

CP determination and tests for CP or P violation by the $V_1 V_2$ decay mode

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A decay mode such as $\phi\phi$, $\Upsilon\Upsilon$, K^*+K^{*-} , or $D^{*+}D^{*-}$ can be used to distinguish between a neutral spin-0 technipion and a neutral spin-0 Higgs particle. By this generalization of $\phi\phi$ parity test, the CP eigenvalue γ_{CP} can be determined for an X particle of any spin J which decays CP invariantly into VV , or $V\bar{V}$, where each vector meson either decays into two spin-0 bosons, or is ω . The absence in a VV , or $V\bar{V}$, decay channel of $\sin 2\phi$ and $\sin\phi$ terms in the azimuthal distribution is due to CP invariance and/or P invariance. For a $V_1 V_2$ decay channel without a $V_1 \leftrightarrow V_2$ exchange property, and in a mode like $K^*+\bar{K}^{*0}$, such terms would imply that P is violated. For a $V_1 V_2$ mode such as $\phi\omega$ where each vector meson is its own antiparticle, such terms would imply that both P and CP are violated; when CP invariance holds, the $\gamma_{CP}(-)^J$ eigenvalue of X can be determined provided certain amplitudes do not accidentally vanish.

I. INTRODUCTION

As a generalization of the $\phi\phi$ parity test,¹⁻⁴ we have found that the VV or $V\bar{V}$ decay mode can be used to distinguish⁵ between a neutral spin-0 technipion and a neutral spin-0 elementary Higgs particle. This is also true for modes where V_1 and V_2 are each their own antiparticle. This type of test requires a measurement of the dependence of the decay correlation function on the azimuthal angle ϕ between the two V , or V and \bar{V} decay planes. Each of these vector mesons either decays into two spin-0 bosons, or is ω . For spin 0, it is possible to use instead a heavy-quark VV mode, such as the two- J/ψ channel or the $\Upsilon\Upsilon$ channel, where each heavy-quark V decays into $e\bar{e}$. The $\Upsilon\Upsilon$ channel or the two- 1^{--} - t -quarkonium channel, for instance, would be favored if Higgs particles or technipions are sufficiently massive and couple more strongly to heavy quarks.

In Sec. II, we first discuss how the CP eigenvalue γ_{CP} can be determined for an X particle of any spin J which decays via a CP -invariant coupling into VV or $V\bar{V}$, or with certain exceptions how $\gamma_{CP}(-)^J$ can be determined from a $V_1 V_2$ mode where each vector meson is its own antiparticle. Then in Sec. III we discuss how it is possible to use the VV decay mode to test for unexpected violations of CP . The absence of $\sin 2\phi$ and $\sin\phi$ terms in the azimuthal distribution, for example, is due to CP invariance and/or P invariance. That is, either CP or P invariance is sufficient for their absence. Such terms, of course, might also occur because of background effects; however, since such situations will vary from one experiment to another, we omit any treatment of background effects in this paper.

In Sec. IV we discuss the general decay correlation function $I(\theta_1, \theta_2, \phi)$ which follows when no invariance principles are used to relate the decay helicity amplitudes describing $X \rightarrow V_1 V_2$. Besides the azimuthal angle ϕ , $I(\theta_1, \theta_2, \phi)$ depends on the two polar angles θ_1 and θ_2 used to define the spin-0-boson three-momenta directions, respectively, in the V_1 and V_2 rest frames. Then in Sec.

V we list two simple tests for CP violation by the $V\bar{V}$ decay mode. The $K^*+\bar{K}^{*0}$ (or $D^{*+}+\bar{D}^{*0}$) type mode can similarly be used to test for a violation of P , and also for a violation of the combined usage of C and isospin invariance. In a $V_1 V_2$ decay mode without a $V_1 \leftrightarrow V_2$ exchange property, and in a mode like $K^*+\bar{K}^{*0}$, $\sin 2\phi$ and/or $\sin\phi$ terms would imply that P is violated. For a $V_1 V_2$ mode like $\phi\omega$, where each vector meson is its own antiparticle, such terms would imply that both P and CP are violated.

Finally, in Sec. VI, we discuss the stronger results which can be used for CP determination when X is spin 0 or 1.

The reader should note that this paper has been organized so that the results for a particular $X \rightarrow V_1 V_2$ decay mode can be easily located.

II. CP DETERMINATION

When V_1 and V_2 are identical particles, or a particle-antiparticle pair, then by CP invariance the nonvanishing helicity amplitudes⁶ describing the decay $X \rightarrow V_1 V_2$ are related by

$$a_{-\lambda_1, -\lambda_2} = \gamma_{CP} a_{\lambda_2 \lambda_1}, \tag{1}$$

where $\gamma_{CP} = CP$ eigenvalue of X . In Table I we show explicitly the relations among these decay helicity amplitudes when $\gamma_{CP} = \pm 1$.

A. VV decay mode

When V_1 and V_2 are identical neutral particles, the decay helicity amplitudes are also related by

$$a_{\lambda_1 \lambda_2} = (-)^J a_{\lambda_2 \lambda_1}. \tag{2}$$

Since from Eqs. (1) and (3)

$$a_{-\lambda_1, -\lambda_2} = \gamma_{CP} (-)^J a_{\lambda_1 \lambda_2}, \tag{3}$$

it is only necessary to substitute γ_{CP} for η in order to convert to a CP test the already existing treatments in the

TABLE I. Relations among the helicity amplitudes describing the decay $X \rightarrow V_1 V_2$ which follow from invariance under CP where $\gamma_{CP} = CP$ eigenvalue of X . Such modes are two identical vector bosons, VV , and a particle-antiparticle pair of vector bosons, $V\bar{V}$.

Nonvanishing amplitudes	CP eigenvalue
$a_{+0} = -a_{0-}$ $a_{0+} = -a_{-0}$ $a_{++} = -a_{--}$	$\gamma_{CP} = -1$
$a_{+0} = a_{0-}$ $a_{0+} = a_{-0}$ $a_{++} = a_{--}$ a_{+-}, a_{-+}, a_{00}	$\gamma_{CP} = +1$

literature on the $\phi\phi$, or another identical-particle pair, decay mode as a parity test ($\eta = \text{parity of } X$ in the case of a parity test). This also means that except in certain circumstances the signature of X can be determined if it is

$(-)^J = +1$. [For $(-)^J = -1$ signature, analysis of only $V\bar{V}$ modes will yield an inconclusive result.⁴]

B. $V\bar{V}$ decay mode

When V_1 and V_2 are a particle-antiparticle pair, Eq. (1) is valid. Such modes include $K^{*+}K^{*-}$, $K^{*0}\bar{K}^{*0}$, $\rho^+\rho^-$, $D^{*+}D^{*-}$, and $D^{*0}\bar{D}^{*0}$. This implies that the decay correlation function depending on the azimuthal angle ϕ and the two polar angles θ_1 and θ_2 (defined, respectively, in the V and \bar{V} rest frames) is³

$$I(\theta_1, \theta_2, \phi) = C(\theta_1, \theta_2) + A(\theta_1, \theta_2)\cos\phi + B(\theta_1, \theta_2)\cos 2\phi. \quad (4)$$

The angles are defined as in Ref. 3 and in Fig. 1 of paper I (Ref. 4).

The coefficients in Eq. (4) depend on the X decay helicity amplitudes. Using Table I, we find

$$B(\theta_1, \theta_2) = \gamma_{CP} \frac{1}{2} |a_{++}|^2 \sin^2\theta_1 \sin^2\theta_2, \quad (5)$$

$$A(\theta_1, \theta_2) = \frac{1}{4} [2 \operatorname{Re}(a_{++} a_{00}^*) - \gamma_{CP} (|a_{+0}|^2 + |a_{0+}|^2)] \sin 2\theta_1 \sin 2\theta_2, \quad (6)$$

and

$$C(\theta_1, \theta_2) = \frac{1}{4} (2|a_{++}|^2 + |a_{+-}|^2 + |a_{-+}|^2) \sin^2\theta_1 \sin^2\theta_2 + |a_{00}|^2 \cos^2\theta_1 \cos^2\theta_2 + \frac{1}{2} (|a_{+0}|^2 + |a_{0+}|^2) (\sin^2\theta_1 \cos^2\theta_2 + \cos^2\theta_1 \sin^2\theta_2). \quad (7)$$

It is useful to further integrate $I(\theta_1, \theta_2, \phi)$. If the entire θ_1, θ_2 acceptance is integrated over,

$$F(\phi) \equiv \int_{-1}^1 d(\cos\theta_1) \int_{-1}^1 d(\cos\theta_2) I(\theta_1, \theta_2, \phi) = 4\bar{C}(1 + \beta \cos 2\phi), \quad (8)$$

where⁷

$$\beta = \frac{2\gamma_{CP} |a_{++}|^2}{2|a_{++}|^2 + 2|a_{+0}|^2 + 2|a_{0+}|^2 + |a_{+-}|^2 + |a_{-+}|^2 + |a_{00}|^2}. \quad (9)$$

If, instead, the quadrants are separately integrated over

$$F_{ab}(\phi) = \bar{C}(1 \pm \alpha \cos\phi + \beta \cos 2\phi), \quad (10)$$

where the upper sign gives F_{11} or $0 \leq \theta_{1,2} \leq \pi/2$ and F_{22} for $\pi/2 \leq \theta_{1,2} \leq \pi$; and the lower sign gives F_{12} for $0 \leq \theta_1 \leq \pi/2$, $\pi/2 \leq \theta_2 \leq \pi$ and F_{21} for $0 \leq \theta_2 \leq \pi/2$, $\pi/2 \leq \theta_1 \leq \pi$. The reason for treating separately these four quadrants is that the sign of the $\cos\phi$ term in Eq. (10) is quadrant dependent. If there are reasons to expect the background to also be quadrant dependent, due to this sign variation sufficient care must be used in employing Eq. (10) to evaluate the α parameter by data from more than one quadrant. Note that Eq. (8) shows that this $\cos\phi$ term cancels out when the entire θ_1, θ_2 acceptance is integrated over. This additional important parameter for $X \rightarrow V\bar{V}$ is

$$\alpha = \frac{2 \operatorname{Re}(a_{++} a_{00}^*) - \gamma_{CP} (|a_{+0}|^2 + |a_{0+}|^2)}{2|a_{++}|^2 + 2|a_{+0}|^2 + 2|a_{0+}|^2 + |a_{+-}|^2 + |a_{-+}|^2 + |a_{00}|^2}. \quad (11)$$

By taking into account the sign change in Eq. (10), α can be determined from all the available data if enough is known about possible background effects.⁸

If the azimuthal angle ϕ and one of the polar angles is integrated over, the distribution in the other polar angle is

$$G(\theta_2) \equiv \int_{-1}^1 d(\cos\theta_1) \int_0^{2\pi} d\phi I(\theta_1, \theta_2, \phi) = 4\pi\bar{C}[1 + \xi_T P_2(\cos\theta_2)], \quad (12)$$

where the coefficient of the second Legendre polynomial is

$$\xi_T = \frac{2|a_{00}|^2 - 2|a_{++}|^2 - |a_{+-}|^2 - |a_{-+}|^2 + |a_{+0}|^2 + |a_{0+}|^2}{2|a_{++}|^2 + 2|a_{+0}|^2 + 2|a_{0+}|^2 + |a_{+-}|^2 + |a_{-+}|^2 + |a_{00}|^2}. \quad (13)$$

In some cases, see Sec. III of paper I and Sec. III below, it is important to note which polar angle has been integrated over in Eq. (12). For $X \rightarrow V\bar{V}$ the same ξ_T should be found when either polar angle is integrated over. As a cross check, the empirical value of ξ_T from the above integrated distribution, Eq. (12), can be compared with an evaluation of this expression, Eq. (13), using the empirical coefficients from the $C(\theta_1, \theta_2)$ distribution, Eq. (7). For the case when X has a spin of $J=0$ or 1 the value of ξ_T can be used in the determination of the CP eigenvalue of X , see Sec. VI. For any J , from Eq. (13) we see $\xi_T > \frac{1}{2}$ implies $|a_{00}| \neq 0$ and $\gamma_{CP} = +1$. If only the azimuthal angle ϕ is integrated over its entire range, then the $C(\theta_1, \theta_2)$ distribution, Eq. (7), is obtained

$$\int_0^{2\pi} d\phi I(\theta_1, \theta_2, \phi) = 2\pi C(\theta_1, \theta_2). \quad (14)$$

The CP eigenvalue of X can be determined from these β and α coefficients:

- (i) When $\beta \neq 0$, the X eigenvalue $\gamma_{CP} = \text{sgn}\beta$.
- (ii) When $\beta = 0$, but $\alpha \neq 0$, then $\gamma_{CP} = -\text{sgn}\alpha$.
- (iii) Should both $\beta = 0$ and $\alpha = 0$, then $\gamma_{CP} = +1$.

A $C(\theta_1, \theta_2)$ distribution results when $I(\theta_1, \theta_2, \phi)$ is integrated over the full azimuthal acceptance. Although this $C(\theta_1, \theta_2)$ distribution is not needed to determine the CP eigenvalue of X , it could be used, perhaps as a check, to determine whether $|a_{00}| \neq 0$, which would imply $\gamma_{CP} = +1$. As in paper I, the symmetry properties of $C(\theta_1, \theta_2)$ can be used to bin all θ_1, θ_2 events into the triangular region $0 \leq \theta_{1,2} \leq \pi/2$, $\theta_1 > \theta_2$, where the $|a_{00}|^2$ coefficient dominates the $C(\theta_1, \theta_2)$ distribution near the $\theta_1 \simeq 0$, $\theta_2 \simeq 0$ vertex. Should $\beta = 0$, $|a_{++}| = 0$ and then the empirical value of α by Eq. (11) gives a further constraint on $C(\theta_1, \theta_2)$,

$$\alpha |a_{00}|^2 + (2\alpha + \gamma_{CP})(|a_{+0}|^2 + |a_{0+}|^2) + \alpha(|a_{+-}|^2 + |a_{-+}|^2) = 0. \quad (15)$$

This α constraint, however, is γ_{CP} dependent which means that for $\gamma_{CP} = +1$, and, then separately for $\gamma_{CP} = -1$, a two-parameter fit must be made to $C(\theta_1, \theta_2)$.

In summary, when the VV or $V\bar{V}$ decay modes are due to strong interactions such that the decay of X is invariant under C and P separately, the decay correlation function $I(\theta_1, \theta_2, \phi)$ provides enough information to determine the

parity η of the decaying X particle. In contrast, when the VV or $V\bar{V}$ decay mode is due to weak interactions such that invariance under C and P separately is violated but the decay of X is still CP invariant, then $I(\theta_1, \theta_2, \phi)$ provides enough information to determine the CP eigenvalue γ_{CP} of X . Consequently, it is *not* possible to use $I(\theta_1, \theta_2, \phi)$ for either of these modes to demonstrate that P or C separately is violated. This is apparent for $V\bar{V}$ from comparison of Eqs. (5)–(7) here with Eqs. (7)–(9) of paper I. For VV it follows since Eqs. (1) and (2) imply (3). However, the presence of $\sin 2\phi$ and/or $\sin\phi$ terms in $I(\theta_1, \theta_2, \phi)$ for a $V_1 V_2$ decay channel without a $V_1 \leftrightarrow V_2$ exchange property and where V_1 and/or V_2 is not its own antiparticle, or for a mode like $K^{*+} \bar{K}^{*0}$, does imply that P is violated in $X \rightarrow V_1 V_2$ as we discuss below in Sec. V.

C. $V_1 V_2$ decay mode: V_1 and V_2 their own antiparticles

When V_1 and V_2 are each their own antiparticle,

$$a_{-\lambda_1, -\lambda_2} = \gamma_{CP} (-)^J a_{\lambda_1 \lambda_2},$$

and it is again only necessary to substitute γ_{CP} for η in Sec. III of paper I in order to use this mode to determine $\gamma_{CP} (-)^J$ for X provided certain amplitudes do not vanish (see paper I). When J is known, this will determine the CP eigenvalue of X . Modes of this type are $\phi\rho^0$, $\omega\rho^0$, and $\phi\omega$.

III. TESTS FOR CP VIOLATION BY THE VV DECAY MODE

When CP invariance is violated in the decay $X \rightarrow V_1 V_2$, where V_1 and V_2 are identical vector bosons, the decay helicity amplitudes are related as shown by Table II and the azimuthal distribution of the decay correlation function $I(\theta_1, \theta_2, \phi)$ can contain $\sin 2\phi$ and $\sin\phi$ contributions. Even if such azimuthal contributions are absent, if certain values of the decay correlation parameters are found, then there are violations of CP invariance. Second, should CP be violated it remains possible, except in certain circumstances, to use an $X \rightarrow VV$ decay mode to show that the signature of X is $(-)^J = +1$.

From Table II, the decay correlation function for $X \rightarrow VV$ is

$$I(\theta_1, \theta_2, \phi) = C_0(\theta_1, \theta_2) + A_0(\theta_1, \theta_2)\cos\phi + A_X(\theta_1, \theta_2)\sin\phi + B_0(\theta_1, \theta_2)\cos 2\phi + B_X(\theta_1, \theta_2)\sin 2\phi, \quad (16)$$

where

$$B_0(\theta_1, \theta_2) = \frac{1}{2} \text{Re}(a_{++} a_{--}^*) \sin^2 \theta_1 \sin^2 \theta_2, \quad (17)$$

$$B_X(\theta_1, \theta_2) = -\frac{1}{2} \text{Im}(a_{++} a_{--}^*) \sin^2 \theta_1 \sin^2 \theta_2, \quad (18)$$

$$A_0(\theta_1, \theta_2) = \frac{1}{4} \text{Re}(a_{++} a_{00}^* + a_{00} a_{--}^* - 2a_{+0} a_{0-}^*) \sin 2\theta_1 \sin 2\theta_2, \quad (19)$$

$$A_X(\theta_1, \theta_2) = -\frac{1}{4} \text{Im}(a_{++} a_{00}^* + a_{00} a_{--}^* - 2a_{+0} a_{0-}^*) \sin 2\theta_1 \sin 2\theta_2, \quad (20)$$

and

$$C_0(\theta_1, \theta_2) = \frac{1}{4} (|a_{++}|^2 + |a_{--}|^2 + 2|a_{+-}|^2) \sin^2 \theta_1 \sin^2 \theta_2 + |a_{00}|^2 \cos^2 \theta_1 \cos^2 \theta_2 + \frac{1}{2} (|a_{+0}|^2 + |a_{-0}|^2) (\sin^2 \theta_1 \cos^2 \theta_2 + \cos^2 \theta_1 \sin^2 \theta_2), \quad (21)$$

TABLE II. Relations among the helicity amplitudes for the decay $X \rightarrow V_1 V_2$ which follow when V_1 and V_2 are identical vector bosons.

Nonvanishing amplitudes	Signature
$a_{+0} = -a_{0+}$ $a_{-0} = -a_{0-}$ $a_{+-} = -a_{-+}$	$(-)^J = -1$
$a_{+0} = a_{0+}$ $a_{-0} = a_{0-}$ $a_{+-} = a_{-+}$ a_{++}, a_{--}, a_{00}	$(-)^J = +1$

where the θ_1 and θ_2 symmetry properties of $C(\theta_1, \theta_2)$ allow the binning of all θ_1, θ_2 events into the triangular region $0 \leq \theta_{1,2} \leq \pi/2$, $\theta_2 < \theta_1$. The distinct coefficients of Eq. (21), then, respectively dominate the empirical $C(\theta_1, \theta_2)$ distribution near the three vertices of this triangular region.

The associated integrated distributions are

$$F(\phi) \equiv \int_{-1}^1 d(\cos\theta_1) \int_{-1}^1 d(\cos\theta_2) I(\theta_1, \theta_2, \phi) \\ = 4\bar{C}(1 + \beta_0 \cos 2\phi + \beta_X \sin 2\phi) \quad (22)$$

when the entire θ_1, θ_2 acceptance is integrated over. We define

$$D_0 = |a_{++}|^2 + |a_{--}|^2 + 2|a_{+0}|^2 + 2|a_{-0}|^2 \\ + 2|a_{+-}|^2 + |a_{00}|^2 \quad (23)$$

so the parameters in Eq. (22) are

$$\beta_0 = 2 \operatorname{Re}(a_{++} a_{--}^*) / D_0, \quad (24)$$

$$\beta_X = -2 \operatorname{Im}(a_{++} a_{--}^*) / D_0. \quad (25)$$

When the quadrants are separately integrated over, analogous to Eq. (10) of Sec. II, we obtain

$$F_{ab}(\phi) = \bar{C}(1 \pm \alpha_0 \cos \phi \pm \alpha_X \sin \phi + \beta_0 \cos 2\phi + \beta_X \sin 2\phi) \quad (26)$$

with the upper signs for F_{11}, F_{22} and the lower signs for F_{12}, F_{21} . The two additional parameters in Eq. (26) are

$$\alpha_0 = \operatorname{Re}(a_{++} a_{00}^* + a_{00} a_{--}^* - 2a_{+0} a_{0-}^*) / D_0, \quad (27)$$

$$\alpha_X = -\operatorname{Im}(a_{++} a_{00}^* + a_{00} a_{--}^* - 2a_{+0} a_{0-}^*) / D_0. \quad (28)$$

When one of the polar angles and the azimuthal angle is integrated over, Eq. (12) of Sec. II is obtained but now the parameter is

$$\xi_T = (2|a_{00}|^2 - |a_{++}|^2 - |a_{--}|^2 - 2|a_{+-}|^2 \\ + |a_{+0}|^2 + |a_{-0}|^2) / D_0. \quad (29)$$

Again, when the azimuthal angle is integrated over, the $C_0(\theta_1, \theta_2)$ distribution of Eq. (21) is obtained just as in Eq. (14).

A. Signature test

From Table II we see that $\beta_0 \neq 0$ and/or $\beta_X \neq 0$ implies that X has the signature $(-)^J = +1$. When both β_0 and β_X vanish, the $C_0(\theta_1, \theta_2)$ distribution of Eq. (20) can be used to determine $|a_{00}|$. If $|a_{00}| \neq 0$, then $(-)^J = +1$ but if $|a_{00}| = 0$ and $\beta_0 = \beta_X = 0$, the signature cannot be obtained from $I(\theta_1, \theta_2, \phi)$.

B. Tests for CP violation

When $\beta_X \neq 0$ and/or $\alpha_X \neq 0$, CP invariance is violated in the decay $X \rightarrow VV$. [Note $\beta_X \neq 0$ if and only if $B_X \neq 0$, and similarly $\alpha_X \neq 0$ if and only if $A_X \neq 0$.]

When it is found that both $\beta_X = 0$ and $\alpha_X = 0$, from comparison of Table II in this paper with Table II of paper I we nevertheless find that if certain values of the parameters occur, then there must be violations of CP invariance. The negative-signature amplitudes in Table II are a generalization of cases I and II of paper I, and similarly the positive-signature amplitudes are a generalization of cases III and IV. For the remainder of this section we assume that $\beta_X = \alpha_X = 0$ and discuss these tests. It is convenient to define the quantities

$$R = |a_{+0}|^2 + |a_{-0}|^2 \quad (30)$$

and

$$S = |a_{++}|^2 + |a_{--}|^2 + 2|a_{+-}|^2 \quad (31)$$

which can be determined from the empirical $C(\theta_1, \theta_2)$ distribution. We consider $\beta_0 = 0$ first and then $\beta_0 \neq 0$.

(i) When $\beta_0 = 0$ and the $C_0(\theta_1, \theta_2)$ distribution indicates that $R = 0$, then if $|a_{00}| \neq 0$ and $\alpha_0 \neq 0$, CP is violated; but if $|a_{00}|^2 \neq 0$ and $\alpha_0 = 0$, or $|a_{00}| \neq 0$ and $S = 0$, CP is not violated. Although $\beta_0 = 0$ and $R = 0$ exclude CP -invariant cases I and III, if $|a_{00}| = 0$, there is no test for CP violation.

(ii) When $\beta_0 = 0$ and $R \neq 0$, we first consider the case $|a_{00}| = 0$.

When also $\alpha_0 = 0$, then $S = 0$ implies CP is violated but the situation is inconclusive if the latter coefficient of $C_0(\theta_1, \theta_2)$ is found not to vanish.

When $\alpha_0 \neq 0$, then $|a_{+0}| \neq 0$ and we can define a real proportionality factor κ by

$$a_{0-} = \kappa a_{+0}, \quad \kappa = \kappa^* \quad (32)$$

and

$$\alpha_0 = \frac{-2\kappa |a_{+0}|^2}{|a_{++}|^2 + |a_{--}|^2 + 2(1+\kappa^2)|a_{+0}|^2 + 2|a_{+-}|^2}. \quad (33)$$

If $S = 0$, then

$$\alpha_0 = -\kappa / (1 + \kappa^2) \quad (34)$$

so when $\alpha_0 \neq \pm \frac{1}{2}$, CP is violated but when $\alpha_0 = \frac{1}{2}$ ($-\frac{1}{2}$), CP is not violated and case I or III (II or IV) occurs. However, if $S \neq 0$, then

$$-1/\alpha_0 = \kappa + (1 + S/2 |a_{+0}|^2) / \kappa \quad (35)$$

and we separate consideration into cases $\alpha_0 < 0$ and $\alpha_0 > 0$. Using Eq. (35) for $\kappa = +1$, we find $\alpha_0 < -\frac{1}{2}$ so we conclude $0 > \alpha_0 > -\frac{1}{2}$ implies CP is violated, but that $\alpha_0 < -\frac{1}{2}$ alone is inconclusive ($\alpha_0 = \pm \frac{1}{2}$ are excluded by $S \neq 0$). But, if we also use the values of R and S , we find

$$\frac{R}{\alpha_0(R+S)} \neq -2 \quad (36)$$

implies CP is violated, but that equality only implies $\kappa = 1$ which is inconclusive. Proceeding similarly for $\alpha_0 > 0$, we find that CP is violated for any $\alpha_0 > 0$, $\alpha_0 \neq 1/2$.

Still considering $\beta_0 = 0$ and $R \neq 0$, we consider the alternate case $|a_{00}| \neq 0$ which requires $\kappa = +1$ in Eq. (32), and $\alpha_0 \leq 0$, if CP is to be good. When $S = 0$ and $\alpha_0 = 0$, CP is violated. But when $S = 0$ and $\alpha_0 \neq 0$, we find

$$-1/\alpha_0 = \kappa + (1 + |a_{00}|^2/2|a_{+0}|^2)/\kappa \quad (37)$$

and $0 > \alpha_0 > -\frac{1}{2}$ implies CP is violated but find that $\alpha_0 < -\frac{1}{2}$ alone is inconclusive. On the other hand, if

$$\frac{R}{\alpha_0(R + |a_{00}|^2)} \neq -2, \quad (38)$$

CP is violated, while equality means CP is good and case IV occurs. However, if $S \neq 0$, we find CP is violated when $\alpha_0 = 0$ and also when

$$\frac{R}{\alpha_0(R+S + |a_{00}|^2)} \neq -2, \quad (39)$$

but that equality in Eq. (39) is inconclusive.

(iii) When $\beta_0 \neq 0$, we know $(-)^J = +1$ and $|a_{++}| \neq 0$ so we can define a convenient real proportionality factor ϵ by

$$a_{--} = \epsilon a_{++}, \quad \epsilon = \epsilon^* \quad (40)$$

Since

$$\beta_0 = 2\epsilon |a_{++}|^2/D_0, \quad (41)$$

we have $\text{sgn}\beta_0 = \text{sgn}\epsilon$.

When $R = 0$ and $|a_{00}| = 0$, $\beta_0 \neq -1$ implies CP is violated whereas $\beta_0 = -1$ implies CP is not violated and case III occurs. For positive β_0 we find $\beta_0 > 1$ implies CP is violated, but that $\beta_0 = +1$ implies CP is not violated and that $0 < \beta_0 < 1$ is inconclusive.

Next for $\beta_0 \neq 0$, $R = 0$, and also $|a_{00}| \neq 0$ we find $\beta_0 < 0$ and

$$I(\theta_1, \theta_2, \phi) = C_0(\theta_1, \theta_2) + A_0(\theta_1, \theta_2)\cos\phi + A_X(\theta_1, \theta_2)\sin\phi + B_0(\theta_1, \theta_2)\cos 2\phi + B_X(\theta_1, \theta_2)\sin 2\phi. \quad (44)$$

The B_0 and B_X coefficients are the same as those in Eqs. (17) and (18). The other coefficients are

$$A_0(\theta_1, \theta_2) = \frac{1}{4} \text{Re}(a_{++}a_{00}^* + a_{00}a_{--}^* - a_{+0}a_{0-}^* - a_{0+}a_{-0}^*) \sin 2\theta_1 \sin 2\theta_2, \quad (45)$$

$$A_X(\theta_1, \theta_2) = -\frac{1}{4} \text{Im}(a_{++}a_{00}^* + a_{00}a_{--}^* - a_{+0}a_{0-}^* - a_{0+}a_{-0}^*) \sin 2\theta_1 \sin 2\theta_2, \quad (46)$$

and

$$C_0(\theta_1, \theta_2) = \frac{1}{4} (|a_{++}|^2 + |a_{--}|^2 + |a_{+-}|^2 + |a_{-+}|^2) \sin^2\theta_1 \sin^2\theta_2 + |a_{00}|^2 \cos^2\theta_1 \cos^2\theta_2 + \frac{1}{2} (|a_{+0}|^2 + |a_{-0}|^2) \sin^2\theta_1 \cos^2\theta_2 + \frac{1}{2} (|a_{0+}|^2 + |a_{0-}|^2) \cos^2\theta_1 \sin^2\theta_2. \quad (47)$$

$$\beta_0 > (1 + |a_{00}|^2/S)^{-1}$$

implies that CP is violated; when

$$0 < \beta_0 < (1 + |a_{00}|^2/S)^{-1},$$

the situation is inconclusive. When

$$\beta_0 = (1 + |a_{00}|^2/S)^{-1},$$

then $\epsilon = +1$, $|a_{+-}| = 0$, and CP is good.

(iv) When $\beta_0 \neq 0$, we can use Eq. (40) and (41) and so now for $R \neq 0$, we first consider the case $|a_{00}| = 0$. When $\alpha_0 = 0$, CP is violated. When $\alpha_0 \neq 0$, we can again define κ via Eq. (32) and find

$$\frac{R}{\alpha_0(R+S)} \neq \pm 2 \quad (42)$$

implies CP is violated but that equality respectively only implies $\kappa = \mp 1$ which is inconclusive. Using Eq. (41), we find

$$\beta_0 > (1 + R/S)^{-1} \quad (43)$$

implies CP is violated. When $\beta_0 = (1 + R/S)^{-1}$, then $\epsilon = 1$ and $|a_{+-}| = 0$, so if $R = -2\alpha_0(R+S)$ as we discussed following Eq. (42), CP is good since case IV can occur. When

$$0 < \beta_0 < (1 + R/S)^{-1},$$

the value of $\epsilon > 0$ is not known so the situation remains inconclusive. If

$$\beta_0 \neq -(1 + R/S)^{-1},$$

then CP is violated whereas equality implies $\epsilon = -1$ and $|a_{+-}| = 0$ so if $R = 2\alpha_0(R+S)$, CP-invariant case III can occur.

When $\beta_0 \neq 0$, $R \neq 0$, and $|a_{00}| \neq 0$, we find $\beta_0 < 0$ and $\beta_0 > 1$ implies CP is violated, but that $0 < \beta_0 < 1$ is inconclusive. (Here α_0 does not provide any useful constraints.)

IV. GENERAL DECAY CORRELATION FUNCTION $I(\theta_1, \theta_2, \phi)$

When no invariance principles are used to relate the nine helicity decay amplitudes describing the decay $X \rightarrow V_1 V_2$, we find the general decay correlation function is

The θ_1, θ_2 symmetry properties of this $C(\theta_1, \theta_2)$ allow the binning of all θ_1, θ_2 events into the square region $0 \leq \theta_{1,2} \leq \pi/2$; the distinct coefficients of Eq. (47) then respectively dominate the empirical $C(\theta_1, \theta_2)$ distribution near the four corners of this region.

Integrated distributions can be defined in the same way as before in Eqs. (22), (26), and (14). We define the sum of the squares of the magnitudes of the nine amplitudes

$$\mathcal{D} = \sum_{\lambda_1, \lambda_2} |a_{\lambda_1 \lambda_2}|^2 \quad (48)$$

and find the parameters appearing in the ϕ distributions are

$$\beta_0 = 2 \operatorname{Re}(a_{++} a_{--}^*) / \mathcal{D}, \quad (49)$$

$$\beta_X = -2 \operatorname{Im}(a_{++} a_{--}^*) / \mathcal{D}, \quad (50)$$

$$\alpha_0 = \operatorname{Re}(a_{++} a_{00}^* + a_{00} a_{--}^* - a_{+0} a_{0-}^* - a_{0+} a_{-0}^*) / \mathcal{D}, \quad (51)$$

$$\alpha_X = -\operatorname{Im}(a_{++} a_{00}^* + a_{00} a_{--}^* - a_{+0} a_{0-}^* - a_{0+} a_{-0}^*) / \mathcal{D}. \quad (52)$$

As in Eq. (12), we define ξ_T polar angle distributions by integrating $I(\theta_1, \theta_2, \phi)$ over ϕ and θ_j , $j=1$ or 2 , so then $\xi_T^{(i)}$, $i \neq j$, is the coefficient of $P_2(\cos \theta_i)$ and find

$$\xi_T^{(1)} - \xi_T^{(2)} = 3(|a_{0+}|^2 - |a_{+0}|^2 + |a_{0-}|^2 - |a_{-0}|^2) / \mathcal{D}, \quad (53)$$

$$\begin{aligned} \xi_T^{(1)} + \xi_T^{(2)} = & (4|a_{00}|^2 - 2|a_{++}|^2 \\ & - 2|a_{--}|^2 - 2|a_{+-}|^2 - 2|a_{-+}|^2 \\ & + |a_{+0}|^2 + |a_{0+}|^2 \\ & + |a_{-0}|^2 + |a_{0-}|^2) / \mathcal{D}. \end{aligned} \quad (54)$$

When $I(\theta_1, \theta_2, \phi)$ is only integrated over the azimuthal angle ϕ , the $C_0(\theta_1, \theta_2)$ distribution of Eq. (47) appears as in Eq. (14).

V. TESTS BY MODES OTHER THAN $V\bar{V}$ FOR VIOLATIONS OF INVARIANCES

We now make use of the results of the preceding section to list some tests for violations of invariance principles.

A. Tests of CP violation by the $V\bar{V}$ decay mode

By comparing Eq. (44) with Eq. (4), which assumed CP invariance, or by comparing their associated integrated distributions, we see the following.

(i) If $\beta_X \neq 0$ and/or $\alpha_X \neq 0$, CP invariance is violated in the decay $X \rightarrow V\bar{V}$.

A second test involving different amplitudes follows by comparing the $C_0(\theta_1, \theta_2)$ distribution of Eq. (47) with the $C(\theta_1, \theta_2)$ distribution of Eq. (7). The difference between these expressions means the following.

(ii) If

$$\int_0^{2\pi} d\phi I(\theta_1, \theta_2, \phi) \quad (55)$$

is found not to be symmetric under $\theta_1 \leftrightarrow \theta_2$ exchange, then

CP must be violated in the decay $X \rightarrow V\bar{V}$. Comparison of Eq. (55) with $C_0(\theta_1, \theta_2)$ of Eq. (47) shows that an equivalent signature of CP violation is to find $\xi_T^{(1)} - \xi_T^{(2)} \neq 0$.

Even if these two tests do not indicate CP violation, by considering Table I, it may still be possible for certain values of the decay parameters to demonstrate that there must be CP noninvariance in the decay. We do not develop this more detailed analysis in this section but note that the analogous systematic analysis was carried out in Sec. III for the $X \rightarrow VV$ mode which has more constraints and which has less background in the case $X \rightarrow \phi\phi$.

Should CP be shown to be violated, the decay correlation function $I(\theta_1, \theta_2, \phi)$ could be used to obtain additional information on the mechanism of violation. For instance, from Sec. IV of paper I we see that test (ii) requires both CP violation and C violation (even though the decay $X \rightarrow V\bar{V}$ might be P invariant). In contrast, test (i) can show CP violation because of P violation although the decay $X \rightarrow V\bar{V}$ might be C invariant. [Bose-Einstein statistics, of course, implies that Eq. (55) is symmetric under $\theta_1 \leftrightarrow \theta_2$ exchange when V_1 and V_2 are identical particles.]

B. Other decay modes

For X an electromagnetic charged state and a decay mode such as $K^* + \bar{K}^* 0$ or $D^* + D^* 0$, where V_1 and \bar{V}_2 (the antiparticle of V_2) are members of the same isospin multiplet and $I_3(V_1) = I_3(\bar{V}_2)$, the above test (i) can show P is violated. Test (ii), as discussed in paper I, can show that there is a violation of the combined usage of C and isospin invariance.

For a $V_1 V_2$ decay mode without a $V_1 \leftrightarrow V_2$ exchange property and where either V_1 or V_2 is not its own antiparticle (see Sec. III of paper I), test (i) would show that parity is violated. Modes useful for demonstrating P violation are $\phi\rho^\pm$, $\omega\rho^\pm$, and ϕK^* , ωK^* , ρK^* , where K^* denotes K^{*+} , K^{*-} , K^{*0} , \bar{K}^{*0} , and the modes obtained by replacing K^* by D^* .

For a $V_1 V_2$ decay mode without a $V_1 \leftrightarrow V_2$ exchange property but where V_1 and V_2 are each its own antiparticle, again from Sec. IV and from Sec. III of paper I, we see that test (i) would show that both P and CP are violated. Modes of this type are $\phi\rho^0$, $\omega\rho^0$, and $\phi\omega$.

VI. CP DETERMINATION FOR $J=0$ AND $J=1$

For $J=0$, $a_{\lambda_1 \lambda_2} \neq 0$ only for $\lambda_1 = \lambda_2$. By the discussion in Sec. II about the VV mode and about modes where V_1 and V_2 are each its own antiparticle, we need only transcribe earlier results^{1,3} for parity determination. The results of Sec. II for the $V\bar{V}$ mode happen to imply the same values and constraints: For a VV or $V\bar{V}$ decay mode or a mode where V_1 and V_2 are each its own antiparticle, if $\gamma_{CP} = -1$ as for a neutral technipion, then

$$\beta = -1, \quad \alpha = 0, \quad \xi_T = -1, \quad (56)$$

whereas if $\gamma_{CP} = +1$ as for a neutral elementary Higgs particle, then

$$0 \leq \beta \leq 1, \quad (57)$$

$$|\alpha| \leq [2\beta(1-\beta)]^{1/2}, \quad (58)$$

$$\xi_T = 2 - 3\beta. \quad (59)$$

For identical vector boson modes such as $\Upsilon\Upsilon$ where each Υ decays into a $\mu^+\mu^-$ or e^+e^- pair, the earlier results¹ on parity determination can similarly be used since for $X \rightarrow VV$ the constraints on the decay helicity amplitudes are the same, assuming Bose-Einstein statistics, whether P or CP invariance is assumed. Consequently, when both polar angles θ_1 and θ_2 are integrated over the full acceptance, the β parameter for $\gamma_{CP} = -1$ as for a neutral technipion is

$$\beta = -\bar{\beta}_0, \quad (60)$$

where $\bar{\beta}_0 = +0.25$ for massless fermions and in general

$$\bar{\beta}_0 = [(m^2 - 4\mu^2)/(2m^2 + 4\mu^2)]^2 \quad (61)$$

with m = mass of V , and μ = mass of fermion. For $\gamma_{CP} = +1$ as for a neutral elementary Higgs boson,

$$0 \leq \beta \leq \bar{\beta}_0. \quad (62)$$

For $J=1$, $a_{\lambda_1\lambda_2} \neq 0$ for

$$|\lambda| = |\lambda_1 - \lambda_2| \leq 1.$$

Again, for the VV mode we need only transcribe earlier results.³ For the $V\bar{V}$ mode⁸

$$\beta = 0, \quad \alpha = -\gamma_{CP}/2, \quad \xi_T = \frac{1}{2}. \quad (63)$$

Using the results of Sec. III, we find that for a $V\bar{V}$ mode, if $\gamma_{CP} = -1$, then

$$-1 \leq \beta \leq 0, \quad (64)$$

$$\alpha = \frac{1}{2}(1 + \beta), \quad (65)$$

and

$$\xi_T = \frac{1}{2}(1 + 3\beta), \quad (66)$$

whereas if $\gamma_{CP} = +1$, then

$$0 \leq \beta \leq 1 \quad (67)$$

and there is a constraint on α of [$C(\theta_1, \theta_2)$ provides coefficients in Eq. (68)]

$$|\alpha(2|a_{++}|^2 + |a_{00}|^2) + (|a_{+0}|^2 + |a_{0+}|^2)(2\alpha + 1)| \leq 2|a_{++}| |a_{00}|. \quad (68)$$

For $J=1$ and a mode where V_1 and V_2 are each their own antiparticle, Eqs. (42)–(46) of paper I apply when η is replaced by γ_{CP} in text of paper I.

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⁵The experimental phenomenology of elementary Higgs particles versus dynamical techniparticles has been discussed by many authors. A classic comprehensive discussion of elementary-Higgs-boson phenomenology is J. Ellis, M. K. Gaillard, and D. V. Nanopoulos, Nucl. Phys. **B106**, 292 (1976). Elementary-Higgs-boson and technipion signatures are compared in M. A. B. Bég, H. D. Politzer, and P. Ramond, Phys. Rev. Lett. **43**, 1701 (1979); K. Lane, in *Proceedings of the 1982 DPF Summer Study on Elementary Particle*

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⁷For several of the integrated distributions in the text, the common normalization factor is $\bar{C} = \mathcal{D}/9$, where \mathcal{D} is given by Eq. (48). When invariance principles are valid, \mathcal{D} can be expressed in terms of the minimum number of independent helicity amplitudes, e.g., in Eq. (8) in Sec. II, $\bar{C} = (2|a_{++}|^2 + 2|a_{+0}|^2 + 2|a_{0+}|^2 + |a_{+-}|^2 + |a_{-+}|^2 + |a_{00}|^2)/9$.

⁸When $\beta=0$, the sign of α is crucial [equivalently the sign of the coefficient in $A(\theta_1, \theta_2)$ of Eq. (6)]. So, it is best to determine α [or the coefficient in $A(\theta_1, \theta_2)$] using as full a θ_1 and θ_2 acceptance range as is permitted by the experimental cuts. Otherwise data is simply wasted.