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Partial-wave analysis of coherent 3π production on nuclei at 200 GeV

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We have carried out a partial-wave analysis (PWA) of three-pion systems produced in the coherent dissociation of π^+ mesons on nuclear targets. The data have been analyzed for copper and lead targets at an incident π^+ energy of 202.5 GeV. This PWA provides further evidence for resonant contributions to $J^P=1^+$ and 0^- waves at 3π masses below 1.5 GeV, which can be plausibly identified with A_1 and π' mesons. The contribution from electromagnetic production of the A_2 has also been extracted, and an estimate for Coulomb production and radiative width of the A_1 has been obtained.

INTRODUCTION

In this paper, we describe a partial-wave analysis (PWA) of three-charged-pion systems produced coherently in π^+ interactions with nuclei:

$$\pi^+ A \to \pi^+\pi^+\pi^- A \ . \tag{1}$$

The data were obtained at Fermilab using Cu and Pb targets at an incident pion energy of 202.5 GeV. Our primary goal in the analysis of reaction (1), which is dominated by diffractive production, was the extraction of the rather small contributions of A_2 and A_1 mesons expected from electromagnetic sources. We have already reported evidence for the Coulomb production of the A_1^+ and A_2^+ (Ref. 1); here we give an account of the technical aspects of our PWA of reaction (1), and present the results for other partial waves that are produced primarily through the strong interaction. Interest in the diffractive channels of reaction (1) has been renewed recently by the reported observation of resonant A_1 and π' behavior in these final states.[2–5] Our results support previous evidence of such resonant behavior in 1^+S and 0^-S partial waves for three-pion masses below 1.5 GeV.

THE EXPERIMENT

The experimental setup and data-taking procedure were described elsewhere.[6] The selection criteria and general features of the data on reaction (1) can be found in Ref. 7. For the present analysis we used 85 200 events on Cu and 30 100 events on Pb, selected within the coherent forward peaks ($t<t^*=0.4A^{-2/3}$ GeV^2), for three-pion masses $m_{3\pi}<1.5$ GeV. The latter cutoff (to be discussed below) was necessitated by our poor acceptance for events of large mass. The selected samples contain a small contamination from incoherent events that originate primarily from production off a scintillation counter located ~ 15 cm downstream of our target. This contamination decreases from $\sim 2\%$ at $m_{3\pi}=1$ GeV to $\sim 0.5\%$ at $m_{3\pi}=1.5$ GeV on Cu, and from 8 to 3% on Pb. The background at larger masses is smaller because the resolution on the reconstructed vertex improves with increasing $m_{3\pi}$. Because the background (target-empty data) exhibited a behavior similar to that observed in Cu and Pb, this contamination was ignored in the PWA.

THREE-PION PWA

Many angular momentum states can contribute to the production of 3π systems. The objective of a PWA is to separate the different contributions and measure their intensities and relative phases as a function of three-pion mass and momentum transfer. In the analysis of reaction (1), we employed the Illinois version of PWA, pioneered by Ascoli and collaborators.[8] Because the principles and practical aspects of this PWA have been discussed extensively in the literature,[9,2,4] we recall here only the most relevant points.

The decay of a state of a given mass $m_{3\pi}$, spin J, parity

P, and a spin projection M into three pions is specified through an isobar prescription of the decay, i.e., using the assumption that the produced system is composed of a single pion and a dipion isobar of internal angular momentum L. For low-mass three-particle systems, a good description of the data can be obtained with only a few isobars. We included in our analysis the 0^+, 1^-, and 2^+ isobars: ϵ with $M_\epsilon=770$ MeV and $\Gamma_\epsilon=400$ MeV, ρ^0 with $m_\rho=769$ MeV and $\Gamma_\rho=154$ MeV, and f^0 with $m_f=1273$ MeV and $\Gamma_f=179$ MeV. The contributions of different angular momentum states to the data are described by a density matrix $\rho_{\alpha\alpha'}$, where the indexes α,α' represent sets of angular momentum quantum numbers J^PLM^η. The reflection parity η is introduced by forming two different linear combinations of states with opposite-sign helicities; in this notation, M assumes only positive values. In the high-energy limit, η coincides with the naturality of the exchange producing the 3π system. No significant contribution from $\eta=-1$ states has been found in this analysis.

The density-matrix elements were measured by performing simultaneous maximum-likelihood fits for a selected set of waves to the angular decay and Dalitz-plot variables of the data. Due to the presence of nontrivial complex Breit-Wigner functions describing the decays of the isobars and as a result of the symmetrization of the amplitudes with respect to two same-charge pions, relative phases between pairs of waves, as well as wave intensities, can be extracted in this analysis. The phases we refer to below are the phases of the nondiagonal elements of the density matrix.

Because of the large number of possible waves and fit parameters involved, the following procedure was used to select waves at different mass intervals. Starting with the small set of partial waves 0^-S0^+, 0^-P0^+, 1^+S0^+ and 1^+P0^+ known to contribute at lower energies over the low-mass range, the waves with $J\leq 3$ $M\leq 2$ were admitted and kept only if their fitted intensities were significantly different from zero and if the fit likelihood improved. As a result, in addition to the above basic set, we also included 1^+D0^+, 1^+S1^+, 1^+P1^+, 2^-P0^+, 2^-S0^+, 2^+D1^+, 1^-P1^+, 3^+P0^+, and 3^+D0^+ waves in the final fits. The 2^-S and 3^+ waves contributed only for $M_{3\pi}>1.2$ GeV.

An important condition of performing a reliable partial-wave analysis is an accurate parametrization of the overall acceptance. In our experiment, the average acceptance for three-pion final states was relatively small; it dropped from 37% at $m_{3\pi}=0.9$ GeV to 20% at $m_{3\pi}=1.5$ GeV. The acceptance corrections were taken into account by applying them to PWA Monte Carlo events, which were generated according to expected theoretical distributions for each given set of waves; the final distributions were then used in fits to the data. The geometric and trigger parts of the acceptance were calculated for each theoretical PWA event by tracing the three pions through a Monte Carlo model of the spectrometer. The model was checked and shown to work very well in the studies of several other processes, including Primakoff production of mesons and beam K decays.[6]

The reconstruction efficiency was studied for events generated with a 3π production Monte Carlo model set up to closely reproduce various distributions of the data.[6] This efficiency, which was found to depend nontrivially on the PWA variables, was parametrized as a function of the four momenta of the final-state particles. To test the quality of our parametrizations, several samples of events were generated according to angular distributions expected from mixtures of partial waves, each with density-matrix elements taken to be approximately equal to those found in lower-energy measurements.[10] These events were then traced through our Monte Carlo model of the spectrometer, and the spectrometer response was generated in an event format identical to that used for our real data. Finally, our data reconstruction software and selection cuts were applied, and the accepted samples of Monte Carlo data were then partial-wave analyzed. The values for the density-matrix elements were thereby recovered and found to agree with the input values within fitting errors. An independent test was offered by the presence of an A_2 signal (2^+D1^+ wave) in the data at a level of a few percent. Despite the smallness of this wave, its phase motion was correctly detected and the intensity distributions as a function of mass and momentum transfer were found to agree (within errors of $\sim 30\%$) with an independent measurement of A_2 production, as observed in the cleaner $\eta\pi$ and KK_s decay modes (see Refs. 1 and 11 for more details). These tests convinced us that our acceptance corrections were reliable. However, due to the poor statistics and small acceptance in the A_3 region, we present the results of our analysis only for the mass range $m_{3\pi}<1.5$ GeV, which includes only the A_1 and A_2 regions.

The PWA fits were performed in the Gottfried-Jackson frame. Because our major interest was in detecting rather small contributions from Coulombic production of the A_1 and A_2 mesons, we used mass bins that were relatively wide (50-MeV bins for mass-dependent fits and 200-MeV bins for t-dependent fits). To minimize the effects of averaging over the bin size, the fits were performed in two stages. First, the approximate mass dependence of partial-wave intensities was determined assuming no variation of density-matrix elements within data bins. In the final fits however, these extracted mass dependences were used to describe the variation of expected distributions within each mass bin. Because the target dependence of the fitted 2^+D1^+ and 1^+S1^+ intensities was needed to check the dominance of the Coulomb-production hypothesis for these waves, the data on Cu and Pb targets were analyzed separately.

RESULTS

General features of the data

The overall 3π mass distribution for Cu data, together with the two main extracted components, the $J^P=0^-$ and 1^+ waves, is shown in Fig. 1. The distributions have been separately corrected for acceptance for each partial wave; the average detection efficiency is shown at the top of the figure.

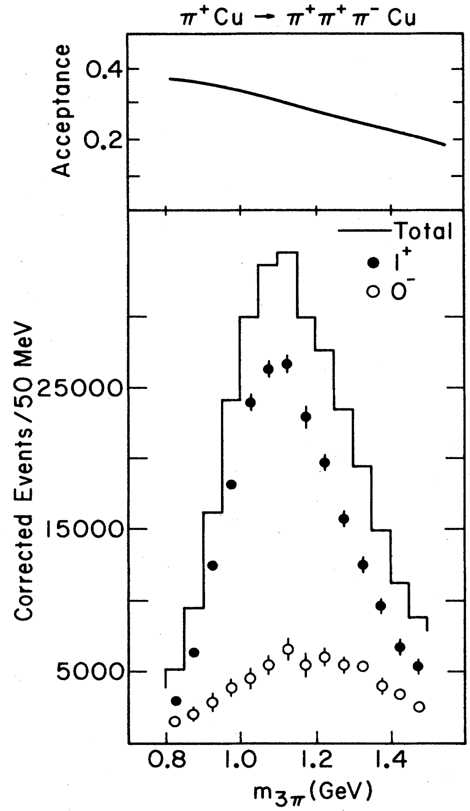

FIG. 1. The overall three-pion mass distribution for Cu data in the coherent region ($t<0.02$ GeV^2); shown separately are the contributions of 1^+ and 0^- waves. All distributions are acceptance corrected. The mass dependence of the average detection efficiency is shown at the top.

The momentum-transfer dependence for the two dominant waves within the coherent forward peaks ($t<t^*$) is shown in Fig. 2 for Cu and Pb targets. Both waves exhibit a simple exponential behavior, characteristic of nuclear form factors.

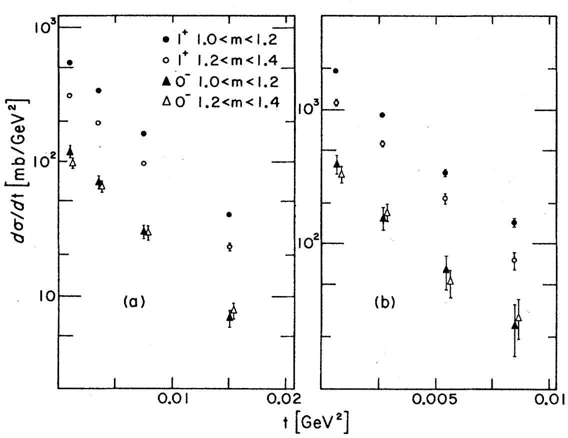

FIG. 2. The t distributions for the 1^+ and 0^- waves for Cu (a) and Pb (b) data, in the coherent regions, for the two mass intervals: $1.0<m<1.2$ GeV (1^+ solid circles, 0^- solid triangles) and $1.2<m<1.4$ GeV (1^+ open circles, 0^- open triangles).

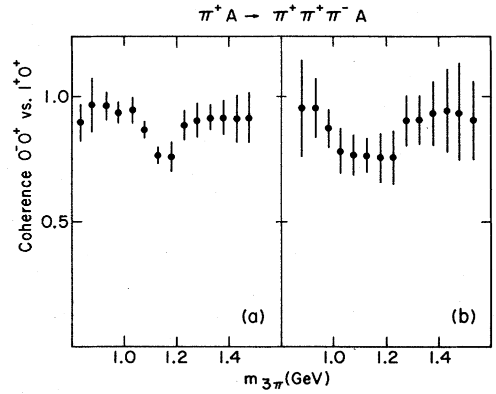

FIG. 3. Mass dependence of the degree of coherence K between 1^+0^+ and 0^-0^+ waves for Cu (a) and Pb (b) data.

Within the forward peaks, the degree of coherence between the 0^- and 1^+ waves is large; the interference, which is displayed in Fig. 3 in terms of the coherence factor K,

$$K=\frac{|\rho_{1^+0^+,0^-0^+}|}{(\rho_{1^+0^+,1^+0^+}\rho_{0^-0^+,0^-0^+})^{1/2}},$$

is almost maximal. Large values of coherence factors, a long-known virtue of forward nuclear production, provide a reliable means for extracting production phases. Nevertheless, interpretation of partial-wave results, and, in particular, conclusions concerning resonant content, is often complicated by ambiguities in the solutions. It was noted in previous PWA's of 3π production data[4,12] that

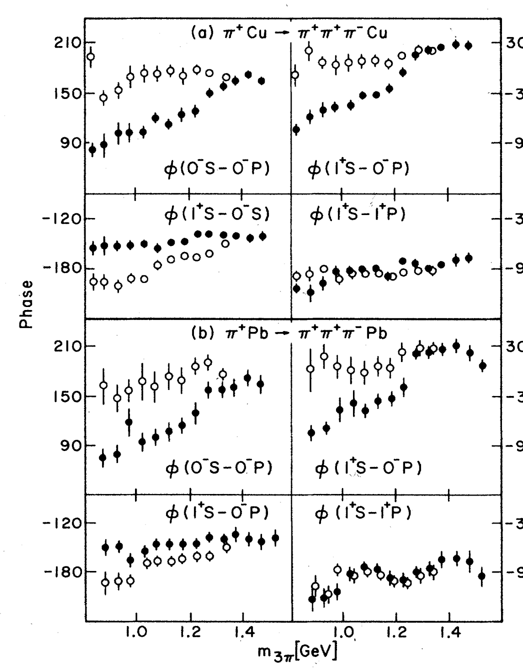

FIG. 4. Relative phases between main waves contributing to Cu (a) and Pb (b) data. Solid circles represent solution I; open circles, solution II.

there existed two solutions (labeled here I and II) with competing values of fit likelihood. Although the wave intensities were rather similar in both solutions, phases between pairs of waves differed significantly, particularly for the 0^-S-0^-P pair.

This ambiguity has also been found in our analysis. Solutions I and II are shown in Fig. 4 for the relative phases between several pairs of waves. Both solutions are quite similar to the results of a PWA at lower energies,[4] where this ambiguity was studied in detail and an argument was given that solution I should be favored. This conclusion was based on an observed instability in solution II caused by sensitivity to variations of the poorly known S-wave $\pi\pi$ phase shift.

The selection of solution I as the correct one is also supported by a recent analysis by the Amsterdam-CERN-Cracow-Munich-Oxford-Rutherford ACCMOR collaboration.[2] Their PWA of very-high-statistics data for 3π production on protons yielded an unambiguous solution corresponding to solution I of Ref. 4 and of our paper. For these reasons we present figures and conclusions based primarily on this solution. This selection does not affect our estimate of the radiative width of the A_1.[1]

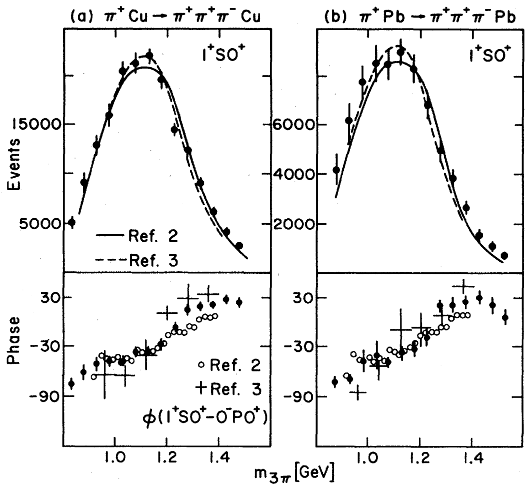

FIG. 5. Mass dependence of the 1^+S0^+ wave intensity and its phase relative to 0^-P0^+ for Cu (a) and Pb (b) data (solid circles). The solid (dashed) lines represent results of a two-component-model fit to the data of Ref. 2 (Ref. 3), normalized to our measurement. Also shown are the actual data of Ref. 2 (open circles) and of Ref. 3 (pluses) for the 1^+S-0^-P phase difference.

The 1^+S0^+ wave

The main interest of 3π PWA's has long focused on the 1^+S0^+ wave and the related A_1 problem. It has recently been observed that this wave has a large phase motion across the A_1 enhancement.[2–4] Model fits to its intensity and phase needed significant contributions of a resonant A_1, in addition to a nonresonant background. Employing a unitarity-corrected two-component model, the A_1 mass has been estimated in the range 1.22—1.28 GeV and its A_1 width at about 300 MeV.[2,3,5]

Our data on the 1^+S0^+ wave are shown in Fig. 5. The mass shape agrees well with fits to low-t proton[2] and lower-energy nuclear[3] data. A fit of the two-component model of Ref. 2 to their measurement of 1^+S intensity in the lowest t bin, $t < 0.05$ GeV^2 is shown as the solid curve in Fig. 5 (after normalization to our data). A fit from Ref. 3 to their Si target data is represented by the dashed curve. In order to describe 1^+S intensities and phases, both fits required significant contributions from a resonant A_1.

In our measurement we observe a rapid phase motion of the 1^+S0^+ wave with respect to 0^-P0^+. This result is compared to the data of Ref. 2 (open circles) and of Ref. 3 (pluses). For the comparison with our Cu (or Pb) data we used the 1^+S-0^-P phase differences obtained in Ref. 3 for the combined Ti and Cu (or Ag, Ta, and Pb) data. Despite some differences, it is clear that all the sets of data indicate strong phase motion of the 1^+S wave. An additional comment is in order here. Because only phase differences are measured in a PWA, clearly, phase motion of a reference wave can affect the interpretation of results, especially in the case of broad structures. Our selection of 0^-P as reference wave follows the choice used in Ref. 2. In those data, the 2^+D1^+ wave was measured over the mass range relevant to our discussion. Because this wave corresponds, essentially, to the A_2 resonance, its phase is known and the phase behavior of other waves can be gauged against it. Thus, it was possible to establish in Ref. 2, that the 0^-P phase was essentially independent of mass across the A_1 bump, while the 1^+S and 0^-S waves had phase motion. According to Ref. 2, the 0^-P phase starts to increase above ~ 1.4 GeV. Consequently, the actual motion of the 1^+S phase may be even larger than given by the measurement relative to 0^-P.

The increase of the 1^+S phase in our data is somewhat larger than found on a proton target,[2] but not quite as large as reported in Ref. 3. Also there is little difference between our 1^+S phase measurements on Cu and Pb targets, so that our data alone cannot establish any clear A dependence of this phase.

We conclude that our data support the results of previous measurements on the intensity and phase of the 1^+S wave, and thus provide further evidence for a resonant A_1. Extraction of detailed values for A_1 parameters must await the completion of a model-dependent analysis of the partial-wave data.

The 0^-S0^+ wave

A resonantlike signal was also detected in our extracted 0^-S wave. The intensity of this wave, presented in Fig. 6 (solid circles), exhibits a clear enhancement at a 3π mass of ~ 1.2 GeV; the phase relative to 0^-P increases across this structure by $\sim 90°$. These observations indicate a strong resonant contribution to this wave. The results of a simple Breit-Wigner fit, and a noninterfering polynomial background, are also shown in the figure (curves). The resonant parameters are estimated to be $m = 1.19 \pm 0.03$ GeV and $\Gamma = 440 \pm 80$ MeV.

A similar signal has been observed previously[3,5] and interpreted as a radial excitation of the pion (π'). Our data

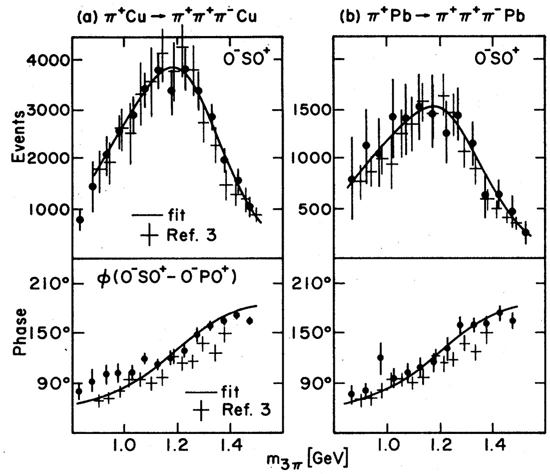

FIG. 6. Mass dependence of the 0^-S wave intensity and its phase relative to 0^-P for Cu (a) and Pb (b) data (solid circles). Solid lines represent a fit to a Breit-Wigner and noninterfering background. The data of Ref. 3 are also shown for comparison (pluses). (The intensity results of Ref. 3 have been normalized to our data.)

(and the extracted π' parameters) agree well with those reported in Ref. 3; the previous results are shown as pluses in Fig. 6. The phase motion of 0^-S in our data is, however, stronger than the one measured on a proton target[2] (not shown). This may be taken as support for an observation made in Ref. 3 that nuclear interactions seem to enhance resonant contributions. The relative 0^-S-1^+S phase is almost constant across the low-mass range (Fig. 4), confirming a similarity in the parameters and structure in these two waves.

Other waves

The strongest of the remaining waves that were included in our analysis are shown in Fig. 7. Neither their intensities nor their phase behaviors indicate the presence of any clear resonant contributions over the mass range of our study. We note, however, that the phase of the 1^+P wave remains almost constant relative to 1^+S; this is similar to results obtained at lower energies. Thus, if, as we just argued, the 1^+S phase has resonantlike motion, then the 1^+P phase must also increase significantly, even though its intensity does not show any apparent resonant peak.

The 1^+D and 2^-S (not shown) waves, although rather small below 1.5 GeV, tend to increase sharply toward the high-mass end of the range of our analysis; this is indicative of more activity in these waves at larger mass values. The 2^-P waves appear much weaker for $m_{3\pi} < 1.5$ GeV (relative to main waves) than in lower-energy measurements.

The two helicity-one waves found to be clearly nonnegligible in our study, namely, $1^+S\,1^+$ and $2^+D\,1^+$, have been discussed in a separate paper,[1] and will therefore not be reproduced here. Both waves were found consistent with a dominant Coulomb-production mechanism, which provided estimates for radiative widths of the A_1 and A_2. We obtained $\Gamma_\gamma = 640 \pm 240$ keV for the A_1, and supported our earlier measurement of $\Gamma_\gamma = 295 \pm 60$ keV for the A_2.[11]

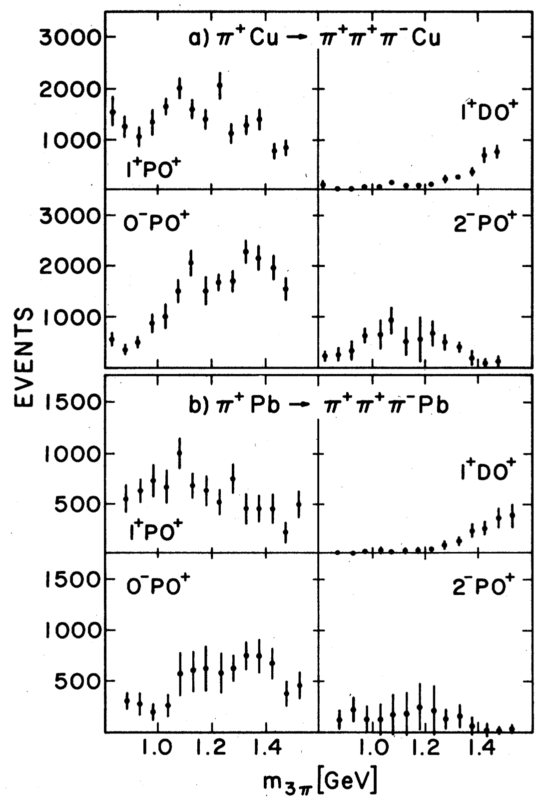

FIG. 7. Mass dependence of the intensities for 1^+P, 1^+D, 0^-P, and 2^-P waves for Cu (a) and Pb (b) data.

SUMMARY

We have carried out a detailed partial-wave analysis of a high-statistics experiment on coherent $\pi^+\pi^+\pi^-$ production on Cu and Pb targets by incident π^+ mesons of 202.5 GeV. For the mass range $m_{3\pi} < 1.5$ GeV our study yielded the following main results.

(i) The phase of the 1^+S0^+ wave increases relative to 0^-P by 110° across the A_1 enhancement. The mass dependence of the intensity of this wave is consistent with model-dependent fits to measurements at lower energy; these required a significant contribution of an A_1 meson of mass of 1.2—1.3 GeV and width about 300 MeV. Thus our data provide confirmatory evidence for a resonant A_1.

(ii) The phase of the 0^-S wave relative to 0^-P increases by $\approx 90°$ across the mass range of our study, and the 0^-S intensity shows a clear enhancement; both results are indicative of a resonance contributing to this wave. The estimated resonant parameters are $M = 1.19 \pm 0.03$ GeV and $\Gamma = 440 \pm 80$ MeV. This resonance may be plausibly interpreted as a radial excitation of the pion.

(iii) The phase motion of both 1^+S and 0^-S waves is larger than measured on a proton target. Taken in conjunction with the results of an independent nuclear target measurement,[3] this may indicate that nuclear production tends to enhance resonant components.

(iv) A clear signal of two helicity-one waves, 1^+S1^+ and 2^+D1^+, has been detected at a level of a few percent.[1] Both waves are consistent with an electromagnetic production mode. The radiative width of the A_1 has been estimated to be $\Gamma_\gamma = 640 \pm 240$ GeV. The results for the A_2 are consistent with a previous measurement of $\Gamma_\gamma = 295 \pm 60$ keV (Ref. 11) and demonstrate reliability of our PWA down to signal intensities of the order of 3%.

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[1] M. Zielinski et al., Phys. Rev. Lett. **52**, 1195 (1984).
[2] C. Daum *et al.*, Phys. Lett. **89B**, 281 (1980); Nucl. Phys. **B182**, 769 (1981).
[3] G. Bellini *et al.*, Phys. Rev. Lett. **48**, 1697 (1982); Nuovo Cimento Lett. **38**, 433 (1983); Nuovo Cimento **79A**, 282 (1984).
[4] J. Pernegr *et al.*, Nucl. Phys. **B134**, 436 (1978).
[5] R. Aaron and R. S. Longacre, Phys. Rev. D **24**, 1207 (1981).
[6] T. Jensen *et al.*, Phys. Rev. D **27**, 26 (1983).
[7] M. Zielinski *et al.*, Z. Phys. C **16**, 197 (1983).
[8] G. Ascoli, *et al.*, Phys. Rev. D **7**, 669 (1973).
[9] J. D. Hansen *et al.*, Nucl. Phys. **B81**, 403 (1974).
[10] T. Roberts *et al.*, Phys. Rev. D **18**, 59 (1978), and private communications from T. Roberts.
[11] S. Cihangir *et al.*, Phys. Lett. **117B**, 119 (1982).
[12] R. L. Schult and H. W. Wyld, Phys. Rev. D **16**, 62 (1977).