Polarized-photon structure function and asymmetry in photon photoproduction process

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The helicity difference of the photon structure function is discussed, using Altarelli-Parisi-type equations and the Nicolaidis technique. The spin asymmetry in the photon photoproduction $\gamma p \to \gamma X$ is then calculated. Also discussed are the contributions originating from three kinds of subprocesses: Compton scattering, photon structure type, and fragmentation type.

I. INTRODUCTION

The quark and gluon distributions within a photon have been discussed. Using the operator-product expansion, Witten has calculated the leading behavior, with respect to Q^2, of the photon structure function.[1] Llewellyn Smith has arrived at the same result using ladder techniques,[2] and Dewitt *et al.* introduced the modification of the Altarelli-Parisi (AP) equations to do this calculation.[3]

The method of Dewitt *et al.* can be expanded to calculate the polarized-photon structure function. In this paper we get the helicity difference of the photon structure function by using the AP-type equations and the Nicolaidis method,[4] and we discuss the asymmetry in large-p_T photon photoproduction $\gamma p \to \gamma x$. The study of the deep-inelastic process is a very useful source of information concerning the short-distance behavior of hadron. This process has been discussed in recent years,[5] and the study of the spin effects in this process is an essential part of the effort towards developing QCD.

In Sec. II we obtain the polarized-photon structure functions. In Sec. III we introduce the formalism of our calculation. The results are in Sec. IV.

II. THE HELICITY DIFFERENCE OF THE PHOTON STRUCTURE FUNCTION

For the quark distribution $G_{q_i/\gamma}(x,Q^2)$ and the gluon distribution $G_{g/\gamma}(x,Q^2)$ within a photon we can write the modified AP-type equations[3]

$$\frac{d\, G_{q_i/\gamma}}{dt} = \frac{\alpha_s(t)}{2\pi}(G_{q_i/\gamma}\otimes P_{qq} + G_{g/\gamma}\otimes P_{qg}) + \frac{\alpha}{2\pi} e_i{}^2 P_{q\gamma}\ , \tag{1}$$

$$\frac{d\, G_{g/\gamma}}{dt} = \frac{\alpha_s(t)}{2\pi}\left[\sum_{j=1}^{2f} G_{q_j/\gamma}\otimes P_{gq} + G_{g/\gamma}\otimes P_{gg}\right]\ , \tag{2}$$

where e_i is the charge of the quark of type i, f is the number of flavors, $\alpha_s(t)=1/bt$, $b=(33-2f)/12\pi$, $t=\ln Q_2/\Lambda^2$. The last term in Eq. (1) cannot be neglected because the quark and gluon densities within a photon, $G_{q_i/\gamma}$ and $G_{g/\gamma}$, are themselves of order $\alpha \simeq \frac{1}{137}$. $P_{q\gamma}$ only differ from the Altarelli-Parisi function[6] P_{qg} by a color factor, and

$$P_{q\gamma} = N[x^2+(1-x)^2]\ ,$$

where N is the number of color replicas for each quark.

For a polarized photon we define the helicity difference distributions as

$$\Delta G_{q_i/\gamma} = G_{q_i(+)/\gamma(+)} - G_{q_i(-)/\gamma(+)}\ , \tag{3}$$

$$\Delta G_{g/\gamma} = G_{g(+)/\gamma(+)} - G_{g(-)/\gamma(+)}\ . \tag{4}$$

We assume that the helicity distributions $G_{q_i(+)/\gamma(+)}$, $G_{q_i(-)/\gamma(+)}$, $G_{g(+)/\gamma(+)}$, and $G_{g(-)/\gamma(+)}$ all satisfy the same equations [(1), (2)], and parity conservation in QCD and QED implies the relations

$$G_{A(+)/B(-)} = G_{A(-)/B(+)}\ ,\quad G_{A(-)/B(-)} = G_{A(+)/B(+)}\ .$$

We then can obtain the master equations for the helicity difference distributions as

$$\frac{d\,\Delta G_{q_i/\gamma}}{dt} = \frac{\alpha_s(t)}{2\pi}(\Delta G_{q_i/\gamma}\otimes \Delta P_{qq} + \Delta G_{g/\gamma}\otimes \Delta P_{qg}) + \frac{\alpha}{2\pi} e_i{}^2 \Delta P_{q\gamma}\ , \tag{5}$$

$$\frac{d\,\Delta G_{g/\gamma}}{dt} = \frac{\alpha_s(t)}{2\pi}\left[\sum_{j=1}^{2f} \Delta G_{q_j/\gamma}\otimes \Delta P_{gq} + \Delta G_{g/\gamma}\otimes \Delta P_{gg}\right]\ , \tag{6}$$

where $\Delta P_{q\gamma} = 6\Delta P_{qg} = N[x^2-(1-x)^2]$.

After separating Eq. (5) as usual into two parts, nonsinglet and singlet quark evolution equations, and going to moments, we can solve Eqs. (5) and (6) easily. The complete asymptotic solutions can then be written as

$$\Delta G_{q_i/\gamma}(n,Q^2)=\frac{\alpha\tilde{d}_{q\gamma}(n)}{\alpha_s(t)}\times\left[\frac{e_i^2-\langle e^2\rangle}{1-\tilde{d}_{qq}(n)}+\langle e^2\rangle\frac{1-\tilde{d}_{gg}(n)}{[1-\tilde{d}^{+}(n)][(1-\tilde{d}^{-}(n)]}\right], \quad (7)$$

$$\Delta G_{g/\gamma}(n,Q^2)=\frac{\alpha\tilde{d}_{q\gamma}(n)2f\langle e^2\rangle\tilde{d}_{gq}(n)}{\alpha_s(t)[1-\tilde{d}^{+}(n)][1-\tilde{d}^{-}(n)]}, \quad (8)$$

where

$$\Delta G_{A/B}(n,Q^2)=\int_0^1 x^{n-1}\Delta G_{A/B}(x,Q^2)\,dx, \quad (9)$$

and

$$\tilde{d}_{qq}(n)=\frac{C_F}{2\pi b}\left[\frac{3}{2}+\frac{1}{n(n+1)}-2\sum_{j=1}^{n}\frac{1}{j}\right], \quad (10)$$

$$\tilde{d}_{gg}(n)=\frac{C_A}{2\pi b}\left[\frac{11}{6}-\frac{2}{3}\frac{T}{C_A}+\frac{4}{n(n+1)}-2\sum_{j=1}^{n}\frac{1}{j}\right], \quad (11)$$

$$\tilde{d}_{gq}(n)=\frac{C_F}{2\pi b}\frac{(n+2)}{n(n+1)}, \quad (12)$$

$$\tilde{d}_{qg}(n)=\frac{T}{2\pi b}\frac{2(n-1)}{n(n+1)}, \quad (13)$$

$$\tilde{d}(n)=\tfrac{1}{2}(\tilde{d}_{gg}(n)\tilde{d}_{qq}(n)\pm\{[\tilde{d}_{gg}(n)-\tilde{d}_{qq}(n)]^2+4\tilde{d}_{qg}(n)\tilde{d}_{gq}(n)\}^{1/2}), \quad (14)$$

$$\tilde{d}_{q\gamma}(n)=\frac{N}{2\pi b}\frac{(n-1)}{n(n+1)} \quad (15)$$

$$\left[C_F=\frac{N^2-1}{2N}\to\frac{4}{3},\ C_A=N\to 3,\ T=\frac{f}{2}\right].$$

In order to recover the x-dependent distribution functions we could perform the inverse Mellin transformation. Instead we use the method proposed in Ref. 4. After factoring out the singular behavior at $x\simeq 0$, the structure functions of photon can be expanded in a series of Jacobian polynomials. Using the formula listed in the Appendix we obtain the five-term helicity difference distributions of quark and gluon within photon as follows:

$$\Delta G_{a/\gamma}=\frac{\alpha}{\pi}\ln\frac{Q^2}{\Lambda^2}\Delta f_{a/\gamma},$$

$$x\Delta f_{q/\gamma}(x,Q^2)=\frac{\alpha}{\pi}\ln\frac{Q^2}{\Lambda^2}(-0.0404+0.138x-0.903x^2+3.0772x^3-2.0916x^4)\ (e_q^2=\tfrac{4}{9}), \quad (16)$$

$$x\Delta f_{q/\gamma}(x,Q^2)=\frac{\alpha}{\pi}\ln\frac{Q^2}{\Lambda^2}(-0.02395+0.2946x-1.304x^2+2.3492x^3-1.2789x^4)\ (e_q^2=\tfrac{1}{9}), \quad (17)$$

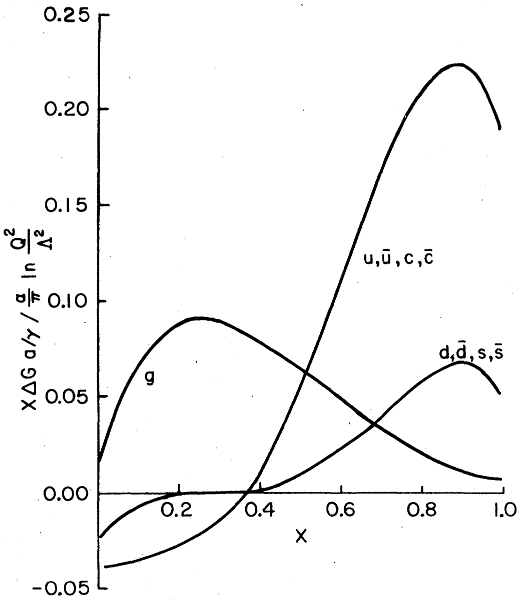


FIG. 1. Helicity difference distributions of quark and gluon within photon.

$$x\Delta f_{g/\gamma}(x,Q^2)=\frac{\alpha}{\pi}\ln\frac{Q^2}{\Lambda^2}(0.0108+0.7944x-2.5452x^2+2.8896x^3-1.1592x^4). \quad (18)$$

Equations (16), (17), and (18) have been shown in Fig. 1. We can also get the helicity difference of the fragmentation (or decay) function $\Delta D_{\gamma/q_i}(x,Q^2)$, and $\Delta D_{\gamma/g}(x,Q^2)$ by using the similar procedure. It must be mentioned that Ref. 7 has also obtained similar results.

III. FORMULA FOR CALCULATING ASYMMETRY

The predictions of QCD for large-momentum-transfer processes, including spin correlations, are based on the QCD factorization theorem[8] which separates the subprocess cross sections from process-independent structure functions and fragmentation functions. The invariant cross section for large-p_T photon photoproduction can be written as

$$E\frac{d\sigma}{d^3p}(s,t,u;\gamma p\to\gamma X)=\sum_{abc}\int dx\,dy\frac{dz}{z^2}G_{a/\gamma}(x,Q^2)\times G_{b/p}(y,Q^2)D_{\gamma/c}(z,Q^2)\frac{\hat{s}}{\pi}\times\frac{d\hat{\sigma}}{d\hat{t}}(ab\to cd)\delta(\hat{s}+\hat{t}+\hat{u}), \quad (19)$$

where s, t, u are the process kinematic invariants and $\hat{s}$, $\hat{t}$, $\hat{u}$ are the subprocess kinematic invariants.

If γ and p are longitudinally polarized, following Ref. 9 we can introduce the helicity difference cross section as

$$\Delta E\frac{d\sigma}{d^3p}=E\frac{d\sigma}{d^3p}(\gamma(+)p(+)\to\gamma X)-E\frac{d\sigma}{d^3p}(\gamma(+)p(-)\to\gamma X)$$
$$=\sum_{abc}\int dx\,dy\frac{dz}{z^2}\Delta G_{a/\gamma}(x,Q^2)\times\Delta G_{b/p}(y,Q^2)D_{\gamma/c}(z,Q^2)\frac{\hat s}{\pi}\Delta\frac{d\hat\sigma}{d\hat t}\ , \tag{20}$$

where

$$\Delta\frac{d\hat\sigma}{d\hat t}=\frac{d\hat\sigma}{d\hat t}(a(+)b(+)\to cd)-\frac{d\hat\sigma}{d\hat t}(a(+)b(-)\to cd)\ . \tag{21}$$

The spin-spin asymmetry can be defined as

$$A=\frac{\Delta E d\sigma/d^3p}{2Ed\sigma/d^3p}\ . \tag{22}$$

At the order α^2 there are three kinds of subprocesses which should be considered (Table I):

(I) Compton subprocess. In this case a and c are photon, and b is quark or antiquark. Equations (19) and (20) can be written as

$$E\frac{d\sigma}{d^3p}=\sum_b\frac{2}{2-x_Te^Y}yG_{b/p}(y,Q^2)\frac{1}{\pi}\frac{d\hat\sigma}{d\hat t}\ , \tag{23}$$

$$\Delta E\frac{d\sigma}{d^3p}=\sum_b\frac{2}{2-x_Te^Y}y\Delta G_{b/p}(y,Q^2)\frac{1}{\pi}\frac{d\hat\sigma}{d\hat t}\ , \tag{24}$$

where

$$y=\frac{x_Te^{-Y}}{2-x_Te^Y}\ ,$$

$$\hat s=ys\ ,\quad \hat t=-\frac{x_T}{2}se^{-Y}\ ,\quad \hat u=-\frac{x_T}{2}sye^{Y}\ ,$$

and

$$x_T=2p_T/\sqrt{s}\ ,$$

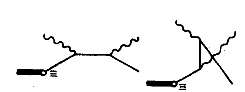
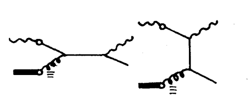
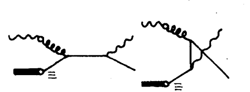
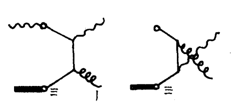
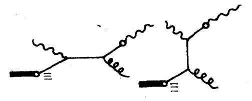
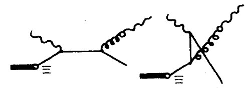
p_T is the transverse momentum of prompt photon, $Y=\ln\tan\theta_{\text{c.m.}}/2$.

(II) Subprocesses due to considering photon structure. Now c is photon and a,b are quark, antiquark, or gluon. The subprocesses $q_ig\to\gamma q_i$ and $\bar q_iq_i\to\gamma g$ are of order $\alpha\alpha_s$, so that the net contribution is of order α^2. Equations (19) and (20) now can be written as

$$E\frac{d\sigma}{d^3p}=\sum_{ab}\int_{x_{\min}}^1dx\left[\frac{2}{2x-x_Te^Y}\right]xyG_{a/\gamma}(x,Q^2)G_{b/p}(y,Q^2)\frac{1}{\pi}\frac{d\hat\sigma}{d\hat t}\ , \tag{25}$$

$$\Delta E\frac{d\sigma}{d^3p}=\sum_{ab}\int_{x_{\min}}^1dx\left[\frac{2}{2x-x_Te^Y}\right]xyG_{a/\gamma}(x,Q^2)\Delta G_{b/p}(y,Q^2)\frac{1}{\pi}\Delta\frac{d\hat\sigma}{d\hat t}\ , \tag{26}$$

TABLE I. Feynman diagram and subprocess cross sections, helicity sum and helicity difference, contributing to photon photoproduction at order $\alpha\alpha_s$.

	Feynman diagram	$d\hat\sigma/d\hat t$	$\Delta d\hat\sigma/d\hat t$
I		$-2\pi\alpha^2e_q^4\frac{1}{\hat s^2}\left[\frac{\hat s}{\hat u}+\frac{\hat u}{\hat s}\right]$	$4\pi\alpha^2e_q^4\frac{1}{\hat s^2}\left[-\frac{\hat s}{\hat u}+\frac{\hat u}{\hat s}\right]$
		$-\frac{\pi}{3}\alpha\alpha_se_q^2\frac{1}{s^2}\left[\frac{\hat t}{\hat s}+\frac{\hat s}{\hat t}\right]$	$\frac{2\pi}{3}\alpha\alpha_se_q^2\frac{1}{s^2}\left[-\frac{\hat s}{\hat t}+\frac{\hat t}{\hat s}\right]$
II		$-\frac{\pi}{3}\alpha\alpha_se_q^2\frac{1}{\hat s^2}\left[\frac{\hat u}{\hat s}+\frac{\hat s}{\hat u}\right]$	$\frac{2\pi}{3}\alpha\alpha_se_q^2\frac{1}{s^2}\left[-\frac{\hat s}{\hat u}+\frac{\hat u}{\hat s}\right]$
		$\frac{8\pi}{3}\alpha\alpha_se_q^2\frac{1}{\hat s^2}\left[\frac{\hat u}{\hat t}+\frac{\hat t}{\hat u}\right]$	$-\frac{16\pi}{9}\alpha\alpha_se_q^2\frac{1}{\hat s^2}\left[\frac{\hat u}{\hat t}+\frac{\hat t}{\hat u}\right]$
		$-\frac{8\pi}{3}\alpha\alpha_se_q^2\frac{1}{\hat s^2}\left[\frac{\hat t}{\hat s}+\frac{\hat s}{\hat t}\right]$	$\frac{16\pi}{3}\alpha\alpha_se_q^2\frac{1}{\hat s^2}\left[-\frac{\hat s}{\hat t}+\frac{\hat t}{\hat s}\right]$
III		$-\frac{8\pi}{3}\alpha\alpha_se_q^2\frac{1}{\hat s^2}\left[\frac{\hat u}{\hat s}+\frac{\hat s}{\hat u}\right]$	$\frac{16\pi}{3}\alpha\alpha_se_q^2\frac{1}{\hat s^2}\left[-\frac{\hat s}{\hat u}+\frac{\hat u}{\hat s}\right]$

where

$$y=\frac{x_T x e^{-Y}}{2x-x_T e^{Y}}\ ,$$

$$\hat{s}=xys\ ,\ \hat{t}=-\frac{x_T}{2}sxe^{-Y}\ ,\ \hat{u}=-\frac{x_T}{2}sye^{Y}\ ,$$

$$x_{\min}=\frac{x_T e^{Y}}{2-x_T e^{-Y}}\ .$$

(III) Subprocesses due to considering the fragmentation. Equations (19) and (20) now can be written as

$$E\frac{d\sigma}{d^3p}=\sum_{ab}\int_{y_{\min}}^{1}dy\ G_{b/p}(y,Q^2)D_{\gamma/c}(z,Q^2)\frac{1}{z}\frac{1}{\pi}\frac{d\hat{\sigma}}{d\hat{t}}\ , \quad (27)$$

$$\Delta E\frac{d\sigma}{d^3p}=\sum_{ab}\int_{y_{\min}}^{1}dy\ \Delta G_{b/p}(y,Q^2)D_{\gamma/c}(z,Q^2)\frac{1}{z}\frac{1}{\pi}\frac{d\hat{\sigma}}{d\hat{t}}\ , \quad (28)$$

where

$$z=\frac{x_T}{2}\left[e^{Y}+\frac{e^{-Y}}{y}\right],$$

$$\hat{s}=ys\ ,\ \hat{t}=-\frac{sy}{(1+ye^{2Y})}\ ,\ \hat{u}=-\frac{sy^2}{(e^{-2Y}+y)}\ ,$$

$$y_{\min}=\frac{x_T e^{-Y}}{2-x_T e^{Y}}\ .$$

IV. CALCULATION AND RESULT

Before doing calculation we should consider two things. First, it is known that the photon structure function contains two parts: the pointlike component and the hadronlike component. For the hadronlike component or nonperturbative hadronic component of photon we will use the vector-dominance model and assume simple structure functions for vector mesons, as many authors have done.[1,10] Moreover we neglect the gluon distribution of the vector-dominance model (VDM) and the Q^2 dependence in the assumed structure function. To get the helicity difference distributions we use the further assumptions that the helicity is conserved at the photon—vector-meson vertex and that all the valence quarks have the same helicity as the vector meson in which they originate.[7] Under these assumptions we have

$$G_{q_i/\gamma}^{\mathrm{VDM}}(x)=\Delta G_{q_i/\gamma}^{\mathrm{VDM}}(x)=\frac{\alpha}{2}\langle x\rangle_{q_i}\frac{1-x}{x}\ , \quad (29)$$

where the average momentum $\langle x\rangle_{q_i}$ carried by the quarks inside vector meson (ρ^0) is taken to

$$\langle x\rangle_{q_i}=\begin{cases}0.25 & (u,\bar{u},d,\bar{d})\\ 0 & (s,\bar{s},c,\bar{c})\end{cases}\ . \quad (30)$$

It is very crude but sufficient for our purpose. The effect of the hadronlike component will mainly be present in the small-x region. Since the pointlike part is proportional to $\ln Q^2/\Lambda^2$, the hadronlike part will be suppressed for very large Q^2.

Second, let us consider the spin dependence of the parton distribution function within proton. We take the following three different models in our calculation:

(1) SU(6) model,[9]

$$\Delta G_{u/p}=0.44G_{u/p},\Delta G_{d/p}=-0.35G_{d/p}\ ,$$

$$\Delta G_{\bar{u}/p}=\Delta G_{\bar{d}/p}=0.13(2-x)(1-x)^{10}\ ,$$

$$\Delta G_{g/p}=0.66(2-x)(1-x)^{6}\ ,$$

$$G_{\bar{u}/p}=G_{\bar{d}/p}=\frac{0.39}{x}(1-x)^{10}[1+(1-x)^2]\ .$$

(2) Carlitz-Kaur model,[11]

$$\Delta G_{u/p}=\cos(2\theta)(G_{u/p}-\tfrac{2}{3}G_{d/p})\ ,$$

$$\Delta G_{d/p}=-\tfrac{1}{3}\cos(2\theta)G_{d/p}\ ,$$

$$\Delta G_{\bar{u}/p}=\Delta G_{\bar{d}/p}=0\ ,$$

$$\Delta G_{g/p}=0.43(2-x)(1-x)^{6}\ ,$$

$$G_{\bar{u}/p}=G_{\bar{d}/p}=0\ ,$$

where the "spin dilute factor"

$$\cos(2\theta)=[1+0.052x^{-1/2}(1-x)^2]^{-1}\ .$$

In SU(6) and the Carlitz-Kaur model the gluon distribution has the simple parametrization

$$G_{g/p}=\frac{1.97}{x}(1-x)^{6}[1+(1-x)^2]$$

based on a QCD bremsstrahlung model.

(3) Modified Sehgal model,[12]

$$\Delta G_{u/p}=0.456G_{u/p}\ ,\ \ \Delta G_{d/p}=-0.315G_{d/p}\ ,$$

$$\Delta G_{\bar{u}/p}=\Delta G_{\bar{d}/p}=0\ ,\ \ \Delta G_{g/p}=0.4xG_{g/p}\ ,$$

$$G_{g/p}=3.5(1-x^6)/x\ ,\ \ G_{\bar{u}/\gamma}=G_{\bar{d}/\gamma}=0\ .$$

For these three models we take the spin-averaged valence-quark distribution as[13]

$$xG_{u/p}=1.79\sqrt{x}\,(1-x^3)(1+2.3x)\ ,$$

$$xG_{d/p}=1.07\sqrt{x}\,(1-x)^{3.1}\ .$$

In Fig. 2 we first show the prediction for the invariant cross section versus $x_T=2p_T/\sqrt{s}$ at $\theta_{\mathrm{c.m.}}=90°$ for a beam energy $\sqrt{s}=40$ GeV. The five-term polynomial formula for the helicity sum of photon structure functions and fragmentation functions in Ref. 7 have been used in our calculation. It is obvious that in the small-x_T region ($x_T<0.3$) the inclusive photon photoproduction is dominated by the photon-structure-type subprocess (II in Table I). But this contribution will decrease in importance rapidly with increasing x_T, and in the large-p_T region ($x_T>0.5$), the contribution from the Compton scattering (I in Table I) becomes dominant. Lastly, the fragmentation-type-subprocess (III in Table I) contribution is always sufficiently small.

As for the helicity sum of the distribution function of

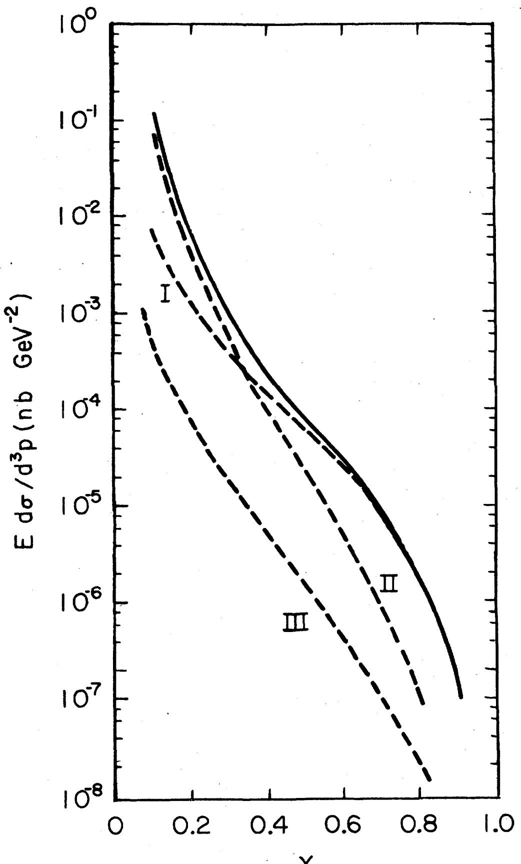


FIG. 2. Cross section in photon photoproduction originating from Compton scattering (I), photon-structure-type subprocesses (II), and fragmentation-type subprocesses (III). Solid curve shows total differential cross section ($\sqrt{s}=40$ GeV, $\theta_{\text{c.m.}}=90°$).

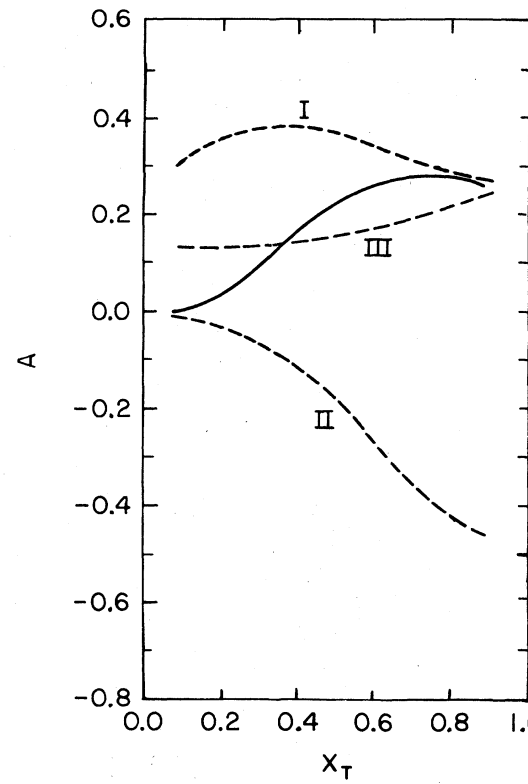


FIG. 3. Asymmetries in photon photoproduction due to Compton scattering (I), photon-structure-type subprocesses (II), and fragmentation-type subprocesses (III) for the SU(6) model. The solid curve shows the total asymmetry ($\sqrt{s}=40$ GeV, $\theta_{\text{c.m.}}=90°$).

the parton within proton, the differences among the above three models are only related to the distributions of sea quark and gluon. And no significant effects originating from these differences have been found in our calculation.

The spin-spin asymmetries in the process of photon photoproduction at order α^2, including the contributions originating from Compton scattering (I), photon-structure-type subprocesses (II), and fragmentation-type subprocesses (III), are shown in Fig. 3 for the SU(6) model, in Fig. 4 for the Carlitz-Kaur model, and in Fig. 5 for the modified Sehgal model. For all spin-dependent parton distributions of proton total asymmetries are positive and their values will increase with increasing x_T. The value of asymmetry ranges between about 0 and 30% for the SU(6) or the modified Sehgal model and between about 0 and 50% for the Carlitz-Kaur model. From our calculations we found that the SU(6) and the modified Sehgal model give similar results. However, the differences between them and the Carlitz-Kaur model are significant.

We now discuss the asymmetry originating from Compton scattering. These values will directly give us the information of spin-dependence parton distributions within proton. For the SU(6) the asymmetry of this subprocess has a value of 30% at $x_T \simeq 0.1$, and then will rise up to its maximum (40%) at $x_T \simeq 0.4$, and after that it will decrease (Fig. 3). The Carlitz-Kaur distribution predicts the value of asymmetry ranging from 25 to 55% [$x_T = 0.1 \sim 0.9$, (Fig. 4)]. The Carlitz-Kaur model has a much larger value of $\Delta G_{u/p}$ for large x. This leads to a larger asymmetry for large x_T.

Next we turn to consider the photon-structure-type subprocesses. For all models discussed above the values of asymmetry originating these kinds of subprocesses are always negative. The reason is as follows. Among three photon-structure subprocesses (II in Table I) the quark annihilation $\bar{q}_i q_i \to \gamma g$ will yield the largest contribution to the helicity difference of invariant cross section, and its value is negative because the factor $\Delta G_{\bar{u}/\gamma} \Delta G_{u/p}$ which appears in the integrand of Eq. (26) has larger positive value for large x, but the subprocess cross section [$\propto -(\hat{u}/\hat{t}+\hat{t}/\hat{u})$] is negative (note: $\hat{s}>0$, $\hat{t}<0$, $\hat{u}<0$ in the physical region).

It is interesting to show that for the photon-structure-type subprocess the value of the invariant cross section is larger in the small-x_T region (Fig. 2), but the (absolute) value of asymmetry is very small (Figs. 3—5). On the other hand, for large x_T this case is just the opposite.

The asymmetry originating from the fragmentation-

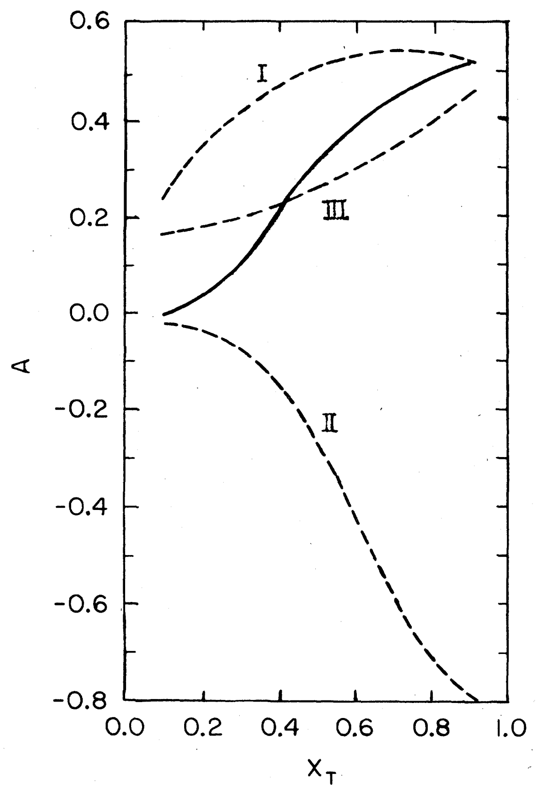


FIG. 4. Asymmetries in photon photoproduction due to Compton scattering (I), photon-structure-type subprocesses (II), and fragmentation-type subprocesses (III) for the Carlitz-Kaur model. The solid curve shows the total asymmetry ($\sqrt{s}=40$ GeV, $\theta_{\rm c.m.}=90°$).

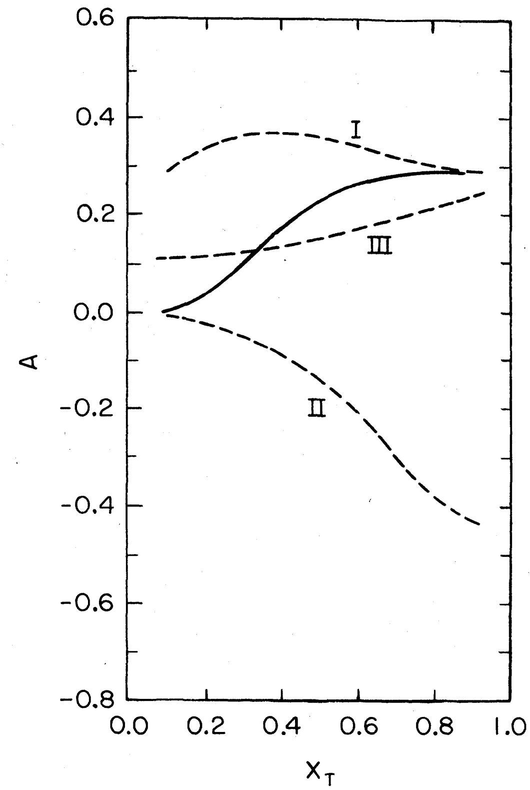


FIG. 5. Asymmetries in photon photoproduction due to Compton scattering (I), photon-structure-type subprocesses (II), and fragmentation-type subprocesses (III) for the modified Sehgal model. The solid curve shows the total asymmetry ($\sqrt{s}=40$ GeV, $\theta_{\rm c.m.}=90°$).

type subprocess is also not small as shown in Figs. 3—5. But we would like to point out again that the cross section originating from this subprocess is very small (Fig. 2).

It should be mentioned that in our calculation we have not adopted scale-violating spinning and nonspinning distribution functions of proton. We may expect that the inclusion of this would not change our results significantly.[9,12] In this case the total energy $\sqrt{s}$ will have no effect on the value of asymmetry.

ACKNOWLEDGMENT

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APPENDIX

In this Appendix we list the formulas which are used in order to get the polynomials in x for helicity difference quark and gluon distributions. Using the method given in Ref. 7 and considering that the rightmost singularity in the complex n plane of the moments in Eqs. (7) and (8) occurs at $n=0.99995\simeq 1$, we have

$$\Delta G_{a/\gamma}(x,Q^2)=\frac{\alpha}{\pi}\ln\frac{Q^2}{\Lambda^2}f_{a/\gamma}(x)\ , \tag{A1}$$

$$xf_{a/\gamma}(x)=\sum_{n=0}^{N} C_n x^n\ , \tag{A2}$$

$$C_n=\sum_{b\geq n}^{N} b_l P(l,n)\ , \tag{A3}$$

$$b_n=\sum_{k=0}^{n} P(n,k) f_{a/\gamma}(k+2)\ , \tag{A4}$$

$$P(n,k)=(-1)^{k+2}(2n+1)^{1/2}\frac{(n+k)!}{(K!)^2(n-K)!}\ , \tag{A5}$$

where $f_{a/\gamma}(n)$ is the moment of $f_{a/\gamma}(x)$:

$$f_{a/\gamma}(n)=\int_0^1 dx\, x^{n-1} f_{a/\gamma}(x)\ . \tag{A6}$$

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