PHYSICAL REVIEW D VOLUME 30, NUMBER 1 1 JULY 1984

Charge commutation relations, asymptotic $SU(2)_L$ symmetry, and the mass of the second Z boson in electroweak gauge theories

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(Received 13 December 1983)

Electroweak gauge theories are discussed, using charge commutation relations and asymptotic symmetry. The mass of the second Z boson (m_2) in the $SU(2)_L\times U(1)_1\times U(1)_2$ models is predicted using the W- and first-Z-boson masses (m_W and m_1) to be $m_2^2=\{3-r+[(3-r)^2+4(r-1)(2-r)/(1-c_\theta^2 r)]^{1/2}\}m_W^2/2$, where $r=(m_1/m_W)^2$ with $1\le r\le 2$ and $c_\theta^2=1-s_\theta^2$ with $s_\theta^2=(37.2\ \mathrm{GeV}/m_W)^2$ defined at zero-four-momentum-transfer-squared limit. At present, m_2 is bounded as $m_2\gtrsim(1.6-1.7)m_W$ by the recent data on s_θ and r from the $p\bar{p}$ collider experiments.

I. INTRODUCTION

After the great success of low-energy phenomenology of the Glashow-Weinberg-Salam (GWS) electroweak model based on the $SU(2)_L\times U(1)_Y$ gauge group,[1] the weak bosons have finally been discovered at the $p\bar{p}$ collider at CERN.[2] The masses of the W boson (m_W) and the Z boson (m_Z) are reported by UA1 (UA2) as[2] $m_W\sim 80$ GeV (~ 81 GeV) and $m_Z\sim 95$ GeV (~ 91 GeV) in good agreement with the predictions of the GWS model. The next experiments at higher energies may disclose how many weak bosons there are in nature. It is, therefore, important to try to predict the masses of the extra weak bosons if they exist. Various alternative gauge models beyond the GWS model with extra weak bosons have been proposed and discussed.[3–14] Among them, the minimal extension of GWS based on $SU(2)_L\times U(1)_1\times U(1)_2$ (Refs. 6–10, 13, 14) is known to imply a light second neutral weak boson. However, its mass has not been predicted because of the complexity due to the Higgs scalars. To get around this problem, we propose the use of charge commutation relations (CR's) present in gauge theories.

A fresh approach using the CR's is carried out by realizing that (1) the breaking of an underlying gauge symmetry such as $SU(2)_L\times U(1)_Y$ can be expressed by the CR's involving the time derivatives of the charges, and (2) the usual Higgs mechanism of spontaneous symmetry breaking permits us to have asymptotic symmetry in the sense that the *linearity* of the transformation under the gauge group is preserved in the asymptotic limit.[15] The latter is compatible with the notion of gauge hierarchy.[16] As to (1), the use of $SU(2)_L$-doublet Higgs scalar will be shown in the GWS model to be equivalent to imposing the CR $[\dot{V}_+,V_+]=0$, where $V_+=V_1+iV_2$ and V_i ($i=1,2,3$) are the generators of $SU(2)_L$. In the $SU(2)_L\times U(1)_1\times U(1)_2$ model, the constraint $[\dot{V}_+,V_+]=0$ together with $[\dddot{V}_+,V_+]=0$ is able to predict the mass of the second Z boson (m_2) as

$$\left[\frac{m_2}{m_W}\right]^2=\frac{1}{2}\left\{3-r+\left[(3-r)^2+\frac{4(r-1)(2-r)}{1-c_\theta^2 r}\right]^{1/2}\right\}, \tag{1.1}$$

where $r=(m_1/m_W)^2$ with $1\le r\le 2$ and $c_\theta^2=1-s_\theta^2$ with $s_\theta^2=(37.2\ \mathrm{GeV}/m_W)^2$. The masses of the W boson and first Z boson (corresponding to the Z boson) are denoted by m_W and m_1, respectively. It should be noted that the CR's are always present in models but their realization depends crucially on the spectra of physical particles contained in the models.

The two points (1) and (2) mentioned above will be shown to be satisfied in the GWS model. However, the proposed method is also useful in the technicolor and preon model with dynamically broken gauge symmetry,[17,18] since CR's do not depend on the detail of models. This CR approach has already been shown[19] to give a good description of hadron masses and couplings. The difference between hadron physics and electroweak physics thus seems to lie only in the choice of underlying group.

This paper is organized as follows: In Sec. II, the main ingredients of our approach are explained in the GWS model. The $SU(2)_L\times U(1)_1\times U(1)_2$ model is then studied and the mass of the second Z boson is expressed in terms of the masses of the W and first Z bosons. Low-energy phenomenology of the $SU(2)_L\times U(1)_1\times U(1)_2$ model is studied in addition in Sec. III. The final section is devoted to a summary.

II. CHARGE-COMMUTATION-RELATION APPROACH TO $SU(2)_L\times U(1)_1\times U(1)_2$

A. Basic ingredients and $SU(2)_L\times U(1)_Y$

Asymptotic symmetry[15] applied to $SU(2)_L$ is as follows. The transformation of the annihilation operator $a_\alpha(\vec{k},\lambda)$, of the *physical* boson with momentum $\vec{k}$, helicity λ, and $\alpha = W^{\pm}, Z, \gamma, \ldots$, under the SU(2)$_L$ generator V_i can be expressed (suppressing λ) by

$$[V_i, a_\alpha(\vec{k})] = i\sum_\beta u_{i\alpha\beta}(\vec{k}) a_\beta(\vec{k}) + \delta u(\vec{k})_i \; . \tag{2.1}$$

On the right-hand side of Eq. (2.1), the first term picks up, in principle, *all* the terms *linear* in the operators a_β of the physical 1^- mesons contained in the models, and the remainder is amalgamated into the term δu. Strictly speaking, a_β should thus involve the 1^- $\bar{q}q$ meson operators as well since, in broken SU(2)$_L$, mixing could arise, though small, even between the 1^- $q\bar{q}$ mesons and the 1^- weak bosons.

Asymptotic symmetry[15] proposes in broken symmetry that as $\vec{k} \to \infty$, then $\delta u(\vec{k}) \to 0$ faster than $1/|\vec{k}|$, i.e., the *physical* operators $a_\alpha(\vec{k})$'s form a linear realization of *broken* SU(2)$_L$ as $\vec{k} \to \infty$. Thus, for $\vec{k} \to \infty$ $a_\beta(\vec{k})$ can be related[15] *linearly* to the hypothetical representation operators of SU(2)$_L$, $a_j(\vec{k})$ ($j = 1,2,3$, and 0, etc.) by (neglecting the mixings between the weak bosons and the 1^- $\bar{q}q$ mesons etc.)

$$a_\beta(\vec{k}) = \sum_j C_{\beta j} a_j(\vec{k}), \quad \vec{k} \to \infty \; , \tag{2.2}$$

where $C_{\beta j}$ defines the mixing parameters. Since mass parameters present in the theory can be neglected in this limit, the four-momentum squared k^2 of $a(\vec{k})$ can be taken to vanish. Therefore, the mixing parameters are defined at the massless limit ($k^2 = 0$).

The above relations are satisfied by electroweak gauge models. In fact, the mixing and diagonalization of the SU(2) triplet $W_\mu^{(3)}$ and singlet B_μ fields considered in the GWS model

$$A_\mu(x) = \sin\theta_W W_\mu^{(3)}(x) + \cos\theta_W B_\mu(x) \; , \tag{2.3a}$$

$$Z_\mu(x) = \cos\theta_W W_\mu^{(3)}(x) - \sin\theta_W B_\mu(x) \; , \tag{2.3b}$$

can be shown to be equivalent to

$$a_A(\vec{k}) = \sin\theta_W a_3(\vec{k}) + \cos\theta_W a_0(\vec{k}) \; , \tag{2.4a}$$

$$a_Z(\vec{k}) = \cosh\lambda(\vec{k}, m_Z)[\cos\theta_W a_3(\vec{k}) - \sin\theta_W a_0(\vec{k})] - \sinh\lambda(\vec{k}, m_Z)[\cos\theta_W a_3^\dagger(-\vec{k}) - \sin\theta_W a_0^\dagger(-\vec{k})] \; , \tag{2.4b}$$

with

$$\cosh\lambda(\vec{k}, m) = \frac{1}{2}\left[\left[\frac{\omega(\vec{k},0)}{\omega(\vec{k},m)}\right]^{1/2} + \left[\frac{\omega(\vec{k},m)}{\omega(\vec{k},0)}\right]^{1/2}\right] , \tag{2.5a}$$

$$\sinh\lambda(\vec{k}, m) = \frac{1}{2}\left[\left[\frac{\omega(\vec{k},0)}{\omega(\vec{k},m)}\right]^{1/2} - \left[\frac{\omega(\vec{k},m)}{\omega(\vec{k},0)}\right]^{1/2}\right] , \tag{2.5b}$$

where $a_i(k)$ ($i = 0,3$) are for the original massless B ($i = 0$) and W_3 ($i = 3$) fields[20] and $\omega(\vec{k}, m) = (|\vec{k}|^2 + m^2)^{1/2}$. Similarly for $W^{(\pm)}$,

$$a_W^{(\pm)}(\vec{k}) = \cosh\lambda(\vec{k}, m_W) a_{1\pm i2}(k) - \sinh\lambda(\vec{k}, m_W) a_{1\pm i2}^\dagger(-\vec{k}) \; . \tag{2.6}$$

From these relations, we find that Eq. (2.2) is recovered as $m_W{}^2/|\vec{k}|^2$ and $m_Z{}^2/|\vec{k}|^2 \to 0$ ($\vec{k} \to \infty$) because $\cosh\lambda(\vec{k}, m) \to 1$, and

$$\sinh\lambda(\vec{k}, m) \to -m^2/4|\vec{k}|^2 \to 0 \; ,$$

where $m = m_W$ or m_Z. Therefore, in this limit, we obtain

$$a_Z(\vec{k}) = \cos\theta_W a_3(\vec{k}) - \sin\theta_W a_0(\vec{k}) \; , \tag{2.7a}$$

$$a_A(\vec{k}) = \sin\theta_W a_3(\vec{k}) + \cos\theta_W a_0(\vec{k}) \; . \tag{2.7b}$$

The CR's of Eq. (2.1) thus exhibit the required property $\delta u(\vec{k}) \to 0$ as $\vec{k} \to \infty$ since $\delta u(\vec{k})$ always contain the terms proportional to sinh (k,m) which vanishes in this limit as $|\vec{k}|^{-2}$.

The procedure (2.2) expresses the gauge hierarchy[16] in terms of a slightly different language—asymptotic symmetry. The linear SU(2)$_L \times$U(1)$_Y$ symmetry is restored (in the sense that $\delta u \to 0$) above the energy scale m_W.

As to the SU(2)$_L$ breaking, we only assume that the breaking interaction belongs to an SU(2)$_L$ triplet, which is true in the GWS model with the SU(2)$_L$-doublet Higgs scalar. Then the "exotic" CR $[\dot{V}_+, V_+] = 0$ mentioned before holds for $\dot{V}$,where $\dot{V} = i[H, V]$. Since we assume, as in the GWS model, that *only* the mixing between W_3 and B is important in Eq. (2.2), we have Eqs. (2.7a) and (2.7b) for A and Z for $\vec{k} \to \infty$. By realizing the CR $[V_i, V_j] = i\epsilon_{ijk} V_k$ in the asymptotic limit using asymptotic SU(2)$_L$, we obtain, for example,

$$\langle Z(\vec{k}') | V_+ | W^-(\vec{k}) \rangle = (2\pi)^3 \delta(\vec{k} - \vec{k}') \sqrt{2} \cos\theta_W$$

for $\vec{k} \to \infty$. The matrix element is evaluated at the zero-four-momentum-transfer-squared limit $q^2 = 0$, i.e.,

$$q^2 \equiv (\vec{k} - \vec{k}')^2 \to (m_W{}^2 - m_Z{}^2)^2/4|\vec{k}|^2 \to 0$$

as $|\vec{k}| \to \infty$. Therefore, our $\sin^2\theta_W$ is the one evaluated at $q^2 = 0$

We now sandwich $[\dot{V}_+, V_+] = 0$ between $\langle W^+(\vec{k})|$ and $|W^-(\vec{k})\rangle$ with $\vec{k} \to \infty$ and obtain

$$\sum_M \langle W^+ | \dot{V}_+ | M \rangle \langle M | V_+ | W^- \rangle - (\dot{V}_+ \Longleftrightarrow V_+) = 0 \; , \quad \vec{k} \to \infty \; . \tag{2.8}$$

Under the present approximation $M = Z$ and A only, we obtain the *first* sum rule,

$$\cos^2\theta_W (m_W{}^2 - m_Z{}^2) + \sin^2\theta_W (m_W{}^2 - m_A{}^2) = 0 \; . \tag{2.9}$$

With the physical photon mass $m_A = 0$, Eq. (2.9) becomes

$$m_W{}^2 = m_Z{}^2 \theta_W \; , \tag{2.10}$$

which is precisely the result of using the SU(2)$_L$ doublet Higgs scalars.

However, we realize that the model actually possesses an infinite number of constraints

$$[V_+^{(n)}, V_+]=0 \ , \quad \text{where} \quad V_+^{(n)}=\frac{d^n V_+}{dt^n} \ , \quad n=1,2,3,\ldots \ . \tag{2.11}$$

If we repeat the same realization for the CR's in Eq. (2.11), we find that it is automatically satisfied for $n=$ even. For $n=$ odd we obtain, including Eq. (2.9),

$$\cos^2\theta_W(m_W{}^2-m_Z{}^2)^n+\sin^2\theta_W(m_W{}^2-m_A{}^2)^n=0 \ , \quad n=1,3,5,\ldots \ . \tag{2.12}$$

Combining the $n=1$ and 3 sum rules, we find a nontrivial solution called the "ideal" solution with $m_A=0$,

$$m_Z{}^2=2m_W{}^2 \ \text{ and } \ \cos^2\theta_W=\tfrac{1}{2} \ . \tag{2.13}$$

In fact, *all* the constraints in Eq. (2.12) are satisfied by Eq. (2.13).

The experimental value of θ_W is $(\cos^2\theta_W)_{\text{expt}}=0.77-0.80$. So something is amiss. However, we have to realize that the diagonalization, Eqs. (2.3a) and (2.3b), is still an approximate one as mentioned just before Eq. (2.2). We have to notice that the effect of the contributions of the neglected states in the intermediate states M in Eq. (2.8) is greatly enhanced as n increases. They take the form $\gamma^2(m_W{}^2-m_M{}^2)^n$, where γ denotes the W_3 component contained in the intermediate physical state M with $\vec{k}\to\infty$. Because of the kinematical factor $(m_W{}^2-m_M{}^2)^n$, even the contributions of the 1^- $q\bar{q}$ mesons could become non-negligible for $n=3$. At this point we have two possibilities: (a) If the minimal model is valid up to the energy scale, say 1 TeV, the above-mentioned mixing between the gauge bosons and 1^- $q\bar{q}$ mesons should already be important at $n=3$, although their effect is certainly negligible at $n=1$ (at most, of order α). (b) If this effect were not still very important at $n=3$, Eq. (2.13) suggests the presence of at least one more weak neutral boson, which can appreciably mix with $W^{(3)}$ and B. This case corresponds to the various alternative gauge models beyond the GWS model with extra weak bosons mentioned before. Present experiments do not exclude this possibility. To pursue the second possibility, we go beyond the GWS model.

B. $SU(2)_L\times U(1)_1\times U(1)_2$

We thus proceed to an $SU(2)_L\times U(1)_1\times U(1)_2$ electroweak model[6–11,13,14] which contains $W_L^{(3)}$ $(\equiv B_3)$, B_1, and B_2 as neutral members in the symmetry limit. According to Eq. (2.2), *physical* states (with $\vec{k}\to\infty$) can be expressed as ($c_\theta=\cos\theta$, $s_\theta=\sin\theta$, $t_\theta=\tan\theta$, $c_W=\cos\theta_W$, etc.)

$$\begin{bmatrix} Z_1 \\ Z_2 \\ A \end{bmatrix}=\begin{bmatrix} c_\alpha, s_\alpha, 0 \\ -s_\alpha, c_\alpha, 0 \\ 0,0,1 \end{bmatrix}\begin{bmatrix} c_\theta W_L^{(3)}-s_\theta c_\phi B_1-s_\theta s_\phi B_2 \\ -s_\phi B_1+c_\phi B_2 \\ s_\theta W_L^{(3)}+c_\theta c_\phi B_1+c_\theta s_\phi B_2 \end{bmatrix} \equiv \begin{bmatrix} \sum_j C_{1j}B_j \\ \sum_j C_{2j}B_j \\ \sum_j C_{3j}B_j \end{bmatrix} \ . \tag{2.14}$$

Two of the three mixing angles (α,θ,ϕ) are specified by three gauge couplings, g_L of $SU(2)_L$ and g_i of $U(1)_i$ $(i=1,2)$, as

$$g_L s_\theta=g_1 c_\theta c_\phi=g_2 c_\theta s_\phi=e. \tag{2.15}$$

Asymptotic realization of Eq. (2.11) in terms of only these weak boson yields using C_{ij} defined in Eq. (2.14),

$$(C_{13})^2(m_W{}^2-m_1{}^2)^n+(C_{23})^2(m_W{}^2-m_2{}^2)^n+(C_{33})^2 m_W{}^{2n}=0 \ , \tag{2.16}$$

with

$$\sum_{j=1}^{3}(C_{ji})^2=1 \ . \tag{2.17}$$

The ideal-like solutions of Eq. (2.16) for *all* values of n are, corresponding to Eq. (2.13), (a) $m_1{}^2=m_2{}^2=2m_W{}^2$ with $s_\theta{}^2=\frac{1}{2}$, (b) $m_1{}^2=m_2{}^2=m_W{}^2$ with $s_\theta{}^2=0$, and (c) $M_1{}^2=m_W{}^2$ and $m_2{}^2=2m_W{}^2$ with $s_\alpha{}^2=t_\theta{}^2$. However, we take a more realistic point of view that only $n=1$ and 3 sum rules are trustworthy when we neglect the mixing between the weak bosons and 1^- $q\bar{q}$ mesons.

From these $(n=1,3)$ we now obtain

$$\begin{aligned}
(C_{13})^2&=m_W{}^2(m_W{}^2-m_2{}^2)(2m_W{}^2-m_2{}^2)\times[m_1{}^2(m_1{}^2-m_2{}^2)(3m_W{}^2-m_1{}^2-m_2{}^2)]^{-1} \ , \\
(C_{23})^2&=m_W{}^2(m_W{}^2-m_1{}^2)(2m_W{}^2-m_1{}^2)\times[m_2{}^2(m_2{}^2-m_1{}^2)(3m_W{}^2-m_1{}^2-m_2{}^2)]^{-1} \ , \\
(C_{33})^2&=(m_W{}^2-m_1{}^2)(m_W{}^2-m_2{}^2)(2m_W{}^2-m_1{}^2-m_2{}^2)\times[m_1{}^2m_2{}^2(3m_W{}^2-m_1{}^2-m_2{}^2)]^{-1} \ ,
\end{aligned} \tag{2.18}$$

with $(C_{13})^2=(c_\theta c_\alpha)^2$, $(C_{23})^2=(c_\theta s_\alpha)^2$, and $(C_{33})^2=s_\theta{}^2$. Thus, θ and α are determined by the weak-boson masses. Since $0\le(C_{i3})^2\le1$ $(i=1,2,3)$, the masses must, in general, satisfy

$$m_1{}^2\le(m_W/c_\theta)^2\le m_2{}^2 \ , \tag{2.19}$$

which is compatible with the result of Ref. 9 and 12. Two different domains are allowed (see Fig. 1):

$$\text{(A)} \ m_W{}^2\le m_1{}^2\le 2m_W{}^2\le m_2{}^2 \ , \tag{2.20a}$$

$$\text{(B)} \ m_1{}^2\le m_W{}^2 \ , \quad m_W{}^2\le m_2{}^2\le 2m_W{}^2 \quad \text{with } 2m_W{}^2\le m_1{}^2+m_2{}^2\le 3m_W{}^2 \ . \tag{2.20b}$$

The GWS model corresponds to the limit of $m_2\to\infty$. The connection between the physical domains (A) and (B) and the ideal-like configurations is illustrated in Fig. 1: (a) is continued to domain (A), (b) to domain (B), and (c) is the transition point from (A) to (B). In Fig. 1, the relation between m_1 and m_2 for given values of $s_\theta{}^2$ is shown. The domain (B) is already excluded by the present experiments.

It should be emphasized that the second-weak-boson mass is predicted by m_W and m_1 (and θ) as from $(C_{33})^2$ of Eq. (2.18)

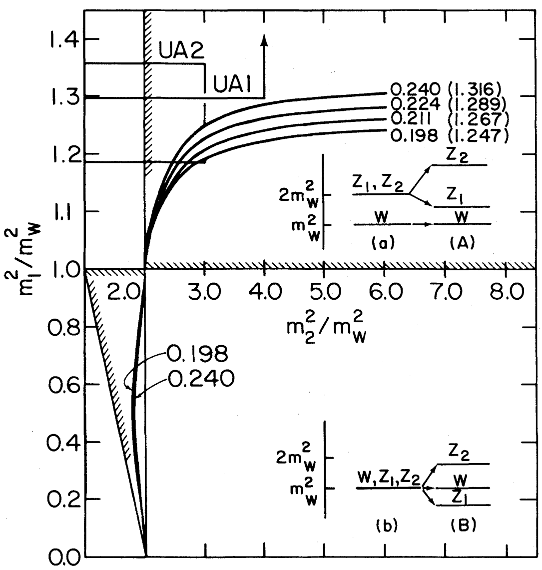

FIG. 1. Relation between m_1 and m_2 for given values of $s_\theta^2=0.198$, 0.211, 0.244, and 0.240. The numbers in the brackets denote the values of $(m_1/m_W)^2$ for $m_2\to\infty$. The experimental data of $(m_1/m_W)^2$ are denoted by UA1 and UA2.[2] Two shaded areas represent the two domains (A) and (B). The passage from "ideal" to real situations is also illustrated.

$$\left[\frac{m_2}{m_W}\right]^2=\frac{1}{2}\left\{3-r+\left[(3-r)^2+\frac{4(r-1)(2-r)}{1-c_\theta^2 r}\right]^{1/2}\right\}, \tag{2.21}$$

where $r\equiv(m_1/m_W)^2$ and $1\le r\le 2$ [domain (B)]. Therefore , in principle, by measuring m_1 and m_W, m_2 can be predicted since s_θ^2 is related to m_W as $m_W=(37.2\ \mathrm{GeV}/s_\theta)$. Our s_θ^2 is evaluated at $q^2=0$ and the observed value of s_θ^2 is related to s_θ^2 evaluated at $q^2=m_W^2$. The relation is given by

$$\sin^2\theta(m_W^2)\simeq\alpha_{\mathrm{EM}}(m_W^2)\sin^2\theta(0)/\alpha_{\mathrm{EM}}(0)$$
$$=1.070\sin^2\theta(0)\ , \tag{2.22}$$

for $\alpha_{\mathrm{EM}}(m_W^2)=(128)^{-1}$,[21] which gives $m_W=[38.5\ \mathrm{GeV}/s_\theta(m_W^2)]$. At present, experiments by UA1 (UA2) indicate $s_\theta^2(m_W^2)=0.226\pm0.011$ (0.014) which leads to

$$s_\theta^2\equiv s_\theta^2(0)=0.211\pm0.010\ (0.013) \tag{2.23a}$$

$$(m_1/m_W)^2=1.385\pm0.088\ (1.268\pm0.084)\ . \tag{2.23b}$$

These data are also shown in Fig. 1. From the UA2 data, the mass of Z_2 is bounded as

$$m_2\gtrsim(1.6\text{–}1.7)m_W \tag{2.24}$$

within one standard deviation. With the precise measurment of the masses of W and Z_1, we are in a position to predict the mass of Z_2.

In the next section, we will demonstrate that the first sum rule ($n=1$) of Eq. (2.16) is equivalent to choosing the $SU(2)_L$-singlet and -doublet Higgs scalars by only examining the effective Lagrangian for small-momentum-transfer interactions. Also discussed is low-energy phenomenology of the $SU(2)_L\times U(1)_1\times U(1)_2$ electroweak gauge model, which also gives a lower bound on m_2.

III. LOW-ENERGY PHENOMENOLOGY OF $SU(2)^L\times U(1)_1\times U(1)_2$

To discuss low-energy phenomenology, we have to derive the effective Lagrangian for small-momentum-transfer interactions. We discuss (A) the low-energy effective Lagrangian expressed in terms of three mixing angles defined in Eq. (2.14) and (m_1/m_W) and (m_2/m_W), (B) the equivalence between the first sum rule ($n=1$) obtained in Eq. (2.16) and the usual result obtained by using the $I=0$ and $\frac{1}{2}$ Higgs scalars, and (C) lower bounds on m_2 derived from low-energy phenomenology.

A. Low-energy effective Lagrangian

The Lagrangian for the $W_{L\mu}^{(i)}$ ($i=1,2,3$) and $B_{i\mu}$ ($i=1,2$) is

$$\mathscr{L}_{\mathrm{int}}=g_L J_{iL}^{\mu} W_{L\mu}^{(i)}+g J_j^{\mu} B_{j\mu}\ , \tag{3.1}$$

where J's are the corresponding currents. To obtain the effective Lagrangian expressed in terms of α, θ, ϕ, m_1/m_W, and m_2/m_W, we follow the method of Refs. 5 and 7. The weak neutral and electromagnetic parts of the Lagrangian are expressed in a convenient form (suppressing the Lorentz indices):[5,7]

$$\mathscr{L}_{\mathrm{int}}=g_L\langle J\mid W\rangle\ , \tag{3.2}$$

where

$$|J\rangle=(J_{3L},J_1 t_\theta/c_\phi,J_2/t_\theta s_\phi)\ , \tag{3.3a}$$

$$|W\rangle=(W_L^{(3)},B_1,B_2)\ . \tag{3.3b}$$

After diagonalization of the neutral fields by Eq. (2.14), we obtain

$$\mathscr{L}_{\mathrm{int}}=g_L\langle J'\mid T_\alpha^{-1}\mid Z\rangle+eAJ_{\mathrm{EM}}\ , \tag{3.4}$$

where

$$T_\alpha=\begin{bmatrix} c_\alpha & s_\alpha \\ -s_\alpha & c_\alpha\end{bmatrix}, \tag{3.5a}$$

$$|J'\rangle=[c_\theta J_{3L}-s_\theta t_\theta(J_1+J_2)\ ,$$
$$t_\theta(c_\phi^2 J_2-s_\phi^2 J_1)/c_\phi s_\phi]\ , \tag{3.5b}$$

$$|Z\rangle=(Z_1,Z_2)\ , \tag{3.5c}$$

$$J_{\mathrm{EM}}=J_{3L}+J_1+J_2\ . \tag{3.5d}$$

For small-momentum-transfer interactions, the effective Lagrangian can be written as

$$\mathscr{L}_{\mathrm{eff}}=4\sqrt{2}G_F m_W^2\langle J'\mid T_\alpha^{-1}M^{-2}T_\alpha\mid J'\rangle\ , \tag{3.6}$$

where

$$M^{-2}=\begin{bmatrix} m_1^{-2} & 0 \\ 0 & m_2^{-2}\end{bmatrix}. \tag{3.7}$$

Following Barger, Ma, and Whisnant,[11] we parametrize $\mathscr{L}_{\rm eff}$ as

$$\mathscr{L}_{\rm eff}=4\sqrt{2}G_F m_W{}^2[(\rho_1 J_{ZL})^2+(\rho_2 J_{ZL}+\eta J_{Z1})^2]\ , \tag{3.8}$$

where

$$J_{ZL}=J_{3L}-s_\theta{}^2 J_{\rm EM}\ , \tag{3.9a}$$

$$J_{Z1}=J_1-c_\phi{}^2 c_\theta{}^2 J_{\rm EM}\ . \tag{3.9b}$$

Since our currents J' are expressed in terms of J_{ZL} and J_{Z1} as

$$J_1'=J_{ZL}/c_\theta\ , \tag{3.10a}$$

$$J_2'=-t_\theta(c_\phi{}^2 J_{ZL}+J_{Z1})/c_\phi s_\phi\ , \tag{3.10b}$$

the effective Lagrangian (3.6) can be transformed into

$$\mathscr{L}_{\rm eff}=4\sqrt{2}G_F m_W{}^2 c_\theta{}^{-2}[A(J_{ZL})^2-2B(J_{Z1}J_{ZL})+C(J_{Z1})^2]\ , \tag{3.11}$$

where

$$A=[D_1-2\Delta(s_\theta/t_\phi)+D_2(s_\theta/t_\phi)^2]\ , \tag{3.12a}$$

$$B=[\Delta-D_2(s_\theta/t_\phi)](s_\theta/s_\phi t_\phi)\ , \tag{3.12b}$$

$$C=D_2(s_\theta/s_\phi t_\phi)^2\ , \tag{3.12c}$$

with

$$D_1=c_\alpha{}^2(m_W/m_1)^2+s_\alpha{}^2(m_W/m_2)^2\ , \tag{3.13a}$$

$$D_2=m_1 \Longleftrightarrow m_2 \ \text{in}\ D_1\ , \tag{3.13b}$$

$$\Delta=c_\alpha s_\alpha[(m_W/m_1)^2-(m_W/m_2)^2]\ . \tag{3.13c}$$

Comparing Eq. (3.11) with Eq. (3.8), we finally find that

$$\rho_1{}^2=(AC-B^2)/Cc_\theta{}^2\ , \tag{3.14a}$$

$$(\rho_2/\rho_1)^2=B^2/(AC-B^2)\ , \tag{3.14b}$$

$$(\eta/\rho_1)^2=C^2/(AC-B^2)\ . \tag{3.14c}$$

B. $[\dot V_+, V_+]=0$ and the representations of the Higgs scalars

For the present purpose, it is enough to express ρ_1 in terms of vacuum expectation values (VEV's) of various Higgs fields. Let us choose under $SU(2)_L\times U(1)_1\times U(1)_2$

$$\begin{aligned}
&\phi_1:\ (\tfrac{1}{2},0,-\tfrac{1}{2}),\ v_1\ ,\\
&\phi_2:\ (\tfrac{1}{2},-\tfrac{1}{2},0),\ v_2\ ,\\
&\phi_3:\ (0,-\tfrac{1}{2},\tfrac{1}{2}),\ v_3\ ,\\
&\Delta_1:\ (1,0,1),\ \delta_1\ ,\\
&\Delta_2:\ (1,1,0),\ \delta_2\ ,
\end{aligned} \tag{3.15}$$

where the v's and the δ's are the VEV's. Then we find that

$$\rho_1{}^2=(v_1{}^2+v_2{}^2+2\delta_1{}^2+2\delta_2{}^2)\times(v_1{}^2+v_2{}^2+4\delta_1{}^2+4\delta_2{}^2)^{-1}\ . \tag{3.16}$$

On the other hand, since

$$A-(B^2/C)=D_1-(\Delta^2/D_2)$$

and

$$D_1D_2-\Delta^2=x_1x_2\ ,$$

where $x_i=(m_i/m_W)^2$, $\rho_1{}^2$ becomes

$$\rho_1{}^2=x_1x_2/D_2c_\theta{}^2\ , \tag{3.17}$$

which finally gives

$$\rho_1=m_W/(c_\alpha{}^2m_1{}^2+s_\alpha{}^2m_2{}^2)^{1/2}c_\theta\ . \tag{3.18}$$

From two equations [(3.16) and (3.18)], we can show that the first sum rule ($n=1$) of Eq. (2.16) derived by $[\dot V_+, V_+]=0$ requires that

$$\rho_1=1\ , \tag{3.19}$$

which in turn chooses $\delta_1=\delta_2=0$ from Eq. (3.16), the absence of the $I=1$ Higgs scalars; therefore, $\rho_1=1$ is equivalent to imposing

$$I=0\ \text{and}\ \tfrac{1}{2} \tag{3.20}$$

for the transformation properties of the Higgs scalars. This is the condition corresponding to $\rho\equiv m_W/m_Zc_\theta=1$ in the GWS model.

By $n=1$ and 3 of Eq. (2.16), other parameters are also calculated to be

$$\begin{aligned}
&\rho_2{}^2=[\Delta-(m_Z{}^2/m_1m_2)^2s_\theta/t_\phi]^2(m_1m_2/m_W{}^2)^2\ ,\\
&\eta=(m_Z{}^2/m_1m_2)(s_\theta/s_\phi c_\phi)\ ,
\end{aligned} \tag{3.21}$$

with $m_Z\equiv m_W/c_\theta$. Since α, θ, and thus Δ, are expressed in terms of (m_1/m_W) and (m_2/m_W) by Eq. (2.14), ρ_2 and η become functions of ϕ only.

C. Lower bounds on m_2 from low-energy phenomenology

Let us examine low-energy phenomenology which also gives the constraint on m_2 although it depends on the details of the model, the quantum number assignment of the leptons and quarks in $SU(2)_L\times U(1)_1\times U(1)_2$. Two familiar models are (1) $SU(2)_L\times U(1)_R\times U(1)_{B-L}$ (Refs. 7–11) where $U(1)_R$ acts on the right-handed states of the leptons and quarks and the electric charge $Q_{\rm EM}$ is defined by

$$V_L^{(3)}+V_R^{(3)}+\frac{B-L}{2}$$

with $V_R^{(3)}$ being the third component of the $SU(2)_R$ charges; and (2) $SU(2)_L\times U(1)_Y\times U(1)_D$ (Refs. 13 and 14), where the leptons and quarks are neutral under $U(1)_D$ and

$$Q_{\rm EM}=V_L^{(3)}+Y/2$$

for the leptons and quarks.

1. $SU(2)_L\times U(1)_R\times U(1)_{B-L}$

Phenomenological analyses on this model have been carried out by various authors.[10,11] The present constraints are[11]

$$0.94 \le \rho_1{}^2+\rho_2{}^2 \le 1.16\ ,\quad \rho_2{}^2 \le 0.15\ ,$$
$$\eta \le 0.5,\quad c_\phi{}^2 c_\theta{}^2 \le 0.7\ . \tag{3.22}$$

with $\rho_1=1$. For example, the lowest m_2 is obtained at $\eta \simeq 0.5$ and $c_\phi{}^2 \simeq 0.5$ once $s_\theta{}^2$ is given:

$$m_2 \simeq 2.0 m_W \tag{3.23}$$

for s_θ determined by Eq. (2.23a). In these ranges of $s_\theta{}^2$, $m_1 \simeq (1.1-1.2)m_W$ and $\rho_2{}^2$ is about 0.02. This lower bound is stronger than Eq. (2.24).

2. $SU(2)_L \times U(1)_Y \times U(1)_D$

Since the current of U(1)$_D$, which is taken to be J_1, can be neglected for the lepton and quark interactions, J_{Z1} becomes $-c_\phi{}^2 c_\theta{}^2 J_{\rm EM}$. It is thus convenient to rewrite $\mathscr{L}_{\rm eff}$ as

$$\mathscr{L}_{\rm eff}=4\sqrt{2}G_F m_W{}^2[(J_{ZL})^2+C_{\rm EM}(J_{\rm EM})^2]\ , \tag{3.24}$$

by requiring that the ν-induced neutral-current interactions be described by the GWS model. This requirement can be fulfilled by

$$\rho_2=0 \quad \text{and} \quad \theta=\theta_W\ . \tag{3.25}$$

since $\rho_1=1$. We thus find that[13,14]

$$C_{\rm EM}=c_W{}^4(m_2{}^2-m_Z{}^2)(m_Z{}^2-m_1{}^2)/m_1{}^2 m_2{}^2\ . \tag{3.26}$$

It is the result of $B=0$ from $\rho_2=0$ and $A=c_\theta{}^2$ from $\rho_1=1$. Therefore, the masses are determined by $s_W{}^2$ and $C_{\rm EM}$. The phenomenologically allowed value of $C_{\rm EM}$ has been given by Dittmann and Hepp[14] as $C_{\rm EM} \lesssim 0.03$. It can be satisfied by

$$m_2 \gtrsim 1.6 m_W\ , \tag{3.27}$$

together with $m_1 \gtrsim 1.1 m_W$ and Eq. (2.23a) for $s_W{}^2$, which is almost the same as Eq. (2.24).

IV. SUMMARY

We have shown that the "exotic" SU(2)$_L$ charge commutation relation $[\dot{V}_+, V_+]=0$ provides

$$\rho \equiv m_W/m_Z c_\theta = 1 \tag{4.1a}$$

for SU(2)$_L \times$U(1)$_Y$ and

$$\rho_1 \equiv m_W/(c_\alpha{}^2 m_1{}^2+s_\alpha{}^2 m_2{}^2)^{1/2} c_\theta = 1 \tag{4.1b}$$

for SU(2)$_L\times$U(1)$_1\times$U(1)$_2$. Having assumed that $[\ddot{V}_+, V_+]=0$ is also well satisfied by the three neutral gauge bosons in SU(2)$_L\times$U(1)$_1\times$U(1)$_2$, we predict the mass of the second Z boson using the masses of W and first Z bosons to be

$$\left[\frac{m_2}{m_W}\right]^2=\frac{1}{2}\left\{3-r+\left[(3-r)^2+\frac{4(r-1)(2-r)}{1-c_\theta{}^2 r}\right]^{1/2}\right\}, \tag{4.2}$$

where $r \equiv (m_1/m_W)^2$ and $1 \le r \le 2$.

We can best summarize, at present, the allowed values of the second-Z-boson mass:

$$m_2 \gtrsim (1.6-1.7) m_W\ , \tag{4.3}$$

from the $\bar{p}p$-collider experiments. If we further specify the models, from the low-energy phenomenology we get a stronger bound

$$m_2 \gtrsim 2.0 m_W \tag{4.4a}$$

for the SU(2)$_L\times$U(1)$_R\times$U(1)$_{B-L}$ model and approximately the same bound

$$m_2 \gtrsim 1.6 m_W \tag{4.4b}$$

for the SU(2)$_L\times$U(1)$_Y\times$U(1)$_D$ model.

ACKNOWLEDGMENTS

The work of M.Y. is supported by National Science Foundation Grant No. 01-5-23536 and the work of M.D.S. is supported by the U.S. Department of Energy. S.O. thanks Toray Science Foundation for support.

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