New paths through the desert: Improving on minimal SU(5)

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We present a model with a particle spectrum identical to minimal SU(5) below the SU(5)-breaking mass scale M_X, where the gauge group is the standard $SU_C(3)\times[SU_L(2)\times U(1)]_{GWS}$ (GWS = Glashow-Weinberg-Salam). However, unlike minimal SU(5), a nonstandard hypercharge embedding into E_6 allows our model to have M_X large enough to agree with present limits on proton decay and simultaneously have a weak mixing angle in agreement with the experimental value extracted from the W and Z^0 masses.

Recent experimental results on $W^{\pm}$ and Z^0 physics and proton decay may be in disagreement with the predictions of the minimal SU(5) grand unified theory.[1] The masses of W and Z^0 have been measured and give the currently most accurate determination of the weak mixing angle[2]

$$\sin^2\theta_W(M_W)\Big|_{\text{expt}}=0.226\pm0.011 ,$$

while a recent careful analysis of minimal SU(5) gives[3]

$$\sin^2\theta_W(M_W)\Big|_{\text{theory}}=0.214^{+0.004}_{-0.003} .$$

The currently quoted experimental lower limit on the proton lifetime is[4]

$$\tau(p\to e^+\pi^0)>1\times10^{32}\ \text{yr} ,$$

while minimal SU(5) predicts[3]

$$\tau(p\to e^+\pi^0)=4.5\times10^{29\pm1.7}\ \text{yr} .$$

If the present experimental results hold up, then minimal SU(5) must be modified.[5] However, since minimal SU(5) has a number of outstandingly successful predictions, it seems sensible to try to keep these successes while attempting to eliminate its few shortcomings. Hence we shall here (i) retain the desert hypothesis, which implies no intermediate mass scales between M_X and M_W; (ii) keep the low-energy (i.e., below M_X) gauge group $SU_C(3)\times[SU_L(2)\times U(1)]_{GWS}$ (GWS = Glashow-Weinberg-Salam) with couplings g_3, g_2, and g_1; and (iii) retain the light-fermion spectrum of the minimal model at low energy, thus no supersymmetry, etc.

At first sight theses constraints seem to imply that a renormalization-group (RG) analysis of g_3, g_2, and g_1 can give nothing but the standard-model predictions.[6] However, there *is* a way to avoid this seemingly inevitable scenario. In fact, we can do this in a way that satisfies constraints (i)—(iii) and at the same time agrees with the experimental values of τ_p and $\sin^2\theta_W$. This is accomplished with an unusual though not unaesthetic charge assignment which relaxes the requirement that the charge operator Q be a generator of SU(5).[7]

In the minimal SU(5) model, g_3, g_2, and g_1 start off equal (up to the appropriate normalization) at M_X, and after evolving according to the standard scenario they give the usual results. We can modify the standard results while keeping the requirements (i)—(iii), if we can give different initial values to the couplings at M_X. In other words, SU(5) can no longer be a complete unification group, but instead only part of a partial unification group which we choose to be $SU(5)\times U_A(1)\times U_B(1)$.[8] [This group can and will be unified directly into a larger simple group E_6 (Refs. 9 and 10).]

The standard charge operator is $Q=T_3+Y$, where Y is an SU(5) generator, $\text{diag}(\frac{1}{3},\frac{1}{3},\frac{1}{3},-\frac{1}{2},-\frac{1}{2})$. Thus the main issue here is whether or not the hypercharge can be embedded into E_6 differently. We will prove in the following that there is an embedding

$$Q=T_3+Y' , \tag{1}$$

where

$$Y'\equiv aA+bB+cY \tag{2}$$

with $(a,b,c)\neq(0,0,1)$. Here A and B are the diagonal generators of E_6 orthogonal to the SU(5) subgroup and will be specified below.

Let us show the existence of the solution. To be specific, we define the charges A and B through the group decomposition

$$E_6\to SO(10)\times U_A(1)\to SU(5)\times U_A(1)\times U_B(1) . \tag{3}$$

Correspondingly, the $\underline{27}$, for example, is decomposed as

$$\begin{aligned}\underline{27}&\to(\underline{1})_4+(\underline{10})_{-2}+(\underline{16})_1\\&\to(\underline{1})_{4,0}+[(\underline{5})_{-2,2}+(\underline{\bar{5}})_{-2,-2}]\\&\quad+[(\underline{1})_{1,-5}+(\underline{\bar{5}})_{1,3}+(\underline{10})_{1,-1}] .\end{aligned} \tag{4}$$

On further reduction to $SU(3)\times SU(2)\times U(1)$, the choice of Eq. (1) with

$$Y'=\tfrac{1}{4}A+\tfrac{1}{20}B-\tfrac{1}{5}Y \tag{5}$$

yields

$$\underline{27}\underset{Y'}{\rightarrow}(\underline{1},\underline{1})_{+1}+[(\underline{3},\underline{1})_{-1/3}+(\underline{1},\underline{2})_{-1/2}+(\underline{\bar{3}},\underline{1})_{-2/3}+(\underline{1},\underline{2})_{-1/2}]$$
$$+[(\underline{1},\underline{1})_{0}+(\underline{\bar{3}},\underline{1})_{1/3}+(\underline{1},\underline{2})_{1/2}+(\underline{3},\underline{2})_{1/6}+(\underline{\bar{3}},\underline{1})_{1/3}+(\underline{1},\underline{1})_{0}]\ , \qquad (6)$$

where the subscripts are Y' charges in (6). This is to be compared with the standard reduction

$$\underline{27}\underset{Y}{\rightarrow}(\underline{1},\underline{1})_{0}+[(\underline{3},\underline{1})_{-1/3}+(\underline{1},\underline{2})_{+1/2}+(\underline{\bar{3}},\underline{1})_{1/3}+(\underline{1},\underline{2})_{-1/2}]$$
$$+[(\underline{1},\underline{1})_{0}+(\underline{\bar{3}},\underline{1})_{1/3}+(\underline{1},\underline{2})_{-1/2}+(\underline{3},\underline{2})_{1/6}+(\underline{\bar{3}},\underline{1})_{-2/3}+(\underline{1},\underline{1})_{1}]\ . \qquad (7)$$

As promised, the light-fermion spectra in (6) and (7) are identical, but they are arranged differently. For example, instead of $\underline{5}+\underline{\bar{5}}$ coming from the $\underline{10}$ of SO(10) pairing off and becoming superheavy as in the standard reduction, we find a triplet and a doublet of the $\underline{10}$ of SO(10) pairing with a triplet and doublet of the $\underline{16}$ to become superheavy. There are other interesting differences, for instance, the e^+ comes from the SO(10) singlet, etc.

If we were to neglect the evolution of the coupling constants, both the embeddings are unitary equivalent, yielding $\sin^2\theta_W=\frac{3}{8}$. However, for the Y' hypercharge of Eq. (5), we can let $\sin^2\theta_W$ differ from $\frac{3}{8}$ by allowing the $SU(5)\times U_A(1)\times U_B(1)$ couplings to run from the E_6 unification scale down to M_X.

Now let us fill in the remaining details of the model. Above the E_6 breaking scale M there are N_g generations of $(\underline{27})_L^F$ left-handed Weyl fermions, N_H Higgs $(\underline{27})_H$'s (which contain the light Higgs doublets) and an adjoint Higgs $(\underline{78})_H$ which contains the $(\underline{24})_H$ to break SU(5). All Higgs particles are light at M. All except the $(\underline{24})_H$ of the $(\underline{78})_H$ will become heavy below M, while the $(\underline{24})_H$ is light until M_X. This requires only a mild tuning of parameters because, as we shall find, M will be at most 3 or 4 orders of magnitude larger than M_X. All the components of $(\underline{27})_L^F$ have A and B charges and hence are massless above M_X.

Remark. Recall that unlike the standard SU(5) model, the $(10)_H$ of SU(5) coming from the 16 of SO(10) has a charge neutral component [at the position where the positron resides the minimal SU(5) 10-plet]. Consequently, the $(16)_H$ of SO(10) is allowed to have a vacuum expectation value (VEV) without disrupting the low-energy $SU(3)\times SU(2)\times U(1)$ theory. Furthermore, this VEV, as well as an SU(5)-singlet VEV from an SO(10) $(16)_H$, are required if we are to break down to $SU(3)\times SU(2)\times U(1)$. Hence we have made the model as realistic as possible by giving VEV's to a $(\underline{10})_{1,-1}$ and a $(\underline{1})_{1,-5}$ Higgs [both of which are contained in a $(27)_H$ of E_6] to break A, B, and Y to the linear combination Y'. The Higgs $\underline{5}$'s that develop vacuum expectation values of order M_W and give masses to the light fermions must come from both the $\underline{10}$ and $\underline{16}$ of SO(10). These will also be correctly included in the b_0's although they make only a small contribution.

The one-loop renormalization-group analysis is straightforward. The β functions are given generically by

$$\beta(g)=-(b_0/16\pi^2)g^3\ ,$$

where

$$b_0^5=\tfrac{55}{3}-2N_g-\tfrac{1}{6}N_5-\tfrac{4}{3}-\tfrac{10}{3}N_{24}^F\ ,$$
$$b_0^A=-48N_g-\tfrac{11}{3}-\tfrac{25}{6}N_5\ ,$$
$$b_0^B=-80N_g-\tfrac{35}{3}-\tfrac{65}{6}N_5\ ,$$
$$b_0^2=11-\tfrac{4}{3}N_g\ , \qquad (8)$$
$$b_0^2=\tfrac{22}{3}-\tfrac{4}{3}N_g-\tfrac{1}{6}N_d\ ,$$
$$b_0^1=-\tfrac{20}{9}N_g-\tfrac{1}{6}N_d\ ,$$

N_5 is the number of Higgs $\underline{5}$'s which contain N_d light Higgs doublets.

We have added extra fermions in Eqs. (8) that are at most as heavy as M_X. To be specific, we add a number N_{24}^F of fermions in the adjoint $(\underline{24})_{0,0}$ representation whose masses are fine-tuned to be $O(M_X)$. (This $\underline{24}$ can come from a $\underline{78}$ of E_6.) The choice of SU(5) representation for the extra fermions is irrelevant although it is important to have zero (or "small") A and B charges. All we need to simultaneously increase *both* $\sin^2\theta_W(M_W)$ and M_X are enough particles (they could also be partly or all scalars) to make α_5 nonasymptotically free (AF) and run downward with decreasing mass scale faster than α_A and α_B.

Figure 1 is a plot of all the running gauge couplings in a model with $N_5=2$, $N_d=1$, and $N_{24}^F=7$. Notice first

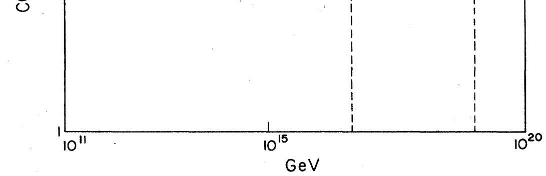

FIG. 1. The running couplings as a function of momentum scale for the case discussed in the text.

TABLE I. The first row in this table shows the value of M_X and $\sin^2\theta_W(M_W)$ in the one-loop approximation with one Higgs doublet in minimal SU(5) Ref. 11. In the second row are the results from the complete two-loop analysis of Ref. 12 for comparison. The remaining seven rows are results of our one-loop analysis for a variety of choices for M, N_d, and N_{78}^F. (We have set $N_5=N_d$ and $N_g=3$ throughout.) τ_p is always at least a factor of 10^4 larger than the one-loop minimal-SU(5) result.

M (10^{14} GeV)	N_{78}^F	N_d	M_X (10^{14} GeV)	$\sin^2\theta_W(M_X)$	$\sin^2\theta_W(M_W)$
	0	1	7.1	0.375	0.207
	0	1	4.1	0.375	0.210
10^5	7	1	388.0	0.405	0.210
10^5	7	3	196.0	0.410	0.221
10^5	7	6	73.2	0.417	0.236
10^4	7	1	193.0	0.396	0.207
10^4	7	3	99.0	0.401	0.218
10^4	8	1	338.0	0.403	0.210
10^4	8	3	188.0	0.409	0.221

that all the couplings are small over the entire range shown and hence perturbation theory is valid. Here, we have used $\Lambda_s=300$ MeV and $\alpha(M_W)=\frac{1}{128}$ as input and have fixed the larger scale M to be m_{Planck} for illustration. All couplings are normalized such that they coincide with α_5 when the evolution is neglected. Starting from the small mass-scale side of the graph, we see α_3 and α_2 converge to α_5 at M_X ($\cong 2\times10^{16}$ GeV), while α_1 crosses α_2 and then α_3 and is larger than either at M_X. Above M_X, α_A and α_B follow very close, nearly parallel, but convergent trajectories, while α_5 rises with a greater slope to meet them at M. At the unification mass M_X, $\sin^2\theta_W=0.405$. This is greater than the minimal SU(5) result of 3/8 and happens because the slope of α_5 is greater than those of α_A and α_B, such that the linear combination Y' in Q starts α_1 off greater than $\alpha_2=\alpha_3$.

Looking in the other direction, as the solution evolves downward in mass scale, $\sin^2\theta_W$ falls at the same rate as in minimal SU(5), since the particle content is the same below M_X; however, the value of M_X we find is somewhat greater than that found in minimal SU(5). This set of circumstances causes two obviously desirable effects. The proton lifetime τ_p and $\sin^2\theta_W(M_W)$ are simultaneously increased. These increases can be adjusted by varying M or by varying the particle content above M_X. Table I gives a sampling of a few such choices.

We find the scheme presented here to improve τ_p and $\sin^2\theta_W(M_W)$ an attractive alternative to that of keeping part of a split fermion multiplet light[5] until M_W, which introduces a fermionic hierarchy problem. It is far from easy to experimentally distinguish the two alternatives. Split multiplet models do lead to a set of discrete predictions for M_X and $\sin^2\theta(M_W)$, while our model allows a continuously adjustable range for these parameters.

To conclude, we emphasize that the chief attraction of the model proposed here is that below M_X none of the advantages of minimal SU(5) are lost (i.e., no intermediate mass scales are introduced between M_X and M_W, no proliferation of undiscovered particles, etc.). On the contrary, the two apparent disagreements with experiment are resolved. There *is* still a desert, but the sands have shifted.

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[1]H. Georgi and S. L. Glashow, Phys. Rev. Lett. **32**, 438 (1974).

[2]G. Arnison *et al.*, Phys. Lett. **129B**, 273 (1983).

[3]W. J. Marciano, in *Proceedings of the Fourth Workshop on Grand Unification, University of Pennsylvania, 1983,* edited by H. A. Weldon, P. Langacker, and P. J. Steinhardt (Birkhäuser, Boston, 1983).

[4]R. M. Bionta *et al.*, Phys. Rev. Lett. **51**, 27 (1983).

[5]One interesting scheme of introducing such modifications has been suggested by P. H. Frampton and S. L. Glashow, Phys. Lett. **131B**, 340 (1983).

[6]H. Georgi, H. Quinn, and S. Weinberg, Phys. Rev. Lett. **33**, 451 (1974).

[7]This would lead to a loss of charge quantization, except that SU(5) and Q will be unified above M_X as will be discussed.

[8]Another model [often called anti-SU(5) in the literature], where the gauge group is SU(5)×U(1) just above M_X that unifies into SO(10) at some larger mass scale with the hypercharge $U_Y(1)$ not a subgroup of SU(5), was first discussed by A. De Rújula, H. Georgi, and S. L. Glashow, Phys. Rev. Lett. **45**, 413 (1980), and further analyzed by H. Georgi, S. L. Glashow, and M. Machacek, Phys. Rev. D **23**, 783 (1981). S. M. Barr, Phys. Lett. **112B**, 219 (1982); R. W. Robinett and J. L. Rosner, Phys. Rev. D **26**, 2396 (1982); H. Georgi, in *Proceedings of the 21st International Conference on High Energy Physics, Paris, 1982,* edited by P. Petiau and M. Porneuf [J. Phys. (Paris) Colloq. **43**, C3 (1982)]; S. M. Barr and S. D. Ellis, Phys. Rev. D **27**, 1190 (1983); Y. Hara, Nucl. Phys. **B214**, 167 (1983); G. Anastaze, J.-P. Derendinger, and F. Buccella, Z. Phys. C **20**, 269 (1983); J.-P. Derendinger, J. E. Kim, and D. V. Nanopoulos, Phys. Lett. **139B**, 170 (1984). The most interesting property of the anti-SU(5) model is that it can be distinguished from minimal SU(5) by its de-

cay modes (we assume gauge vector dominated decay modes). The model we are about to present can be distinguished from both minimal SU(5) and anti-SU(5) by decay-mode measurements. Consequently, our model will be of interest in its own right.

[9]Grand unification based on the exceptional group E_6 was first discussed by F. Gürsey, P. Ramond, and P. Sikivie, Phys. Lett. **60B**, 177 (1976); F. Gürsey and M. Sardaroglu, Lett. Nuovo Cimento **21**, 28 (1978); Y. Achiman and B. Stech, Phys. Lett. **77B**, 389 (1978).

[10]While other higher-rank groups such as SU(N) or SO(N) do allow embeddings of hypercharge that differs from the standard assignment, the scheme described below that preserves conditions (i)—(iii) is unique to E_6 [with the exception of the anti-SU(5) model (see Ref. (8)]. We have shown this elsewhere. [T. W. Kephart and N. Nakagawa, Phys. Lett. **141B**, 329 (1984).

[11]See, e.g., M. B. Einhorn and D. R. T. Jones, Nucl. Phys. **B196**, 475 (1982) and references therein.

[12]W. J. Marciano and A. Sirlin, in *Second Workshop on Grand Unification, Ann Arbor, Michigan, 1981,* edited by J. P. Leveille, L. R. Sulak, and D. G. Unger (Birkhäuser, Boston, 1981).