Pion contributions to baryon magnetic moments

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The addition of pion contributions to symmetric-quark-model predictions of baryon magnetic moments is shown to lead to considerable improvement in quantitative agreement with experiment. The added variation of the nucleon quark moment in the Ξ hyperons, interpreted as a relativistic effect, removes all salient difficulties in reconciling the quark model with accurate magnetic-moment measurements. The pion probability required in the nucleon wave function is found to be $(18\pm7)\%$.

I. INTRODUCTION

The simple quark model that pictures baryons as three quarks bound in relatively similar states has been challenged by recent accurate magnetic-moment measurements.[1,2] Although the quark model predicts the rough magnitudes and signs of the eight measured moments, it fails in more quantitative tests. This is seen most clearly in magnetic-moment sum rules[2-5] that cancel out nonstatic magnetic-moment contributions to the extent that baryon wave functions are flavor independent. These sum rules show relatively large deviations, especially considering that the magnetic moments are only affected to second order in any perturbation.

The most general sum rule is[3]

$$\Sigma^+ - \Sigma^- + \Xi^- - \Xi^0 = p - n = 4.70 \ (4.04\pm0.07) \ , \quad (1)$$

where the baryon symbols represent their magnetic moments and the hyperon moment sum is given in parentheses (here and elsewhere values are in nuclear magnetons). The difficulties with the magnetic moments can be pinpointed by taking appropriate differences of baryon moments to isolate quark-moment contributions. Three combinations indicate particularly large symmetry breaking. These are the sum rule[3]

$$d = \Xi^0 - \Xi^- = p + 2n = -1.03(-0.56\pm0.04) \quad (2)$$

and the quark-moment contributions[4]

$$s = -\Sigma^+ - 2\Sigma^- = -0.18\pm0.10 \quad (3)$$

and

$$s - d = 3(p - \Sigma^+) = 1.23\pm0.06 \ . \quad (4)$$

Equations (2) and (3) depend on the further assumption that $u = -2d$, relating the nucleon-quark-moment contributions. In each case, the quark-moment contributions in Eqs. (2)—(4) are for the unlike quark in the baryon. These quark-moment contributions are to be compared to the contributions $d \simeq -1.0$ and $s \simeq -0.6$ from other baryon combinations.[2]

Relativistic corrections to quark moments could not alone account for the discrepancies in Eqs. (1)—(4) because they could not be expected to affect the heavy-s-quark contribution enough to correct Eqs. (3) and (4). The magnitude of the effect on baryon magnetic moments of configuration mixing (of higher orbital states), suggested by dynamical quark-model calculations with large symmetry breaking, is much too small to resolve Eqs. (1)—(4).[6,7] Meson contributions (or q-$\bar{q}$ sea contributions), to the extent they are SU(3) (flavor) symmetric, would not affect the discrepancy of Eq. (1), but might modify the other relations.

Pion contributions have been suggested as a symmetry-breaking correction to baryon magnetic moments by several authors[8-10] and pion terms have been included in a specific form in the cloudy bag quark model.[11] Pions, because of their anomalously light mass in the meson octet, would be expected to dominate meson exchange currents and thus break the SU(3) (flavor) symmetry of exchange moments. This would break all the moment sum rules. It would further invalidate Eqs. (2) and (3) since the assumption $u = -2d$ would be affected by pion clouds producing anomalous nucleon quark moments.

In this paper, pion magnetic-moment contributions are treated in a semiphenomenological way to see whether their inclusion can reconcile the (otherwise) symmetric quark model with baryon-magnetic-moment measurements. It is found that their inclusion considerably improves the agreement with experiment and, with the addition of a reasonable relativistic effect, the difficulties presented by Eqs. (1)—(4) can be resolved.

II. EXPERIMENT AND THE STATIC QUARK MODEL

Seven baryon moments are now known quite accurately and the (Σ,Λ) transition moment has also been measured. The experimental values[1,2] for these are listed in Table I along with the best two-parameter (d and s) fit in the static quark model (with $u = -2d$).[12] Because the baryon moments are known to such good accuracy an arbitrary theoretical error of 0.05 has been folded with the experimental error on each moment. This tends to weight each of the pure baryon moments equally. It is felt that this a reasonable expectation for theoretical accuracy in the type of analysis presented here. The quark model input parameters are also listed in Table I with errors determined by an increase in χ^2 of χ_{min}^2/DF (degrees of freedom), al-

TABLE I. Fits to baryon magnetic moments (in nuclear magnetons) for three different models with each contribution to χ^2 shown in parentheses. The pion moment is fixed at 6.7 with other parameters shown in the table. The experimental error has been folded with a theoretical error of 0.05 for the χ^2 fit.

Baryon	Experiment	Quark model (QM)	QM+pion	QM+π+$d(\Xi)$	Pion contribution
p	2.793	2.70 (5)	2.69 (4)	2.72 (2)	+0.26
n	−1.913	−1.80 (5)	−1.99 (2)	− 1.98 (2)	−0.34
Λ^0	−0.613±0.004	−0.60 (0)	−0.64 (0)	−0.67 (1)	−0.02
Σ^+	2.38±0.02	2.59 (15)	2.44 (1)	2.44 (1)	+0.05
Σ^-	−1.10±0.05	−1.01 (2)	−1.05 (0)	− 1.01 (2)	−0.12
Ξ^0	−1.250±0.014	−1.36 (4)	−1.32 (2)	−1.22 (0)	0.00
Ξ^-	−0.69±0.04	−0.46 (13)	−0.52 (7)	−0.62 (1)	0.01
(Σ,Λ)	$-1.82^{+0.18}_{-0.25}$	−1.57 (2)	−1.62 (1)	−1.62 (1)	−0.19
χ^2/DF		45/6	19/5	11/4	

Parameters

d		−0.90±0.04	−0.82±0.05	−0.82±0.04
s		−0.57±0.08	−0.54±0.05	−0.62±0.05
$d(\Xi)$				−0.59±0.13
$P(\pi)$			0.17±0.08	0.18±0.07
R(decuplet/octet)			0.25	0.50

though χ^2 is so large that these may not be too meaningful.

It can be seen from Table I that the static quark model cannot match the experimental differences $(\Xi^0-\Xi^-)$, $(p-\Sigma^+)$, and $(-\Sigma^+-2\Sigma^-)$, as was also evident from Eqs. (2)–(4). The Σ^+ and Ξ^- have the largest discrepancies and the correlated differences in Table I are even worse.

III. PION CONTRIBUTIONS

The pion contribution can be included in a semi-phenomenological calculation by parametrizing the pion component of baryon wave functions and the effective orbital magnetic moment of the predominantly p-wave pions. The baryon magnetic moments are then given by the expectation value of the magnetic-moment operator with these wave functions. Imposition of isotopic-spin conservation at the baryon and at the quark level requires only one parameter to describe the pion component of all baryon wave functions. The procedure is illustrated below for the proton.

The "physical" proton wave function $|p\rangle$ is given by

$$|p\rangle = \alpha p + (\beta/3)(p\pi^0 - \sqrt{2}n\pi^+)(\uparrow Y_1^0 - \sqrt{2}\downarrow Y_1^1)$$
$$+ (\delta/6)(\sqrt{3}\Delta^{++}\pi^- - \sqrt{2}\Delta^+\pi^0 + \Delta^0\pi^+)$$
$$\times(\sqrt{3}\mathcal{X}_{3/2}Y_1^{-1} - \sqrt{2}\mathcal{X}_{1/2}Y_1^0 + \mathcal{X}_{-1/2}Y_1^1) , \quad (5)$$

where p, n, Δ represent quark-model states, $\mathcal{X}_m$ is the spin-$\frac{3}{2}$ wave function, and Y_1^m is the pion orbital wave function. The expansion coefficients, β and δ, are determined by matrix elements of the pion-emission operator

$$\theta_\pi = \gamma \sum_{i=1}^{3} \vec{\sigma}^i \cdot \hat{p}_\pi \vec{\tau}^i \cdot \vec{\phi} \quad (6)$$

between quark-model states. In Eq. (6), the sum on i is

over the three quarks, $\hat{p}_\pi$ is the direction of the emitted pion, and $\vec{\phi}$ is the pion field operator. The coefficient γ is not the quark-pion coupling constant, but is meant to include an overlap integral between quark-model states and the averaged energy denominator that would occur in either a perturbation-theory or dispersion-theory calculation of $|p\rangle$. For any quark-model baryon state B, the corresponding physical state is

$$|B\rangle = (\alpha + \gamma\theta_\pi)B . \quad (7)$$

It is sufficient to evaluate matrix elements only for π^0 emission with $\hat{p}_\pi$ in the z direction. This matrix element is, for any baryon pair B and B',

$$(B'\pi^-\uparrow Y_1^0 | B\uparrow) = (B'\pi^0\uparrow Y_1^0, \theta_\pi B\uparrow)$$
$$= \gamma(B'\uparrow, \sum_i \sigma_z^i \tau_3^i B\uparrow) . \quad (8)$$

It can be taken from the corresponding quark-model magnetic-moment formula[13] by simply making the substitutions

$$\mu_u \to \gamma, \quad \mu_d \to -\gamma, \quad \mu_s \to 0 . \quad (9)$$

For the proton this leads to

$$(p\pi^0\uparrow Y_1^0 | p\uparrow) = 5\gamma/3 \quad (10)$$

and

$$(\Delta^+\pi^0\mathcal{X}_{1/2}Y_1^0 | p\uparrow) = 4\sqrt{2}\gamma'/3 . \quad (11)$$

Comparison of Eqs. (10) and (11) with Eq. (5) for $1p\rangle$ leads to

$$\beta = 5\gamma, \quad \delta = 4\sqrt{2}\gamma' . \quad (12)$$

In the simple quark model considered here, the coefficient γ would be the same for all octet baryons, but it is not as reasonable to expect this to extend to the decuplet

baryons. This is because their considerably higher masses than their octet partners would increase the energy denominators and decrease the overlap integrals in any calculation. For this reason a separate coefficient γ' has been introduced in Eqs. (11) and (12), with the expectation that it should be smaller than γ, but the same for all decuplet baryons.

The magnetic-moment operator is

$$\vec{\mu}_{\mathrm{op}} = \sum_{i=1}^{3} \vec{\sigma}^{\,i} \mu_i + \vec{L}_\pi M I_3 \ , \tag{13}$$

where the μ_i are quark moments, M is the effective $L=1$, π^+ orbital moment, and I_3 is the isotopic-spin operator for pions. The baryon magnetic moments, including pion contributions, are the expectation values of $\vec{\mu}_{\mathrm{op}}$ in physical states $|B\rangle$, such as $|p\rangle$ of Eq. (5). The results for the seven measured baryon moments and the (Σ,Λ) transition moment are given in the Appendix. Before being compared to experiment, the Λ moment and (Σ,Λ) transition moment have to be corrected for (Σ,Λ) mixing,[14] which changes μ_Λ by about -0.04 and $\mu(\Sigma,\Lambda)$ by about $+0.01$. The pions affect the baryon magnetic moments in three ways: (1) The pion orbital moment, (2) the "recoil" baryon moments, and (3) the decrease in "bare" (pure quark) baryon probability represented for the proton by $\alpha^2 = 1 - \beta^2 - \delta^2$ in Eq. (5).

It is reasonable to relate the nucleon quarks by $u = -2d$ here, because pion effects which are probably the main cause of anomalous moments are explicitly included. Then the baryon moments depend on three basic magnetic moments (d,s,M). The pion wave function is described by the probability that the physical nucleon contains one pion

$$P = (25\gamma^2 + 32\gamma'^{\,2})/9 \tag{14}$$

and the ratio of decuplet to octet coupling squared

$$R = (\gamma'/\gamma)^2 \ . \tag{15}$$

This makes five parameters (d,s,M,P,R) to describe the eight measured moments.

It turns out, however, that there is really only one effective parameter for the pion contribution. First, the fit is relatively insensitive to the decuplet ratio R in the range $0.1-0.6$. (Note, however, that $R=0$ is excluded.) This is seen in Fig. 1(a) where $\chi^2(R)$ is plotted against R. Second, while χ^2 does show sensitivity to the pion moment M for small M, this sensitivity is not as great for $M \gtrsim 5$. Also, the product $P \times M$ of pion probability in the nucleon and the pion moment is relatively constant at about 1.3 ± 0.1 as M varies above 5. This is shown in Fig. 2(a) where P and $P \times M$ are plotted against M. Thus, adding the pion degree of freedom adds only one real new parameter $P \times M$ to the fit.

The result of this fit is shown in Table I for the choice of parameters $R=0.25$ and $M=6.7$. This value of M is taken from the proton to pion mass ratio. The fit could be improved somewhat by increasing M to unrealistic values as shown in Fig. 1(a), but this is probably not significant. (χ^2 asymptotically approaches 12 in the unphysical limit $M \to \infty$, $P \to 0$.)
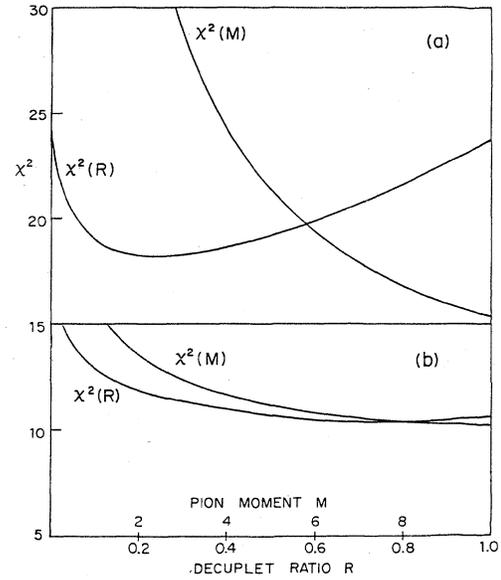

FIG. 1. (a) χ^2 as a function of the pion magnetic moment M (in nuclear magnetons) and the squared ratio R of the decuplet to the octet coupling coefficients ($R = \gamma'^{\,2}/\gamma^2$). $\chi^2(M)$ is plotted for fixed $R=0.25$ and $\chi^2(R)$ for fixed $M=6.7$. (b) $\chi^2(M)$ (with $R=0.50$) and $\chi^2(R)$ (with $M=6.7$) including the additional parameter $d(\Xi)$.

It can be seen from Table I that the fit is improved considerably by the pion contribution. The difficulty with the Σ moments is cleared up, but the contributions to χ^2 from the nucleon and Ξ moments are still large. The small magnitude of the $(\Xi^0 - \Xi^-)$ difference compared to the combination $(p + 2n)$ in Eq. (2) still cannot be ac-
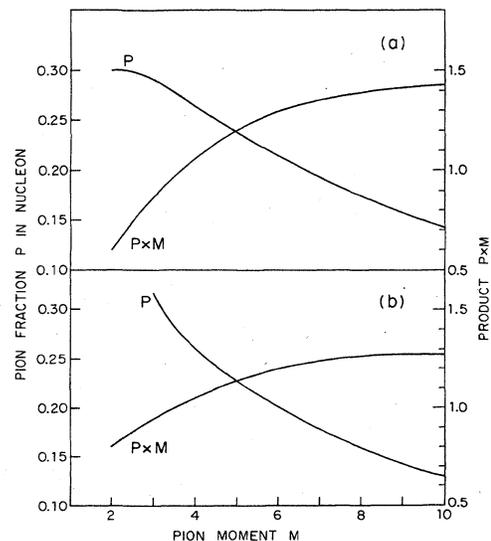

FIG. 2. (a) The pion probability P in the nucleon wave function and the product $P \times M$ as functions of the pion moment M (in nuclear magnetons). (b) P and $P \times M$ including the additional parameter $D(\Xi)$.

counted for. This is because the pion contribution (shown in the last column of Table I) is very small for the Ξ moments.

The results found here for pion effects are quite similar to those found in an explicit calculation in the cloudy bag model (CBM). The CBM magnetic moment predictions lead to $\chi^2 = 23$ for the seven pure baryon moments in Table I but adjustment of the CBM input could undoubtedly lower χ^2. It is difficult to determine the number of effective parameters that are implicit in the CBM prediction because several parameter choices are made and the center-of-mass correction in the CBM is ambiguous.

IV. THE ($\Xi^0 - \Xi^-$) DIFFERENCE

One possibility to understand the small ($\Xi^0 - \Xi^-$) difference might be to include K-meson effects, which have been left out because of the high K mass compared to the pion. However, any K-meson contribution, as given by Ref. 9, would affect ($\Xi^0 - \Xi^-$) in the wrong direction, increasing the magnitude of the difference. Because of this, the magnitude of K-meson effects in fitting the baryon moments would be constrained to be small (as expected because of the large K mass) and they have not been included. The K-meson effect has also been found to be small in bag-model estimates.[8,15]

Relativistic effects have been suggested previously[5] as a likely mechanism to understand the small magnitude of ($\Xi^0 - \Xi^-$) since they would be most effective on the light d and u quarks in the singlet spin state for which the symmetry-breaking spin-spin interaction is the largest. These are just the quarks isolated in Eq. (2). A simple way to parametrize the symmetry breaking of expected relativistic effects is to introduce a new quark moment $d(\Xi)$ for the nucleon quark in the Ξ.[16] The results of this fit are indicated in the column "QM $+\pi + d(\Xi)$" of Table I.[17] The dependence of this fit on the pion parameters M and R (with $P \times M \simeq 1.2$) shown in Fig. 1(b) is very slight (for $M > 5$), so this is effectively a four-parameter fit $[d, d(\Xi), s, P]$ with $M = 6.7$ and $R = 0.5$ chosen as reasonable fixed values.

The fit to baryon moments including the relativistic $d(\Xi)$ as well as pion contributions removes all salient dif-

ficulties of the quark model in describing baryon magnetic moments. It is not unreasonable to expect that the remaining discrepancies could be corrected by minor, but model-dependent, modifications of each baryon wave function. The $\chi^2/DF = 11/4$ for this fit is still not good, but could be interpreted as indicating an increase in the ascribed theoretical error from 0.05 to 0.09 for each baryon.

The more reasonable χ^2 including $d(\Xi)$ and the lack of any particular bad moment, means that the quark-model parameters for this fit may be meaningful. This is further supported by the stability in corresponding parameters between the two different pion fits. The large difference between d and $d(\Xi)$ indicates that relativistic effects are large for the nucleon quarks so that their magnetic moments give no good indication of their masses, except that they are small. There is no indication from the fit that the heavier s quark is relativistic so that its mass may be estimated from its Dirac moment $\mu = q/2m$, resulting in $m_s = 500 \pm 30$ MeV.

V. CONCLUSIONS

It can be seen from the fits in Table I that pion contributions to baryon magnetic moments considerably improve the quark-model predictions of these moments, particularly when a reasonable relativistic effect is included for the nucleon quarks in the Ξ hyperons. The pion effects alone remove the Σ problem of Eqs. (3) and (4) and the addition of the relativistic moment $d(\Xi)$ clears up the Ξ problem of Eq. (2). No salient discrepancy is left between the modified quark model and experimental magnetic moments. The percentage of pion component needed in the physical nucleon is rather large, being about 20% in probability, which is large enough to affect other quark-model predictions.

ACKNOWLEDGMENTS

APPENDIX

The physical baryon magnetic moments μ_B are given by

$$\mu_p = p + 50g(-5p - n + 2M) + 32g'(-9p - 20d + M) - 640(gg')^{1/2}d , \tag{A1}$$

$$\mu_n = n + 50g(-5n - p - 2M) + 32g'(-9n + 5d - M) + 640(gg')^{1/2}d , \tag{A2}$$

$$\mu_{\Sigma^+} = \Sigma^+ + 4g[-37\Sigma^+ - 4\Sigma^0 - 3\Lambda + 14M - 4\sqrt{3}(\Sigma,\Lambda)] + 8g'[-6\Sigma^+ - M + \tfrac{5}{3}(-5d + 2s)]$$
$$- \tfrac{16}{3}(gg')^{1/2}(23d + 16s) , \tag{A3}$$

$$\mu_{\Sigma^-} = \Sigma^- + 4g[-37\Sigma^- - 4\Sigma^0 - 3\Lambda - 14M + 4\sqrt{3}(\Sigma,\Lambda)] + 8g'[-6\Sigma^- + M + \tfrac{5}{3}(d + 2s)]$$
$$+ \tfrac{16}{3}(gg')^{1/2}(13d - 16s) , \tag{A4}$$

$$\mu_{\Xi^0} = \Xi^0 + 2g(-5\Xi^0 - \Xi^- + 2M) + 8g'(-9\Xi^0 + 10s - 2M) + 32(gg')^{1/2}s , \tag{A5}$$

$$\mu_{\Xi^-} = \Xi^- + 2g(-5\Xi^- - \Xi^0 - 2M) + 8g'(-9\Xi^- - 5d + 10s + 2M) + 32(gg')^{1/2}(s + d) , \tag{A6}$$

$$\mu_\Lambda = \Lambda + 12g(-9\Lambda - \Sigma^+ - \Sigma^0 - \Sigma^-) + 24g'(-9\Lambda - 5d + 5s) + 96(gg')^{1/2}(d + 2s) \ , \tag{A7}$$

$$\mu_{\Sigma\Lambda} = [(1 - 108g - 216g')^{1/2}(1 - 132g - 48g')^{1/2} - 12(gg')^{1/2}](\Sigma, \Lambda) + 8\sqrt{3}g(\Sigma^- - \Sigma^+ - 4M)$$
$$+ 16\sqrt{3}g'(5d - M) + 80\sqrt{3}(gg')^{1/2}d \ . \tag{A8}$$

In the above equations, the baryon symbols refer to static-quark-model magnetic moments. The decuplet moments and decuplet transition moments have been replaced by the static-quark-model values in terms of the quark moments d and s (with $u = -2d$). The meson coupling parameters are given by

$$g = \gamma^2/9 \text{ and } g' = \gamma'^2/9 \ . \tag{A9}$$

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[16] Any other mechanism that reduced $d(\Xi)$ leaving other quark moments unchanged, would have the same effect.

[17] The best fit using $d(\Xi)$ without pion contributions leads to $\chi^2/DF = 32/5$ with the Σ^+ contributing 17 to χ^2.