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Curvature effects in interferometry

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The phase shift in interferometry due to the space-time curvature is computed in nonrelativistic and relativistic physics. This treatment is applied to an interferometer analogous to the perfect crystal interferometer used in the Colella-Overhauser-Werner experiment with thermal neutrons, an interferometer consisting of only horizontal mirrors, suitable for ultracold neutrons, and optical interferometry. Experiments are proposed to detect the effect of the tidal forces of the Earth, Moon, and the Sun, for the first time in interferometry. The phase shift in neutron interferometry due to terrestrial objects is considered and an experiment is proposed with superfluid helium as the source of gravity. This would provide, for the first time, direct evidence concerning the gravitational interaction between two quantum-mechanical states. The possibility of detecting purely general-relativistic effects due to curvature, by means of neutron interferometry, is also explored.

I. INTRODUCTION

An elegant experiment by Colella, Overhauser, and Werner[1] (COW) has demonstrated the influence of gravity in the interference of thermal neutrons by using a Laue-type interferometer which was first developed by Bonse and Hart[2] and applied to neutron interferometry by Rauch, Treimer, and Bonse.[3] This experiment, however, detects only the effect of the acceleration due to gravity g and therefore tests only the equivalence principle in Newtonian physics at the quantum-mechanical level. The phase shift due to gravity, in this experiment, has been computed general relativistically.[4–7] While these treatments should, in principle, contain the higher-order corrections, including curvature effects, no clear statement of the lowest-order curvature effects are found in the literature, to my knowledge.

These effects are important because, as I shall show in the present paper, if the interferometric experiment is redone with ultracold neutrons[8,9] (UCN), or even with thermal neutrons on a larger scale, then the curvature effects of the Earth, Moon, and the Sun will be of observable magnitude. Such effects would represent genuine gravitational effects in quantum interference as opposed to a mere test of the equivalence principle.

At present, however, neutron interferometry is only sensitive to the nonrelativistic component of the above-mentioned curvature effects. I therefore obtain these effects in Sec. II A, for a massive particle, using only Newtonian physics. But unlike in the previous nonrelativistic treatment,[1] I obtain corrections to the COW phase shift to the fifth order in g, since these effects are comparable to the curvature terms when UCN are used. In Sec. II B, I shall then obtain the curvature effects within the framework of general relativity for a massive and massless particle, and discuss the relativistic contributions. For an experiment done with UCN, owing to the extreme slowness of the neutrons (~ 6 m/sec), the effect of bending of the beams which was utterly negligible for the COW experiment[10,11] would no longer be negligible. This effect is considered in Sec. II C.

In Sec. III, the problem of bending of the beams is overcome altogether by means of an experimental arrangement having only horizontal mirrors. The phase shift due to gravity for this particular arrangement, including curvature effects and higher order effects in g, is then computed.

In Sec. IV, I shall consider the phase shift in interferometry due to terrestrial objects, such as an iron ball, which is observable. Such an experiment, in which the curvature effects are not negligible, would provide the first test of the inverse-square law of gravity at the quantum-mechanical level for distances of the order of a meter. An experiment is then proposed with superfluid helium as the source of gravity. This has the remarkable feature that it would provide the first experimental evidence of the gravitational interaction between two quantum-mechanical states. If the experiment would disagree with the result obtained here, using semiclassical gravity, then it would give the first direct experimental evidence of quantum gravity.

In Sec. V, neutron-interferometric experiments to detect the curvature effects due to the Earth, Moon, and the Sun are proposed. One of these proposed experiments, which may be done on a satellite, has the advantage that no correction needs to be made for the parabolic trajectories of the neutron beams and, more generally, it eliminates all effects that depend on g. In Sec. VI, the curvature effects are again discussed from a general relativistic point of view and it is concluded that neutron interferometry may be very useful as a serious probe of gravity.

For simplicity, the phase shift due to the Earth's rotation, which has been computed previously,[4,5] is disregarded in Secs. II—V and this phase shift must therefore be added to the results given therein. But this effect is implicit in the treatment in Sec. VI. Also the curvature effects due to the intrinsic spin of the particle, computed previously,[7] are disregarded in the present paper since they are negligible.

II. HIGHER-ORDER CORRECTIONS TO THE COW EXPERIMENT

In the COW experiment,[1] a beam of neutrons with fairly well-defined energy-momentum is split at a point 0 and the two beams travel along paths OAC and OBC around a parallelogram $OACB$, by means of suitable reflections by crystal planes, and interfere at C, in the presence of gravity. For simplicity, I shall assume $OACB$ to be a rigid rectangle in a plane with OB vertical, as determined by a plumb line. But the results that are obtained in this section can easily be generalized to more general cases. Also the relativistic treatment, given in Sec. II B, will be valid for both massive and massless particles.

A. Nonrelativistic treatment

Assuming that the WKB approximation is valid, each beam has a well-defined frequency ω and wave number κ which satisfy

$$\hbar\omega=\frac{\hbar^2\kappa^2}{2m}+mV\ , \tag{2.1}$$

where m is the mass of the particle and V is the gravitational potential. Since ω is constant for both beams, if $\kappa=\kappa_0$ and $V=0$ at 0 (the beam splitter), then

$$\kappa=\kappa_0\left[1-\frac{2m^2V}{\hbar^2\kappa_0{}^2}\right]^{1/2}. \tag{2.2}$$

Suppose that the interferometer is in the x-z plane of a Cartesian coordinate system with O,A,B,C having coordinates $(0,0),(s,0),(0,r),(s,r)$, respectively. Since OB is vertical, $\partial V/\partial x(0,0)=0$. Let

$$g\equiv\frac{\partial V}{\partial z}(0,0),\quad V_{xx}\equiv\frac{\partial^2 V}{\partial x^2}(0,0),\quad V_{zz}\equiv\frac{\partial^2 V}{\partial z^2}(0,0)$$

and

$$V_{xz}\equiv\frac{\partial^2 V}{\partial x\partial z}(0,0)\ .$$

Then g is the acceleration due to gravity at the origin and

$$V=zg+\frac{x^2}{2}V_{xx}+\frac{z^2}{2}V_{zz}+xzV_{xz}\ , \tag{2.3}$$

neglecting third and higher derivatives in V.

Ignoring, for the present, the effect of bending of the beams, the phase shift is given by

$$\Delta\phi=\int_{(0,0)}^{(0,r)}\kappa\,dz+\int_{(0,r)}^{(s,r)}\kappa\,dx-\int_{(0,0)}^{(s,0)}\kappa\,dx-\int_{(s,0)}^{(s,r)}\kappa\,dz\ . \tag{2.4}$$

From (2.2),

$$\kappa=\kappa_0-\frac{m^2V}{\hbar^2\kappa_0}-\frac{m^4V^2}{2\hbar^4\kappa_0{}^3}-\frac{m^6V^3}{2\hbar^6\kappa_0{}^5}-\frac{5}{8}\frac{m^8V^4}{\hbar^8\kappa_0{}^7}-\frac{7}{8}\frac{m^{10}V^5}{\hbar^{10}\kappa_0{}^9}+O(V^6)\ . \tag{2.5}$$

From (2.4), (2.5), and (2.3), we finally obtain the nonrelativistic phase shift

$$\Delta\phi_{\rm NR}=-\frac{m^2rsg}{\hbar^2\kappa_0}-\frac{m^4r^2sg^2}{2\hbar^4\kappa_0{}^3}-\frac{m^6r^3sg^3}{2\hbar^6\kappa_0{}^5}-\frac{5m^8r^4sg^4}{8\hbar^8\kappa_0{}^7}-\frac{7m^{10}r^5sg^5}{8\hbar^{10}\kappa_0{}^9}-\frac{m^2rs}{2\hbar^2\kappa_0}[rV_{zz}-sV_{xx}+(s-r)V_{xz}]\ , \tag{2.6}$$

neglecting higher-order terms. The reason for expanding to $O(g^5)$ will become clear in Sec. II B.

The first term in (2.6) is the well-known COW phase shift, $\Delta\phi_{\rm COW}$ while the next four terms are higher-order corrections in g to this phase shift. The last term in (2.6) is the nonrelativistic phase shift due to space-time curvature. Now $V=V_E+V_c$, where V_E is the Earth's gravitational potential and V_c is the centrifugal potential due to the Earth's rotation. Since $V_c \ll V_E$, we shall neglect V_c in evaluating the second derivatives of V. Therefore, if the Earth's potential is idealized to be spherically symmetric so that $V_E=-GM/R$, where M and R are the mass and radius of the Earth, then

$$V_{xx}\simeq\frac{g}{R},\quad V_{zz}\simeq-\frac{2g}{R}\ \text{and}\ V_{xz}\simeq 0\ . \tag{2.7}$$

B. Relativistic treatment

In the general relativistic treatment one must use, instead of (2.1), the relativistic eikonal equation $g_{\mu\nu}k^\mu k^\nu=m^2c^2/\hbar^2$, where $g_{\mu\nu}$ is the pseudo-Riemannian metric of space-time and the wave vector k^μ is defined by $k_\mu=-\partial_\mu\phi$, where ϕ is the phase of the wave function, in the WKB approximation. Suppose that the gravitational field is stationary and that the four-velocity field of the apparatus is parallel to a Killing field ξ^μ of the gravitational field. Choose ξ^μ such that $\lambda\equiv\xi^\mu\xi_\mu$ is 1 at the beam splitter. The eikonal equation can then be written as[5]

$$\frac{\omega_0{}^2}{\lambda c^2}-\kappa^2=\frac{m^2c^2}{\hbar^2}\ , \tag{2.8}$$

where κ is the "wave number" and ω_0 is the "frequency" at the beam splitter.[5] Suppose $\kappa=\kappa_0$ at the beam splitter and $\lambda=1+\frac{2V}{c^2}$, with $V=0$ at the beam splitter. This V may be interpreted as the Newtonian potential. It then follows from (2.8) that, for a massive particle ($m\neq 0$),

$$\kappa=\frac{mc}{\hbar}\left[\lambda^{-1}\left[1+\frac{\hbar^2\kappa_0^2}{m^2c^2}\right]-1\right]^{1/2}.$$

Hence,

$$\kappa=\kappa_0\left[1-\frac{m^2V\zeta}{\hbar^2\kappa_0^2}\left[1+\frac{m^2V\zeta}{2\hbar^2\kappa_0^2}+\frac{m^4V^2\zeta^2}{2\hbar^4\kappa_0^4}-\frac{2m^2V^2\zeta}{\hbar^2\kappa_0^2c^2}-\frac{2V}{c^2}+\frac{4V^2}{c^4}\right]\right]+O(V^4)\,, \quad (2.9)$$

where

$$\zeta=1+\frac{\hbar^2\kappa_0^2}{m^2c^2}=\left[1-\frac{v^2}{c^2}\right]^{-1},$$

v being the speed of the beam at the beam splitter. In the nonrelativistic limit, $c\rightarrow\infty$ so that $\zeta\rightarrow 1$ and the limit of (2.9) is the same as (2.5).

If curvature effects are neglected, then κ at the upper beam is obtained by setting $V=gr$ in (2.9). Then the phase shift is

$$\Delta\phi_0=(\kappa-\kappa_0)s$$
$$=-\frac{m^2grs\zeta}{\hbar^2\kappa_0}\left[1+\frac{m^2gr\zeta}{2\hbar^2\kappa_0^2}+\frac{m^4g^2r^2\zeta^2}{2\hbar^4\kappa_0^4}-\frac{2m^2g^2r^2\zeta}{\hbar^2\kappa_0^2c^2}-\frac{2gr}{c^2}+\frac{4g^2r^2}{c^4}\right]+O(g^4)\,. \quad (2.10)$$

The $O(g)$ part of (2.10) is the sum of the COW phase shift and the relativistic correction found previously.[5] However, for an experiment in the Earth's gravitational field, (2.10) would be grossly inadequate because the curvature effects are larger than the higher-order corrections to $\Delta\phi_{\rm COW}$ in (2.10). It is therefore necessary to use (2.4) again to obtain the phase shift, which would include curvature contributions.

It is clear from the above considerations that the relativistic phase shift can be expanded in powers of the dimensionless parameters

$$\alpha\equiv\frac{m^2gr}{\hbar^2\kappa_0^2}\simeq\frac{gr}{v^2},\quad \beta\equiv\frac{\hbar^2\kappa_0^2}{m^2c^2}\simeq\frac{v^2}{c^2},$$

$$\gamma\equiv\frac{gr}{c^2},\quad \delta\equiv\frac{r}{R},\ \text{and}\ \delta'=\frac{s}{R},$$

where R is the distance of the interferometer from the Earth's center. The nonrelativistic phase shift (2.6) depends on α,δ and δ' with δ,δ' determining the contribution due to curvature. The relativistic correction to this phase shift is determined by β and γ. Since we are here concerned primarily with the curvature contribution, we may terminate the power series when the terms become negligible compared to $O(\delta)$ and $O(\delta')$ terms. The terms that are kept, of course, would depend on the relative values of α, β, γ, δ and δ', which, in turn, are determined by the experiment being performed. Suppose that $r,s\simeq 10$ cm for the interferometer. For the thermal neutrons used in the COW experiment $v\simeq 2.74\times 10^3$ m/sec. Hence $\alpha=1.3\times 10^{-7}$, $\beta=8.3\times 10^{-11}$, $\gamma=1.1\times 10^{-17}$, and $\delta=1.6\times 10^{-8}=\delta'$. For ultracold neutrons, $v\simeq 6$ m/sec, so that $\alpha=2.8\times 10^{-2}$ and $\beta=4\times 10^{-16}$, while γ, δ, and δ' are the same as above.

It therefore follows that, for thermal neutrons, $\Delta\phi/\Delta\phi_{\rm cow}$ may be expanded to $O(\alpha)$ and $O(\delta)$, since $O(\beta)$ and $O(\gamma)$ terms are negligible, i.e., the relativistic corrections are negligible. Since

$$\Delta\phi_{\rm cow}=-\frac{m^2rsg}{\hbar^2\kappa_0}=-\alpha\kappa_0 S\,,$$

$\Delta\phi$ is then expanded to $O(\alpha^2)$ and $O(\delta)$. This $\Delta\phi$ is then the same as (2.6) with $O(g^3)$ terms neglected. For ultracold neutrons $\Delta\phi/\Delta\phi_{\rm cow}$ may be expanded to $O(\alpha^4)$ and $O(\delta)$, since again $O(\beta)$ and $O(\gamma)$ terms are negligible. Then $\Delta\phi$ contains terms up to $O(\alpha^5)$ and $O(\delta)$, i.e., it is the same as the $\Delta\phi_{\rm NR}$ of (2.6). To summarize, the relativistic corrections to the COW phase shift are negligible if one is interested only in the lowest-order curvature contribution. Hence the above treatment fills an important gap in the earlier treatments[4–6] which took the relativistic corrections into account, but which did not treat the curvature effects obtained above.

But it should be noted that in the above treatment the effect of spatial curvature was neglected. The contribution to the phase shift due to this general relativistic effect is $\sim\kappa_0\Delta l$, where Δl is the difference in the path lengths of the interfering beams due to the spatial curvature, l denoting the typical linear dimension of the interferometer ($r,s\sim l$). Then the ratio of this term to the first term in (2.6) is

$$a\sim(\hbar^2\kappa_0^2\Delta l/m^2gl^2)\simeq(v^2/gl)(\Delta l/l)\,.$$

But

$$(\Delta l/l)\sim(GMl^2/R^3c^2)=gl^2/Rc^2$$

in the Earth's gravitational field. Therefore, $a\sim(v^2/c^2)(l/R)$. But the curvature contribution in the last term in (2.6) is smaller than the first term by the factor $\sim l/R$. Hence the contribution due to spatial curvature is smaller than the above curvature contribution by the factor $\sim(v^2/c^2)$ and can therefore be neglected in neutron interferometry. But it cannot be neglected for interferometry with massless particles for which $v=c$.

For a massless particle, the phase shift can, in principle, be obtained from (2.8) by setting $m=0$. Then $\omega_0=c\kappa_0$ and $\kappa=\kappa_0/\sqrt{\lambda}$ which, of course, represents the "gravitational red-shift." Therefore,

$$\kappa=\kappa_0\left[1-\frac{V}{c^2}+\frac{3V^2}{2c^4}-\frac{5V^3}{2c^6}\right]+O(V^4)\,.$$

This result can also be obtained from (2.9) by taking the limit $m\rightarrow 0$, so that $m^2\zeta/\hbar^2\kappa_0^2\rightarrow 1/c^2$. By taking the same limit in Eq. (2.10), we get the corresponding result for a massless particle:

$$\Delta\phi_0 = -\frac{\kappa_0 grs}{c^2}\left[1-\frac{3}{2}\frac{gr}{c^2}+\frac{5}{2}\frac{g^2r^2}{c^2}\right]+O(g^4)\ ,$$

where curvature effects have been neglected. However, $\gamma/\delta = g/Rc^2 \simeq 1.7\times10^{-24}$, where γ and δ are defined above. Hence, $O(g^2)$ term is negligible compared to the curvature contribution. The relevant phase shift can therefore be obtained by substituting $\kappa \simeq \kappa_0(1-V/c^2)$ in (2.4), with V given by (2.3). This gives, assuming the interferometer to be a rectangle,

$$\Delta\phi = -\frac{\kappa_0 grs}{c^2} - \frac{\kappa_0 rs}{2c^2}[rV_{zz} - sV_{xx} + (s-r)V_{xz}]\ . \tag{2.11}$$

But, in general, the interferometer would fail to be a rectangle due to spatial curvature which, as already mentioned, makes a contribution to the phase shift which is comparable to the second term in (2.11). However, as seen below, curvature contributions are negligible in optical interferometry and it is therefore not necessary to compute this contribution.

It is clear that the phase shift (2.11) is smaller than the phase shift (2.6) for thermal neutron by $\sim(\hbar\omega_0/mc^2)(v/c)\sim10^{-14}$ for visible light. Hence there seems to be no hope of detecting gravitational effects in optical interferometry, unless the experiment is done on a large scale, say, by using laser beams between satellites that form the corners of an interferometer or by using long optical fibers. Even then, only the first term in (2.11) can at present be detected.

C. Bending effects

The above treatment has ignored the effect of bending of the beams due to gravity which could be important if the higher-order effects due to gravity are to be tested by means of neutron interferometry. As already mentioned, the relativistic corrections, in this case, are negligible compared to curvature contributions. Also the contribution to the bending effect due to the curvature is higher order compared to the lowest-order curvature contribution obtained above. Hence, the bending effect can be treated nonrelativistically, neglecting also the curvature contributions to it.

It is easy to obtain the general form of the lowest-order contribution due to the bending of the horizontal beams. If there is no bending then the phase difference along the lower horizontal beam is $\kappa_0 s$. Since the bending results in the wave fronts moving parallel to themselves to first order in g, the change in phase of this beam should be $O((gs)^2)$, i.e., it should be proportional to $\kappa_0 s(gs)^2$. Hence the phase shift due to bending, which should be dimensionless, should be

$$\Delta\phi_{\text{bending}} = p\frac{\kappa_0 g^2 l^3}{v^4} = p\frac{m^4 g^2 l^3}{\hbar^4\kappa_0^3}\ ,$$

where p is a dimensionless number which must be determined by a more detailed calculation and l is the distance between successive crystal planes in a Laue-type interferometer.[3]

To obtain a more exact procedure that would give this bending effect, take the gradient of (2.1), while noting that $\omega = -\partial\phi/\partial t$ and $\kappa^2 = \vec{k}\cdot\vec{k}$, where $\vec{k} = \nabla\phi$. This gives

$$-\hbar\frac{\partial\vec{k}}{\partial t} = \frac{\hbar^2}{m}(\vec{k}\cdot\nabla)\vec{k} + m\nabla V\ . \tag{2.12}$$

Hence, on substituting the velocity $\vec{v}\equiv\hbar\vec{k}/m$,

$$\frac{d\vec{v}}{dt} \equiv \frac{\partial\vec{v}}{\partial t} + (\vec{v}\cdot\nabla)\vec{v} = -\nabla V = \vec{g}\ .$$

Hence the integral curves of $\vec{v}$ or $\vec{k}$ are just the classical trajectories for a particle freely falling in a Newtonian gravitational field. It is then straightforward to determine them and the phase difference for a given geometry of the interferometer. But it is, of course, necessary to take into consideration the reflection conditions at the crystal when computing the phase shift.

The change in the interference conditions within the crystal[12] due to gravity results in a phase shift of order g, according to a recent study.[13] This must be added to the $O(g^2)$ phase shift due to the bending of beams between the crystal slabs to get the total phase shift due to bending. In Secs. III and V, we shall discuss ways of getting around the bending effects so that it is not necessary for us to compute the phase shift due to bending explicitly.

III. INTERFEROMETRY WITH HORIZONTAL AND VERTICAL MIRRORS

An idea which not only gets around the problem of the bending of the beams due to gravity, but which actually takes advantage of it, is to use only horizontal and vertical mirrors. Then, assuming that the collision at each mirror is elastic and neglecting curvature effects, $|k_x|$ is constant for each beam, where the x axis is horizontal while the z axis is vertical and the interferometer plane is in the x-z plane. Hence, if each beam travels the same horizontal distance then the phase shift, in a stationary situation, is

$$\Delta\phi = \oint\vec{k}\cdot d\vec{r} = \oint k_z dz\ , \tag{3.1}$$

where the integral is around the interfering beams. Also, when curvature effects are neglected, k_z is a function of z only. Hence the problem of computing $\Delta\phi$ is purely *one dimensional*, so that the bending of beams causes no additional problem. Interferometers having this feature have been considered by Steyerl *et al.*[8] and Chiu and Stodolsky.[11] Such an interferometer is particularly well suited for UCN for which the bending effects are huge because of their low velocity ($\sim$6 m/sec). However, the previous treatments have ignored the curvature effects which, as already seen, are certainly significant for UCN.

We shall therefore compute the phase shift, including the curvature effects for a simple interferometer of the above type in which only three horizontal mirrors M_0, M_1, and M_2 are used (Fig. 1). Also the treatment here will be nonrelativistic because, as shown in Sec. II B, the special relativistic correction is negligible compared to the nonrelativistic lowest-order curvature contribution. The incoming beam is split into two by M_0 and the two beams, after reflections at M_1 and M_2, are recombined at M_0 again. Suppose that M_1 is at a height H_1 above M_0

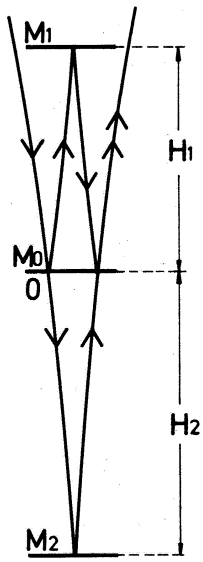

FIG. 1. Schematic diagram of an interferometer with only horizontal mirrors M_0, M_1, M_2, which would be suitable for detecting gravitational effects using ultracold neutrons.

and M_2 is at a height H_2 below M_0. From (2.12), for a stationary situation $(\partial\vec{k}/\partial t)=\vec{0}$),

$$(\vec{k}\cdot\nabla)k_x=-\frac{m^2}{\hbar^2}\partial_x V=-\frac{m^2}{\hbar^2}\left[V_x+xV_{xx}+zV_{zx}\right], \quad (3.2)$$

$$(\vec{k}\cdot\nabla)k_z=-\frac{m^2}{\hbar^2}\partial_z V=-\frac{m^2}{\hbar^2}\left[V_z+xV_{xz}+zV_{zz}\right], \quad (3.3)$$

where

$$V_x=\partial_x V(0),\quad V_z=\partial_z V(0)=g\ ,$$

$$V_{xx}=\partial_x{}^2V(0),\quad V_{xz}=\partial_x\partial_z V(0)\ ,$$

$V_{zz}=\partial_z{}^2V(0)$, the origin 0 is chosen to be the point where the incoming beam strikes M_0 and the higher-order derivatives in the Taylor expansion in (3.2) and (3.3) are neglected. Since the 0z axis is vertical (as determined by a Plumb line), $V_x=0$. Also from (2.7) $V_{xz}=V_{zx}\simeq 0$. For simplicity, we shall consider an experiment in which the incoming beam is nearly vertical so that the horizontal distance x traveled by the beams is negligible compared to the vertical distance. This also has the advantage that, since the area enclosed by the interfering beams is very small, the Sagnac phase shift due to the Earth's rotation is negligible. A vertical incoming beam can be obtained, of course, by suitably reflecting the horizontal beam that comes from the reactor. Then (3.2) and (3.3) become

$$(\vec{k}\cdot\nabla)k_x\simeq 0,\quad (\vec{k}\cdot\nabla)k_z\simeq-\frac{m^2}{\hbar^2}(g+zV_{zz})\ . \quad (3.4)$$

Hence k_x is constant for each beam and therefore (3.1) is valid. Also, from (3.4),

$$k_z\partial_z k_z\simeq-\frac{m^2}{\hbar^2}(g+zV_{zz})\ ,$$

which, on integration, yields

$$k_z{}^2=k_{z0}{}^2-\frac{m^2}{\hbar^2}(2gz+z^2V_{zz})\ ,$$

where k_{z0} is a positive constant. This, of course, represents conservation of energy under the above assumptions, with $k_z=k_{z0}$ at the origin. Hence

$$k_z=\pm k_{z0}\left[1-\frac{m^2}{\hbar^2k_{z0}{}^2}\left[gz+\frac{z^2}{2}V_{zz}\right]-\frac{m^4g^2z^2}{2\hbar^4k_{z0}{}^4}-\frac{m^6g^3z^3}{2\hbar^6k_{z0}{}^6}-\frac{5m^8g^4z^4}{8\hbar^8k_{z0}{}^8}-\frac{7m^{10}g^5z^5}{8\hbar^{10}k_{z0}{}^{10}}\right], \quad (3.5)$$

where the $\pm$ sign in front of k_{z0} corresponds to the direction of the beam being up or down and in the power series expansion in g, only the terms that are significant compared to the curvature term containing V_{zz}, for UCN, are retained. Hence, from (3.1), the phase shift is

$$\Delta\phi=2k_{z0}(H_1-H_2)-\frac{m^2g}{\hbar^2k_{z0}}\left[H_1{}^2+H_2{}^2\right]-\frac{m^4g^2}{3\hbar^4k_{z0}{}^3}\left[H_1{}^3-H_2{}^3\right]-\frac{m^6g^3}{4\hbar^6k_{z0}{}^5}\left[H_1{}^4+H_2{}^4\right]-\frac{m^8g^4}{4\hbar^8k_{z0}{}^7}\left[H_1{}^5-H_2{}^5\right]-\frac{7m^{10}g^5}{24\hbar^{10}k_{z0}{}^9}\left[H_1{}^6+H_2{}^6\right]-\frac{m^2V_{zz}}{3\hbar^2k_{z0}{}^2}\left[H_1{}^3-H_2{}^3\right]. \quad (3.6)$$

By varying H_1 and H_2, (3.6) can be experimentally verified, in principle.

IV. GRAVITATIONAL INFLUENCE OF TERRESTRIAL OBJECTS ON NEUTRON INTERFEROMETRY AND GRAVITATIONAL INTERACTION BETWEEN QUANTUM-MECHANICAL STATES

Any experiment that is sensitive to the effects of the Earth's tidal force may also be expected to be sensitive to the gravitational influence of terrestrial objects. This can be realized by noting that the tidal force per unit mass on the apparatus is $\sim 2GMl/R^3\sim\frac{8}{3}\pi G\rho l$ where M, ρ, and R are the mass, density, and the radius of the Earth and l is the relevant linear dimension of the apparatus: In the case of the neutron interferometer l is the separation between two of its arms. The Newtonian gravitational force per unit mass on the apparatus due to a nearby iron ball, is $\sim GM_I/R_I{}^2=\frac{4}{3}\pi G\rho_I R_I$ where M_I, ρ_I, and R_I are the mass, density, and radius of the ball. Hence the ratio of the two forces is $\rho_I R_I/2\rho l$. Since $(\rho_I/\rho)=1.4$, it follows that the two forces are comparable when $(R_I/l)\sim 1$.

An experiment to measure the gravitational force due to terrestrial objects, such as an iron ball, in neutron interferometry, and thereby test the inverse square law of gravity at the quantum-mechanical level, has been proposed previously.[14] A more precise value for the phase shift than the one given previously, can be obtained from (2.6) or (3.6), where obviously the terms in g of second and higher order are negligible. But the curvature terms are not negligible and, in fact, it may be necessary to extend (2.6) or (3.6) to include terms that depend on the third-and higher-order derivatives of V, for this type of experiment, which is easily done using the general procedure given. For an iron ball of radius 1 m and an interferometer with $r \simeq 0.2$ m, $s \simeq 0.2$ m, the phase shift, from (2.6), is about 3.5×10^{-4} radians for thermal neutrons and for $H_1 = H_2 = 0.2$ m, from (3.6), it is 0.46 radians for UCN, if the interferometer is almost at the surface of the ball. In both cases, this effect may be observable, as will be seen from the limits on sensitivity discussed in Sec. V. In performing this experiment, the ball may be vibrated (shaken) with known frequency, so that the corresponding phase shift, which varies in time with the same frequency, can be easily isolated.

However, in this and all other proposed gravitational experiments, as well as the COW experiment that has been performed, the source of gravity has been "classical," i.e., the atoms constituting the source (e.g., the Earth, iron ball) are in different mixed states that are so localized that each atom may be treated as classical as far as the gravitational field it produces is concerned. We shall therefore consider the case of the gravitational field produced by quantum fluids. For instance, if some superfluid helium is brought near the interferometer then, since each helium atom is in the same pure quantum-mechanical state (spread out over macroscopic distances), the resulting phase shift would provide information about *the gravitational interaction between two quantum-mechanical states,* namely, that of the helium atom and the neutron.

This phase shift can be immediately computed if it is assumed that semiclassical gravity, described by

$$G^{\mu\nu} = 8\pi G \langle \Psi | \hat{T}^{\mu\nu} | \Psi \rangle \tag{4.1}$$

is a good approximation in this case, where $G^{\mu\nu}$ is the Einstein tensor found using the classical metric $g_{\mu\nu}$, $\hat{T}^{\mu\nu}$ is the quantum stress-energy tensor operator, and $|\Psi\rangle$ is the quantum-mechanical state of the superfluid helium. Then the treatment given in Secs. II and III would apply to this experiment with the mass of the superfluid replacing the mass of the Earth. If superfluid helium-4 (density $= 122$ kg/m^3) is contained in a sphere of radius 1 m then the phase shift for an interferometer of linear dimension 0.2 m is 5.5×10^{-6} for thermal neutrons and 7.2×10^{-3} radians for UCN. The latter phase shift is of observable magnitude, as will be seen in Sec. V.

It is interesting to note that Page and Geilker[15] have performed an experiment that refutes (4.1) in the Everett interpretation of quantum mechanics. Nevertheless, intuitively (4.1) appears to be a very good approximation in the above experiment. Indeed, any violation of the prediction of (4.1) in the experiment proposed above would provide the first direct evidence of quantum gravity. This would imply that the gravitational field of a macroscopic "classical" source of gravity that seems to obey Einstein's field equations is only an approximation to the gravitational field of quantum-mechanical states. One could look for such a quantum-gravitational effect by doing the experiment, described above, with superfluid helium and then heating the liquid helium until it becomes normal fluid. If this results in a phase shift then that would mean that the gravitational interaction of the neutron with the quantum-mechanical state of superfluid helium is different from its interaction with the normal fluid, which is a classical source of gravity.

V. DETECTION OF THE TIDAL FORCES OF THE EARTH, MOON, AND THE SUN USING NEUTRON INTERFEROMETRY

If the COW experiment is performed using a Laue-type crystal interferometer,[2,3] for which $r \sim 10$ cm and $s \sim 10$ cm, then the last term in (2.6) due to the Earth's tidal force, on using (2.7) is 1.4×10^{-5} for thermal neutrons ($\kappa_0 \simeq 4.35\times 10^{10} m^{-1}$). At present a phase shift of 2.5×10^{-3} radians can be observed and a sensitivity of 4×10^{-4} radians is achievable.[16] Hence to detect the above predicted effect, either the sensitivity has to be increased by an order of magnitude and/or the interferometer must be built on a larger scale. The biggest possible perfect crystal interferometer that can be built is about 50 cm$\times$50 cm (Ref. 17) which we shall assume to be the case from now on. The above phase shift is then 1.7×10^{-3} radians which is observable. For an interferometric experiment with UCN ($\kappa_0 \simeq 9\times 10^7 m^{-1}$) for which $r \sim 50$ cm, $s \sim 50$ cm, the above phase shift due to the Earth's tidal force is 0.82 radians, which is also observable.

The major problem in detecting the above effect, in both experiments, is that the remaining part of the phase shift that depends on g is much bigger. Of course, the detection of the higher-order terms in g would itself be interesting. The $O(g^2)$ term in (2.6), for instance, is 4.8×10^{-3} radians for thermal neutrons which is observable. However, for UCN it is 5×10^5 radians which is huge. Hence, in order to detect curvature effects, it would be necessary to compensate for the effect that depends on g.

An elegant way of eliminating the effect that depends on g, to all orders in g, is to do the experiment on a satellite that is not acted upon by any force other than the gravitational field. Then the center of mass C of the satellite would move along the geodesic to a very good approximation. So if the interferometer described in Secs. II or III is placed in the satellite with 0 at C then the effective $g=0$ at 0, so that only the last term in (2.6) is nonzero. Of course, in this case also there is no contribution due to the bending of beams. As the interferometer is now rotated about an axis through 0 (e.g., 0x or 0z), there would be a variation in phase shift which is predicted by the last term in (2.6). Also as the interferometer is translated so that 0 is at (X,Y,Z) with respect to coordinate axes centered at C, then g (at 0) becomes

$$-\left[X\frac{\partial^2 V}{\partial x^2}+Y\frac{\partial^2 V}{\partial y\partial x}+Z\frac{\partial^2 V}{\partial z\partial x}\,,\quad X\frac{\partial^2 V}{\partial x\partial y}+Y\frac{\partial^2 V}{\partial y^2}+Z\frac{\partial^2 V}{\partial z\partial y}\,,\quad X\frac{\partial^2 V}{\partial x\partial z}+Y\frac{\partial^2 V}{\partial y\partial z}+Z\frac{\partial^2 V}{\partial z^2}\right],$$

to the lowest order, where all derivatives are evaluated at C. Hence there would be a phase shift due to this g which is given in (2.6) for a special choice of axes.

Another advantage of doing the experiment on a satellite is that it eliminates the problem of the influence of terrestrial objects on the phase shift. It would be desirable to make the satellite spherically symmetric so that no correction needs to be made for the influence of the gravitational field of the satellite in interference.

I now show that the phase shifts in neutron interferometry due to the curvature effects of the Sun and the Moon are also measurable. In this case it is *not* necessary to put the interferometer in a satellite. It will be sufficient to keep it fixed on the Earth and look for a phase shift that oscillates in time with periods of about half a day, since obviously this phase shift is correlated with the direction of the Sun or the Moon relative to the Earth.

Since the center of mass of the Earth is freely falling to a good approximation, only the tidal forces due to the Sun and the Moon, with respect to the center of mass of the Earth, would contribute to their phase shift. The gravitational potential due to the Sun is $V_s = -GM_s/R_s$ where M_s is the mass of the Sun and R_s is the distance from the Sun. Now choose coordinate axes with origin at the center of the earth so that the z axis is toward the center of the sun. Then, at the center of the Earth,

$$\frac{\partial^2 V_s}{\partial x^2} = \frac{\partial^2 V_s}{\partial y^2} = \frac{GM_s}{R_s{}^3}, \quad \frac{\partial^2 V_s}{\partial z^2} = -\frac{2GM_s}{R_s{}^3}$$

and

$$\frac{\partial^2 V_s}{\partial x \partial y} = \frac{\partial^2 V_s}{\partial y \partial z} = \frac{\partial^2 V_s}{\partial z \partial x} = 0 .$$

Therefore the tidal acceleration, due to the Sun at a point $(\tilde{x}, \tilde{y}, \tilde{z})$ on the earth's surface, is

$$\vec{g}_s = -\left[\tilde{x}\frac{\partial^2 V_s}{\partial x^2}, \tilde{y}\frac{\partial^2 V_s}{\partial y^2}, \tilde{z}\frac{\partial^2 V_s}{\partial z^2} \right]$$

$$= -\frac{GM_s}{R_s{}^3}(\tilde{x}, \tilde{y}, -2\tilde{z}) .$$

We shall assume, for simplicity, that the experiment is performed during spring or autumn when the Earth's axis of rotation is perpendicular to the line joining the centers of mass of the earth and Sun, i.e., the z axis. We shall then choose the y axis along the Earth's axis of rotation. If the apparatus is at latitude ψ, then the unit vector along the radius through the apparatus is $\vec{n} = (\cos\psi \sin\phi, \sin\psi, \cos\psi \cos\phi)$, where ϕ is the angle through which the Earth has rotated from the position in which the apparatus is closest to the Sun, for which $\tilde{x} = 0$. Also, then

$$\vec{g}_s = -\frac{GM_s R}{R_s{}^3}(\cos\psi \sin\phi, \sin\psi, -2\cos\psi \cos\phi) ,$$

where R is the radius of the Earth.

Therefore, as the earth rotates, the component of $\vec{g}_s$ along the y axis is constant while the interferometer, which is fixed with respect to the Earth, also gets rotated about the y axis. It would therefore be convenient to orient the Laue-type interferometer so that two of the arms are parallel to the y axis. A unit vector which is perpendicular to these two arms in the plane of the interferometer is $\vec{m} = (\sin(\phi+\beta), 0, \cos(\phi+\beta))$, where β is the angle between the plane of the interferometer and the z axis when $\phi = 0$. Hence the part of the phase shift that varies as the earth rotates is

$$\Delta\phi_s = \frac{m^2 A}{\hbar^2 \kappa_0}(\vec{g}_s \cdot \vec{m})$$
$$= \frac{m^2 AGM_s R}{\hbar^2 \kappa_0 R_s{}^3}\cos\psi[2\cos\phi \cos(\phi+\beta) - \sin\phi \sin(\phi+\beta)] , \qquad (5.1)$$

to lowest order, where A is the area enclosed by the interfering beams. When the interferometer is in a vertical plane, $\beta = 0$. Then the difference between the phase shifts at noon ($\phi = 0$) and sunset or sunrise ($\phi = \pm\pi/2$) is $(3m^2 AGM_s R \cos\psi / \hbar^2 \kappa_0 R_s{}^3)$. This is $\sim 10^{-3}\cos\psi$ radians for thermal neutrons. For UCN, on substituting $\vec{g}_s \cdot \vec{m}$ in the g in (3.6), this phase shift is 1.1 $\cos\psi$ radians for $H_1 = H_2 = 50$ cm. Hence, in both cases, the phase shift due to the tidal force of the Sun is measurable.

A calculation analogous to the above, for the tidal force due to the Moon, shows that the corresponding phase shift varies during a quarter of a day by about 2.6×10^{-3} radians if thermal neutrons are used and about 2.5 radians if UCN are used. The effect in both cases is measurable. The tidal forces due to the Moon and the Sun can be detected quite simply by fixing the interferometer with respect to the Earth and looking for a phase shift that varies with time, with a period of about half a day and then repeating the experiment for different values of the angle β that was defined above.

VI. ON THE POSSIBILITY OF TESTING GENERAL RELATIVISTIC EFFECTS

From a general relativistic point of view the curvature effects discussed in Secs. II–V are due to the "electric" components R_{0i0j} of the curvature. For a stationary, spherically symmetric gravitational field, $R_{0i0j} = -(1/c^2)\partial^2 V/\partial x^i \partial x^j$ in an appropriate coordinate system, to leading order, where V is the Newtonian potential. Thus the proposed experiments of Sec. V would measure these curvature components R_{0i0j}. However, the confirmation of these effects in neutron interference would not constitute a test of general relativity since the same effects can be obtained for a massive particle using Newtonian physics as shown in Sec. II A. On the other hand, the phase shift for light obtained in Sec. II B cannot be obtained using Newtonian physics; so the detection of the above curvature components, using optical interferometry, may be regarded as a test of general relativity. However, as already concluded, it would be very difficult to do such an experiment.

But the detection of the "magnetic" components R_{0ijk} or the components R_{ijkl}, which have not been discussed above, using neutron interferometry, would constitute a test of general relativity, since these components have no Newtonian analog. To discuss, systematically, the contributions due to all the curvature components, it is convenient to use the proper reference frame of the interferometer, which is a generalization of the Fermi-normal coordinate system that undergoes acceleration and rotation. In this coordinate system, the metric coefficients are,[18] to second order in the spatial coordinates x^i,

$$\begin{aligned} g_{00} &= 1+\frac{2}{c^2}a_j x^j+\frac{1}{c^4}(a^j x^j)^2+\frac{1}{c^2}(\Omega^j x^j)^2 \\ &\quad -\frac{\Omega^2}{c^2}x^j x^j + R_{0j0k}x^j x^k \,, \\ g_{0i} &= \epsilon_{ijk}\frac{\Omega^j}{c}x^k - \tfrac{2}{3}R_{0jik}x^j x^k \,, \\ g_{ij} &= -\delta_{ij}+\tfrac{1}{3}R_{iljm}x^l x^m \,, \end{aligned} \tag{6.1}$$

where a^i and Ω^i are the acceleration and angular velocities of the apparatus relative to the local inertial frame. If the apparatus is fixed with respect to the Earth, then $-a^i$ is the acceleration due to gravity and Ω^i is the angular velocity of the Earth. If the apparatus is fixed with respect to a freely falling satellite that is made nonrotating by means of gyroscopes then $a^i=0$ and $\Omega^i=0$.

On defining $h_{\mu\nu}\equiv g_{\mu\nu}-\eta_{\mu\nu}$, where $\eta_{\mu\nu}=\mathrm{diag}(1,-1,-1,-1)$, the change in phase along each beam due to the gravitational field to first order in $h_{\mu\nu}$ is[4,6] $\frac{1}{2}\int h_{\mu\nu}p^\mu(dx^\nu/dt)dt$ where p^μ is the "energy-momentum" and the integral is along the unperturbed classical trajectory. It follows then from (6.1) that the contribution of R_{0ijk} to the phase shift is decreased by $O(v/c)$ while the contribution of R_{ijkl} is decreased by $O(v^2/c^2)$ compared to the contribution of R_{0i0j}, where v is the velocity of the interfering particle. In the case of optical interferometry, however, since $v=c$, all three types of curvature components will contribute equally to the phase shift. If the Earth's gravitational field is approximated by the Schwarzschild metric, then the nonzero components of R_{0i0j} and R_{ijkl} have the same order of magnitude. But the contribution of the latter is much smaller than the contribution of the former to the phase shift of neutrons because v^2/c^2 is 8.4×10^{-11} for thermal neutrons and 4×10^{-16} for UCN.

According to Mashhoon and Theiss[19] there is a surprisingly large contribution to the R_{0i0j} components, in a frame that is parallel transported along a geodesic in the gravitational field of a rotating spherical object, due to the angular momentum of this object. This effect, due to the Earth's gravitational field, can be detected in principle as follows: Suppose that a satellite is freely falling around the Earth in a near Earth circular orbit of radius R whose plane makes an angle α with the equatorial plane. $0x^1x^2x^3$ is a Cartesian frame at the center of mass 0 of the satellite, which is assumed, to follow a geodesic. There are gyroscopes along the axes $0x^1, 0x^2, 0x^3$ so that the Cartesian frame is parallel transported along the world line of 0. Also a clock at 0 measures proper time τ. Suppose that when $\tau=0$, $0x^1$ points away from the center of the Earth and $0x^3$ is in the direction of motion of 0. The interferometer may be placed on the $0x^2$ axis at a distance D from 0 which is much bigger than the size of the interferometer. Suppose also that the interferometer is in the $0x^1x^2$ plane, symmetrically about the $0x^2$ axis, so that the phase shift due to the usual tidal acceleration of a spherically symmetric gravitational field would be constant in time. But there would then be, at the interferometer, according to the results in Ref. 19, an additional tidal acceleration $g_M\simeq(3DJ\omega_0/MR^2)\sin\alpha\,\sin\omega\tau\cos\omega_0\tau$ to the lowest order in G, in the $0x^1$ direction, where $\omega_0=(GM/R^3)^{1/2}, \omega=\omega_0(1-3GM/c^2R)^{-1/2}$ and M,J are the mass and angular momentum of the Earth. If $D=10$ m and $\alpha=\pi/2$ radians (polar orbit), then $g_M=-5.4\times10^{-7}[\sin2\omega_0\tau+2(\omega-\omega_0)\tau\cos^2\omega_0\tau]$, neglecting $O((\omega-\omega_0)^2)$. The phase shift due to g_M, to lowest order, is $m^2 g_M A/\hbar^2\kappa_0=8\times10^{-4}[\sin2\omega_0\tau+2(\omega-\omega_0)\tau\cos^2\omega_0\tau]$ radians for thermal neutrons, where m,κ_0 are the mass and the wave number of the neutron and the area A enclosed by the beams is assumed to be 0.25 m^2. For UCN, if $H_1=H_2=0.5$ m in (3.6) then the phase shift is 0.37 $[\sin2\omega_0\tau+2(\omega-\omega_0)\tau\cos^2\omega_0\tau]$ radians. Both phase shifts are of observable magnitude; however it would be difficult to isolate the part of the phase shift proportional to $\sin2\omega_0\tau$, from the Newtonian tidal force effects which also have the same frequency $2\omega_0$. Indeed, it is necessary to orient the axes at $\tau=0$, as described above, to an accuracy of less than 0.02 radians in order that this term is not masked by a component of the Newtonian tidal acceleration. If this accuracy cannot be achieved, then one can only hope to detect the part of the phase shift proportional to $(\omega-\omega_0)\tau$. But since $(\omega-\omega_0)\simeq1.3\times10^{-12}$ sec^{-1}, one has to wait for a time interval $\tau\sim10^{12}$ sec in order that this secular term is of observable magnitude for thermal neutrons. One can also use other accelerometers, instead of the neutron interferometer, in the experiment described above.

The Earth's gravitational field also has nonzero R_{0ijk} components because of its Lense-Thirring field.[20] The phase shift due to these components is one contribution to the phase shift[4–6] $(mc/\hbar)\oint g_{0i}dx^i$ in neutron interferometry, where the integral goes around the interfering beams and g_{0i} are given by (6.1). But this is too small to be detected. However an experiment was proposed by the author[14] to detect the Lense-Thirring field of the Earth by placing the interferometer on a platform which is made nonrotating relative to the distant stars by means of telescopes. Then the interferometer is rotating relative to the local inertial frames with angular velocity $\vec{\Omega}$ (say). According to general relativity,[21]

$$\begin{aligned}\vec{\Omega} &= -(1/2c^2)\vec{f}\times\vec{u}-(3GM/2r^3c^2)\vec{r}\times\vec{u} \\ &\quad +(GI/r^3c^2)[\vec{\Omega}_E-(3\vec{\Omega}_E\cdot\vec{r}/r^2)\vec{r}] \,,\end{aligned}$$

which must be substituted into (6.1), where M, R, I and $\vec{\Omega}_E$ are the mass, radius, moment of inertia, and angular velocity of the Earth, $\vec{r}$ is the position vector, and $\vec{u}$ is the velocity of the interferometer relative to axes fixed to the center of the Earth and $\vec{f}$ is its acceleration relative to local inertial frames. The first two terms in $\vec{\Omega}$ are due to the Thomas precession and the geodetic precession, respectively. The last term is due to the Lense-Thirring field. The phase shift in neutron interferometry due to the g_{0i} components, to the lowest order, is $2m\Omega_n A/\hbar$, where Ω_n is the component of $\vec{\Omega}$ normal to the plane of the interferometer. The contribution to the phase shift due to the last term of g_{0i} in (6.1) is negligible compared to the above contribution.

For an experiment performed in the laboratory, $r=R$, $\vec{u}=\vec{\Omega}_E\times\vec{r}$, and $\vec{f}=-\vec{g}$, where the acceleration due to gravity $\vec{g}\simeq-(GM/R^3)\vec{r}$. Then

$$\vec{\Omega}=-(8GM/5Rc^2)\vec{\Omega}_E+(4GM/5R^2c^2)\Omega_E\sin\psi\vec{r}\ ,$$

where ψ is the latitude. The phase shift is

$$(8GMmA/5\hbar c^2R)(-2\Omega_{En}+\Omega_E\sin\psi\cos\theta)\ ,$$

where θ is the angle between the normal to the plane of the interferometer and the vertical and Ω_{En} is the component of $\vec{\Omega}_E$ normal to the interferometer plane. If the experiment is performed at the South pole ($\psi=-\pi/2$) or the North pole ($\psi=\pi/2$), then there would be no contribution due to the Thomas precession or the geodetic precession, since $\vec{v}=0$. Then the corresponding phase shift $\Delta\phi_{\mathrm{LT}}=-(8GMm\Omega_{En}A/5\hbar c^2R)$ is entirely due to the Lense-Thirring precession. When $A=0.25m^2$, $\Delta\phi_{\mathrm{LT}}=3.2\times10^{-7}$ radians, which is three orders of magnitude smaller than the currently observable phase shift.

If the experiment is performed in an orbiting satellite, then $\vec{f}=0$. Also for a circular orbit $|\vec{u}|\simeq(GM/r)^{1/2}$. Suppose, again, that the interferometer is placed on a platform which is made nonrotating relative to the distant stars by means of telescopes. For a polar orbit, if the plane of the interferometer is parallel to the plane of the orbit, then only the geodetic precession will contribute to the phase shift, which is then $\Delta\phi_G\simeq3(GM/r)^{3/2}(mA/\hbar rc^2)$. For an orbit at an altitude of about 500 km, and for an interferometer with $A=0.25m^2$, $\Delta\phi_G=8.5\times10^{-6}$ radians. If the interferometer plane is perpendicular to the orbital plane, then there would be only the Lense-Thirring phase shift $(4GMmA/5\hbar c^2r)(-\Omega_{En}+3\Omega_E\sin\psi\cos\theta)$. If the plane of the interferometer is perpendicular to the Earth's axis of rotation ($\Omega_{En}=\Omega_E,\theta+\psi=\pi/2$) then this phase shift is $1.6\times10^{-7}(-1+3\sin^2\psi)$ radians for a near Earth orbit ($r\simeq R$).

An important feature of this proposed experiment is that the use of the telescopes makes it nonlocal. It is for this reason that the above phase shift, due to $\vec{\Omega}$, is much bigger than the contribution from the curvature components R_{0ijk}. This phase shift is independent of the velocity of the beam relative to the interferometer, apart from relativistic corrections that are negligible for neutrons. But this experiment may be done with UCN on a larger scale than if thermal neutrons are used. Also by using a higher intensity beam, e.g., interferometry with helium atoms,[14] it may be possible to increase the sensitivity. The high intensity of the beams which is possible in optical interferometry, enables a phase shift $\sim10^{-9}$ radians to be detectable. But the above phase shift gets reduced for light by the factor $(\hbar\omega/mc^2)\sim10^{-9}$. The performance of the proposed experiments in Secs. IV and V, which can be done much more easily than the experiments proposed in this section, would enable a fresh assessment of the feasibility of the latter. Also, the former feasible experiments to detect the curvature effects, together with a test of the equivalence principle in a relativistic context, can be regarded as an indirect test of general relativity at the quantum-mechanical level. Recently, tests of the relativistic equivalence principle were proposed for charged particle interferometry[22] in the presence of the electromagnetic and gravitational fields. The latter effects seem to be too small to be measured in electron interferometry. It is desirable to develop a proton interferometer, using which the new effect[22] in the presence of the electric and gravitational fields may be measurable.

In conclusion, it may be noted that the results of this paper suggest that the COW experiment is just the first of a series of experiments that can be done to detect the influence of gravity, which would make interferometry a serious probe of gravity and not just an amusing example of the application of quantum mechanics to the gravitational field.

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