in the measurements of M_{W_1} and M_{Z_1}. A reasonable scenario would be that no deviation from $SU(2)\times U(1)$ is observed at the 1-GeV level (with the uncertainties dominated by m_0). In that case, the mixing angles would be bounded by 0.07, 0.03, and 0.01 for $m_i = 200$, 500, or 1000 GeV.

The advantage of these limits is their generality; more stringent limits can often be found in specific models. For example, there are a number of limits on the mixing angle ζ between the charged bosons $W_L^\pm$ and $W_R^\pm$ in the $SU(2)_L \times SU(2)_R \times U(1)$ model. If the electron neutrino is Dirac, then the recent measurement at TRIUMF[12] of the e^+ spectrum in polarized μ^+ decay yields $M_{W_R} > 380$ GeV and $|\zeta| < 0.045$ for infinite M_{W_R}, while earlier muon- and β-decay measurements imply[13] $|\zeta| < 0.06$. If the right-handed neutrino is Majorana and massive, these limits do not apply. However, the success of current-algebra predictions for nonleptonic kaon and hyperon decays suggests[14] $M_{W_R} > 300$ GeV and $|\zeta| < 0.004$, but this involves some hadronic uncertainties and assumes Cabibbo-type mixing angles for the right-handed currents. Wolfenstein[15] has recently argued on the basis of universality between the quark and lepton vector currents that $|\zeta| < 0.005$, again assuming Cabibbo mixing angles for the right-handed quarks. The K_L-K_S mass difference requires[16] $M_{W_R} > 1.6$ TeV for small mixing angles. Finally, the y distributions in νN and $\bar{\nu} N$ deep-inelastic scattering imply[17] $|\zeta| \lesssim 0.10$, independent of the nature of ν_R and the right-handed mixings. Limits on the mixing between the neutral bosons in $SU(2)_L \times SU(2)_R \times U(1)$ are much weaker.[2,18]

Even with existing data the bounds in (11) and (12) provide nontrivial constraints on the mixing of the W_0 and Z_0 with heavier bosons. Improved determinations of M_{W_1}, M_{Z_1}, and $\sin^2\theta_W$ should tighten these constraints considerably [assuming that no deviation from $SU(2) \times U(1)$ is observed]. In the special case of $W_L^{\pm}$-$W_R^{\pm}$ mixing, more stringent limits can be derived by searching for the effects of right-handed currents. The present bounds are much more general, however: they depend only on the observed and predicted masses m_1 and m_0 and on the assumption that the W_0 and Z_0 are the lightest bosons in the absence of mixing. No assumptions concerning the number of new bosons, the gauge group, or the couplings of the new bosons to fermions are needed, and the only assumptions on the Higgs representations concern their $SU(2) \times U(1)$ properties. The only real loophole, which applies to the Z but not the W, concerns the value of m_0^2. One could evade the limits for the mixing of the Z_0 by postulating the existence of new Higgs representations [e.g., an SU(2) triplet with $Y/2 = 1$] with large enough vacuum expectation values to raise m_0^2 appreciably. The closeness of m_1^2 to the canonical value would then be due to an accidental compensation between the effects of the new Higgs representations and the mixing with the heavy bosons. Although very unnatural, such models are hard to rigorously exclude.

The inequalities in this paper may, in principle, be applied to other Hermitian matrices, such as the mass-squared matrices $mm^\dagger$ and $m^\dagger m$ relevant to left- and right-handed fermions, respectively. However, such bounds are only useful if one has an independent prediction for m_0^2.

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