On the determination of isospin-0 nucleon-nucleon elastic-scattering amplitudes

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At angles and energies where the isospin-1 (pp) nucleon-nucleon elastic-scattering amplitudes are known, a considerable amount of information on the isospin-0 nucleon-nucleon amplitudes can be obtained from np spin-parameter measurements with polarized neutron beam and polarized proton targets. Data at $\theta_{\text{c.m.}}$ and $\pi-\theta_{\text{c.m.}}$ can be combined to give pure isospin-0 spin parameters and quantities which are expressible as products of isospin-0 and isospin-1 amplitudes. This information is summarized in preparation for the analysis of such data from LAMPF and for the planning of similar experiments at other accelerators. The special case of the isospin-0 amplitudes at $\theta_{\text{c.m.}}=90°$ is also considered, and a number of consistency checks for such data are derived.

INTRODUCTION

With the assumption of parity and time-reversal invariance, there are five complex $I=1$ (isospin-1 or pp) elastic-scattering amplitudes. An overall phase is not directly measurable except near 0°, where it can be obtained relative to the Coulomb phase. There have been several analyses and suggested schemes to determine these amplitudes in a model-independent fashion for beam momenta up to 6 GeV/c (Refs. 1—7). These schemes involve measurement at each angle and energy of at least nine different pp elastic spin parameters, some of which include the determination of the spin of an outgoing proton. The good event rates with a carbon polarimeter are usually small because of the second scattering. Furthermore, at proton momenta above about 2—3 GeV/c, the analyzing power of a carbon polarimeter becomes small at small scattering angles in the carbon (where the cross section is large), making such measurements prohibitive. Thus, in the foreseeable future, model-independent amplitude determinations at high energies will be restricted to small four-momentum transfer squared ($|t|\lesssim 1.0$ GeV$^2/c^2$), where the spin of the low-momentum recoil protons can be measured to good precision.

In the np system, similar constraints apply. With the additional assumption of isospin invariance, there are also five amplitudes for the $I=0$ system. Polarized proton targets and carbon polarimeters to measure the spin of the outgoing protons can be used for np experiments as well as for pp measurements. Polarized neutron beams can be produced in a number of ways, for example, from stripping of polarized deuterons that have been accelerated from an ion source, or from production by unpolarized protons on a target at a nonzero angle, or from the $^2\text{H}(p_{\text{pol}},n_{\text{pol}})$ reaction at 0° using a polarized proton beam.

In both the $I=0$ and $I=1$ systems, the amplitudes at $\pi-\theta_{\text{c.m.}}$ are determined by the amplitudes at $\theta=\theta_{\text{c.m.}}$. On the other hand, certain experimental setups allow the determination of more than one spin parameter by performing measurements at both θ and $\pi-\theta$. For example, this is true with a polarized beam of either protons or neutrons, an unpolarized or polarized proton target, and a carbon polarimeter. Measurements of the outgoing protons' spin made at laboratory angles corresponding to θ and $\pi-\theta$ give independent results in general (see Refs. 1 and 8). With no measurement of the outgoing particles' spin, and with either polarized or unpolarized beam and target, only one spin parameter can be obtained for pp scattering (because the beam and target particles are identical), whereas two can be obtained for np scattering. Therefore, np scattering results at one energy on the differential cross section $d\sigma/d\Omega$, the polarization parameter P, and the four nonzero spin-spin parameters with polarized beam and target, C_{NN}, C_{SS}, C_{LL}, and $C_{LS}=C_{SL}$, will give 12 different spin data at angles for which measurements at both θ and $\pi-\theta$ exist.

The purpose of this paper is to describe the $I=0$ amplitude determination under the assumptions that the $I=1$ amplitudes are known and that the six np spin parameters mentioned above are measured at both θ and $\pi-\theta$. The next section deals with the case of a general angle θ, and the following section with the special case of $\theta=\pi/2$.

AMPLITUDES AT A GENERAL ANGLE

For the purposes of this paper, it is convenient to use the amplitudes

$$\begin{aligned}
\phi_s&=(\phi_1-\phi_2)/2\ ,\\
\phi_t&=(\phi_1+\phi_2)/2\ ,\\
\phi_T&=(\phi_3-\phi_4)/2\ ,\\
\phi_\tau&=(\phi_3+\phi_4)/2\ ,\\
\phi_5&=\phi_5\ ,
\end{aligned}\tag{1}$$

which are defined in terms of the helicity amplitudes of Goldberger, Grisaru, MacDowell, and Wong,[9,10]

$$\begin{aligned}\phi_1&=\langle ++\,|\,++\rangle\,,\\ \phi_2&=\langle ++\,|\,--\rangle\,,\\ \phi_3&=\langle +-\,|\,+-\rangle\,,\\ \phi_4&=\langle +-\,|\,-+\rangle\,,\\ \phi_5&=\langle ++\,|\,+-\rangle\,.\end{aligned} \tag{2}$$

The amplitude ϕ_s contains only spin-singlet contributions, ϕ_T and ϕ_τ contain only spin-triplet partial waves, and ϕ_t and ϕ_5 contain only coupled spin-triplet terms (see Ref. 11). The relations between the amplitudes at θ and $\pi-\theta$ are particularly simple,[10]

$$\begin{aligned}\phi_{s,I}(\pi-\theta)&=(-1)^{I+1}\phi_{s,I}(\theta)\,,\\ \phi_{t,I}(\pi-\theta)&=(-1)^{I+1}\phi_{t,I}(\theta)\,,\\ \phi_{T,I}(\pi-\theta)&=(-1)^{I+1}\phi_{T,I}(\theta)\,,\\ \phi_{\tau,I}(\pi-\theta)&=(-1)^{I}\phi_{\tau,I}(\theta)\,,\\ \phi_{5,I}(\pi-\theta)&=(-1)^{I}\phi_{5,I}(\theta)\,.\end{aligned} \tag{3}$$

The np amplitudes are given by

$$\phi_{K,np}=(\phi_{K,0}+\phi_{K,1})/2\,, \tag{4}$$

where $K=s$, t, T, τ, and 1–5.

In the notation[12] $(B,T;S,R)$ where B is the spin direction of the beam, T of the target, S of the scattered, and R of the recoil particles, then the six nonzero spin parameters which do not involve measurement of the final particles' spin are[13–15]

$$\begin{aligned}d\sigma/d\Omega&=(0,0;0,0)=|\phi_s|^2+|\phi_t|^2+|\phi_T|^2+|\phi_\tau|^2+2|\phi_5|^2\,,\\ P\,d\sigma/d\Omega&=(N,0;0,0)=(0,N;0,0)=2\,\mathrm{Im}(\phi_t^*\phi_5-\phi_T\phi_5^*)\,,\\ C_{LS}d\sigma/d\Omega&=(L,S;0,0)=(S,L;0,0)=C_{SL}d\sigma/d\Omega=2\,\mathrm{Re}(\phi_t^*\phi_5-\phi_T\phi_5^*)\,,\\ C_{NN}d\sigma/d\Omega&=(N,N;0,0)=-|\phi_s|^2+|\phi_t|^2+|\phi_T|^2-|\phi_\tau|^2+2|\phi_5|^2\,,\\ C_{SS}d\sigma/d\Omega&=(S,S;0,0)=-|\phi_s|^2+|\phi_t|^2-|\phi_T|^2+|\phi_\tau|^2\,,\\ C_{LL}d\sigma/d\Omega&=(L,L;0,0)=-|\phi_s|^2-|\phi_t|^2+|\phi_T|^2+|\phi_\tau|^2\,.\end{aligned} \tag{5}$$

The spin directions are $\vec{N}$=normal to the scattering plane, $\vec{L}$=longitudinal, and $\vec{S}=\vec{N}\times\vec{L}$; 0 denotes unpolarized or a spin that is not measured.

From Eq. (5), it can be seen that the quantities $d\sigma/d\Omega$, $C_{NN}d\sigma/d\Omega$, $C_{SS}d\sigma/d\Omega$, and $C_{LL}d\sigma/d\Omega$ are all sums of the magnitudes of the amplitudes squared. The $I=1$ spin parameters come from the pp observables, such as

$$(d\sigma/d\Omega)_{I=1}=|\phi_{s,1}|^2+|\phi_{t,1}|^2+|\phi_{T,1}|^2+|\phi_{\tau,1}|^2+2|\phi_{5,1}|^2=(d\sigma/d\Omega)_{pp}\,. \tag{6}$$

Similar relations hold for all $I=1$ spin parameters. Using Eqs. (3) and (4), the $I=0$ spin parameters can be derived from np and pp measurements:

$$\begin{aligned}(d\sigma/d\Omega)_{I=0}(\theta)&=|\phi_{s,0}|^2+|\phi_{t,0}|^2+|\phi_{T,0}|^2+|\phi_{\tau,0}|^2+2|\phi_{5,0}|^2\\ &=2(d\sigma/d\Omega)_{np}(\theta)+2(d\sigma/d\Omega)_{np}(\pi-\theta)-(d\sigma/d\Omega)_{pp}(\theta)\,,\\ (C_{\alpha\alpha}d\sigma/d\Omega)_{I=0}(\theta)&=2(C_{\alpha\alpha}d\sigma/d\Omega)_{np}(\theta)+2(C_{\alpha\alpha}d\sigma/d\Omega)_{np}(\pi-\theta)-(C_{\alpha\alpha}d\sigma/d\Omega)_{pp}(\theta)\,,\end{aligned} \tag{7}$$

where $\alpha=N$, S, or L. Combining the results from Eqs. (5), (6), and (7) at an arbitrary angle θ, then the following relations hold for either $I=0$ or $I=1$ amplitudes:

$$\begin{aligned}|\phi_{s,I}|^2&=(1-C_{NN}-C_{SS}-C_{LL})(d\sigma/d\Omega)/4\,,\\ |\phi_{\tau,I}|^2&=(1-C_{NN}+C_{SS}+C_{LL})(d\sigma/d\Omega)/4\,,\\ |\phi_{t,I}|^2+|\phi_{5,I}|^2&=(1+C_{NN}+C_{SS}-C_{LL})(d\sigma/d\Omega)/4\,,\\ |\phi_{T,I}|^2+|\phi_{5,I}|^2&=(1+C_{NN}-C_{SS}+C_{LL})(d\sigma/d\Omega)/4\,,\end{aligned} \tag{8}$$

where the subscript I on the spin parameters has been suppressed.

A different combination of np spin observables can give interference terms between $I=0$ and $I=1$:

$$\begin{aligned}(d\sigma/d\Omega)_{\mathrm{int}}(\theta)&=2(d\sigma/d\Omega)_{np}(\theta)-2(d\sigma/d\Omega)_{np}(\pi-\theta)\\ &=2\,\mathrm{Re}(\phi_{s,1}^*\phi_{s,0}+\phi_{t,1}^*\phi_{t,0}+\phi_{T,1}^*\phi_{T,0}+\phi_{\tau,1}^*\phi_{\tau,0}+2\phi_{5,1}^*\phi_{5,0})\,,\end{aligned} \tag{9}$$

and similarly for

$$(C_{\alpha\alpha}d\sigma/d\Omega)_{\rm int}(\theta)=2(C_{\alpha\alpha}d\sigma/d\Omega)_{np}(\theta)-2(C_{\alpha\alpha}d\sigma/d\Omega)_{np}(\pi-\theta)\ ,$$

for $\alpha=N$, S, or L. In analogy to Eq. (8),

$$\begin{aligned}
&\mathrm{Re}(\phi^*_{s,1}\phi_{s,0})=(1-C_{NN}-C_{SS}-C_{LL})(d\sigma/d\Omega)/8\ ,\\
&\mathrm{Re}(\phi^*_{\tau,1}\phi_{\tau,0})=(1-C_{NN}+C_{SS}+C_{LL})(d\sigma/d\Omega)/8\ ,\\
&\mathrm{Re}(\phi^*_{t,1}\phi_{t,0}+\phi^*_{5,1}\phi_{5,0})=(1+C_{NN}+C_{SS}-C_{LL})(d\sigma/d\Omega)/8\ ,\\
&\mathrm{Re}(\phi^*_{T,1}\phi_{T,0}+\phi^*_{5,1}\phi_{5,0})=(1+C_{NN}-C_{SS}+C_{LL})(d\sigma/d\Omega)/8\ ,
\end{aligned}\tag{10}$$

where the subscript int on the spin parameters has been suppressed. The $I=1$ amplitudes are assumed to be known. Therefore the magnitudes of $\phi_{s,0}$ and $\phi_{\tau,0}$ can be determined as well as their components along $\phi_{s,1}$ and $\phi_{\tau,1}$, respectively. From the six spin observables being considered, one can solve a quadratic equation for $\phi_{s,0}$, $\phi_{\tau,0}$, up to a twofold ambiguity in each. (The signs of $\mathrm{Im}\phi^*_{s,1}\phi_{s,0}$ and $\mathrm{Im}\phi^*_{\tau,1}\phi_{\tau,0}$ cannot be determined.) Because $P\,d\sigma/d\Omega$ and $C_{LS}d\sigma/d\Omega$ do not contain ϕ_s or ϕ_τ, it is not possible to resolve the ambiguities without observing the spin of one of the final-state nucleons.

In the same manner that Eqs. (7) and (9) were derived,

$$\begin{aligned}
(P\,d\sigma/d\Omega)_{I=0}(\theta)&=2\,\mathrm{Im}(\phi^*_{t,0}\phi_{5,0}-\phi_{T,0}\phi^*_{5,0})\\
&=2(P\,d\sigma/d\Omega)_{np}(\theta)-2(P\,d\sigma/d\Omega)_{np}(\pi-\theta)-(P\,d\sigma/d\Omega)_{pp}(\theta)\ ,
\end{aligned}\tag{11}$$

and

$$\begin{aligned}
(P\,d\sigma/d\Omega)_{\rm int}(\theta)&=2(P\,d\sigma/d\Omega)_{np}(\theta)+2(P\,d\sigma/d\Omega)_{np}(\pi-\theta)\\
&=2\,\mathrm{Im}(\phi^*_{t,1}\phi_{5,0}+\phi^*_{t,0}\phi_{5,1}-\phi_{T,1}\phi^*_{5,0}-\phi_{T,0}\phi^*_{5,1})\ .
\end{aligned}\tag{12}$$

Identical equations hold if P is replaced by C_{LS} and if the imaginary part is replaced with the real part in the previous equations.

Assuming the $I=1$ amplitudes are known, then the six real numbers corresponding to the amplitudes $\phi_{t,0}$, $\phi_{T,0}$, and $\phi_{5,0}$ can be found using Eqs. (8), (10), (11), and (12). Equations (10) and (12) (including C_{LS}) give four linear relations in the six unknowns. Equations (8) and (11) give four additional quadratic relations. In general, this system can be solved with up to a twofold ambiguity. Therefore, if the $I=1$ amplitudes are known, and if the *np* elastic-scattering spin observables $d\sigma/d\Omega$, P, C_{LS}, C_{NN}, C_{SS}, and C_{LL} are measured at some energy for both θ and $\pi-\theta$, then the $I=0$ amplitudes can be obtained in a model independent fashion with up to an eightfold discrete ambiguity. This ambiguity consists of two values for $\phi_{s,0}$, two values for $\phi_{\tau,0}$, and two sets of values $(\phi_{t,0}, \phi_{T,0}, \phi_{5,0})$.

The measurement of *np* observables such as $D_{NN}=(0,N;0,N)$ or $K_{LS}=(L,0;0,S)$, etc., would give additional information on the $I=0$ amplitudes. However, mixtures of these observables are needed to obtain pure $I=0$ or interference terms, such as

$$\begin{aligned}
2\left[K_{NN}\frac{d\sigma}{d\Omega}\right]_{np}(\theta)-2\left[D_{NN}\frac{d\sigma}{d\Omega}\right]_{np}(\pi-\theta)&=\left[K_{NN}\frac{d\sigma}{d\Omega}\right]_{\rm int}(\theta)\\
&=2\,\mathrm{Re}(-\phi^*_{\tau,1}\phi_{s,0}-\phi_{s,1}\phi^*_{\tau,0}+\phi^*_{T,1}\phi_{t,0}+\phi_{t,1}\phi^*_{T,0}+2\phi^*_{5,1}\phi_{5,0})
\end{aligned}\tag{13}$$

and

$$2F_{np}(\theta)-2G_{np}(\pi-\theta)-F_{pp}(\theta)=F_{I=0}(\theta)=2\,\mathrm{Re}(\phi_{T,0}\phi^*_{\tau,0}+\phi_{s,0}\phi^*_{t,0})\ ,\tag{14}$$

where

$$F=D_{LS}\sin\theta_R-D_{LL}\cos\theta_R\ ,$$

$$G=K_{LS}\sin\theta_R-K_{LL}\cos\theta_R\ ,$$

and θ_R is the laboratory angle of the recoil particle. In the particular case where both D_{NN} and K_{NN} are measured at θ and $\pi-\theta$, in addition to the six observables with polarized beam and/or polarized target, then unique $I=0$ amplitudes can be obtained and they will be overconstrained. Furthermore, in that case the five transversity amplitudes[7,15,16] ($\psi_{K,I}$ for $K=1,5$ and $I=$ isospin) can be used to derive the amplitudes if desired. The magnitudes $|\psi_{K,I}|^2$ and interference terms $\mathrm{Re}(\psi^*_{K,1}\psi_{K,0})$ are expressible in terms of $d\sigma/d\Omega$, P, C_{NN}, D_{NN}, and K_{NN}, while the other observables C_{SS}, C_{LS}, and C_{LL}, would be used to eliminate the 32-fold discrete ambiguity corresponding to the unknown signs of $\mathrm{Im}(\psi^*_{K,1}\psi_{K,0})$.

AMPLITUDES AT $\theta_{\rm c.m.}=90°$

The conditions given in Eq. (3) require two $I=1$ and three $I=0$ elastic-scattering amplitudes to vanish at $\theta=\pi/2$ (Ref. 8), namely, $\phi_{s,0}$, $\phi_{t,0}$, $\phi_{T,0}$, $\phi_{\tau,1}$, and $\phi_{5,1}$. Various techniques to obtain the $I=1$ amplitudes at $\theta=\pi/2$ have been discussed in the literature.[16–20] Here it is desired to concentrate on the $I=0$ amplitudes at 90°.

From Eq. (8) and the vanishing of the three $I=0$ amplitudes at $\theta=\pi/2$, the following relations can be derived:

$$1=C_{NN}+C_{SS}+C_{LL} \ , \qquad C_{SS}=C_{LL}=1/2(1-C_{NN}) \ , \tag{15}$$

where the $I=0$ subscripts have been suppressed. The first equation above is analogous to the well known $I=1$ relation at 90° that follows immediately from Eq. (8) and $\phi_{\tau,1}=0$:

$$1=C_{NN,pp}-C_{SS,pp}-C_{LL,pp} \ . \tag{16}$$

The magnitudes of the two nonzero $I=0$ amplitudes are (omitting $I=0$ subscripts)

$$\begin{aligned} |\phi_\tau|^2 &= (1-C_{NN})(d\sigma/d\Omega)/2 \\ &= C_{LL}\, d\sigma/d\Omega \ , \\ |\phi_5|^2 &= (1+C_{NN})(d\sigma/d\Omega)/4 \\ &= (1-C_{LL})(d\sigma/d\Omega)/2 \ . \end{aligned} \tag{17}$$

These magnitudes are shown from the preliminary data of Ref. 21 in Fig. 1. The phase of $\phi_{\tau,0}$ cannot be determined from the measurements. The amplitude $\phi_{5,0}$ can be uniquely determined from the relations

$$\begin{aligned} (P\, d\sigma/d\Omega)_{np} &= \tfrac{1}{2}\mathrm{Im}(\phi_{t,1}^{*}\phi_{5,0}-\phi_{T,1}\phi_{5,0}^{*}) \ , \\ (C_{LS}\, d\sigma/d\Omega)_{np} &= \tfrac{1}{2}\mathrm{Re}(\phi_{t,1}^{*}\phi_{5,0}-\phi_{T,1}\phi_{5,0}^{*}) \ . \end{aligned} \tag{18}$$

Therefore, in principle, only $d\sigma/d\Omega$, P, and C_{LS} for np scattering are required to obtain the magnitudes of $\phi_{\tau,0}$ and $\phi_{5,0}$ and the phase of $\phi_{5,0}$. A consistency check on the experimental measurements of $(d\sigma/d\Omega)_{pp}$, $(d\sigma/d\Omega)_{np}$, P_{np}, $C_{LS,np}$, $C_{\alpha\alpha,pp}$, and $C_{\alpha\alpha,np}$, (for $\alpha=N$ or L or S) is provided by the evaluation of $|\phi_{5,0}|$ by Eq. (17) and independently by Eq. (18).

Several other relations between np and pp spin observables follow from Eqs. (15) and (16), and the definitions in Eq. (7) [see also Ref. 15, Eq. (7.8) for additional relations]:

$$\begin{aligned} C_{SS,np} &= C_{LL,np}+(C_{NN,pp}-1-2C_{LL,pp})\frac{(d\sigma/d\Omega)_{pp}}{(d\sigma/d\Omega)_{np}}\Big/4 \ , \\ 2C_{SS,np} &= 1-C_{NN,np}+(3C_{NN,pp}-3-2C_{LL,pp})\frac{(d\sigma/d\Omega)_{pp}}{(d\sigma/d\Omega)_{np}}\Big/4 \ , \\ 2C_{LL,np} &= 1-C_{NN,np}+(C_{NN,pp}-1+2C_{LL,pp})\frac{(d\sigma/d\Omega)_{pp}}{(d\sigma/d\Omega)_{np}}\Big/4 \ . \end{aligned} \tag{19}$$

These relations should be useful tests for systematic errors in the nucleon-nucleon data base at lower energies.

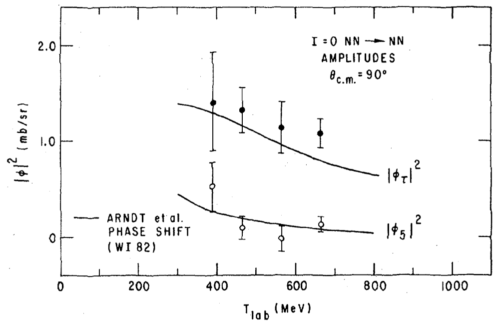

FIG. 1. Experimental isospin-0 nucleon-nucleon elastic-scattering amplitudes at $\theta_{\mathrm{c.m.}}=90°$. These are derived from the data of Ref. 21 using Eq. (17) in the text. For comparison, phase-shift predictions of Arndt *et al.* (Ref. 22) are also shown.

CONCLUSIONS

Measurements of the six nonzero np elastic-scattering spin observables $d\sigma/d\Omega$, P, C_{LS}, C_{NN}, C_{SS}, and C_{LL} at one energy and both angles θ and $\pi-\theta$ permit the determination of the $I=0$ amplitudes at θ (assuming the $I=1$ amplitudes are known). These are good measurements at high energies, since the experimentally difficult np elastic-scattering experiments are not compromised further at large angles by low rates in a carbon polarimeter whose analyzing power drops with increasing proton momentum. However, up to eightfold discrete ambiguities may be present in the $I=0$ amplitudes. To resolve these ambiguities, spin observables where the beam or target is polarized and where the spin of the outgoing protons is measured would be required.

At $\theta_{\mathrm{c.m.}}=90°$, there are only two $I=0$ amplitudes. Measurements of $(d\sigma/d\Omega)_{np}$, P_{np}, and $C_{LS,np}$, combined with the knowledge of the 90° $I=1$ amplitudes, would permit the determination of one $I=0$ amplitude and the magnitude of the other amplitude. The determination of the remaining phase would require experiments with a polarized beam or target and a polarimeter to measure the spin of one of the outgoing particles. Various relations between the $I=0$ spin parameters at $\theta=\pi/2$ were derived, as well as relations between np and pp spin observables.

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