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Twist-four effects on the asymmetry in polarized-electron–deuteron scattering and $\sin^2\theta_W$

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(Received 19 January 1984; revised manuscript received 20 August 1984)

The twist-four, spin-two quantum-chromodynamic corrections to the asymmetry parameter in polarized-electron–deuteron scattering and their effect on $\sin^2\theta_W$ have been calculated using the operator-product expansion of the product of the weak and electromagnetic currents. The coefficients in the expansion were determined using perturbation-theory techniques, and the deuteron matrix elements of the operators were evaluated using the MIT bag model. The higher-twist effects decrease the value of $\sin^2\theta_W$, as determined from polarized-electron–deuteron scattering, similar to the electroweak radiative corrections, but by less than 1%.

In the standard electroweak theory[1] the fundamental parameter $\sin^2\theta_W$ is of considerable interest and a great deal of effort has been devoted to its precise determination.[2] The asymmetry in polarized-electron–deuteron scattering, arising from the interference between electromagnetic and weak interactions, depends on $\sin^2\theta_W$, and polarized-electron scattering data[3] have been used to measure $\sin^2\theta_W$. However, in extracting $\sin^2\theta_W$ from these data one must consider, in addition to the radiative electroweak corrections,[4] the effects of quantum chromodynamics[5] (QCD).

We have calculated the nonperturbative twist-four, spin-two QCD corrections to the asymmetry parameter and $\sin^2\theta_W$ and have found their effect is, typically, to decrease $\sin^2\theta_W$ less than 1%. After a brief review of the asymmetry in polarized-electron–deuteron scattering we present our calculation of the twist-four, spin-two QCD effects and finally compute the magnitude of these QCD corrections to $\sin^2\theta_W$ for the present data.

The asymmetry parameter is defined in terms of the longitudinally-polarized-electron scattering inclusive cross sections on deuterium,[6] $e_{R,L}(k)+\mathrm{D}(p)\rightarrow e'(k')+\text{anything}$:

$$A=\frac{d\sigma_R-d\sigma_L}{d\sigma_R+d\sigma_L} \quad . \tag{1}$$

This parity-nonconserving asymmetry arises in the standard electroweak theory from the interference between the electromagnetic and weak neutral currents; i.e., the γ and Z^0 exchange diagrams.

The contribution coming from the axial-vector coupling at the electron vertex is of the form[7]

$$\frac{A_{VV}(Q^2,x,y)}{Q^2}\propto\frac{\int e^{iq\cdot z}\langle D|[j^\mu(z)J^\nu(0)+J^\mu(z)j^\nu(0)]|D\rangle\, d^4z\, l_{\mu\nu}}{\int e^{iq\cdot z}\langle D|j^\mu(z)j^\nu(0)|D\rangle\, d^4\, l_{\mu\nu}} \quad , \tag{2}$$

where j^μ and J^μ are the usual electromagnetic and vector weak neutral currents, respectively, in the standard electroweak theory and $l_{\mu\nu}=\mathrm{Tr}(k'\gamma_\mu k\gamma_\nu)$. The kinematical variables are defined as usual: $q=k-k'$, $Q^2=-q^2$, $\nu=p\cdot q$, $x=Q^2/2\nu$, and $y=p\cdot q/p\cdot k=(E-E')/E$. In the quark-parton model, direct calculations show

$$\frac{A_{VV}(Q^2,x,y)}{Q^2}=-\frac{G}{\sqrt{2}\pi\alpha}\frac{9}{20}(1-\tfrac{20}{9}\sin^2\theta_W) \quad . \tag{3}$$

Similarly, the contribution to the asymmetry parameter coming from the vector coupling at the electron vertex is of the form

$$\frac{A_{VA}(Q^2,x,y)}{Q^2}\propto\frac{\int e^{iq\cdot z}\langle D|j^\mu(z)J_5^\nu(0)|D\rangle\, d^4z\, l^5_{\mu\nu}}{\int e^{iq\cdot z}\langle D|j^\mu(z)j^\nu(0)|D\rangle\, d^4z\, l_{\mu\nu}} \quad , \tag{4}$$

where J_5^μ is the axial-vector weak neutral current in the standard electroweak theory and $l^5_{\mu\nu}=\mathrm{Tr}(k'\gamma_\mu k\gamma_\nu\gamma_5)$. In the quark-parton model one can easily show that

$$\frac{A_{VA}(Q^2,x,y)}{Q^2}=-\frac{G}{\sqrt{2}\pi\alpha}\frac{9}{20}(1-4\sin^2\theta_W\left[\frac{1-(1-y)^2}{1+(1-y)^2}\right] \quad . \tag{5}$$

To calculate the twist-four, spin-two QCD corrections to the quark-parton-model result for $A_{VV}(Q^2,x,y)$ we define the operator

$$V_{\mu\nu}=i\int e^{iq\cdot z}T[j_\mu(z)J_\nu(0)+J_\mu(z)j_\nu(0)]d^4z \quad . \tag{6}$$

In the physical region

$$A_{VV}(Q^2,x,y)\propto\frac{1}{2\pi}\mathrm{Im}\langle D|V_{\mu\nu}|D\rangle l^{\mu\nu} \quad ; \tag{7}$$

that is, the numerator in Eq. (2) above. Using the Wilson operator-product expansion[8] (OPE) for the bilocal four-quark operators, which are of the typical form

$$\bar{q}_i(z)\gamma_\mu q_i(z)\bar{q}_j(0)\gamma_\nu q_j(0) \quad ,$$

where i and j denote u and d quarks, one obtains an expansion in terms of local operators with Q^2-dependent coefficients that obey the renormalization-group equation. We shall choose the renormalization point μ^2 at Q^2 in these equations for the coefficients in the OPE, which amounts to neglecting their anomalous dimensions and allows the coefficients to be calculated using perturbative QCD techniques. One finds both diagonal contributions, involving two-quark-gluon operators, and nondiagonal contributions, involving four-quark operators. These can both be conveniently expressed in terms of the traceless, antisymmetric basis of operators $O_l^{\mu\nu}$, containing no contracted covariant derivatives, due to Jaffe and Soldate:[9]

$$V_{\mathrm{diag}}^{\mu\nu}=\frac{g^2}{q^6}\left(T^{\mu\nu}_{\mu_1\mu_2}[\tfrac{1}{4}O_7^{\mu_1\mu_2}(0)+\tfrac{5}{2}O_9^{\mu_1\mu_2}(0)]+(q^\mu q^\nu-q^{\mu\nu}q^2)\frac{q_{\mu_1}q_{\mu_2}}{q^2}[-\tfrac{3}{2}O_7^{\mu_1\mu_2}(0)+O_9^{\mu_1\mu_2}(0)]\right) \quad , \tag{8}$$

and

$$V^{\mu\nu}_{\mathrm{nondiag}}=\frac{4g^2}{q^6}T^{\mu\nu}_{\mu_1\mu_2}[O_2^{\mu_1\mu_2}(0)-2O_4^{\mu_1\mu_2}(0)+O_6^{\mu_1\mu_2}(0)] \quad . \tag{9}$$

In Eqs. (8) and (9) we have defined

$$T^{\mu\nu}_{\mu_1\mu_2}=\delta^\mu_{\mu_1}\delta^\nu_{\mu_2}q^2-(\delta^\nu_{\mu_1}q^\nu+\delta^\nu_{\mu_1}q^\nu)q_{\mu_2}+q^{\mu\nu}q_{\mu_1}q_{\mu_2} \quad , \tag{10}$$

and $g^2=4\pi\alpha_s(Q^2)$ is the QCD running coupling constant. We have verified that these results agree with both the previous calculations of the higher-twist effects in electroproduction[9] and neutrino neutral-current scattering.[10]

In the case of $A_{VA}(Q^2,x,y)$ one can show there are no twist-four, spin-two interference terms and, consequently, the quark-parton-model result, Eq. (5), is unaffected.

The calculation is further simplified since the twist-four corrections to electroproduction,[9] although model dependent, are nevertheless certainly small[11] ($<2\%$). Therefore, these higher-twist corrections to the denominator of Eq. (2) can be neglected in comparison with the twist-four, spin-two effects in the numerator. Their effects on $\sin^2\theta_W$ is unimportant since A is itself quite small.

To calculate the matrix element of $V_{\mu\nu}$ between deuteron states (spin-averaged), we shall assume quark-confinement models for which the quark wave function is of the form

$$q(\vec{r})=\begin{pmatrix} f(r) \\ \vec{\sigma}\cdot\hat{r}g(r)\end{pmatrix}\chi \quad , \tag{11}$$

where χ is a two-component spinor; for example, the MIT bag model.[12] Direct calculation shows that the spatial part of the matrix element $\langle D|V_{\mu\nu}|D\rangle$ can be expressed in terms of the two integrals

$$I_1=\int[|f(r)|^2+|g(r)|^2]^2d^3r \quad , \tag{12}$$

and

$$I_2=\int|f(r)|^2|g(r)|^2d^3r \quad . \tag{13}$$

The matrix elements of the six different spin-, color-, and flavor-dependent four-quark local operators needed in the evaluation of $\langle D|V_{\mu\nu}|D\rangle$ are among the set of nine such matrix elements that have previously been given in the calculations of the twist-four, spin-two corrections to the neutrino neutral-current cross section.[10] Matrix elements involving gluon fields will be neglected since they enter the OPE with coefficients smaller by an order of magnitude and, in addition, the gluon content of a nucleon (at rest) is also rather small.

Combining the above results one finds the total asymmetry $A=A_{VV}+A_{VA}$ to be

$$\begin{aligned}\frac{A(Q^2,x,y)}{Q^2}=-\frac{G}{\sqrt{2}\pi\alpha}(\tfrac{9}{20})\Bigg\{\Bigg[1+\frac{1-(1-y)^2}{1+(1-y)^2}+\frac{\alpha_s(Q^2)}{Q^4}\frac{M}{x[u(x)+d(x)]}(-\tfrac{85}{9}I_1+\tfrac{440}{27}I_2)\Bigg]\\+\sin^2\theta_W\Bigg[-\tfrac{20}{9}-4\frac{1-(1-y)^2}{1+(1+y)^2}+\frac{\alpha_s(Q^2)}{Q^4}\frac{M}{x[u(x)+d(x)]}(+\tfrac{524}{27}I_1-32I_2)\Bigg]\Bigg\} \quad ,\end{aligned} \tag{14}$$

where $u(x)$ and $d(x)$ are the u and d (valence) quark distributions in nucleons and M is the nucleon mass. The x and y dependence of the $\alpha_s(Q^2)/Q^4$ correction terms in Eq. (14) follows directly from multiplying the leptonic tensors with the twist-four, spin-two operators, as described above. Of course, to obtain the complete twist-four corrections (for all x and y) the contributions of all the higher spins must be calculated, which is clearly an intractable task. However, for the moderate values of x and y where the existing data[3] have been taken, keeping the spin-two contributions should be a reasonably good approximation.

To investigate the effect on the determination of $\sin^2\theta_W$ of the twist-four, spin-two corrections to the polarized-electron–deuteron asymmetry parameter $A(Q^2,x,y)$, we have used Eq. (14) to calculate

$$\delta\sin^2\theta_W=-\frac{\alpha_s(Q^2)}{Q^4}\frac{M}{x[u(x)+d(x)]}[(\tfrac{85}{9}I_1-\tfrac{440}{27}I_2)-(\tfrac{524}{27}I_1-32I_2)\sin^2\theta_W]\left[\tfrac{20}{9}+4\frac{1-(1-y)^2}{1+(1-y)^2}\right]^{-1} \quad . \tag{15}$$

The value $\sin^2\theta_W = 0.224 \pm 0.020$ has been determined from the polarized-electron–deuteron data[3] taken typically in the kinematic region $Q^2 = 1.67\ \mathrm{GeV}^2$, $x = 0.24$, and $y = 0.19$ ($E \simeq 20$ GeV). In this regime[13] $x[u(x)+d(x)] \simeq 0.9$. Taking $\alpha_s(Q^2) = 0.27$ and using the MIT-bag-model values $I_1 = 20.36\times10^{-4}\ \mathrm{GeV}^3$ and $I_2 = 3.21\times10^{-4}\ \mathrm{GeV}^3$ one then finds from Eq. (15) that $\delta\sin^2\theta_W \simeq -0.0003$. Thus, the twist-four, spin-two effects on the asymmetry in polarized-electron–deuteron scattering result in a very small decrease ($< 1\%$) in the value of $\sin^2\theta_W$, similar to the electroweak corrections. We have, therefore, explicitly shown here that these twist-four, spin-two effects are not unexpectedly large and, in fact, are negligible in comparison with other uncertainties in $\sin^2\theta_W$, at present. It is interesting to note that the higher-twist corrections to $\sin^2\theta_W$, as determined from neutrino neutral-current scattering,[10] decrease the value of $\sin^2\theta_W$ by about 1%.

We thank Chris Quigg for his kind hospitality in the Theoretical Physics Department at Fermilab. One of us (S.F.) is also grateful to Dennis Sivers and Alan White for the opportunity to visit the Theoretical High Energy Physics Division of Argonne National Laboratory. This work was supported in part by the National Science Foundation under Grants No. PHY-82-09145 and No. YOR-81-020.

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