Chiral symmetry and the "penguin" interaction

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It is shown how the "penguin" interaction, proposed by Shifman, Vainshtein, and Zakharov in the theory of nonleptonic weak decays, is consistent with the constraints of chiral symmetry. For the K-π transition this involves including a diagram which was missed in previous work. The formulas for the $K \to 2\pi$ matrix elements are found to be a factor of 2 larger than the original estimates using the vacuum-saturation method. In addition the equivalence of a procedure using a normal-ordered operator and the more standard technique is demonstrated.

I. INTRODUCTION

One of the most interesting, and controversial, developments in the theory of low-energy weak interactions has been the suggestion of the importance of the "penguin diagram," Fig. 1, in nonleptonic $\Delta S=1$ processes, as first introduced by Shifman, Vainshtein, and Zakharov[1] (SVZ). Despite many investigations,[2,3] it is still not clear whether or not it is this diagram which is responsible for the observed $\Delta I=\frac{1}{2}$ rule in kaon and hyperon decays. In addition to disagreements on the strength of this process, there is, throughout the literature, considerable confusion and error in the discussion of the chiral properties of the penguin interaction. For example, there has recently appeared a paper[4] claiming that the vacuum-insertion method of evaluating the penguin operator is inconsistent with PCAC (partial conservation of axial-vector current) and therefore wrong. The present paper will not be concerned with the controversy over the strength of the interaction, although the results bear on that question, but will primarily attempt to elucidate correctly the chiral properties.

In the remainder of this section I will discuss the constraints of chiral symmetry while in Sec. II the vacuum-insertion method will be applied. In particular it will be shown that this technique is completely consistent with PCAC when one includes a new diagram, previously missed, in the $K\to\pi$ transition. In addition it will be seen that the full result is a factor of 2 larger than obtained in the original method.

In the application of the MIT bag model to this problem,[2] the author and his colleagues have taken a different direction in treating the chiral properties. That our work has caused confusion is evidenced by the various comments which we have received. In fact, however, the "anomalous commutator" of our paper is equivalent to the new diagram mentioned above, and our work yields the same answer for $K\to 2\pi$ as would a more conventional approach. I clarify the equivalence of the two methods in Sec. III.

These questions arise because the penguin operator appears superficially to have a different chiral structure than do the other parts of the weak interaction. The fundamental Lagrangian for the weak and electromagnetic interactions is constructed so that only left-handed fermions feel the weak force. The flavor properties of the interaction are therefore those of only the left-handed fermions. The $\Delta S=1$ transition can be $\Delta I=\frac{1}{2}$ or $\frac{3}{2}$ or in SU(3) can belong to the octet or 27-plet. If one allows separate left-handed and right-handed chiral SU(3) transformations, the weak Hamiltonian must transform as $(8_L,1_R)$ or $(27_L,1_R)$. The penguin operator appears at first to be somewhat different from other weak operators in that right-handed fermions are explicitly present. If, for the purposes of this paper, we restrict our attention to the dominant $\Delta I=\frac{1}{2}$, octet pieces in the interaction, then the effective Hamiltonian including QCD corrections[5] is

$$H_w \simeq \frac{G_F}{2\sqrt{2}}\cos\theta_1\sin\theta_1\cos\theta_3(c_1\mathcal{O}_1+c_5\mathcal{O}_5)\ , \tag{1}$$

where

$$\begin{aligned}\mathcal{O}_1&=\bar{d}\gamma_\mu(1+\gamma_5)u\ \bar{u}\gamma^\mu(1+\gamma_5)s-\bar{u}\gamma_\mu(1+\gamma_5)u\ \bar{d}\gamma^\mu(1+\gamma_5)s\ ,\\ \mathcal{O}_5&=\bar{d}\gamma_\mu(1+\gamma_5)t^As[\bar{u}\gamma^\mu(1-\gamma_5)t^Au+\bar{d}\gamma^\mu(1-\gamma_5)t^Ad+\bar{s}\gamma^\mu(1-\gamma_5)t^As]\ ,\end{aligned} \tag{2}$$

with t^A being the (color) SU(3) matrices, with $\mathrm{Tr}\,t^At^B=2\delta^{AB}$. The operator $\mathcal{O}_1$ is representative of the usual "non-penguin" pieces of the weak interaction and explicitly contains only left-handed quarks. $\mathcal{O}_5$ is the dominant operator due to the penguin diagram. The right-handed quarks enter in a flavor-singlet combination and therefore, despite naive appearances, this does have the same chiral properties as $\mathcal{O}_1$, i.e., $(8_L,1_R)$. (I will as-

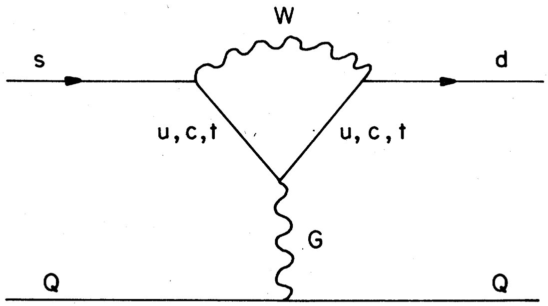

FIG. 1. The penguin diagram in $\Delta S=1$ nonleptonic decays.

sume that the normal-ordering prescription of Ref. 2 is *not* applied, and return to this topic later.) We therefore expect the *same* behavior under chiral symmetry for both $\mathscr{O}_1$ and $\mathscr{O}_5$. It has not been apparent in the literature how this comes about.

There are two constraints on kaon decay amplitudes which we deal with. The first follows from SU(3) alone and states that the $K\rightarrow 2\pi$ amplitude must vanish in the SU(3) limit.[6] The second is a consequence of chiral SU(3) [or vector SU(3) plus chiral SU(2)] which states that kaon amplitudes are all related in a special way and must be at least first order in the products of four-momenta (i.e., in terms like $k\cdot q$). It is well known that, even using chiral SU(2), the soft-pion theorems for kaon decays are inconsistent if the amplitude is independent of momentum. Expanding the amplitudes to first order, requiring consistency in the soft-pion limits, and applying the SU(3) theorem mentioned above, leads to a unique form of the amplitudes.[2] Equivalently, early work by Cronin[7] using effective Lagrangians shows that this form emerges given only the $(8_L, 1_R)$ transformation property. The appropriate forms are

$$\begin{aligned}
&\langle 0 \,|\, H_w \,|\, K^0(k^2=0)\rangle = 0 \,, \\
&\langle \pi^-(k) \,|\, H_w \,|\, K^-(k)\rangle = -2F_\pi g k^2 \,, \\
&\langle \pi^+(q_+)\pi^-(q_-) \,|\, H_w \,|\, K_S^0(k)\rangle = ig(2k^2-q_+{}^2-q_-{}^2) \,, \\
&\langle \pi^+(q_1)\pi^-(q_-)\pi^+(q_2) \,|\, H_w \,|\, K^+(k)\rangle \\
&\qquad = \frac{2g}{3F_\pi}[k^2+\tfrac{3}{2}(s_3-s_0)] \,,
\end{aligned} \tag{3}$$

where

$s_3 \equiv (k-q_-)^2, \;\; s_0 = m_K{}^2/3 - m_\pi{}^2$

and with all other amplitudes related to these by the $\Delta I=\frac{1}{2}$ rule. It is the structure of these amplitudes and the relationship between them which one must reproduce if one is to be consistent with PCAC. Equation (3) will therefore be our criterion for PCAC consistency.

II. VACUUM SATURATION

In this method, one evaluates matrix elements involving four quark fields by breaking the amplitudes into the product of two-quark matrix elements through insertion of the vacuum state in all possible ways between the quark bilinears. To discuss this technique one needs the various two-quark matrix elements. I will assume SU(3) invariance in defining the various constants. For vector and axial-vector currents, we have the usual definitions:

$$\begin{aligned}
&\langle \pi^+(q) \,|\, \bar{u}\gamma_\mu s \,|\, \bar{K}^0(k)\rangle = (k+q)_\mu \,, \\
&\langle \pi^-(q) \,|\, \bar{d}\gamma_\mu\gamma_5 u \,|\, 0\rangle = i\sqrt{2}F_\pi q_\mu \,, \\
&\langle \pi^+(q_+)\pi^0(q_0) \,|\, \bar{\mu}\gamma_\mu d \,|\, 0\rangle = \frac{1}{\sqrt{2}}(q_- - q_0)_\mu \,, \\
&\langle 0 \,|\, \bar{d}\gamma_\mu\gamma_5 s \,|\, \bar{K}^0(k)\rangle = i\sqrt{2}F_\pi k_\mu \,.
\end{aligned} \tag{4}$$

We do not have as direct a measure of the constants involved in matrix elements of scalar and pseudoscalar densities. Here if we assume that there is no momentum dependence in the amplitudes we obtain

$$\begin{aligned}
&\langle \pi^+ \,|\, \bar{u}s \,|\, \bar{K}^0\rangle = A \,, \\
&\langle \pi^- \,|\, \bar{d}\gamma_5 u \,|\, 0\rangle = i\sqrt{2}F_\pi A \,, \\
&\langle \pi^+\pi^0 \,|\, \bar{u}d \,|\, 0\rangle = -\frac{1}{\sqrt{2}}A \,, \\
&\langle 0 \,|\, \bar{d}\gamma_5 s \,|\, \bar{K}^0\rangle = -i\sqrt{2}F_\pi A \,, \\
&\langle 0 \,|\, \bar{d}d \,|\, 0\rangle = \langle 0 \,|\, \bar{s}s \,|\, 0\rangle = -2F_\pi{}^2 A \,.
\end{aligned} \tag{5}$$

All vertices can be written in terms of single amplitude A by using the soft-pion theorems of PCAC and current algebra. If one treats the chiral-symmetry breaking due to quark masses

$$\mathscr{H}_{\text{mass}} = m_u\bar{u}u + m_d\bar{d}d + m_s\bar{s}s \tag{6}$$

to first order in the masses, then A gives the relationship between quark masses and meson masses

$$\begin{aligned}
&m_\pi{}^2 = \langle \pi^+ \,|\, \mathscr{H}_{\text{mass}} \,|\, \pi^+\rangle \,, \\
&m_K{}^2 = \langle K \,|\, \mathscr{H}_{\text{mass}} \,|\, K\rangle \,,
\end{aligned}$$

and therefore

$$A = \frac{m_{K^+}{}^2 - m_{\pi^+}{}^2}{m_s - m_d} = \frac{m_\pi{}^2}{m_u + m_d} = \frac{m_{K^+}{}^2}{m_s + m_u} \tag{7}$$

with the quark masses clearly being the current-algebra masses. In what follows we will see that one is forced to consider the momentum dependence of these amplitudes. For later convenience, the parametrization becomes

$$\begin{aligned}
&\langle \pi^+(q) \,|\, \bar{u}s \,|\, \bar{K}^0(k)\rangle = A\left[1 \pm \frac{(k-q)^2}{\Lambda^2}\right] \,, \\
&\langle \pi^-(q) \,|\, \bar{d}\gamma_5 u \,|\, 0\rangle = i\sqrt{2}F_\pi A\left[1 \pm \frac{q^2}{\Lambda^2}\right] \,, \\
&\langle \pi^+(q_+)\pi^0(q_0) \,|\, \bar{u}d \,|\, 0\rangle = -\frac{1}{\sqrt{2}}A\left[1 \pm \frac{(q_+ + q_0)^2}{\Lambda^2}\right] \,, \\
&\langle 0 \,|\, \bar{d}\gamma_5 s \,|\, \bar{K}^0(k)\rangle = -i\sqrt{2}F_\pi A\left[1 \pm \frac{k^2}{\Lambda^2}\right] \,, \\
&\langle 0 \,|\, \bar{d}d \,|\, 0\rangle = \langle 0 \,|\, \bar{s}s \,|\, 0\rangle = -2F_\pi{}^2 A \,.
\end{aligned} \tag{8}$$

The treatment of the vertices as form factors would suggest the choice of the minus sign in the momentum-dependent terms.

In the $K\rightarrow 2\pi$ transition, the vacuum-saturation result for $\mathscr{O}_1$ is well known and has the appropriate form

$$\langle \pi^+(q_+)\pi^-(q_-)\,|\,\mathcal{O}_1\,|\,\bar{K}^0(k)\rangle = \tfrac{2}{3}[\,\langle \pi^-(q_-)\,|\,\bar{d}\gamma_\mu\gamma_5 u\,|\,0\rangle\langle \pi^+(q_+)\,|\,\bar{u}\gamma_\mu s\,|\,\bar{K}^0(k)\rangle$$
$$-\langle \pi^+(q_+)\pi^-(q_-)\,|\,\bar{u}\gamma_\mu u\,|\,0\rangle\langle 0\,|\,\bar{d}\gamma_\mu\gamma_5 s\,|\,\bar{K}^0(k)\rangle]$$
$$=\frac{2\sqrt{2}}{3}iF_\pi[(k^2-q_+{}^2)+(q_+{}^2-q_-{}^2)]\,, \tag{9}$$

which leads to

$$\langle \pi^+(q_+)\pi^-(q_-)\,|\,\mathcal{O}_1\,|\,K_S^0(k)\rangle = \frac{i2F_\pi}{3}(2k^2-q_+{}^2-q_-{}^2)\,. \tag{10}$$

The overall factor of $\frac{2}{3}$ above comes from color factors associated with the two orderings in $\mathcal{O}_1$, i.e., from

$$\langle \pi^-\,|\,\bar{d}_i\gamma_\mu\gamma_5 u_j\,|\,0\rangle = i\sqrt{2}F_\pi\frac{\delta_{ij}}{3}p_\mu\,, \tag{11}$$

where i and j are color labels. Use has been made of the Fierz-rearrangement property

$$\bar{\psi}_{1i}\gamma_\mu(1+\gamma_5)\psi_{2i}\bar{\psi}_{3j}\gamma^\mu(1+\gamma_5)\psi_{4j} = \bar{\psi}_{1i}\gamma^\mu(1+\gamma_5)\psi_{4j}\bar{\psi}_{3j}\gamma^\mu(1+\gamma_5)\psi_{2i}\,. \tag{12}$$

With the penguin transition one also has matrix elements of S,P densities, with the Fierz relation

$$\bar{\psi}_{1_i}\gamma_\mu(1+\gamma_5)\psi_{2_j}\bar{\psi}_{3_k}\gamma^\mu(1-\gamma_5)\psi_{4_l} = -2\bar{\psi}_{1_i}(1-\gamma_5)\psi_{4_l}\bar{\psi}_{3_k}(1+\gamma_5)\psi_{2_j}\,. \tag{13}$$

The result is

$$\langle \pi^+(q_+)\pi^-(q_-)\,|\,\mathcal{O}_5\,|\,\bar{K}^0(k)\rangle = [\,\langle \pi^+(q_+)\pi^-(q_-)\,|\,\bar{u}_i\gamma_\mu u_j+\bar{d}_i\gamma_\mu d_j\,|\,0\rangle\langle 0\,|\,d_k\gamma_\mu\gamma_5 s_l\,|\,\bar{K}^0(k)\rangle$$
$$+2\langle \pi^-(q_-)\,|\,\bar{d}_i\gamma_5 u_l\,|\,0\rangle\langle \pi^+(q_+)\,|\,u_k s_j\,|\,\bar{K}^0(k)\rangle$$
$$-2\langle \pi^+(q_+)\pi^-(q_-)\,|\,\bar{d}_i d_l\,|\,0\rangle\langle 0\,|\,\bar{d}_k\gamma_5 s_j\,|\,\bar{K}^0(k)\rangle]t_{ij}^A t_{kl}^A \tag{14}$$

$$=\tfrac{32}{9}[\,\langle \pi^-(q_-)\,|\,\bar{d}\gamma_5 u\,|\,0\rangle\langle \pi^+(q_+)\,|\,\bar{u}s\,|\,\bar{K}^0(k)\rangle$$
$$-\langle \pi^+(q_+)\pi^-(q_-)\,|\,\bar{d}d\,|\,0\rangle\langle 0\,|\,\bar{d}\gamma_5 s\,|\,\bar{K}^0(k)\rangle]\,. \tag{15}$$

If this is evaluated with no momentum dependence the amplitude will vanish, as was noted in the original work of SVZ. Allowing momentum dependence

$$\langle \pi^+\pi^-\,|\,\mathcal{O}_5\,|\,\bar{K}^0\rangle = \tfrac{32}{9}i\sqrt{2}F_\pi\left[A^2\left[1\pm\frac{q_-{}^2}{\Lambda^2}\right]\left[1\pm\frac{(k-q)^2}{\Lambda^2}\right]-A^2\left[1\pm\frac{(q_++q_-)^2}{\Lambda^2}\right]\left[1\pm\frac{k^2}{\Lambda^2}\right]\right]$$

$$=\mp\tfrac{32}{9}\sqrt{2}iF_\pi A^2 2(k^2-q_-{}^2) \tag{16}$$

or

$$\langle \pi^+\pi^-\,|\,\mathcal{O}_5\,|\,K_s\rangle = \mp i\frac{64}{9}\frac{F_\pi A^2}{\Lambda^2}(2k^2-q_-{}^2-q_+{}^2)+\mathcal{O}\left[\frac{1}{\Lambda^4}\right]\,. \tag{17}$$

This discussion is similar in spirit to the original work of SVZ; however the final result is a factor of 2 larger than would be obtained in their method. This difference is due to the fact that SVZ used momentum dependence in scalar amplitude

$$\langle \pi\,|\,\bar{u}s\,|\,\bar{K}^0\rangle$$

but not in pseudoscalar ones

$$\langle \pi\,|\,\bar{d}\gamma_5 u\,|\,0\rangle\,.$$

The procedure used in the present paper is the correct one if soft-pion theorems are to be satisfied. In $K\to 2\pi$, the penguin operator has the form required by PCAC.

Most of the confusion in the literature occurs in the treatment of the $K\to\pi$ transition, and it appears that everyone has missed an important diagram in the evaluation of the penguin term. For the nonpenguin operator we have, as usual,

$$\langle \pi^-(k)\,|\,\mathcal{O}_1\,|\,K^-(k)\rangle = \tfrac{2}{3}\langle \pi^-\,|\,\bar{d}\gamma_\mu\gamma_5 u\,|\,0\rangle\langle 0\,|\,\bar{u}\gamma_\mu\gamma_5 s\,|\,K^-\rangle$$
$$=-\tfrac{4}{3}F_\pi{}^2k^2\,. \tag{18}$$

In the case of $\mathscr{O}_5$ there are two contributions, given pictorially in Fig. 2:

$$\begin{aligned}\langle \pi^-(k) \,|\, \mathscr{O}_5 \,|\, K^-(k)\rangle &= -2t_{ij}^A t_{kl}^A[-\langle \pi^- \,|\, \bar{d}_i\gamma_5 u_l \,|\, 0\rangle\langle 0 \,|\, \bar{u}_k\gamma_5 s_j \,|\, K^-\rangle \\ &\quad + \langle \pi^- \,|\, \bar{d}_i s_l \,|\, K^-\rangle(\langle 0 \,|\, \bar{d}_k d_j \,|\, 0\rangle + \langle 0 \,|\, \bar{s}_k s_j \,|\, 0\rangle)] \\ &= \tfrac{32}{9}(\langle \pi^- \,|\, \bar{d}_i\gamma_5 u \,|\, 0\rangle\langle 0 \,|\, \bar{u}\gamma_5 s \,|\, K^-\rangle - 2\langle \pi^- \,|\, \bar{d}s \,|\, K^-\rangle\langle 0 \,|\, \bar{d}d \,|\, 0\rangle)\,. \end{aligned} \tag{19}$$

In most past investigations only Fig. 2(a) has been included. Again, if one uses vertices independent of momentum, the amplitude vanishes. Including momentum dependence, one has

$$\begin{aligned}\langle \pi^-(k) \,|\, \mathscr{O}_5 \,|\, K^-(k)\rangle &= \tfrac{32}{9}\times 2F_\pi{}^2A^2\left[\left[1\pm\frac{k^2}{\Lambda^2}\right]^2-1\right] \\ &= \pm\frac{128}{9}\frac{F_\pi{}^2A^2}{\Lambda^2}k^2 + O\left[\frac{k^4}{\Lambda^4}\right]. \end{aligned} \tag{20}$$

Again the consistency with PCAC is evident if one compares Eqs. (3), (17), and (20). The new diagram, Fig. 2(b), has played a crucial role in obtaining this consistency as it cancels out the constant term which would have disagreed with Eq. (3). Note that in $K\to 2\pi$, any diagram similar to Fig. 2(b) which contains $\langle 0 \,|\, \bar{q}q \,|\, 0\rangle$ vanishes for the on-shell amplitude. Such diagrams only contribute to off-shell processes, such as $K\to\pi$.

It is from the outset clear that the vacuum-saturation technique applied to $\mathscr{O}_5$ *should* be consistent with PCAC, because the operator has a clear chiral property and there is nothing intrinsic to the vacuum-saturation method which violates PCAC. The above calculation has, I hope, made clear in detail how such consistency arises.

Even though it is somewhat outside the main development of this paper, it is appropriate to comment on the magnitude of the penguin contribution. Combining all terms we have

$$\begin{aligned}\langle \pi^+\pi^- \,|\, H_w \,|\, K_s\rangle &= \frac{G_F}{2\sqrt{2}}\cos\theta_1\cos\theta_3\sin\theta_1\frac{2F_\pi}{3} \\ &\quad \times(2k^2-q_+{}^2-q_-{}^2)\left[C_1\mp\frac{32}{3}\frac{A^2}{\Lambda^2}C_5\right] \\ &= 5.3\times 10^{-8}m_K\left[C_1\mp\frac{32}{3}\frac{A^2}{\Lambda^2}C_5\right]. \end{aligned} \tag{21}$$

Experimentally the desired answer is

$$\langle \pi^+\pi^- \,|\, H_w \,|\, K_s\rangle = \pm 7.78\times 10^{-7}m_K \tag{22}$$

while QCD short-distance analysis[1,6] yields the coefficients

$$\begin{aligned} &C_1 = 2.4\,, \\ &C_5 \approx 0.02\text{—}0.1\,. \end{aligned} \tag{23}$$

The coefficient C_5 has a rather strong dependence on the renormalization point, and there are legitimate questions as to whether or not the penguin diagram is short-distance dominated.

To evaluate the amplitude A one must evaluate the quark masses appropriate for the scale at which one chooses the renormalization point. Here we will repeat Weinberg's method,[8] including observations made in Ref. 2. Weinberg ascribes $\Delta S=1$ mass splittings to the quark masses treated to first order (here neglecting isospin violation)

$$\begin{aligned} m_\Lambda - m_N &= m_s\langle \Lambda \,|\, \bar{s}s \,|\, \Lambda\rangle - m_d\langle N \,|\, \bar{d}d \,|\, N\rangle \\ &\approx (m_s-m_d)Z_m \\ &\approx m_s Z_m \\ &\approx 150\text{ MeV} \end{aligned} \tag{24}$$

with

$$Z_m = \langle \Lambda \,|\, \bar{s}s \,|\, \Lambda\rangle\,.$$

At this stage, most authors absorb Z_m into the mass, i.e., $m_s^* = Z_m m_s = 150$ MeV. However, we are after m_s not m_s^* and therefore this is not appropriate here. At low energies it is certainly not correct to set $Z_m=1$. Z_m may be calculated at the hadronic scale using quark models. In the bag model[9]

$$Z_m = \frac{\int d^3x(u^2-l^2)}{\int d^3x(u^2+l^2)} = 0.48 \tag{25}$$

with $u\,(l)$ being the upper (lower) component of the quark wave function. In potential models

$$Z_m = \frac{\langle \bar{u}(p)u(p)\rangle}{\langle u^+(p)u(p)\rangle} = 1-\left\langle \frac{\vec{p}^{\,2}}{E(E+m)}\right\rangle\,. \tag{26}$$

If I use the model of Isgur and Karl[10] with $\alpha^2=0.17$ GeV^2, I estimate

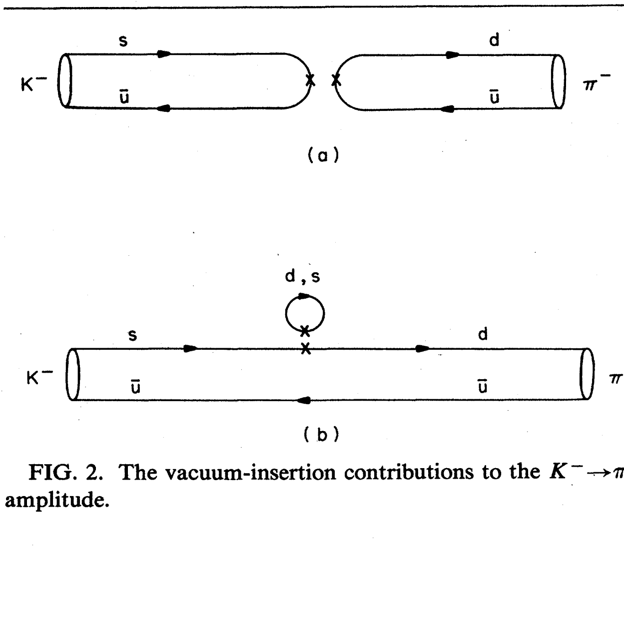

FIG. 2. The vacuum-insertion contributions to the $K^-\to\pi^-$ amplitude.

$$\left\langle \frac{\vec{p}^{\,2}}{E(E+m)} \right\rangle \approx 0.6 ,$$
$$Z_m \approx 0.4 . \tag{27}$$

These estimates indicate that at the hadronic scale

$$m_s \approx 150 \text{ MeV}/Z_m \approx 300\text{—}400 \text{ MeV}$$

and

$$A = \frac{m_K{}^2}{m_s + m_d} \approx \frac{0.25 \text{ GeV}^2}{0.15 \text{ GeV}} Z_m \approx 670\text{—}800 \text{ MeV} . \tag{28}$$

A direct calculation of A in the bag model yields a somewhat smaller result:[9]

$$A = 430 \text{ MeV} . \tag{29}$$

Let us here take the most optimistic value $A = 800$ MeV.

The value of Λ, which governs the momentum dependence of the chiral amplitudes, is also needed. SVZ[1] choose $\Lambda \approx m_\sigma \approx 700$ MeV. Manohar and Georgi[11] would argue that this chiral scale is $4\pi F_\pi \approx 1$ GeV, a result which is consistent with a recent analysis of this scale in $K \to 3\pi$ data.[12] The range $700 \to 1000$ MeV is probably a reasonable range.

Combining all these threads, one has the optimistic estimate

$$\langle \pi^+\pi^- \mid H_w \mid K^0 \rangle \approx 5.3\times 10^{-8} m_K (C_1 \mp 10 C_5)$$
$$\approx 5.3 \times 10^{-8} m_K (2.4 \pm 1) , \tag{30}$$

where the factor in parentheses should be 15 to match the experimental value. Despite a large color factor, the perturbative QCD coefficient of the penguin operator appears too small, in this evaluation, to explain the $\Delta I = \frac{1}{2}$ rule.

III. NORMAL ORDERING

In Ref. 2, Donoghue, Golowich, Holstein, and Ponce used an operator for the penguin interaction which was normal ordered with respect to the true vacuum:

$$\tilde{\mathcal{O}}_5 = \mathcal{O}_5 + \tfrac{32}{9} \langle 0 \mid \bar{d}d \mid 0 \rangle \bar{d}(1+\gamma_5)s$$
$$+ \tfrac{32}{9} \langle 0 \mid \bar{s}s \mid 0 \rangle \bar{d}(1-\gamma_5)s . \tag{31}$$

This is defined so that the off-diagonal mass term which would arise from the $s \to d$ matrix element is absent. The new term changes the chiral properties but does not affect the physical decay rates. The "anomalous" piece which occurs in the commutator when taking a soft-pion limit has the same effect as Fig. 2(b), which is absent for the normal-ordered operator. The purpose of this section is to clarify this situation.

The fact that the new term should not change any of the physical decay amplitudes is known from work which dates back to a 1959 paper by Feinberg, Kabir, and Weinberg.[13] The new term is like a piece of the s-d mass matrix, and is added to make the matrix diagonal. However, such mass terms cannot lead to physical decay amplitudes. It has been shown that one obtains the same answer whether or not one first diagonalizes the mass matrix, as long as one deals with one-mass-shell states.[13] Therefore the new piece can, and does, modify $K \to \pi$ but not $K \to 2\pi$.

The current algebra commutator for the standard penguin operator $\mathcal{O}_5$, which has not been normal ordered, satisfies

$$[F_3^5, \mathcal{O}_5] = [F_3, \mathcal{O}_5] . \tag{32}$$

This is a statement of the left-handed chiral property. However, upon normal ordering we have

$$[F_3^5, \tilde{\mathcal{O}}_5] = [F_3, \tilde{\mathcal{O}}_5] - \tfrac{16}{9} \langle 0 \mid \bar{d}d \mid 0 \rangle \bar{d}(1+\gamma_5)s . \tag{33}$$

The other difference between the two operators is that for $\mathcal{O}_5$ one must evaluate both Figs. 2(a) and 2(b), while for $\tilde{\mathcal{O}}_5$ Fig. 2(b) is absent because of the normal-ordering prescription. Therefore, one may calculate a physical amplitude such as $K \to \pi^0\pi^0$ by use of PCAC as follows:

$$\lim_{q_1 \to 0} \langle \pi^0(q_1)\pi^0(q_2) \mid \mathcal{O}_5 \mid K^0(k) \rangle$$
$$= \frac{-i}{F_\pi} \langle \pi^0(k) \mid [F_3^5, \mathcal{O}_5] \mid K^0(k) \rangle$$
$$= \frac{-i}{2F_\pi} \langle \pi^0(k) \mid \mathcal{O}_5 \mid K^0(k) \rangle$$
$$= \frac{-i}{2F_\pi} [\text{Fig.2(a)} + \text{Fig.2(b)}] . \tag{34}$$

However, use of the normal-ordered operator yields

$$\lim_{q_1 \to 0} \langle \pi^0(q_1)\pi^0(q_2) \mid \tilde{\mathcal{O}}_5 \mid K^0(k) \rangle = \frac{-i}{F_\pi} [\langle \pi^0(k) \mid [F_3, \tilde{\mathcal{O}}_5] \mid K^0(k) \rangle - \tfrac{16}{9} \langle 0 \mid \bar{d}d \mid 0 \rangle \langle \pi^0(k) \mid \bar{d}s \mid K^0(k) \rangle]$$
$$= \frac{-i}{2F_\pi} [\langle \pi^0(k) \mid \tilde{\mathcal{O}}_5 \mid K^0(k) \rangle - \tfrac{32}{9} \langle 0 \mid \bar{d}d \mid 0 \rangle \langle \pi^0(k) \mid \bar{d}s \mid K^0(k) \rangle]$$
$$= \frac{-i}{2F_\pi} [\text{Fig.2(a)} - \tfrac{32}{9} \langle 0 \mid \bar{d}d \mid 0 \rangle \langle \pi^0(k) \mid \bar{d}s \mid K^0(k) \rangle] . \tag{35}$$

When explicitly evaluated the same $K \to 2\pi$ amplitude is obtained in each case. The result would be that quoted in Sec. II [Eq. (21)] if the vacuum-saturation method is used. The bag model provides an alternative technique of evaluation, but yields a result larger than the vacuum-saturation method.

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