Natural class of non-Peccei-Quinn models

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We discuss a class of models, not based on the Peccei-Quinn mechanism, that solve the strong *CP* problem. These are a generalization of a model of Nelson. Improved conditions are given for such models. These conditions are natural and in many simple models are automatically satisfied.

I. INTRODUCTION

One of the most subtle and intriguing puzzles in the theory of *CP* violation is the so-called strong *CP* problem which is that of explaining in a natural way the apparent absence of *CP*-violating effects in the strong interactions. It was once thought that the QCD Lagrangian automatically conserved *CP*. The discovery of instanton effects[1] showed that this is not so. The parameter that measures the magnitude of *CP*-violating effects in QCD is called $\bar{\theta}$ and is given by

$$\bar{\theta}=\theta_{\rm QCD}+\theta_{\rm QFD}\ . \tag{1}$$

$\theta_{\rm QCD}$ is the vacuum angle of QCD or, equivalently, the coefficient of the term $(1/32\pi^2)G^{a*}_{\mu\nu}G^{a\mu\nu}$ in the QCD Lagrangian ($G^a_{\mu\nu}$ is the gluon field-strength tensor). $\theta_{\rm QFD}$ is the phase of the determinant of the quark mass matrix, that is, $\theta_{\rm QFD}=\arg\det M_Q$. Under chiral rotations of the quark fields $\theta_{\rm QFD}$ and $\theta_{\rm QCD}$ may change but $\bar{\theta}$ is an invariant, physically observable quantity. $\bar{\theta}$, being an angle, may in principle take values between 0 and 2π, but is experimentally known[2] to be less than or of the order of 10^{-9}. Since $\bar{\theta}=0$ does not correspond to an enlarged symmetry of the theory this constitutes a severe fine-tuning problem.

There are two known ways to resolve this problem. The generally favored of these is the Peccei-Quinn mechanism.[3] This is so familiar that we will not undertake to explain it here. The Peccei-Quinn mechanism is unquestionably a very elegant and economical solution to the strong *CP* problem. Nevertheless, there are two reasons for exploring the alternatives. First, elegance alone is not enough to guarantee the correctness of any idea, so it is important to know what the viable theoretical alternatives are. Second, the Peccei-Quinn models are already under some pressure from astrophysical constraints. The scale of breaking of the Peccei-Quinn symmetry (call it $M_{\rm PQ}$) must be larger than about 10^9 GeV or else axions couple to ordinary matter strongly enough that the emission of axions from stars will be too copious and affect stellar evolution in ways inconsistent with observation.[4] On the other hand, $M_{\rm PQ}$ must be less than about 10^{12} GeV or else the energy trapped in the cosmological axion field will be large enough to overclose the Universe. This is the axion energy problem.[5] Therefore, there is necessarily a new intermediate scale in such models. Furthermore, unless some care is taken to avoid them, Peccei-Quinn models will lead to cosmological domain walls.[6] These could be red-shifted away in an inflationary epoch which occurs after the Peccei-Quinn symmetry breaks; however, such an inflation would necessitate generating the baryon asymmetry at temperatures below $M_{\rm PQ}$ (which is rather low). The problem of domain walls can be avoided in other ways,[6] but these involve constraints on the structure of the Peccei-Quinn models.

Heretofore, no simple or particularly plausible alternative to the Peccei-Quinn mechanism existed. A class of models based on spontaneous *CP* breaking which solves the strong *CP* problem indeed exists in the literature.[7] However, these models suffer from a number of difficulties. In the first place, it is necessary to impose in them some symmetry for no purpose other than to render $\theta_{\rm QFD}$ zero at the tree level. These are not symmetries one would have suspected to exist for any other reason. In the second place, all of the models of this type constructed in the past involve *CP* being broken at the same scale as $\mathrm{SU(2)\times U(1)}$. This leads to a (probably) insurmountable problem with cosmological domain walls. It also makes it difficult if not impossible to generate an adequate baryon asymmetry. Furthermore, to accomplish this *CP* breaking requires the low-energy $\mathrm{SU(2)\times U(1)}$-breaking Higgs sector to be nonminimal. [This is so because to violate *CP* one needs to have some nontrivial *relative* phase between two (or more) vacuum expectation values.] This leads to unacceptable flavor-changing neutral-current interactions mediated by scalars, unless the Higgs masses are quite large—in the TeV range. A final problem with these models is that they were found to be difficult to unify.[8]

Recently, however, a much superior version of these models with spontaneously broken *CP* has been proposed.[9] It suffers from none of the above difficulties. Far from being contrived to solve the strong *CP* problem, it was in fact put forth for other reasons[10] and only afterwards discovered to resolve this problem. This model was generalized in Ref. 11. In that paper it was observed that any model satisfying two simple criteria would solve the strong *CP* problem and have the same desirable features as the model of Ref. 9. A calculation of $\bar{\theta}$ in this general class of models was presented in Ref. 12.

In the present paper we have several further remarks to make on this new class of models. In the first place, we wish to present a similar but considerably less restrictive set of conditions than that presented in Ref. 11. Second, we will discuss these conditions somewhat more fully than was done in that paper and make clear in particular that these conditions are technically natural and quite simple to implement. Finally, we wish to dwell on some virtues of these models not previously emphasized.

II. THE NEW CLASS OF MODELS

A. Superheavy fermions

A key element of the new class of models discovered in Refs. 9 and 11 is the existence of superheavy fermions. Such fermions are a common feature of grand unified models. (They appear, for example, in various types of models which naturally give rise to a light-fermion-mass hierarchy. They are also usual in Peccei-Quinn models which manage to avoid the Sikivie domain-wall problem by having the Peccei-Quinn symmetry break down to a unique vacuum[6].) In fact, the Georgi "survival hypothesis"[13] leads one to suspect that such fermions may be present. According to the survival hypothesis, if a unified group G breaks at superlarge mass scales down to $SU(3)\times SU(2)\times U(1)$, then all fermions which *can* have G-invariant masses *will* naturally have such masses (unless some unbroken symmetry prevents it) and these masses will be superlarge. Thus, if we have a unified model whose fermions are in a set of representations $S=F+R$, where R is a *real* set under G and F is a complex set under G, then at low energies only a set of fermions with the quantum numbers of F will remain light. The remaining set with the quantum numbers of R will become superheavy. This is because a G-invariant mass term of the form $M_R(RR)$ can exist due to the fact that R is a real representation and $R\times R$ contains a singlet. Therefore, if F contains fermions with the quantum numbers of the n_f "families" (i.e., of the observed low-mass fermions), then—no matter what R contains so long as it is real under G—one will obtain the correct low-energy light fermion spectrum. [A familiar example of this is the right-handed neutrino which is present in each family in SO(10) models. If SO(10) breaks to SU(5) this particle, being an SU(5) singlet, can and naturally does acquire a superlarge mass.[14]] An important point to emphasize is that the particular fermions that remain light may be mixtures of fermions in F and in R. If there are superlarge mass terms coupling F to R, then these mixings can be quite large.

B. Conditions for solving the strong *CP* problem

Let us consider a general grand unified model with a gauge group G. Let us suppose *CP* is a symmetry of the Lagrangian. Then $\theta_{\rm QCD}=0$ and there will be no *CP*-violating phases in any of the couplings of the theory. Let us distinguish the fermions into two sets F and R, F containing fermions with the quantum numbers of n_f families and R being a real set. This in itself is no restriction on the model but is, as we have tried to make clear, the general fermion content of any model with the correct low-energy spectrum. [F may contain some "extra" particles besides the observed families; e.g., in SO(10), F may consist of $\underline{16}$'s which have right-handed neutrinos; or in E(6), F may consist of $\underline{27}$'s.] Now let us suppose that R is composed of a set of fermion representations, C, and its conjugate set, $\overline{C}$. So $R=C+\overline{C}$. Then if two simple conditions are satisfied, $\overline{\theta}$ will be *naturally* zero at the tree level. These conditions are the following. (Condition 1 is slightly different and less restrictive than the corresponding condition stated in Ref. 11.)

Condition 1. At the tree level there are no Yukawa or mass terms coupling F fermions to $\overline{C}$ fermions, or C fermions to C fermions.

Condition 2. The *CP*-violating phases appear at the tree level only in those Yukawa terms that couple F fermions to $R=(C+\overline{C})$ fermions.

We will discuss the "naturalness" (in the technical *and* the ordinary senses) of these conditions below. First, let us see what they imply for *CP* violation.

If there were no F-R couplings, then the R fermions would be superheavy and the light families would be purely in F. We require, however, that there be scalars which couple F to C and which acquire superlarge vacuum expectation values (VEV's). The light fermions will then be mixtures of F and R. It is through such mixing that *CP*-violating phases show up in the low-energy effective theory. Since the F-R terms have *CP*-violating phases, the fermions that end up being light are linear combinations of fields in F and fields in R with *complex* coefficients. At low energies the effective theory, therefore, looks just like the Kobayashi-Maskawa model. To see why $\overline{\theta}=0$ at the tree level, let us first examine the tree-level mass matrix, M_-, of the charge $-\frac{1}{3}$ quarks, q_-, and the charge $+\frac{1}{3}$ antiquarks, q^c_-). Such quarks and antiquarks are to be found in F, C, and C. Schematically,

$$q^c_- M_- q_- = (q^c_-(F)_i, q^c_-(C)_j, q^c_-(\overline{C})_k) \times \begin{pmatrix} (\lambda v)_{ii'} & 0 & (f\omega)_{ik'} \\ (f'\omega')_{ji'} & M_{jj'} & 0 \\ 0 & 0 & M_{kk'} \end{pmatrix} \begin{pmatrix} q_-(F)_{i'} \\ q_-(\overline{C})_{j'} \\ q_-(C)_{k'} \end{pmatrix} . \tag{2}$$

The λ, f, and f' are Yukawa coupling-constant matrices. The v, ω, and ω' are vacuum expectation values of scalar fields. The $M_{jj'}$ and $M_{kk'}$ are *square* matrices. This is obvious from the fact that there are as many q_- in $\overline{C}$ as there are q^c_- in C. The zero entries vanish by condition 1. The $\det M_-$ is given by

$$\det M_- = \det\,(\lambda v)_{ii'} \det M_{jj'} \det M_{kk'} . \tag{3}$$

Notice that the matrices $f\omega$ and $f'\omega'$ do not contribute to $\det M_-$. [Each term in $\det M_-$ must have as a factor an element from every column of $M_{jj'}$, and thus one from every row of $M_{jj'}$ as well, and hence none from $(f'\omega')_{ji'}$. A similar argument applies for $(f\omega)_{ik'}$.] But by condition 2 the only *CP*-violating phases in M_- appear in $(f\omega)_{ik'}$ and $(f'\omega')_{ji'}$. The same reasoning applies to the mass matrix M_+ or the charge $+\frac{2}{3}$ quarks, and also to the mass matrices of other colored fermions. Therefore,

$$\overline{\theta}^{\,\rm tree} = \theta^{(\rm tree)}_{\rm QFD} + \theta^{(\rm tree)}_{\rm QCD} = 0 . \tag{4}$$

To summarize the situation then, *CP* violation shows up in the F-R terms that intermix light and superheavy quarks, and then filters down, as it were, to the light-quark mass matrices. Yet an inspection such as we have done of the *entire* mass matrix of the quarks, both light and superheavy, reveals that $\det M_Q^{(\text{tree})} = \text{real}$.

C. The naturalness of the conditions

Let us now discuss the naturalness of the two conditions. Looking first at condition 2, how is it possible (or even meaningful) to say that there are no *CP*-violating phases in the tree-level F-F or R-R couplings?

First, we show that the tree-level F-F couplings naturally, indeed *automatically,* satisfy condition 2 if there is a *minimal light Higgs content.* The scalar(s) that couple F to F at the tree level contain the Weinberg-Salam Higgs doublet(s). By minimal light Higgs content we mean simply that there is only *one* such multiplet and therefore only *one* light Higgs doublet. This doublet has nonzero hypercharge, of course, since it breaks $SU(2)\times U(1)$. (Its hypercharge is $Y/2 = \frac{1}{2}$.) Therefore, by a global hypercharge rotation the phase of its vacuum expectation value, v, can be changed arbitrarily. In particular, it can be set to zero. From this it is clear that this phase cannot contribute to any physical, *CP*-violating quantity like $\bar{\theta}$. (If we did not set the phase of v to zero we would simply find that it contributed oppositely to $\arg\det M_-$ and $\arg\det M_+$ and canceled out in $\bar{\theta}$.)

If there is more than one light Higgs doublet ("non-minimal Higgs") coupling F to F, then in general these may have VEV's with nontrivial *relative* phases which cannot be absorbed by a hypercharge rotation. In this case condition 2 might not be satisfied.

Now, let us consider the tree-level R-R terms. These are either explicit masses, which are necessarily *CP*-invariant because $\mathscr{L}$ is, or spontaneous masses arising from C-$\bar{C}$ terms (by condition 1). These latter, in some models, could contain *CP*-violating phases. However, generally speaking, this will not be so in simple models. Suppose, for instance, that a model contains a single Higgs boson in the adjoint representation. [Such fields are used commonly to break SU(5) down to $SU(3)\times SU(2)\times U(1)$.] The adjoint *is* in the product $C\times\bar{C}$. However, since the adjoint is a real representation, it is easy to see that its VEV cannot have a *CP*-violating phase. In this case the R-R terms would automatically satisfy condition 2. It is also perfectly possible to have a realistic, phenomenologically satisfactory model without *any* Higgs field in the product $C\times\bar{C}$. In this case also, condition 2 is automatically fulfilled. In order to violate condition 2, in fact, a model must have a somewhat complicated Higgs structure. The scalar fields coupling C to $\bar{C}$ must be complex so that they *can* have phases, and there must be enough of them so that there is at least one phase that cannot be "rotated away." There seems no particular reason from the point of view of phenomenology to introduce such a complicated Higgs structure.

To recapitulate, in order to have condition 2 satisfied—not merely naturally—but automatically, it generally suffices to have a simple and economical Higgs structure.

It is easy to see as well how condition 1 may be satisfied naturally. Namely, there should not be any Higgs bosons in the model which are in the Kronecker products $F\times\bar{C}$ and $C\times C$. Since F, C, and $\bar{C}$ would in general consist of different sets of representations this constraint is not very restrictive. This should become more apparent after we discuss some examples.

III. EXAMPLES

A. Nelson's $SU(5)\times SO(3)_{\text{family}}\times U(1)_{\text{global}}\times CP$ model

This model[9,10,12] is the first of this type discovered. The fermions are in the following multiplets:

$$F:\ (10,3,0)+(\bar{5},3,0)\ ,\qquad R\begin{cases} C:\ (\overline{10},1-1)+(5,1,-1),\\ \bar{C}:\ (10,1,1)+(\bar{5},1,1)\ . \end{cases} \tag{5}$$

The numbers represent the SU(5) and $SO(3)_{\text{family}}$ representations and $U(1)_{\text{Global}}$ charges, respectively. So there are three light families and one vectorlike, superheavy family. The Weinberger-Salam Higgs multiplet is a (5,1,0). The SU(5) breaking is, as usual, done by an adjoint: (24,1,0). The breaking of *CP* and of the family group are both done by a number of (1,3,1) Higgs multiplets. These couple the F to the R fermions. (One may have additional Higgs multiplets as well, or a somewhat different set.) Condition 1 is trivially satisfied since the model has no scalars in the Kronecker products $F\times\bar{C}$ and $C\times C$.

The light Higgs content is minimal. That is, there is only one $(5,1,0)_H$. (The subscript H will denote Higgs multiplets.) This is the only scalar whose VEV couples F to F at the tree level. This VEV, which breaks $SU(2)\times U(1)$, can be made real by a global hypercharge rotation. So no *CP*-violating phase appears at the tree level in the F to F couplings in accordance with condition 2. Moreover, the only tree-level R to R couplings are bare-mass terms of the form $m(\overline{10},1,-1)\times(10,1,1)$ and $m'(5,1,-1)\times(\bar{5},1,1)$, and couplings of the adjoint Higgs multiplets of the form $g(\overline{10},1,-1)\times(10,1,1)\times(24,1,0)_H$ and $g'(5,1,-1)\times(\bar{5},1,1)\times(24,1,0)_H$. m and m' are real by the *CP* invariance of $\mathscr{L}$, as are g and g'. And the components of $(24,1,0)_H$ which develop VEV's are Hermitian and can have no *CP*-violating phase. Thus, condition 2 is satisfied *automatically.* To violate condition 2 there would have to be either several $(5,1,0)_H$'s, several $(24,1,0)_H$'s, or, in some way, a more complicated Higgs structure. Because it satisfies our two conditions, $\bar{\theta}^{(\text{tree})}=0$. The reader is referred to Refs. 9. and 12 for more details of this model.

B. An $SO(10)\times CP$ example

This model shows that our conditions can be satisfied quite naturally *without any global symmetries* (other than *CP*). The fermions are in the following sets of representations:

$$F:\ 16_i,\ \ i=1,2,\ldots,n_f\qquad R\begin{cases} C:\ \overline{126},\\ \bar{C}:\ 126\ . \end{cases} \tag{6}$$

The Higgs representations are chosen to be 10_H, 16_H, and 45_H. The 10_H contains the Weinberger-Salam doublet. The 16_H's and 45_H break SO(10) down to $SU(3)\times SU(2)\times U(1)$. The 16_H's also break *CP* (which is why there must be more than one of them) and couple *F* to *R*. There are no scalars in the products $F\times\bar{C}$ ($\underline{16}\times\underline{126}=\overline{\underline{144}}+\overline{\underline{672}}+\overline{\underline{1200}}$), or $C\times C$ ($\overline{\underline{126}}\times\overline{\underline{126}}=\underline{54}+\underline{945}+$ larger representations). So condition 1 is automatically satisfied by SO(10) invariance. Moreover, condition 2 is satisfied for the same reasons given in the previous example: (a) there is a minimal light Higgs content (only one 10_H), and (b) the only Higgs coupling *C* to $\bar{C}$ is a single 45_H which has a real VEV.

In Ref. 11 we presented a set of conditions which differed slightly from those given in this paper. Condition 1 of Ref. 11 required that there be no $SU(2)\times U(1)$-breaking VEV's (at tree level) coupling *F* to *R* or *R* to *R*. The superiority of the present set of conditions is well illustrated using this SO(10) model (which was also used as an example in Ref. 11). To satisfy the present set of conditions no other restrictions have to be imposed on this model. On the other hand, to satisfy the conditions of Ref. 11 one has to require that the VEV's of the 16_H do not break $SU(2)\times U(1)$ since the 16_H couple *F* to *R* at the tree level. This is not natural unless, say, a global symmetry is imposed. For in the Higgs potential there would be (without such a symmetry to prevent it) an SO(10)-invariant term $16_H\times 16_H\times 10_H$. If we decompose this under an SU(5) subgroup of SO(10) we find that this term contains $\bar{5}(16_H)\times 1(16_H)\times 5(10_H)$, in an obvious notation. $\langle 5(10_H)\rangle$ is nonvanishing and breaks $SU(2)\times U(1)$. $\langle 1(16_H)\rangle$ also is nonvanishing. This acts as a linear term in the potential for $\langle 5(16_H)\rangle$ and *forces* it to be nonvanishing as well. Then $\langle 16_H\rangle$ will break $SU(2)\times U(1)$. This illustrates a general phenomenon: it is difficult without global symmetries to prevent particular Higgs multiplets from developing $SU(2)\times U(1)$-breaking VEV's if $SU(2)\times U(1)$ *is* spontaneously broken. However, our new set of conditions simply obviates this problem and can be satisfied without introducing any *ad hoc* symmetries.

C. An $SO(10)\times G_{\text{family}}\times CP$ example

By this time the idea should be so familiar that we will just state the particle content of the model. The fermions are

$$F:\ (16,p)\ ,\qquad R\begin{cases}\bar{C}:\ (\overline{16},1),\\ \bar{C}:\ (16,1)\ .\end{cases} \tag{7}$$

The scalars are $(10,q)_H$, $(45,1)_H$, and (more than one) $(1,\bar{p})_H$. The (10,*q*) does the $SU(2)\times U(1)$ breaking at low scales. It couples the *F* to *F*, and hence *q* must be in $\bar{p}\times\bar{p}$. The $(45,1)_H$ does the superheavy breaking of SO(10) (other Higgs representations may be necessary for this as well). And the *F*-*R* mixing and *CP* breaking is done by the $(1,\bar{p})_H$. If condition 1 is to be satisfied it must be that $q\neq 1$ [or else $(10,q)_H$ will couple *C* to *C*] and that $q\neq\bar{p}$ (or else it will couple *F* to *C*).

These examples show the "usefulness" of family symmetries (whether global or local). Without such a family symmetry the possibilities for the representations in *C* and $\bar{C}$ are slightly more constrained. One could not have $C=$real as then the existence of tree-level $F\bar{C}$ couplings follow from the existence of tree-level *FC* couplings. Nor could $C=F$ or $\bar{F}$ as then the existence of *FF* couplings implies *CC* and $\bar{C}\bar{C}$ couplings. That is why $C=\overline{\underline{126}}$ was the smallest representation one could have used in example B. It is the smallest complex representation of SO(10) other than $\underline{16}$ and $\overline{\underline{16}}$. But with a family symmetry present condition 1 can be satisfied even with very small representations, as examples A and C illustrate. If nature is parsimonious with fermions, this would be an argument, perhaps, for family symmetry.

IV. MINIMAL LIGHT HIGGS REPRESENTATIONS

A. Necessary in Nelson-type models to solve strong *CP* problems

We showed in Sec. II that if there is only *one* Higgs representation whose VEV contributes to the *F*-*F* terms in M_Q, then one can naturally satisfy condition 2. This in particular means there is only *one* Weinberg-Salam doublet. Now suppose, on the contrary, that there are two or more such Higgs representations. Call them h_i ($i=1,2,\ldots$). Call the several Higgs representations whose VEV's couple *F* to *R*, and break *CP*, H_k ($k=1,2,\ldots$). Unless there were some *ad hoc* symmetry to prevent it the Higgs potential would contain terms of the form $(h_i^\dagger h_j)(H_k^\dagger H_l)$. These will perforce lead to *CP*-violating relative phases between the VEV's of the h_i. Thus, condition 2 would be violated. We conclude that it is a *necessary* condition (barring *ad hoc* symmetries) in Nelson-type models that only *one* Higgs VEV couples *F* quarks to *F* quarks at the tree level.

B. Sufficient to avoid flavor-changing Higgs exchange

Normally if light-quark masses come from the VEV's of several (more than one) Higgs representations there will be unacceptably large flavor-changing amplitudes from Higgs exchange, unless the Higgs mass is quite large ($\gtrsim 1$ TeV). If these several [$SU(2)\times U(1)$-breaking] VEV's are denoted v_i, and the corresponding Yukawa-coupling matrices in the effective low-energy theory are denoted $f^{(i)}$, then the light-quark mass matrices are given by

$$M_Q=\sum_i f^{(i)}v_i\ . \tag{8}$$

Thus, diagonalizing M_Q does not in general diagonalize the $f^{(i)}$ and the several light Higgs bosons will have flavor-changing couplings. (In Peccei-Quinn models one can have different Higgs bosons contribute to up and down quark masses. This kind of nonminimal Higgs content avoids flavor-changing problems but requires some global symmetry, generally). Consider now a Nelson-type model which has only a single VEV coupling *F* to *F* at the tree level. We will argue that there is no problem with Higgs-mediated flavor changing. Whatever $SU(2)\times U(1)$-breaking VEV's that appear in *F*-*R* or *R*-*R* terms lead only to order $(M_w/M_{\rm GUT})$ mixings of F and R fermions. [$SU(2)\times U(1)$-breaking VEV's are of order M_w, while the R fermions have order $M_{\rm GUT}$ bare masses.] Neglecting effects of this order, the light-quark-mass at the tree level comes only from a *single* $SU(2)\times U(1)$-breaking VEV that couples F to F. Therefore, there is no flavor-changing Higgs exchange at the tree level.

C. Realistic light fermion masses

Ordinarily it is required that there be several Higgs bosons contributing to the quark and lepton masses in grand unified models to arrive at a realistic light-fermion mass spectrum. This is necessary, in particular, to avoid such relations as $m_{d(\text{bare})}=m_{e(\text{bare})}$ which follow from SU(5) in the minimal Higgs scheme. However, if there is large mixing of light and superheavy fermions, as there is in our *CP* models, then SU(5) breaking that occurs in the superheavy sector of the fermion mass matrix will show up in the masses of the light fermions. So that even with minimal light Higgs content the bad relations like $m_{d(\text{bare})}=m_{e(\text{bare})}$ can be broken, and, indeed, quite complicated patterns of light fermion masses can be achieved, as is illustrated in Ref. 10. In fact, this is a significant argument in favor of such light-superheavy fermion mixing in unified models.

In summary, then, a minimal light Higgs content helps to solve *two* potential difficulties: flavor-changing effects mediated by Higgs exchange and the strong *CP* problem. In other words, one of the restrictions required by condition 2 is in any event strongly suggested by phenomenology.

V. THE VALUE OF $\bar\theta$ AND BARYON ASYMMETRY

$\bar\theta$ vanishes at the tree level in these models but will receive finite contributions from radiative corrections to the fermion mass matrix. A typical diagram of such a correction is shown in Fig. 1. An instance of such a diagram in the model of Sec. III B is displayed in Fig. 2. This contributes to the $F\bar{C}$ elements of M_Q which vanish at the tree level by condition 1. Diagrams of this type will give contributions to $\bar\theta$ of order

$$\delta_1\bar\theta=(1/16\pi^2)f^2(\text{phase}) ,$$

where f is a typical F-R Yukawa coupling of a superheavy scalar. This is—equivalently—of order $(M_f/M_b)^2$, where $M_{f(b)}$ are superheavy fermion (boson) masses. For $\bar\theta$ to be less than or of the order of 10^{-9}, one must have that f or M_f/M_b is less than or of the order of 10^{-3}. This is technically natural as emphasized by Nelson,[12] but perhaps awkward. For while there may be very small Yukawa couplings in nature (at least effectively),

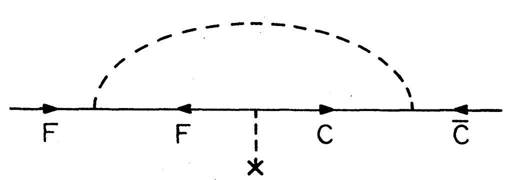

FIG. 1. A one-loop contribution to $\bar\theta$ in Nelson-type models.

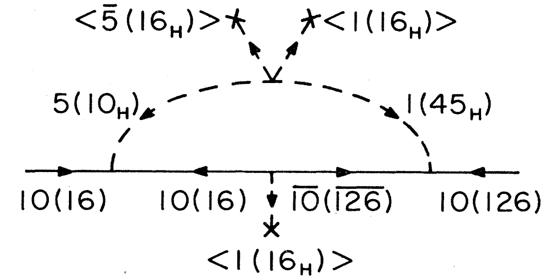

FIG. 2. An instance of a one-loop contribution to $\bar\theta$ in example B of Sec. III.

such as $f_e\sim m_e/300$ GeV$\sim10^{-6}$ or $f_s\sim m_s/300$ GeV$\sim10^{-4}$, the heaviness of the t quark suggests that this is not always the case.

Even if such contributions to $\bar\theta$ are made arbitrarily small there are still other contributions which have been estimated by Nelson[12] to be of the order of 10^{-11}. This number may be taken to be a lower bound on $\bar\theta$ in these models.

Segrè[15] has shown that there are contributions to n_B/s in models of this type which come from diagrams (see Fig. 3) very similar to those that contribute to $\bar\theta$. If all baryon asymmetry came from such diagrams (essentially the decays of superheavy colored Higgs bosons directly into quarks) then, as estimated by Segrè, $n_B/s\lesssim10^{-2}\bar\theta$. This is rather uncomfortably close as experimentally $n_B/s\cong(3-10)\times10^{-11}$. Segrè makes the point, however, that there are other sources of baryon asymmetry in these models which could be more important.

Here we would like to point out one such potentially large contribution to n_B/s. It is essentially that discussed in Ref. 16. Consider the decays of superheavy colored scalars. These will decay, not only directly into light fermions, but also into other, lighter scalars. If the self-couplings of the superheavy scalars are typically larger than their Yukawa couplings (which we know from the bounds on $\bar\theta$ must be $\lesssim10^{-3}$), then one would expect them to decay *predominantly* into lighter scalars. Because such decay amplitudes contain *CP*-violating pieces (due to the superlarge, *CP*-violating VEV's of some of the Higgs scalars) one would expect asymmetries to develop between various species of scalars and their antiparticles (see Fig. 4). Eventually the lightest of the color-triplet superheavy scalars will decay into colored fermions (energetically, there are no other channels open.) As observed in Ref. 16, a scalar's decay modes may, on average, be preferentially

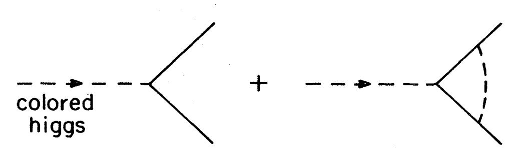

FIG. 3. Typical baryon-number-violating decay amplitudes of superheavy colored Higgs bosons. The interference between these amplitudes leads to a baryon asymmetry. The magnitude of this effect is related to the magnitude of the contributions to $\bar\theta$ shown in Fig. 1 as pointed out by Segrè.

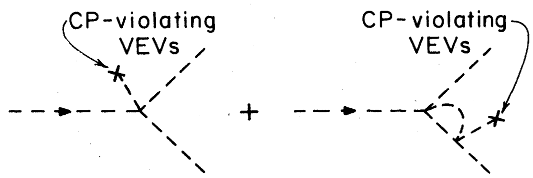

FIG. 4. Typical amplitudes that lead to asymmetries between colored Higgs scalars and their antiparticles. When these later decay into quarks and antiquarks a baryon asymmetry can result.

into quarks rather than antiquarks (or vice versa). The antiparticle decays, of course, would have the opposite preference. The asymmetry between a scalar and its antiparticle would then be converted ultimately into an asymmetry in baryon number. Various decay modes would presumably contribute to n_B/s with various signs so that some cancellation will result. However, the essential point (emphasized in Ref. 16) is that the resulting asymmetry is controlled not by Yukawa couplings but by scalar self-couplings (see Fig. 4). Thus, this contribution to n_B/s is not tied to the value of $\bar{\theta}$ and may be quite large in principle.

If, nevertheless, the mechanism discussed by Segré turns out to be the dominant contribution to n_B/s, it would argue for a large value of $\bar{\theta}$ around the present bound of about 10^{-9}. If the baryogenesis mechanism we have suggested here is dominant, then $\bar{\theta}$ could be as low as 10^{-11} which is the bound derived by Nelson. With $\bar{\theta}$ in the range

$$10^{-11} \lesssim \bar{\theta} \lesssim 10^{-9} , \tag{9}$$

one expects[17] that the neutron electric dipole moment will be in the range

$$5\times 10^{-27} e \text{ cm} \lesssim D_n \lesssim 5\times 10^{-25} e \text{ cm} . \tag{10}$$

This is to be compared to the contribution to D_n by $\bar{\theta}$ in Peccei-Quinn models which is expected[18] to be of order 10^{-31} e cm.

VI. CONCLUSIONS

We have shown that any model satisfying two conditions has $\bar{\theta}=0$ at the tree level. These conditions may be satisfied in a technically natural way. In fact, we argue that with a sufficiently economical Higgs structure they are automatically satisfied. These models have *CP* broken spontaneously at unification scales, thus there is no problem with domain walls which may be inflated away. We have shown that there is no difficulty with flavor-changing neutral currents. A large baryon asymmetry is a natural feature of these models as shown by Segrè. No *ad hoc* symmetries (either discrete or continuous global symmetries) are needed in these models. $\bar{\theta}$ is predicted to lie in the range 10^{-9} to 10^{-11} which is larger than the Peccei-Quinn prediction and accessible to experiment in the foreseeable future. These models can thus be distinguished experimentally from Peccei-Quinn models by a larger neutron electric dipole moment and by the absence of axions.

APPENDIX: LIGHT MIRROR FERMIONS

It may be asked whether this mechanism for solving the strong *CP* problem *requires* that the set F, which has the same quantum numbers as the light fermions, is a complex representation of the gauge group. There are various attractive theoretical ideas for which it is necessary that light fermions be in a real representation, implying the existence of "mirror" families[19] with $V+A$ weak interactions. In such a case we could replace F by a set $F+\bar{F}$. If we still imposed our two conditions, then the mass matrix M_- would look, schematically, like

$$q^c_- M_- q_- = (F\bar{F}C\bar{C}) \begin{bmatrix} V_L & 0 & 0 & \omega \\ 0 & V_R & u & 0 \\ \omega' & 0 & M & 0 \\ 0 & u' & 0 & M \end{bmatrix} \begin{bmatrix} F \\ \bar{F} \\ \bar{C} \\ C \end{bmatrix} . \tag{A1}$$

The determinant of this *would* depend on ω,ω', and hence be *CP* violating. However, to rule out the disastrous superheavy F-$\bar{F}$ masses (whose presence is otherwise suggested by the survival hypothesis), it is necessary to have some unbroken global or discrete symmetry.[19] This same symmetry will also rule out the $\overline{FC}$ entries in the mass matrix which we have denoted u, u' in Eq. (A1). (We are assuming that the CC and FC couplings *are* present.) That is, the symmetry which forbids $F+\bar{F}$ from acquiring superheavy masses also prevents the $\bar{F}$ from mixing with the superheavy fermions. This will lead to $\bar{\theta}^{\text{tree}}=0$ as before. To illustrate this let us add some $\overline{16}$ mirrors to the second model of Sec. III. Suppose we still allow $\overline{126}_f\times 126_f$ bare-mass terms and $(16_f\times\overline{126}_f)\times 16_H$ and $(16_f\times 16_f)\times 10_H$ couplings so that the ordinary families mix with the superheavies. If a symmetry rules out $16_f\times\overline{16}_f$ bare masses it will rule out the $(\overline{16}_f\times 126_f)\times\overline{16}_H$ couplings as well. [An example of such a symmetry is given in Ref. 19 for an SO(16) model. If we embed SO(10) in SO(16) there is a discrete group transformation (called K in Ref. 19) under which $F\rightarrow -iF$, $\bar{F}\rightarrow i\bar{F}$, and $R\rightarrow -R$. If we impose as well a discrete symmetry D under which $F\rightarrow iF$, $\bar{F}\rightarrow i\bar{F}$, and $R\rightarrow -R$, with the scalars transforming suitably, then DK need not be broken and will allow RR and FR couplings but forbid $\bar{F}F$.]

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