Vector-pseudoscalar coupling as a model of nonlinear confinement

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(Received 10 April 1984; revised manuscript received 13 July 1984)

We investigate a Lagrangian with fourth-order terms of the form of vector-pseudoscalar couplings as a model for the description of interactions with confinement of c-number spinorial fields. This Lagrangian is shown to belong to a class of fourth-order Lagrangians leading to kink or particlelike solutions. The masses obtained for the baryonlike and spin-one mesonlike solutions coincide with those found in other previous models, but the mass of the spin-zero mesonlike solution turns out to be 228 MeV.

I. INTRODUCTION

The possibility of using the intrinsic properties of nonlinear equations for seeking an explanation of some of the characteristics of confinement has been studied lately.[1] The solutions of the nonlinear equations associated with the system of three Dirac fields ψ_k, $k=1,2,3$, and three fields ϕ_k, $k=1,2,3$, charged conjugated to the ψ_k were studied in two recently proposed models[2] (V model and S-P model) of interactions of c-number fields. These equations were obtained from the Lagrangian density

$$L=L_1+L_2+L_3\ , \tag{1a}$$

$$L_1=\sum [L_D(\psi_k)+L_{\bar{D}}(\phi_k)]\ , \tag{1b}$$

$$L_3=\lambda \sum_{i<j} (\bar{\chi}_{ij}\chi_{ij})^2\ , \tag{1c}$$

where $L_D(\psi)$ and $L_{\bar{D}}(\phi)$ are the usual linear Lagrangian densities with a change of sign in the derivative terms in $\partial_\mu\phi$, and χ_{ij} denotes $\chi_{ij}=\chi_i-\chi_j$, $\chi_i=\psi_i+\gamma^5\phi_i$. As far as L_2 is concerned, it was of the form

$$L_2=\frac{\lambda}{3}[V_\mu(\psi)V^\mu(\psi)+V_\mu(\phi)V^\mu(\phi)+4V_\mu(\psi)V^\mu(\phi)]\ , \tag{1d}$$

for the V model and

$$L_2=\frac{\lambda}{3}\{[S^2(\psi)+S^2(\phi)+4S(\psi)S(\phi)] -[P^2(\psi)+P^2(\phi)+4P(\psi)P(\phi)]\}\ , \tag{1e}$$

for the S-P model. We have denoted by S, P, and V^μ the bilinear forms

$$S(\psi)=\sum \bar{\psi}_k\psi_k,\quad P(\psi)=\sum \bar{\psi}_k\gamma^5\psi_k\ ,\qquad V^\mu(\psi)=\sum \bar{\psi}_k\gamma^\mu\psi_k\ , \tag{2}$$

and the analogous forms of ϕ_k.

The fourth-order terms in the Lagrangian L are grouped in two parts: L_2 and L_3. The first one, L_2, represents the two-body forces between the fields introducing the nonlinearities in the equation causing the existence of particlelike or kink solutions. The second one, L_3, prevents the existence of solutions violating triality. The presence of the covariant forms P or V^μ is needed in order to introduce spin-dependent forces splitting the masses of the scalar and vector mesons. This result would not be obtained using only the covariant form S. Moreover, the form P alone does not guarantee the existence of particlelike solutions.

These two models must be considered as two different first approximations to the problem, not only by their classical character previous to the quantization, but fundamentally because both make use of only one kind (flavor) of constituent field. A more realistic SU(2) model must be built over the one that looks more accurate in its physical predictions. Curiously enough, although they are different concerning the corresponding associated nonlinear problem, they agree for the value of the mass of the scalar mesonlike solution that was close to that of the η in both cases. This gives rise to the question of whether this agreement is a general feature characterizing all these nonlinear models. Consequently it is worthwhile to study other possible fourth-order confining Lagrangians before attacking the SU(2) generalization.

II. VECTOR-PSEUDOSCALAR COUPLING

We present a new model, from now on called the V-P model, based on the Lagrangian L_2,

$$L_2=\frac{\lambda}{3}\{[V_\mu(\psi)V^\mu(\psi)+V_\mu(\phi)V^\mu(\phi)+4V_\mu(\psi)V^\mu(\phi)] -[P^2(\psi)+P^2(\phi)+4P(\psi)P(\phi)]\} \tag{3}$$

with L_1 and L_3 as before. The field equations are

$$(i\gamma^\mu\partial_\mu-m)\psi_a+\tfrac{2}{3}\lambda\{[V_\mu(\psi)+2V_\mu(\phi)]\gamma^\mu\psi_a-[P(\psi)+2P(\phi)]\gamma^5\psi_a\}+2\lambda\sum(\bar{\chi}_{ak}\chi_{ak})\chi_{ak}=0\ , \tag{4a}$$

$$(-i\gamma^\mu\partial_\mu-m)\phi_a+\tfrac{2}{3}\lambda\{[V_\mu(\phi)+2V_\mu(\phi)]\gamma^\mu\phi_a-[P(\phi)+2P(\psi)]\gamma^5\phi_a\}+2\lambda\sum(\bar{\chi}_{ak}\chi_{ak})\chi_{ak}=0\ . \tag{4b}$$

As was proved in Ref. 1, the most important qualitative feature is the appearance of constraint equations between the fields. They are introduced by L_3, and in general place restrictions on the existence of solutions in such a way that the only permitted ones are those verifying triality.

A. Baryonlike solutions

The baryonlike solutions are given by those solutions of the field equations with the additional condition of vanishing conjugate fields $\phi_k=0$, $k=1,2,3$. The terms introduced in (4b) by L_3 impose on the system the constraint of the equality of the other three fields $\psi_k=\psi$, $k=1,2,3$, reducing (4a) to only one equation, the one associated to the one field Lagrangian L,

$$L=L_D(\psi)+\lambda[(\bar{\psi}\gamma_\mu\psi)(\bar{\psi}\gamma^\mu\psi)-(\bar{\psi}\gamma^5\psi)^2] . \quad (5)$$

Finkelstein[3] proved that the most general form of the Lagrangian density of a Dirac field with a fourth-order self-coupling is

$$L=L_D(\psi)+\lambda[(\bar{\psi}\psi)^2+z(\bar{\psi}\gamma^5\psi)^2] , \quad (6)$$

where z is a real parameter. The Lagrangian (5) corresponds to the value $z=0$ and therefore is equivalent to a pure scalar self-coupling of the form $(\bar{\psi}\psi)^2$ which has been studied by Soler[4] and Finkelstein.[5] For the S-wave spinors $\psi_\uparrow$ and $\psi_\downarrow$ (see Appendix) the Lagrangian (6) leads to the nonlinear coupled radial equations:

$$G'+(1+\Omega+F^2-MG^2)F=0 , \quad (7a)$$

$$F'+\frac{2}{\rho}F+(1-\Omega+MF^2-G^2)G=0 , \quad (7b)$$

where the coefficient M is given by $M=1-2z/3$ and therefore, for (5), takes the value $M=1$. We obtain, by integration of the densities associated to (5), that the energy, norm, and spin have the values

$$E=3\left[\frac{2\pi}{\lambda m}\right]\mathscr{E},\quad \mathscr{E}=\Omega I_1+\tfrac{1}{2}I_2 , \quad (8a)$$

$$N=3\left[\frac{2\pi}{\lambda m^2}\right]I_1 , \quad (8b)$$

$$\vec{S}=\left[0,0,\pm\frac{N}{2}\right] , \quad (8c)$$

where I_1 and I_2 are the integrals

$$I_1=\int_0^\infty (F^2+G^2)\rho^2 d\rho , \quad (9a)$$

$$I_2=\int_0^\infty (F^2-G^2)^2\rho^2 d\rho . \quad (9b)$$

B. Antibaryonlike solutions

There is a charge-conjugate solution to the previous one with $\psi_k=0$, $\phi_k=\phi$, $k=1,2,3$. For the S-wave spinors $\phi_\uparrow$ and $\phi_\downarrow$ (see Appendix) we obtain the same radial equations. This solution has the same energy (8a) and spin (8c) but opposite baryonic norm (8b) as consequence that the conserved current is

$$j^\mu=\sum(\bar{\psi}_k\gamma^\mu\psi_k-\bar{\phi}_k\gamma^\mu\phi_k) . \quad (10)$$

C. Mesonlike solutions

Let us now consider the two-field solutions with zero baryonic norm. They are of the form $\psi_i=\psi$, $\phi_i=\phi$, $\psi_j=\psi_k=\phi_j=\phi_k=0$, where (i,j,k) is any permutation of (1,2,3). For parallel spins $\psi=\psi_\uparrow$, $\phi=\phi_\uparrow$, the radial equations are of the form (7) with $M=1$ just the same as for the baryonic solutions. The energy, norm, and spin are

$$E=2\left[\frac{2\pi}{\lambda m}\right]\mathscr{E},\quad \mathscr{E}=\Omega I_1+\tfrac{1}{2}I_2 , \quad (11a)$$

$$N=N(\psi)+N(\phi)=0,\quad N(\psi)=\frac{2\pi}{\lambda m^2}I_1 , \quad (11b)$$

$$\vec{S}=(0,0,N(\psi)) . \quad (11c)$$

The energy of this solution is $\frac{2}{3}$ times that of the baryon and the baryonic norm is zero. If m and λ are chosen in such a way that $N(\psi)=\hbar$, the baryon has spin $\frac{3}{2}$ and this meson has spin one. For antiparallel spins $\psi=\psi_\uparrow$, $\phi=\phi_\downarrow$ and we obtain radial equations of the form (7) with $M=-\frac{5}{3}$ corresponding to $z=4$. The energy, norm, and spin are

$$E=2\left[\frac{2\pi}{\lambda m}\right]\mathscr{E},\quad \mathscr{E}=\Omega I_1+\tfrac{1}{2}I_2+\tfrac{8}{3}I_3 , \quad (12a)$$

$$N=N(\psi)+N(\phi)=0,\quad N(\psi)=\frac{2\pi}{\lambda m^2}I_1 , \quad (12b)$$

$$\vec{S}=(0,0,0) , \quad (12c)$$

$$I_3=\int_0^\infty F^2G^2\rho^2 d\rho , \quad (12d)$$

corresponding to a spin-zero meson.

The different values found for the parameter z in (5) together with those obtained in other models are given in Table I.

D. Study of the radial equations

No analytic solutions to the system (7) are known but some of its qualitative properties will indicate to us how to obtain numerically the solutions that tend to zero as $\rho\to\infty$. For those Ω such that $\Omega^2<1$, there exist solutions that behave at infinity as $F\to 0$, $G\to\pm\sqrt{1-\Omega}$. This asymptotic behavior seems to depend on the two initial data $F(0)$ and $G(0)$, but as the term F/ρ in (7b) imposes $F(0)=0$, the only real dependence is in $G(0)$. Those values of $G(0)$ that generate the separatrices between the

TABLE I. Values of z in the three models (V, S-P, V-P).

	V model	S-P model	V-P model
Baryon or spin-one meson	1	-1	0
Spin-zero meson	$\frac{11}{3}$	$\frac{1}{3}$	4

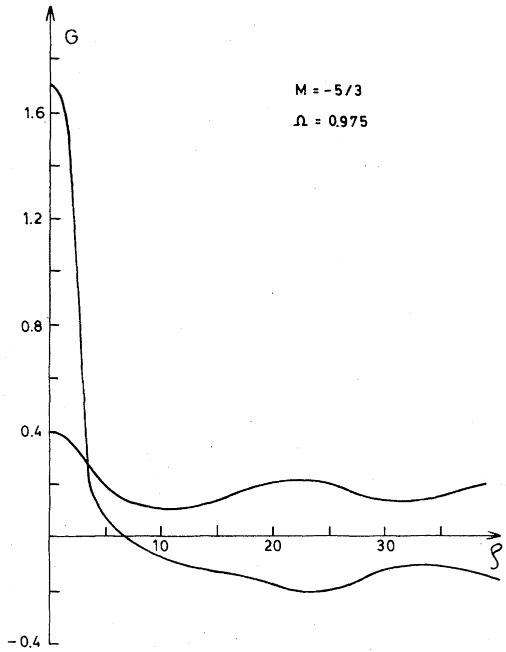

FIG. 1. Behavior of the function $G(\rho)$ for two different values of the initial data $G(0)$ for the values $M=-\frac{5}{3}$, $\Omega=0.975$.

positive and negative behavior (see Fig. 1) are the only initial data that lead to square-integrable solutions.

According to the relation between the nodeless solutions and the corresponding values of $G(0)$, two types of equations are possible whose basic representatives are those associated with the Finkelstein-Soler[4] (FS) and to the Dirac-Weyl[6] (DW) models characterized by the values

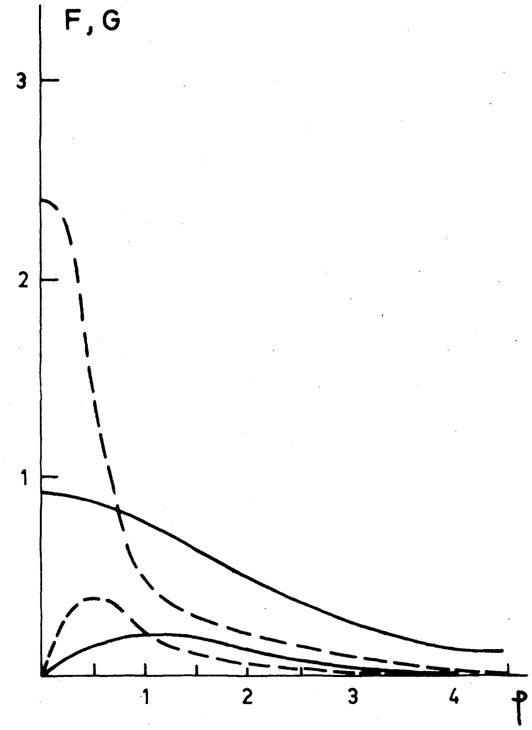

FIG. 3. Radial functions F and G vs ρ in the cases of the spin-one meson (solid curve) and the spin-zero meson (dashed curve).

$z=0$ and $z=1$, respectively.

For $M=1$, as was said above, the solutions are known. The energy (8a) as a function of Ω has only one minimum located at the point $\Omega=0.936$ with value $\mathscr{E}=3.7569$. For $M=-\frac{5}{3}$ the solutions show characteristics according to

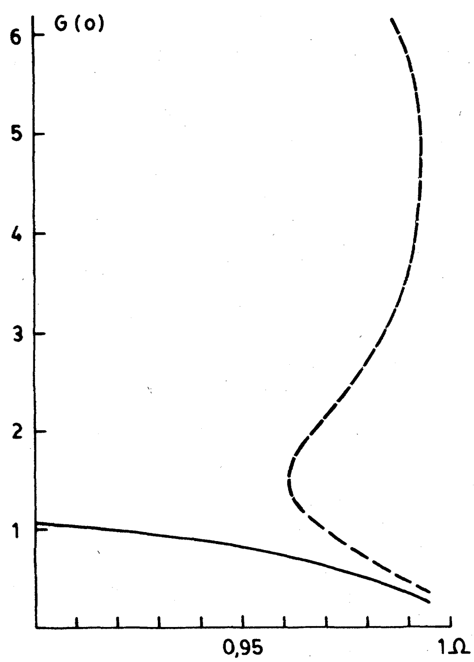

FIG. 2. Value of $G(0)$ for the nodeless solutions as a function of Ω for $z=0$ (solid curve) and $z=4$ (dashed curve).

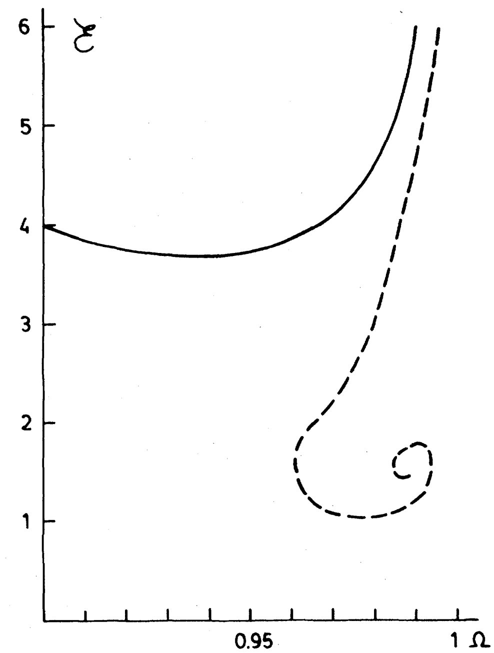

FIG. 4. Shape of the curves $\mathscr{E}(\Omega)$ corresponding to the spin-one meson (solid curve) and the spin-zero meson (dashed curve).

the DW model, that is to say, there exist nodeless solutions only for those values of Ω that are greater than a certain Ω_m that in this case is obtained to be $\Omega_m = 0.961$. Moreover, for some of these Ω such that $\Omega > \Omega_m$ there are several values of $G(0)$ leading to square-integrable solutions. As a consequence for $M = -\frac{5}{3}$, contrary to what happens for $M=1$, the curve of the energy is concentrated on the narrow interval $0.961 < \Omega < 1$ and has the very curious spiral form associated with the DW behavior. It takes the minimum value $\mathscr{E} = 1.0710$ for $\Omega = 0.975$. In Fig. 2 we plot $G(0)$ against Ω for the two cases. The functions $F(\rho)$, $G(\rho)$, and $\mathscr{E}(\Omega)$ are plotted in Figs. 3 and 4.

E. Physical interpretation

If we take $m = 390$ MeV and $\lambda = 23/m^2$ the rest mass E, baryonic norm N, spin S, and mean-square radius of the three solutions are the following.

Baryon or antibaryon:

$$E = 1200 \text{ MeV}, \quad S = \tfrac{3}{2}\hbar ,$$

$$N = \pm\hbar, \quad \langle r^2 \rangle^{1/2} = 1.65 \text{ fm} .$$

Spin-one meson:

$$E = 800 \text{ MeV}, \quad S = \hbar ,$$

$$N = 0, \quad \langle \vec{r}^{\,2} \rangle^{1/2} = 1.65 \text{ fm} .$$

Spin-zero meson:

$$E = 228 \text{ MeV}, \quad S = 0 ,$$

$$N = 0, \quad \langle r^2 \rangle^{1/2} = 1.78 \text{ fm} .$$

The characteristics of the baryon and of the spin-one meson are very similar to those of the $\Delta(\frac{3}{2}+)$ and of the ω or the ρ^0. The value of the mass of the spin-zero meson is less straightforward and deserves a further analysis. These results are compared with those previously found with other Lagrangians in Table II.

TABLE II. Results in V, S-P, and V-P models.

	V model	S-P model	V-P model
m (MeV)	286	393	390
λm^2	6.49	28.47	23
Mass of the baryon (MeV)	1200	1200	1200
Mass of the spin-one meson (MeV)	800	800	800
Mass of the spin-zero meson (MeV)	582	552	228

III. SUMMARY AND CONCLUSIONS

To sum up, as Table II shows, the above-quoted agreement between the V and S-P models is not a general characteristic of all these Lagrangian densities and therefore the prediction for the pseudoscalar-meson mass depends on the interactions represented in L_2.

The fact that the baryonlike solutions are associated with a self-coupling of the form $(\bar{\psi}\psi)^2$ leading to the simplest value of the parameter M in Eqs. (7), that is to say $M=1$, gives a very attractive aspect to this new model from a mathematical point of view. However, it seems that the lower value obtained for the lowest pseudoscalar mass is too big for the pion.

The interpretation of the mesonlike solutions presents the difficulties of working with only one kind of field corresponding, for example, to the u quark. This nonexistence of flavor causes us not to have enough quantum numbers to allow for the whole mass spectrum and consequently, it constrains the resulting solutions to be taken as representative of all the associated multiplets. Also, while for the 1^- meson octet the two particles with a $\bar{u}u$ contribution to the wave function, the ω and the ρ^0, have similar values for their masses, this is not the case for the η and the π^0 in the 0^- octet. The two models studied in Ref. 2 give solutions associated to the η; this new model changes this result and gives instead a value closer to the π^0. The V-P model is not so accurate as the other two but, what is very important, it proves that this nonlinear approach admits different nonequivalent models.

It is to be expected that the SU(2) generalizations of the three models, obtained by incorporating τ matrices in L_2 coupling the u and d components, will solve this problem by introducing new types of particlelike solutions characterized by its isospin and it will permit us to select the model whose spectrum is closest to physical reality.

ACKNOWLEDGMENTS

I would like to thank A. F. Rañada for fruitful discussions. This work is part of a program partially supported by CAICYT (Comisión Asesora de Investigacion Cientifica y Tecnica).

APPENDIX

In this appendix we give the concrete form of the different S-wave spinors used throughout the paper. The spin-up and spin-down fields ψ are

$$\psi_\uparrow = \begin{bmatrix} g \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ if \begin{bmatrix} \cos\theta \\ \sin\theta e^{i\phi} \end{bmatrix} \end{bmatrix} e^{-i\omega t} ,$$

$$\psi_\downarrow = \begin{bmatrix} g \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\ if \begin{bmatrix} \sin\theta e^{-i\phi} \\ -\cos\theta \end{bmatrix} \end{bmatrix} e^{-i\omega t} , \qquad \text{(A1)}$$

and the corresponding fields ϕ are

$$\phi^{\uparrow}=\begin{bmatrix} g\begin{bmatrix}0\\1\end{bmatrix} \\ if\begin{bmatrix}-\sin\theta e^{-i\phi}\\ \cos\theta\end{bmatrix}\end{bmatrix}e^{i\omega t}\,,$$

$$\phi_{\downarrow}=\begin{bmatrix} g\begin{bmatrix}1\\0\end{bmatrix} \\ -if\begin{bmatrix}\cos\theta\\ \sin\theta e^{i\phi}\end{bmatrix}\end{bmatrix}e^{i\omega t}\,. \tag{A2}$$

For the purpose of best simplifying the nonlinear equations, it is advantageous to express the equations in terms of dimensionless variables defined by

$$\Omega=\omega/m\,,\quad \rho=mr\,,\quad (F,G)=\left[\frac{2\lambda}{m}\right]^{1/2}(f,g)\,. \tag{A3}$$

It is in these variables that Eqs. (7) are formulated.

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