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Confined quark systems, and state vectors with proper relativistic properties

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The three-momentum eigenstate which is usually used in the mean-field approach to confined quark systems is modified so as to have proper relativistic properties. We investigate structures of current matrix elements between the state vectors so constructed, which serves to clarify the meanings of various models and approximate calculations previously proposed.

I. INTRODUCTION

The underlying theory of the hadronic strong interaction is now widely accepted to be quantum chromodynamics (QCD). Yet the present level of understanding the confinement or the formation of confined quark systems is unsatisfactory. This gives a methodological significance of various intermediate-step approaches to the confinement, such as the MIT or SLAC bag models,[1,2] the mean-field approach,[3–6] and the soliton bag model.[7,8] In order to assert the phenomenological validity of such approaches, and also in order to extend the region of their application to hadronic phenomena, it is necessary to calculate related physical quantities in a relativistically invariant way.

The purpose of the present paper is to construct a formalism which allows a relativistically covariant evaluation of matrix elements between hadron states which are confined quark (and antiquark) systems. The basic idea has been proposed about ten years ago.[3] Nevertheless, we think that it is now necessary to reexhibit the essential part of the original work[3] and also to extend our idea so as to allow a wider application. It will help us to understand systematically various methods which have been proposed by many authors.[6,9–12]

Sections II and III are devoted to constructing the momentum eigenstate of a confined quark system with the proper relativistic properties, and in Sec. IV we discuss the meanings of various models and approximate calculations previously proposed.

II. MOMENTUM EIGENSTATES WITH LORENTZ BOOST

In order to recover the translational invariance in the mean-field model, Bando *et al.*[3] constructed the momentum eigenstate by superposing states which are composed of quarks (and antiquarks) confined in a certain central potential as follows:

$$|\vec{p},\beta\rangle^0 = N_\beta(p)\int d\vec{S}\, e^{i\vec{p}\cdot\vec{S}}\,|\vec{S},\beta\rangle\ , \tag{1}$$

where β means a set of quantum numbers necessary to specify the state; $\vec{S}$ is the center of the confining potential. This method is familiar in nuclear physics, and is often called the "generator-coordinate method."[13] If $|\vec{p},\beta\rangle^0$ is the low-lying baryon ($\frac{1}{2}^+$), $|\vec{S},\beta\rangle$ is assumed to be given by

$$|\vec{S},\beta\rangle = \sum_{ABC} C^\beta_{ABC}\, a_A(\vec{S})^\dagger a_B(\vec{S})^\dagger a_C(\vec{S})^\dagger\,|h(\vec{S})\rangle\ , \tag{2}$$

where $a_A(\vec{S})^\dagger$ is the creation operator of a quark with a set of quantum numbers A (its flavor and color part is denoted by $\underline{A}$) and is given by

$$a_A(\vec{S}) = \int d\vec{x}\, U_{a,A}(\vec{x}-\vec{S})^\dagger \psi_{a,\underline{A}}(x)\ . \tag{3}$$

$\psi_{,\underline{A}}(x)$ is the quark field and $U_{,A}(\vec{x}-\vec{S})$ is the ground-state positive-energy solution of the Dirac equation of potential W

$$\left[-E_A\gamma_4+\vec{\gamma}\frac{\partial}{\partial\vec{x}}+W(\vec{x}-\vec{S})\right]U_{,A}(\vec{x}-\vec{S})=0\ . \tag{4}$$

a is the Dirac-spinor index. C^β_{ABC}'s are appropriate SU(6) numerical coefficients including the color degree of freedom. $|h(\vec{S})\rangle$ is the "hadronic" vacuum,[3] which satisfies for arbitrary (not always ground state) A and B

$$a_A(\vec{S})\,|h(\vec{S})\rangle = b_B(\vec{S})\,|h(\vec{S})\rangle = 0\ . \tag{5}$$

$b_B(\vec{S})$ is the annihilation operator of the antiquark mode B.

Note that Eq. (1) recovers only the rotational and translational invariances in the three-dimensional space. It is, however, not manifestly covariant under rotations including the time axis.[14]

Before entering into detailed considerations, it is worthwhile to note a remark in order to clarify the following arguments. We consider matrix elements of, e.g., a current $J_\rho(x)$ between two states written as

$$^{0}\langle \vec{q},\beta \,|\, J_\rho(0) \,|\, \vec{p},\gamma \rangle^{0} = N_\beta(q) N_\gamma(p) \int d\vec{r}\, e^{-i(\vec{q}+\vec{p})\cdot\vec{r}/2} \left\langle \vec{r}/2,\beta \left| \int d\vec{x}\, e^{i\vec{k}\cdot\vec{x}} J_\rho(\vec{x},0) \right| -\vec{r}/2,\gamma \right\rangle, \quad \vec{k}=\vec{q}-\vec{p} \ . \tag{6}$$

In the "local approximation,"[3] which has been adopted in most of our previous papers,[3–5] we assume dominance of contributions from cases where the two centers of the confining potential (or "bag centers") on the right-hand side (RHS) of (6) coincide. (As to considerations without recourse to this approximation, see Refs. 15 and 16.) When we take the normalization conditions as

$$^{0}\langle \vec{q},\beta \,|\, \vec{p},\gamma \rangle^{0} = (2\pi)^3 2E_\beta(p)\delta(\vec{p}-\vec{q})\delta_{\beta\gamma} \ , \tag{7a}$$

$$\int d\vec{x}\, U_{,A}(\vec{x})^\dagger U_{,B}(\vec{x}) = \delta_{AB} \ , \tag{7b}$$

$$\{ a_A(\vec{S}), a_B(\vec{S})^\dagger \} = \delta_{AB} \ , \tag{7c}$$

and write

$$N_\beta(p) = [2E_\beta(p)/v_\beta(p)]^{1/2} \ , \tag{8}$$

we obtain

$$v_\beta(p) = \int d\vec{r}\, \langle \vec{r},\beta \,|\, \vec{r}=\vec{0},\beta \rangle e^{-i\vec{p}\cdot\vec{r}} \ . \tag{9}$$

Hence, in the local approximation, we have

$$(6) \simeq [2E_\beta(q) 2E_\gamma(p)]^{1/2} \left\langle \vec{r}=\vec{0},\beta \left| \int d\vec{x}\, e^{i\vec{k}\cdot\vec{x}} J_\rho(\vec{x},0) \right| \vec{r}=\vec{0},\gamma \right\rangle \ , \tag{10}$$

which leads to what has been examined in the usual bag-model calculations.

Now, in order to consider the covariant description, let us define first the state vector

$$|\, \vec{p},\beta \rangle \equiv \mathcal{N}_N(p) \int d\vec{S}\, e^{i\vec{p}\cdot\vec{S}} e^{-i\vec{P}\cdot\vec{S}} U(\vec{v}_p) \,|\, \vec{r}=\vec{0},\beta \rangle \ , \tag{11}$$

where $\vec{P}$ is the translation operator, $U(\vec{v}_p)$ the Lorentz-boost operator; for simplicity we consider the nucleon state, and the momentum $\vec{p}$ and the velocity $\vec{v}_p$ are related to each other by

$$\vec{p} = \frac{m\vec{v}_p}{(1-v^2)^{1/2}}, \quad m = \text{nucleon mass} \ . \tag{12}$$

The Lorentz boost considered here is

$$(x_\rho) \to (x'_\rho) = (L(\vec{v})_{\rho\lambda} x_\lambda) = \left[\vec{x}_\perp, \frac{1}{(1-v^2)^{1/2}}(x_{\parallel} + v x_0), \frac{i}{(1-v^2)^{1/2}}(v x_{\parallel} + x_0) \right] \ , \tag{13}$$

where $\perp$ and $\parallel$ mean the components of $\vec{x}$ perpendicular and parallel to $\vec{v}$, respectively. Thus, the momentum and energy operators satisfy

$$(U(\vec{v}) P_\rho U(\vec{v})^{-1}) = (P_\lambda L(\vec{v})_{\lambda\rho}) = \left[\vec{P}_\perp, \frac{1}{(1-v_p{}^2)^{1/2}}(P_{\parallel} - v P_0), \frac{i}{(1-v^2)^{1/2}}(-v P_{\parallel} + P_0) \right] \ . \tag{14}$$

The current matrix element $\langle \vec{q},\beta \,|\, J_\rho(0) \,|\, \vec{p},\gamma \rangle$ is written as

$$\langle \vec{q},\beta \,|\, J_\rho(0) \,|\, \vec{p},\gamma \rangle$$
$$= \mathcal{N}_N(q)\mathcal{N}_N(p) \int d\vec{r}\, e^{-i(\vec{q}+\vec{p})\cdot\vec{r}/2} \left\langle \vec{S}=\vec{0},\beta \left| U(\vec{v}_q)^{-1} e^{i\vec{P}\cdot\vec{r}/2} \int d\vec{x}\, e^{i\vec{k}\cdot\vec{x}} J_\rho(\vec{x},0) e^{i\vec{P}\cdot\vec{r}/2} U(\vec{v}_p) \right| \vec{S}=\vec{0},\gamma \right\rangle \ . \tag{15}$$

For the Lorentz boost (13), we have

$$U(\vec{v}) = e^{-ib\hat{v}\cdot\vec{K}} \tag{16}$$

and

$$U(\vec{v})\psi(\vec{x},0)U(\vec{v})^{-1} = T(\vec{v})^{-1}\psi(L(\vec{v})x)\,|_{x_0=0} = T(\vec{v})^{-1}\psi(\vec{x}_\perp, x_{\parallel}\cosh b, x_{\parallel}\sinh b) \ , \tag{17}$$

where $\vec{K}$ is the generator of the Lorentz boost,

$$\tanh b = v, \quad T(\vec{v}) = \cosh(b/2) + \hat{v}\cdot\vec{\alpha}\sinh(b/2), \quad \hat{v} = \vec{v}/v, \quad \vec{\alpha} = i\gamma_4\vec{\gamma} \quad (\gamma_\rho^\dagger = \gamma_\rho) \ .$$

For simplicity, we take $\vec{q} = -\vec{p} = \vec{k}/2$ (i.e., the Breit frame). Then, for the quark current

$$J_\rho^{\underline{A}\,\underline{B}}(x) = \bar{\psi}_{,\underline{A}}(x) O_\rho \psi_{,\underline{B}}(x) \ ,$$

we obtain in the local approximation

$$\langle \vec{q},\beta \,|\, J_p^{\underline{A}\,\underline{B}}(0) \,|\, \vec{p}=-\vec{q},\gamma\rangle \simeq 2E_N(q)\int d\vec{x}\, e^{i\vec{k}\cdot\vec{x}} \sum_{(A-\underline{A},B-\underline{B})}\sum_{(A_{2,3}B_{2,3})} 9C^{\beta}_{AA_2A_3}C^{\gamma}_{BB_2B_3}$$
$$\times\langle h(\vec{0})\,|\,a_{A_3}a_{A_2}U(\vec{v}_q)^{-1}U(\vec{v}_p)a^{\dagger}_{B_2}a^{\dagger}_{B_3}\,|\,h(\vec{0})\rangle$$
$$\times\,\overline{U}_{,A}(\vec{x}_{\perp},(\cosh b_q)x_{||},0)T(\vec{v}_q)^{-1}O_pT(\vec{v}_p)$$
$$\times\, U_{,B}(\vec{x}_{\perp},(\cosh b_q)x_{||},0)e^{-2iE_0(\sinh b_q)x_{||}}\ , \qquad (18)$$

where E_0 is the energy of the ground-state quark. To the first order of $\vec{k}/m$, we obtain

$$(18)\simeq 2m\int d\vec{x}\sum_{(A-\underline{A},B-\underline{B})}\sum_{(A_{2,3}B_{2,3})} 9C^{\beta}_{AA_2A_3}C^{\gamma}_{BB_2B_3}\langle h(0)\,|\,a_{A_3}a_{A_2}U(\vec{v}_q)^{-1}U(-\vec{v}_q)a^{\dagger}_{B_2}a^{\dagger}_{B_3}\,|\,h(\vec{0})\rangle$$
$$\times\,\overline{U}_{,A}(\vec{x},0)\left[\left[1+i\vec{k}\cdot\vec{x}-i\frac{E_0}{m}\vec{k}\cdot\vec{x}\right]O_p-\frac{1}{4m}\{\vec{k}\cdot\vec{\alpha},O_p\}\right]U_{,B}(\vec{x},0)\ . \qquad (19)$$

When we take $J_p^{\underline{A}\,\underline{B}}$ to be the electromagnetic quark current, we obtain as the nucleon magnetic moment the same result as derived by Guichon,[10] that is, $-iE_0\vec{k}\cdot\vec{x}/m$ and $-\vec{k}\cdot\vec{\alpha}/4m$ terms in (19) leads to the "retardation" and the "spin-precession"[9] contributions, respectively, in addition to the ordinary one coming from the $+i\vec{k}\cdot\vec{x}$ term in (19).

It is worthwhile to note the following properties of $|\,\vec{p},\beta\rangle$. Equation (11) is rewritten as

$$|\,\vec{p},\beta\rangle=\mathcal{N}_N(p)(2\pi)^3\delta(\vec{P}-\vec{p})U(\vec{v}_p)\,|\,\vec{r}=\vec{0},\beta\rangle\ , \qquad (20)$$

which shows directly that $|\,\vec{p},\beta\rangle$ is the momentum eigenstate. The condition for the energy eigenvalue to be $E(p)=m/(1-v_p{}^2)^{1/2}$ is proved to be

$$P_0\,|\,\vec{r}=\vec{0},\beta\rangle=M\,|\,\vec{r}=\vec{0},\beta\rangle \qquad (21)$$

with $M=m$.

Proof. By using (14) and (21), $U(\vec{v}_p)$ is shown to satisfy

$$\begin{aligned} mU(\vec{v}_p)\,|\,\vec{r}=\vec{0},\beta\rangle &= U(\vec{v}_p)P_0\,|\,\vec{r}=\vec{0},\beta\rangle \\ &=(PL(\vec{v}_p))_0U(\vec{v}_p)\,|\,\vec{0},\beta\rangle \\ &=\frac{1}{(1-v_p{}^2)^{1/2}}(-v_pP_{||}+P_0) \\ &\quad\times U(\vec{v}_p)\,|\,\vec{0},\beta\rangle\ , \end{aligned} \qquad (22)$$

from which one obtains

$$P_0U(\vec{v}_p)\,|\,\vec{r}=\vec{0},\beta\rangle =[\vec{v}_p\cdot\vec{P}+E(p)(1-v_p{}^2)]U(\vec{v}_p)\,|\,\vec{r}=\vec{0},\beta\rangle\ . \qquad (23)$$

Therefore,

$$\begin{aligned} P_0\,|\,\vec{p},\beta\rangle &=[\vec{v}_p\cdot\vec{P}+E(p)(1-v_p{}^2)]\,|\,\vec{p},\beta\rangle \\ &=E(p)\,|\,\vec{p},\beta\rangle.\ \ \text{(Q.E.D.)} \end{aligned} \qquad (24)$$

This suggests that, under the condition (21) and with an appropriate choice of $\mathcal{N}_N(p)$, the state vector $|\,\vec{p},\beta\rangle$ has the proper transformation properties under the Lorentz boost. In order to make the situation more clear, we consider in Sec. III what relation the state vector $|\,\vec{p},\beta\rangle$ has to

$$|\,\vec{p},\beta\rangle\rangle\equiv U(\vec{v}_p)\,|\,\vec{p}=\vec{0},m,\beta\rangle\rangle\ , \qquad (25)$$

where $|\,\vec{p}=\vec{0},m,\beta\rangle\rangle$ is the state vector with momentum and energy eigenvalues $(\vec{0},m)$; $|\,\vec{p},\beta\rangle\rangle$ has the eigenvalues

$$(\vec{p}=m\vec{v}_p/(1-v_p{}^2)^{1/2},E(p))\ .$$

In the above consideration, we *assume* that we can construct the state satisfying Eq. (21) approximately well in the Hartree-Fock approach.[3–6] The reason for demanding the condition Eqs. (21) or (24), as opposed to the weaker expectation-value requirement of Refs. 2 or 12, will be explained in Sec. IV.

III. RELATION OF $|\,\vec{p},\beta\rangle$ TO $|\,\vec{p},\beta\rangle\rangle$

Let us take $|\,\vec{p},=\vec{0},m,\beta\rangle\rangle$ to be $|\,\vec{p}=\vec{0},\beta\rangle$ (or equivalently $|\,\vec{p}=\vec{0},\beta\rangle^0$), which is allowed so far as the condition (21) is satisfied. Then, $|\,\vec{p},\beta\rangle\rangle$ is given by

$$|\,\vec{p},\beta\rangle\rangle=\mathcal{N}_N(0)U(\vec{v}_p)\int d\vec{S}\,e^{-i\vec{P}\cdot\vec{S}}\,|\,\vec{r}=\vec{0},\beta\rangle\ . \qquad (26)$$

The RHS is easily rewritten as

$$\text{RHS}=\mathcal{N}_N(0)(1-v_p{}^2)^{1/2}\int d\vec{S}\exp[-i(\vec{P}-\vec{v}_pP_0)\cdot\vec{S}]U(\vec{v}_p)\,|\,\vec{r}=\vec{0},\beta\rangle \quad [\text{from (14)}]$$
$$=\mathcal{N}_N(0)(1-v_p{}^2)^{1/2}\int d\vec{S}\exp[-i\vec{P}_\perp\cdot\vec{S}_\perp-i(P_{\parallel}-p_{\parallel})(1-v_p{}^2)S_{\parallel}]U(\vec{v}_p)\,|\,\vec{r}=\vec{0},\beta\rangle \quad [\text{from (23)}]$$
$$=\frac{\mathcal{N}_N(0)}{(1-v_p{}^2)^{1/2}}\frac{1}{\mathcal{N}_N(p)}\,|\,\vec{p},\beta\rangle\ . \tag{27}$$

Thus we see that, when we set

$$\mathcal{N}_N(p)=\mathcal{N}_N(0)/(1-v_p{}^2)^{1/2}\ , \tag{28}$$

$|\,\vec{p},\beta\rangle\rangle$ which is obtained by boosting a certain superposition of the "bag" states with different centers, is equivalent to the state $|\,\vec{p},\beta\rangle$ which is obtained by boosting first the bag state with its center at the origin of coordinate and then taking the projection to the momentum eigenstate, Eq. (20). It may be superfluous to add that when we take the normalization as

$$\langle\,\vec{q},\beta\,|\,\vec{p},\gamma\,\rangle=(2\pi)^3 2E(p)\delta(\vec{q}-\vec{p})\delta_{\beta\gamma} \tag{29}$$

and set

$$\mathcal{N}_N(p)=(2E(p)/V_N(p))^{1/2}\ , \tag{30}$$

we have

$$V_N(p)=V_N(0)(1-v_p{}^2)^{1/2}\ . \tag{31}$$

IV. DISCUSSIONS AND FINAL REMARKS

We have investigated the Lorentz boosting of hadron states composed of quarks (and antiquarks) with the aim of constructing the relativistically covariant description. Once we know the general structure of the state vector (11), we have an overall view which makes transparent the meanings of various formalisms and approximation methods previously proposed concerning effects due to c.m. motion and/or Lorentz boost.[1,6,9–12] We have already mentioned in Sec. II an example on this point in connection with recoil corrections to the magnetic moment. We will further add three examples.

First, Tegen *et al.*[6] have proposed a mean-field approach to the confinement, and used a wave function which describes hadrons as confined systems and has a separated c.m. part. It is easy to find the relation between the nucleon wave function used by Tegen *et al.*[6] and ours by introducing an appropriate "$\vec{x}$ representation" of the state (1) for the nucleon, $|\,N,\vec{p},\beta\rangle^0$. This can be done as follows:

the $\vec{x}$ representation of $|\,N,\vec{p},\beta\rangle^0\equiv\Phi(\vec{S},\{\vec{x}a\underline{A}\}\,|\,N,\vec{p},\beta)$

$$\equiv\left\langle h(\vec{S})\left|\prod_{j=1}^{3}\psi_{a_j,\underline{A}_j}(\vec{x}_j,0)^{(+)}\right|N,\vec{p},\beta\right\rangle^0 \tag{32a}$$

$$=-\sum_{\{B_j\}}C^{\beta}_{B_1B_2B_3}\,\epsilon\begin{bmatrix}a_1\underline{A}_1,\ a_2\underline{A}_2,\ a_3\underline{A}_3\\ b_1\underline{B}_1,\ b_2\underline{B}_2,\ b_3\underline{B}_3\end{bmatrix}N_\beta(p)(2\pi)^{-9/2}$$
$$\times\int\prod_{\mu=0}^{3}d\vec{p}_\mu\,\delta\left[-\sum_{\nu=0}^{3}\vec{p}_\nu+\vec{p}\right]\exp i\left[\sum_{j=1}^{3}\vec{p}_j\cdot\vec{x}_j+\vec{p}_0\cdot\vec{S}\right]\left[\prod_{k=1}^{3}U_{b_k,B_k}(\vec{p}_k)\right]v_h(\vec{p}_0)\ , \tag{32b}$$

where $\psi_{a,\underline{A}}(x)^{(+)}$ means the positive-energy part of the quark field; $\epsilon(:\,:\,:)$ is the sign function; $U_{b,B}(\vec{p})$ and $v_h(\vec{p})$ are defined by

$$U_{b,B}(\vec{p})\equiv(2\pi)^{-3/2}\int d\vec{x}\,e^{-i\vec{p}\cdot\vec{x}}U_{b,B}(\vec{x})\ , \tag{33}$$

$$v_h(\vec{p})\equiv\int d\vec{x}\,e^{-i\vec{p}\cdot\vec{x}}\langle h(\vec{x})\,|\,h(\vec{0})\rangle\ . \tag{34}$$

If $v_h(\vec{p}_0)=(2\pi)^3\delta(\vec{p}_0)$, i.e., $\langle h(\vec{r})\,|\,h(\vec{0})\rangle=1$, (32b) reduces to the wave function given by Tegen *et al.* [Eq. (5) in Ref. 6(a) and Eq. (10) in Ref. 6(b)].

Second, a remark should be given on the wave-packet formalism proposed by Donoghue and Johnson.[12] They expand the bag state as

$$|\,\vec{S}=\vec{0},\beta\rangle=\int\frac{d\vec{p}}{(2\pi)^3}\left[\frac{v_\beta(p)}{2E_\beta(p)}\right]^{1/2}|\,\vec{p},\beta\rangle\ , \tag{35}$$

which is formally equivalent to the one obtained from (1). Using the normalizations (7a) and

$$\langle\vec{S}=\vec{0},\beta\,|\,\vec{S}=\vec{0},\beta\rangle=1\ , \tag{36}$$

one obtains

$$\frac{1}{(2\pi)^3}\int d\vec{p}\,v_\beta(p)=1\ , \tag{37}$$

which allows us to calculate physical quantities as "expectation values" with effects coming from c.m. motion. We cannot, however, convert $|\,\vec{p},\beta\rangle$ defined by (11) into the one corresponding to (35), and the condition (21) contradicts in general with (35), because (35) cannot be an eigenstate of P_0, but has only the expectation value of P_0 as

$$\langle \vec{S}=\vec{0},\beta \mid P_0 \mid \vec{S}=\vec{0},\beta\rangle = \frac{1}{(2\pi)^3}\int d\vec{p}\, v_\beta(p) E_\beta(p) = \langle E_\beta(p)\rangle \; . \tag{38}$$

Here, as mentioned in the last paragraph of Sec. II, we will comment on the meaning for demanding the condition, Eqs. (21) or (24). In usual treatments such as in Refs. 2 or 12, the on-shell relation between momentum and energy is realized as a relation between expectation values. But, when we wish to calculate matrix elements of physical quantities beyond the static properties, where the motion of hadrons plays an essential role, we must prepare the momentum and energy eigenstates $\langle \vec{q},\gamma \mid$ and $\mid \vec{p},\beta\rangle$ satisfying Eq. (24). We see from considerations in Secs. II and III that the simplest condition leading to

$$P_0 \mid \vec{p}=\vec{0},\beta\rangle = m \mid \vec{p}=\vec{0},\beta\rangle \quad \text{with } m = \text{relevant hadron mass} \tag{39}$$

is to assume the bag state $\mid \vec{r},\beta\rangle$ satisfies Eq. (21), and that the state (25)

$$\mid \vec{p}\,\rangle\rangle = U(\vec{v}_p) \mid \vec{p}=\vec{0},\beta\rangle$$

has the form of Eq. (11), useful to understand the meanings of investigations proposed in other papers. It is an important problem how to realize Eqs. (21) and (39) approximately well; this will be shown to be possible if the Hartree-Fock (HF) approach[3–6] well describes behaviors of the ground-state quarks bound in hadrons and the HF potential is suitably chosen, which is to be explained in a forthcoming paper.

Lastly, after the main part of this paper had been completed, we noticed the interesting paper written by Betz and Goldflam,[11] who considered the boosting problem in the soliton bag model. Their calculations are based on the assumption that the usual static-bag wave function can be identified with the zero-momentum eigenstate. But in our case, we impose the condition (21) on $\mid \vec{r}=\vec{0},\beta\rangle$, and after projecting the zero-momentum state, take the boosting, i.e., $U(\vec{v}_p)\delta(\vec{P}) \mid \vec{r}=\vec{0},\beta\rangle$. We obtain, however, results similar to those obtained by Betz and Goldflam[11] in the local approximation; e.g., the current matrix element is written in our formalism as (15), which is rewritten in the local approximation as

$$(15) \simeq [2E_\beta(q) 2E_\gamma(p)]^{1/2} \left\langle \vec{S}=\vec{0},\beta \left| U(\vec{v}_q)^{-1} \int d\vec{x}\, e^{i\vec{k}\cdot\vec{x}} J_\rho(\vec{x},0) U(\vec{v}_p) \right| \vec{S}=\vec{0},\gamma \right\rangle \; . \tag{40}$$

This corresponds to Eq. (50) in Ref. 11. The same logical relation as the one between (6) and (10) exists between (15) and (40).

Details of the formalism on the basis of the Hartree-Fock approach and general treatment of calculating matrix elements and investigations of their structures will be considered in a forthcoming paper.

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[1] A. Chodos, R. L. Jaffe, K. Johnson, C. B. Thorn, and V. F. Weisskopf, Phys. Rev. D **9**, 3471 (1974); A. Chodos, R. L. Jaffe, K. Johnson, and C. B. Thorn, *ibid.* **10**, 2599 (1974); T. DeGrand, R. L. Jaffe, K. Johnson, and J. Kiskis, *ibid.* **12**, 2060 (1975). See further, e.g., K. Johnson, in *Fundamentals of Quarks Models,* proceedings of the 17th Scottish University Summer School in Physics, 1976, edited by I. M. Barbour and A. T. Davis (SUSSP, Edinburgh, 1977); see also references therein.

[2] W. A. Bardeen, M. S. Chanowitz, S. D. Drell, M. Weinstein, and T. M. Yan, Phys. Rev. D **11**, 1094 (1975).

[3] M. Bando, T. Kugo, and S. Tanaka, Prog. Theor. Phys. **53**, 544 (1975); M. Bando, S. Tanaka, and M. Toya, *ibid.* **55**, 169 (1976).

[4] M. Bando, H. Sugimoto, and M. Toya, Prog. Theor. Phys. **59**, 480 (1978); **59**, 903 (1978); **62**, 168 (1979); H. Arisue, M. Bando, H. Sugimoto, and M. Toya, *ibid.* **62**, 1340 (1979); M. Bando, H. Sugimoto, and M. Toya, *ibid.* **63**, 1452 (1980).

[5] Y. Abe, M. Bando, T. Okazaki, and K. Fujii, Phys. Rev. D **19**, 3470 (1979); Y. Abe, K. Fujii, T. Okazaki, M. Bando, H. Arisue, and M. Toya, Prog. Theor. Phys. **64**, 1363 (1980); Lett. Nuovo Cimento **30**, 393 (1981); H. Bando and M. Bando, Phys. Lett. **100B**, 228 (1981); M. Bando, T. Okazaki, and K. Fujii, Z. Phys. C **12**, 17 (1982); K. I. Aoki and M. Bando, Prog. Theor. Phys. **70**, 259 (1983); **70**, 272 (1983); K. I. Aoki and M. Bando, Phys. Lett. **126B**, 101 (1983); T. Okazaki and K. Fujii, Phys. Rev. D **27**, 188 (1983); H. Sawayanagi, K. Fujii, T. Okazaki, and S. Okubo, *ibid.* **27**, 2107 (1983).

[6] (a) R. Tegen, R. Brockman, and W. Weise, Z. Phys. A **307**, 339 (1982); (b) R. Tegen, M. Schedl, and W. Weise, Phys. Lett. **125B**, 9 (1983); (c) W. Weise, in *Quarks, Mesons, and Isobars in Nuclei,* proceedings of the Fifth Topical School, Motril, Granada, Spain, 1982, edited by R. Guardiola and A. Polls (World Scientific, Singapore, 1982).

[7] R. Freidberg and T. D. Lee, Phys. Rev. D **15**, 1694 (1977); **16**, 1096 (1977); **18**, 2623 (1978).

[8] R. Goldflam and L. Wilets, Phys. Rev. D **25**, 1951 (1982).

[9] W.-Y. P. Hwang, Z. Phys. C **16**, 327 (1983); I. Picek and D. Tadić, Phys. Rev. D **27**, 665 (1983).

[10] P. A. M. Guichon, Phys. Lett. **129B**, 108 (1983).

[11] M. Betz and R. Goldflam, Phys. Rev. D **28**, 2848 (1983).

[12] J. F. Donoghue and K. Johnson, Phys. Rev. D **21**, 1975

(1980); C. W. Wong, *ibid.* **24**, 1416 (1981).

[13]R. E. Peierls and J. Yoccoz, Proc. Phys. Soc. London **A70**, 381 (1957).

[14]The soliton model first developed in Ref. 7 seems also suitable to construct a covariant description, which will be discussed elsewhere.

[15]T. Hsu, Y. Zhunge-le, and L. Lian-sou, Report No. HZPP-80-2, 1980 (unpublished); T. Hsu, Y. Zhung-le, and J. Zhong, Report No. HZPP-81-3, 1981 (unpublished).

[16]T. Okazaki and K. Fujii, Phys. Rev. D **27**, 188 (1983).