Effective potential for chiral supersymmetric models

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The effective potential for chiral supersymmetric models is discussed within the framework of the Wess-Zumino model. The effective potential is calculated to two loops using two different renormalization schemes. In a modified minimal-subtraction scheme the effective potential displays pathological behavior such as multivaluedness, complexity, and negativity. It is shown that these features arise from the existence of ghosts in this scheme. A second renormalization procedure, which preserves positivity of kinetic terms in the effective action, is then examined. It is found that quantum corrections in this scheme are small and that supersymmetry is not radiatively broken to the two-loop level. These results are compared with those obtained for O(N)-symmetric chiral models.

I. INTRODUCTION

Supersymmetric Lagrangians exhibit several features which are not present in nonsupersymmetric theories and which can cause complications in the calculation of the effective potential. For example, the supersymmetric effective potential, like the vacuum expectation value of the supersymmetric Hamiltonian, should be positive semidefinite. Other such characteristics are the existence of auxiliary fields and the decreased number of renormalization constants. Because of these features, there are a number of caveats which must be observed in supersymmetric effective-potential calculations. In this paper these pitfalls are discussed within the framework of the Wess-Zumino model.[1] The details of our previous two-loop calculation[2] of the effective potential for the Wess-Zumino model are given during the course of this discussion.

Early calculations[3–5] showed that the effective potential for the Wess-Zumino model vanishes to all orders at the same points as the tree potential. O'Raifeartaigh and Parravicini[5] showed that the one-loop effective potential is non-negative. Their proof, however, depended on substituting into the effective potential the tree-level expression for the auxiliary field. This technique was also used in a recent two-loop calculation by Miller.[6] To be consistent it is necessary to solve for the auxiliary field at the order that the effective potential is calculated. The auxiliary field as a function of the physical field acquires radiative corrections.

The effective potential for a Wess-Zumino model with a global O(N) symmetry has also been studied. These calculations were done at the one-loop level with the auxiliary field solved at the same level. Zanon[7] found that the effective potential in terms of the physical fields is negative at one of its stationary points. Other authors[8,9] examined this model and found that the effective potential is not only negative but is also double valued for a range of values of the physical fields. Amati and Chou[10] showed that these properties result from an improper renormalization procedure. The renormalization prescription of Zanon and others led to negative kinetic energy terms. Amati and Chou found a renormalization scheme which avoids this problem and showed that the resulting one-loop effective potential is non-negative.

The Wess-Zumino effective potential is a function of two variables, $V_{\text{eff}}=V_{\text{eff}}(a,f)$. These variables, a and f, are related to the scalar component fields in the Wess-Zumino model. The component fields in the Wess-Zumino model are a physical scalar field A, an auxiliary scalar field F, and a spinor field ψ. The Lagrangian does not contain kinetic terms for the field F. Thus, F may be eliminated from the Lagrangian by using its Euler-Lagrange equation. The term "on-shell" is used if, in a calculation, the equation of motion is used to eliminate F. If F is not eliminated the term "off-shell" is used. The easiest way to compute the effective potential is off-shell. To see this, consider the generating functional for connected Green's functions $W[J]$ defined by

$$\exp\{-W[J]\}\equiv\int dA\,dF\,d\psi\exp\left\{-\int d^4x[\mathscr{L}(A,F,\psi)+AJ_A+FJ_F+\psi J_\psi]\right\},\tag{1.1}$$

where $\mathscr{L}$ is the Lagrangian and J_A, J_F, and J_ψ are the sources associated with the fields A, F, and ψ, respectively. The effective action is then defined as the Legendre transform of W:[11]

$$\Gamma[a(x),f(x),\hat{\psi}(x)] \equiv W[J_A,J_F,J_\psi]-\int d^4x\left[\frac{\delta W}{\delta J_A}J_A+\frac{\delta W}{\delta J_F}J_F+\frac{\delta W}{\delta J_\psi}J_\psi\right], \tag{1.2}$$

where

$$a(x)=\frac{\delta W}{\delta J_A}\ , \tag{1.3}$$

$$f(x)=\frac{\delta W}{\delta J_F}\ , \tag{1.4}$$

$$\hat{\psi}(x)=\frac{\delta W}{\delta J_\psi}\ . \tag{1.5}$$

Equations (1.3)—(1.5) are to be solved for J_A, J_F, and J_ψ in terms of a, f, and $\hat{\psi}$. The resulting expressions for the sources are then substituted into (1.2) in order to obtain a function of a, f, and $\hat{\psi}$.

The effective potential is defined by

$$V_{\rm eff}(a,f)\int d^4x=-\Gamma(a,f,0)\ , \tag{1.6}$$

where a and f are constant fields. Note that (1.6) is defined with $\hat{\psi}=0$. This is because the minimum of the effective potential gives the ground-state configuration of the fields and $\langle\hat{\psi}\rangle_0\neq 0$ violates Lorentz invariance. Explicit supersymmetry invariance is lost by setting $\hat{\psi}=0$. To demonstrate that the effective potential is supersymmetric one must first calculate $V_{\rm eff}(a,f,\hat{\psi})$ with $\hat{\psi}$ a constant spinor field and then show that it is invariant under supersymmetry transformations. If the vacuum is to be Lorentz invariant the minimum of $V_{\rm eff}(a,f,\hat{\psi})$ must be in the a-f plane with $\hat{\psi}=0$. Here we bypass these steps, assume from the start that $\langle\hat{\psi}\rangle_0=0$, and use (1.6) as the definition of the effective potential.

Calculating the effective potential on-shell is equivalent to integrating (1.1) over F [this can be done since the integrand in (1.1) is Gaussian in F] before forming the effective action. The effective action, however, will not be a function of $a(x)$ and $\hat{\psi}(x)$ only. The source J_F survives integration over F and complicates further calculations. The resulting expression is equivalent to the effective potential calculated in this paper, but is in a less useful form.

It may also be possible to compute the effective potential on-shell setting $J_F=0$ but imposing the supersymmetric Ward-Takahashi identities at each loop order of the calculation. This point is presently under investigation.

The method of computing the effective potential which will be used in this paper is to first compute $V_{\rm eff}(a,f)$ as defined in (1.6) (in nonsupersymmetric theories one stops here), eliminate f by solving

$$\frac{\delta V(a,f)}{\delta f}=0 \tag{1.7}$$

for $f(a)$, and then identifying

$$V_{\rm eff}(a)\equiv V_{\rm eff}[a,f(a)] \tag{1.8}$$

as the effective potential for the physical field a.

The unrenormalized $V_{\rm eff}(a,f)$ is discussed in Sec. II. A modified minimal-subtraction renormalization scheme is used and the resulting $V_{\rm eff}(a)$ is discussed in Sec. III. The Wess-Zumino effective potential in this scheme shows the same peculiar properties seen in the O(N)-symmetric model of Zanon. The causes for this behavior are studied.

In Sec. IV a renormalization prescription which is equivalent to that of Amati and Chou, and which is easily extended to higher order, is adopted. The one-loop and two-loop $V_{\rm eff}(a)$ are discussed and are shown to be positive semidefinite. Various results concerning renormalization, β functions, and approximations are collected in the appendices.

II. DETAILS OF THE MODEL

The Lagrangian density for the Wess-Zumino model is, in the notation of Ref. 12,

$$\mathscr{L}=\int d^4\theta\{\phi_0^\dagger\phi_0-\tfrac{1}{2}m_0[\phi_0\phi_0\delta^2(\bar\theta)+\phi_0^\dagger\phi_0^\dagger\delta^2(\theta)] -(1/3!)\lambda_0[\phi_0\phi_0\phi_0\delta^2(\bar\theta)+\phi_0^\dagger\phi_0^\dagger\phi_0^\dagger\delta^2(\theta)]\}\ . \tag{2.1}$$

The θ's are anticommuting Grassmann variables, m_0 is a bare mass, λ_0 is a bare coupling constant, and the superfield ϕ_0 is given in terms of component fields by

$$\phi_0(x,\theta,\theta')+A(x)+i\theta\sigma^m\bar\theta\partial_m A(x)+\tfrac{1}{4}\theta\theta\bar\theta\bar\theta\Box A(x) +\sqrt{2}\theta\psi(x)-(i/\sqrt{2})\theta\theta\partial_m\psi(x)\sigma^m\bar\theta+\theta\theta F(x)\ . \tag{2.2}$$

This model requires only one renormalization constant, Z, to remove all the cutoff-dependent terms which arise.[13] The renormalized Lagrangian is obtained by rescaling

$$\phi_0=Z^{1/2}\phi_R\ , \tag{2.3}$$

$$m_0=Z^{-1}m_R\ , \tag{2.4}$$

$$\lambda_0=Z^{-3/2}\lambda_R\ . \tag{2.5}$$

Re-express (2.1) in terms of renormalized ϕ_R, m_R, and λ_R, and drop the subscript R, to obtain

$$\mathscr{L}=\int d^4\theta\{Z\phi^\dagger\phi-\tfrac{1}{2}m[\phi\phi\delta^2(\bar\theta)+\phi^\dagger\phi^\dagger\delta^2(\theta)] -(1/3!)\lambda[\phi\phi\phi\delta^2(\bar\theta)+\phi^\dagger\phi^\dagger\phi^\dagger\delta^2(\theta)]\}\ . \tag{2.6}$$

The only difference in form between (2.1) and (2.6) is that the kinetic $\phi^\dagger\phi$ term in (2.6) is multiplied by the wave-function renormalization Z.

To compute the effective potential use Jackiw's theorem[11] and shift the superfield ϕ to

$$\phi\to\phi+\phi_{\rm cl}\ , \tag{2.7}$$

where

$$\phi_{\rm cl}\equiv a+\theta\theta f\ , \tag{2.8}$$

with a and f the (constant) fields conjugate to the sources J_A and J_F. The fermionic field conjugate to the source J_ψ is assumed to vanish. The vacuum-bubble method will be used, so all terms linear in ϕ are dropped (for a discussion of the vacuum-bubble method versus the auxiliary-field tadpole method in the two-loop calculation see Ref. 6). The Lagrangian can now be split:

$$\mathscr{L}=\mathscr{L}_{\rm cl}+\mathscr{L}_q\ , \tag{2.9}$$

where $\mathscr{L}_{\rm cl}$ is just $\mathscr{L}$ with ϕ replaced by $\phi_{\rm cl}$ and where the quantum Lagrangian is

$$\begin{aligned}\mathscr{L}_q=\int d^4\theta\{ &Z\phi^\dagger\phi-\tfrac{1}{2}(m'+\theta\theta\lambda f)\phi\phi\delta^2(\bar\theta)\\ &-\tfrac{1}{2}(m'^*+\bar\theta\bar\theta\lambda f^*)\phi^\dagger\phi^\dagger\delta^2(\theta)\\ &-(1/3!)\lambda[\phi\phi\phi\delta^2(\bar\theta)+\phi^\dagger\phi^\dagger\phi^\dagger\delta^2(\theta)]\}\ ,\end{aligned} \tag{2.10}$$

where

$$m'\equiv m+\lambda a\ . \tag{2.11}$$

Integrating over the θ's, $\mathscr{L}_{\rm cl}$ and $\mathscr{L}_q$ can be expressed in terms of component fields. The tree-level effective potential is just (recall that a and f are constant fields)

$$\begin{aligned}V^{(0)}=-\mathscr{L}_{\rm cl}=&-Zf^*f+m(af+a^*f^*)\\ &+\tfrac{1}{2}\lambda(aaf+a^*a^*f^*)\ ,\end{aligned} \tag{2.12}$$

and

$$\begin{aligned}\mathscr{L}_q=&Z(F^*F+A^*\Box A-i\bar\psi\bar\sigma^m\partial_m\psi)-m'(AF-\tfrac{1}{2}\psi\psi)\\ &-m'^*(A^*F^*-\tfrac{1}{2}\bar\psi\bar\psi)-\tfrac{1}{2}\lambda f(AA+A^*A^*)\\ &-\tfrac{1}{2}\lambda(AAF+A^*A^*F^*-\psi\psi A-\bar\psi\bar\psi A^*)\ .\end{aligned} \tag{2.13}$$

$\mathscr{L}_q$ can be written

$$\mathscr{L}_q=\mathscr{L}_0+\mathscr{L}_I\ , \tag{2.14}$$

where

$$\mathscr{L}_I\equiv-\tfrac{1}{2}\lambda(AAF+A^*A^*F^*-\psi\psi A-\bar\psi\bar\psi A^*) \tag{2.15}$$

is the interaction part of $\mathscr{L}_q$. $\mathscr{L}_0$ is quadratic in the fields:

$$\mathscr{L}_0=\frac{1}{2}\begin{bmatrix}A\\A^*\\F\\F^*\end{bmatrix}^T M\begin{bmatrix}A\\A^*\\F\\F^*\end{bmatrix}+\frac{1}{2}\begin{bmatrix}\psi^\alpha\\ \bar\psi_{\dot\alpha}\end{bmatrix}^T M'\begin{bmatrix}\bar\psi^{\dot\beta}\\ \psi_\beta\end{bmatrix}\ , \tag{2.16}$$

where

$$M=\begin{bmatrix}-\lambda f & Z\Box & -m' & 0\\ Z\Box & -\lambda f^* & 0 & -m'^*\\ -m' & 0 & 0 & Z\\ 0 & -m'^* & Z & 0\end{bmatrix}\ , \tag{2.17}$$

$$M'=\begin{bmatrix}-iZ\sigma^m_{\alpha\dot\beta}\partial_m & m'\delta_\alpha{}^\beta\\ m'^*\delta^{\dot\alpha}{}_{\dot\beta} & -iZ\bar\sigma^{m\dot\alpha\beta}\partial_m\end{bmatrix}\ . \tag{2.18}$$

The component propagators can be read from M^{-1} and M'^{-1}:

$$M^{-1}=\frac{1}{\Delta}\begin{bmatrix}\lambda f^* & \Box-|m'|^2 & m'^*(\Box-|m'|^2) & \lambda f^*m'\\ \Box-|m'|^2 & \lambda f & \lambda f m'^* & m'(\Box-|m'|^2)\\ m'^*(\Box-|m'|^2) & \lambda f m'^* & \lambda f m'^* m'^* & -\lambda^2|f|^2+\Box(\Box-|m'|^2)\\ \lambda f^* m' & m'(\Box-|m'|^2) & -\lambda^2|f|^2+\Box(\Box-|m'|^2) & \lambda f^* m' m'\end{bmatrix} \tag{2.19}$$

and

$$M'^{-1}=\frac{1}{\Sigma}\begin{bmatrix}i\bar\sigma^{n\dot\beta\gamma}\partial_n & m'\delta^{\dot\beta}{}_{\dot\gamma}\\ m'^*\delta_\beta{}^\gamma & i\sigma^n_{\beta\dot\gamma}\partial n\end{bmatrix}\ , \tag{2.20}$$

where

$$\Delta\equiv\det M=(Z^2\Box-|m'|^2)^2-Z^2\lambda^2|f|^2 \tag{2.21}$$

and

$$\Sigma=-\det M'=-(Z^2\Box-|m'|^2)\ . \tag{2.22}$$

The wave-function renormalization, Z, was set to unity in (2.19) and (2.20) since higher-order terms are not needed in the two-loop calculation. The factors of Z which appear in the determinants (2.21) and (2.22), however, will be used (see Appendix A).

Using the component propagators and the decomposition

$$\phi(y)=A(y)+\sqrt{2}\theta\psi(y)+\theta\theta F(y)\ , \tag{2.23}$$

where

$$y^m\equiv x^m+i\theta\sigma^m\bar\theta\ ,$$

the following Euclidean superfield propagators may be obtained:

$$\begin{aligned}\langle 0|T[\phi(y,\theta)\phi(y',\theta')]|0\rangle&\\ =\frac{-\Sigma}{\Delta}\Bigg[&m'^*(\theta-\theta')^2-\frac{\lambda f^*}{\Sigma}-\theta\theta\theta'\theta'\frac{\lambda f m'^* m'^*}{\Sigma}\\ &+\theta\theta'\frac{2\lambda^2 f^* f m'^*}{\Sigma^2}\Bigg]\delta^{(4)}(y-y')\ ,\end{aligned} \tag{2.24}$$

$$\begin{aligned}\langle 0|T[\phi(y,\theta)\phi^\dagger(y',\bar\theta')]|0\rangle&\\ =\Bigg[&\frac{-\Sigma}{\Delta}\exp(-2i\theta\sigma^m\bar\theta'\partial_m)\\ &+\frac{1}{\Delta}\Bigg[-2i\theta\sigma^m\bar\theta'\frac{\lambda^2 f^* f}{\Sigma}\partial_m\\ &+\theta\theta\lambda f m'^*+\bar\theta'\bar\theta'\lambda f^* m'\\ &-\theta\theta\bar\theta'\bar\theta'\lambda^2 f^* f\Bigg]\Bigg]\delta^{(4)}(y-y')\ .\end{aligned} \tag{2.25}$$

The δ function can be expanded:

$$\delta^{(4)}(y-y')=\exp[+i(\theta\sigma^m\bar{\theta}-\theta'\sigma^m\bar{\theta}')\partial_m(x)]\delta(x-x') . \tag{2.26}$$

The tree-level effective potential is given in (2.12). The one-loop contribution is given by Jackiw's theorem as[3–5,14,15]

$$V^{(1)}=\frac{\hbar}{2}\operatorname{tr}\ln\left[\frac{\Delta}{\Sigma^2}\right] . \tag{2.27}$$

The evaluation of this is discussed in Appendix A. After regularization (2.27) becomes

$$V^{(1)}(a,f)=\left[\frac{\hbar}{16\pi^2}\right]\frac{1}{4}\left[-\left[\frac{4}{\epsilon}+3-2\gamma\right]\lambda^2 f^* f + m_1{}^4\ln\left[\frac{m_1{}^2}{\mu^2}\right]+m_2{}^4\ln\left[\frac{m_2{}^2}{\mu^2}\right] - 2m'^4\ln\left[\frac{m'^2}{\mu^2}\right]\right] , \tag{2.28}$$

where $\epsilon=4-n$, μ is the renormalization point, γ is Euler's constant, and

$$m_1{}^2\equiv|m'|^2+\lambda|f| , \tag{2.29}$$

$$m_2{}^2\equiv|m'|^2-\lambda|f| . \tag{2.30}$$

The two-loop contribution to the effective potential[2,6] is

$$V^{(2)}=\frac{\hbar^2}{2}\left[\frac{\lambda}{3!}\right]^2 \times\int d^8Z d^8Z'\langle 0|T\{[\phi^3(Z)\delta^2(\bar{\theta})+\phi^{\dagger 3}(Z)\delta^2(\theta)] \times[\phi^3(Z')\delta^2(\bar{\theta}') +\phi^{\dagger 3}(Z')\delta^2(\theta')]\}|0\rangle , \tag{2.31}$$

which can be evaluated using the superfield propagators (2.24) and (2.25). Alternatively, integrating (2.31) over the θ and θ' Grassmann variables, $V^{(2)}$ becomes

$$V^{(2)}=\frac{\hbar^2}{2}\int d^4x d^4x'\langle 0|T[\mathscr{L}_I(A(x),F(x),\psi(x)) \times\mathscr{L}_I(A(x'),F(x'),\psi(x'))]|0\rangle , \tag{2.32}$$

where $\mathscr{L}_I$ is defined in (2.15). $V^{(2)}$ can then be evaluated using the component field propagators defined in (2.19)—(2.22). Either approach gives

$$V^{(2)}(a,f)=\frac{\lambda^2\hbar^2}{8}\int\frac{d^4p_1}{(2\pi)^4}\frac{d^4p_2}{(2\pi)^4}\frac{d^4p_3}{(2\pi)^4}\delta^4(p_1+p_2+p_3)(2\pi)^4 \times\frac{1}{\Delta_1\Delta_2\Delta_3}\left[(f^*m'^*m'^*+fm'm')\left[2\lambda^3 f^* f-4\lambda\frac{\Delta_2\Delta_3}{\Sigma_2\Sigma_3}+4\lambda\Sigma_2\Sigma_3\right] -4\lambda^2 f^* f(\Sigma_1\Sigma_2+2\Sigma_1 m'^* m')+4p_3{}^2\Sigma_1\Sigma_2\Sigma_3+8p_1\cdot p_3\frac{\Sigma_1\Delta_2\Delta_3}{\Sigma_2\Sigma_3}\right] , \tag{2.33}$$

where

$$\Sigma_i\equiv p_i{}^2+|m'|^2 , \tag{2.34}$$

$$\Delta_i\equiv\Sigma_i{}^2-\lambda|f|^2 . \tag{2.35}$$

To simplify this expression, and since we are mainly interested in the qualitative features of (2.33) as a function of a and f, we choose a and f to be real. In this case (2.33) can be expressed as (keep in mind that these expressions are Euclidean)

$$V^{(2)}(a,f)=\frac{\lambda^2\hbar^2}{8}\int\frac{d^4p_1}{(2\pi)^4}\frac{d^4p_2}{(2\pi)^4}\frac{d^4p_3}{(2\pi)^4}\delta^4(p_1+p_2+p_3)(2\pi)^4 \times\{D_1(m_1)D_2(m_1)+2D_1(m_1)D_2(m_2)+D_1(m_2)D_2(m_2)+4D_1(m')D_2(m') -4D_1(m')D_2(m_1)-4D_1(m')D_2(m_2) +m'^2[-3D_{123}(m_1,m_1,m_1)-D_{123}(m_1,m_2,m_2)+8D_{123}(m',m',m_1)] -2m_1{}^2D_{123}(m',m',m_1)-2m_2{}^2D_{123}(m',m',m_2)\} , \tag{2.36}$$

where

$$D_i(a)=\frac{1}{p_i^2+a^2}\ , \tag{2.37}$$

$$D_{123}(a,b,c)=D_1(a)D_2(b)D_3(c)\ . \tag{2.38}$$

The integrals in (2.36) are discussed in Appendix B and the renormalization is discussed in Appendix C. The finite part of (2.36) is given for different renormalization schemes in the relevant sections of the text.

The powers of $\hbar$ in (2.27) and (2.33) will be maintained during renormalization as a bookkeeping device, and the limit $\hbar \to 1$ will be taken in the final result. The wave-function renormalization, Z, is also expanded in a power series in $\hbar$:

$$Z=1+\hbar Z_1+\hbar^2 Z_2+\cdots\ . \tag{2.39}$$

Note that both $V^{(1)}$ and $V^{(2)}$ vanish at $f=0$ (i.e., $m_1=m_2=m'$). Thus, the two-loop corrected effective potential will vanish at $f=0$ as does the tree-level potential.

III. $V_{\rm eff}$ IN A MODIFIED MINIMAL-SUBTRACTION SCHEME

$V^{(1)}$ and $V^{(2)}$ as given in (2.28) and (2.33) can be expanded in a power series in λf. The first term in each power series is order $\lambda^2 f^2$. Thus, the identities

$$\left.\frac{\partial^2 V}{\partial a \partial f}\right|_{f=0}=m\ , \tag{3.1}$$

$$\left.\frac{\partial^3 V}{\partial^2 a \partial f}\right|_{f=0}=\lambda \tag{3.2}$$

are automatically satisfied at the two-loop level. Therefore it seems reasonable to use a modified minimal-subtraction scheme and set

$$Z_1=-\left[\frac{\lambda^2}{16\pi^2}\right]\frac{1}{4}\left[\frac{4}{\epsilon}+3-2\gamma\right] \tag{3.3}$$

and

$$Z_2=-\left[\frac{\lambda^2}{16\pi^2}\right]^2\left[\frac{1}{\epsilon^2}+\frac{(1-\gamma)}{\epsilon}+\tfrac{1}{4}(3-5\gamma+2\gamma^2)\right] \tag{3.4}$$

(see Appendix C), where the wave-function renormalization is defined

$$Z=1+Z_1+Z_2+\cdots \tag{3.5}$$

($\hbar$ has been set to 1).

The effective potential in this scheme is given by

$$V_{\rm eff}=C(\widetilde{V}^{(0)}+\widetilde{V}^{(1)}+\widetilde{V}^{(2)}+\cdots)\ , \tag{3.6}$$

where

$$C=\begin{cases}\dfrac{m^4}{\lambda^2}, & m\neq 0\ ,\\[2mm] \dfrac{\mu^4}{\lambda^2}, & m=0\ ,\end{cases} \tag{3.7}$$

$$\widetilde{V}^{(0)}=-y^2+yx^2-\xi(m)y\ , \tag{3.8}$$

$$\widetilde{V}^{(1)}=B[(x^2+y)^2\ln(x^2+y)+(x^2-y)^2\ln(x^2-y)-2x^4\ln x^2]\ , \tag{3.9}$$

$$\begin{aligned}\widetilde{V}^{(2)}=2B^2\{&[(x^2+y)\ln(x^2+y)+(x^2-y)\ln(x^2-y)-2x^2\ln x^2]^2-8x^4\ln x^2+2(x^2+y)^2\ln^2(x^2+y)\\&+2(x^2-y)^2\ln^2(x^2-y)+2x^2[(x^2+y)\ln^2(x^2+y)+(x^2-y)\ln^2(x^2-y)]\\&-4(\tfrac{3}{2}-\gamma)[(x^2-y)^2\ln(x^2+y)+(x^2-y)^2\ln(x^2-y)-2x^4\ln x^2-\tfrac{1}{2}y(x^2+y)\ln(x^2+y)+\tfrac{1}{2}y(x^2-y)\ln(x^2-y)]\}.\end{aligned} \tag{3.10}$$

The notation used here is

$$x=\begin{cases}\dfrac{m'}{m}=\dfrac{\lambda a}{m}+1, & \text{for } m\neq 0\ ,\\[2mm] \dfrac{m'}{\mu}=\dfrac{\lambda a}{\mu}, & \text{for } m=0\ ,\end{cases} \tag{3.11}$$

$$y=\begin{cases}\dfrac{\lambda f}{m^2}, & \text{for } m\neq 0\ ,\\[2mm] \dfrac{\lambda f}{\mu^2}, & \text{for } m=0\ ,\end{cases} \tag{3.12}$$

$$\xi(m)=\begin{cases}1 & \text{for } m\neq 0\ ,\\ 0 & \text{for } m=0\ ,\end{cases} \tag{3.13}$$

and

$$B=\frac{1}{4}\frac{\lambda^2}{16\pi^2}\ . \tag{3.14}$$

The effective potential defined here is a complicated but well-behaved real function of x and y if $x^2\pm y>0$. For $x^2\pm y<0$ it is complex. In $V_{\rm eff}(x,y)$ the variable y plays the role of auxiliary field and x plays the role of physical field. To find the effective potential in terms of x the equation

$$\frac{\partial \widetilde{V}(x,y)}{\partial y}=0\ , \tag{3.15}$$

where

$$\tilde{V} \equiv \tilde{V}^{(0)} + \tilde{V}^{(1)} + \tilde{V}^{(2)} , \tag{3.16}$$

must be solved for y as a function of x. This trajectory $y(x)$ is then substituted in $\tilde{V}$ to get

$$\tilde{V}(x) \equiv \tilde{V}(x,y(x)) . \tag{3.17}$$

At the tree level

$$y^{(0)}(x) = \tfrac{1}{2}[x^2 - \xi(m)] \tag{3.18}$$

and

$$V^{(0)}(x) = \tfrac{1}{4}[x^2 - \xi(m)]^2 . \tag{3.19}$$

From this point the mass will be set to zero in order to simplify the discussion. The features which will be described for the $m=0$ case will also appear in the $m \neq 0$ case, but the $m \neq 0$ case displays additional features which are not essential to the present discussion. These additional features will be described later in Sec. IV.

Consider the one-loop $\tilde{V}(x)$ with $m=0$. By substituting

$$y = \alpha x^2 \tag{3.20}$$

into (3.15) the equation

$$(1-2\alpha) + 2B[\,(1+\alpha)\ln(1+\alpha) - (1-\alpha)\ln(1-\alpha) + 2\alpha \ln x^2 + \alpha] = 0 \tag{3.21}$$

is obtained. A number of interesting observations may be made from Eq. (3.21). First, $\alpha = 1$ [i.e., the trajectory $y(x)$ goes into a region of the x-y plane where $\tilde{V}(x,y)$ is complex] at

$$x_{(+1)} = \frac{1}{\sqrt{2}} \exp\left[\frac{1}{4}\left[\frac{1}{2B} - 1\right]\right] , \tag{3.22}$$

and $\alpha = -1$ at

$$x_{(-1)} = \frac{1}{\sqrt{2}} \exp\left[\frac{1}{4}\left[\frac{3}{2B} - 1\right]\right] . \tag{3.23}$$

Note that $x_{(+1)} < x_{(-1)}$. From the limiting cases of Eq. (3.21) it can be seen that $\alpha \to 0$ from the positive side as $x \to 0$ and that $\alpha \to 0$ from the negative side as $x \to \infty$. By substituting the asymptotic value of x as a function of α from Eq. (3.21) into $\tilde{V}^{(0)} + \tilde{V}^{(1)}$ it can be seen that for large x the one-loop effective potential is negative and becomes arbitrarily large in magnitude as $x \to \infty$. This behavior is shown graphically in Figs. 1 and 2.

The one-loop trajectory, $y(x)$, has two branches (labeled I and II in Fig. 1). Branch I runs into the $y = x^2$ parabola at $x_{(+1)}$ and branch II begins at the $y = -x^2$ parabola at $x_{(-1)}$. There is a range of values of x between these two branches of $y(x)$ for which the effective potential is complex. Because of the shape of branch II, there is a region where the one-loop effective potential is double valued (see Fig. 2).

It is interesting to note that similar behavior has been observed in O(N)-symmetric ϕ^4 theories in the large-N limit.[16] In these calculations nonpropagating auxiliary fields were included in the Lagrangian as a computational trick.

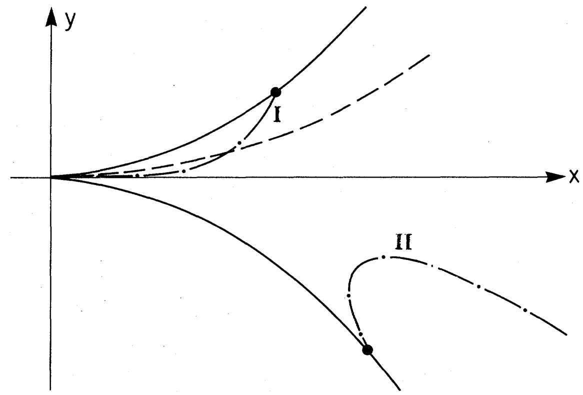

FIG. 1. The trajectory $y(x)$ in the tree-level (dashed) and one-loop (dashed-dotted) approximations in the modified minimal-subtraction scheme. The one-loop trajectory has two branches, I and II. Solid parabolas are $y = \pm x^2$. Units are arbitrary.

Supersymmetric models with a global O(N) symmetry also display this unusual behavior.[7–10] It is possible in these models for the negative branch of the effective potential to have a stationary point, i.e., a value of x where

$$\frac{\partial}{\partial x} \tilde{V}(x,y(x)) = 0 . \tag{3.24}$$

It is these points which have been interpreted as an indication of the spontaneous breaking of supersymmetry. In the Wess-Zumino model there is no stationary point on the negative branch of V_{eff} for $B < \frac{1}{4}$ (i.e., $\lambda < 4\pi$).

Adding one-loop corrections creates major qualitative changes in the effective potential. Two-loop corrections again make qualitative changes. Results of numerical calculations of $y(x)$ are shown in Figs. 3 and 4. The derivative in (3.15) diverges as y approaches $\pm x^2$. Thus, the trajectory $y(x)$ cannot run up against the parabola $y = \pm x^2$ as it did in the one-loop case. Instead the trajectory tracks along these parabolas approaching them asymptotically. This is shown in Fig. 3. The two-loop trajectory has a

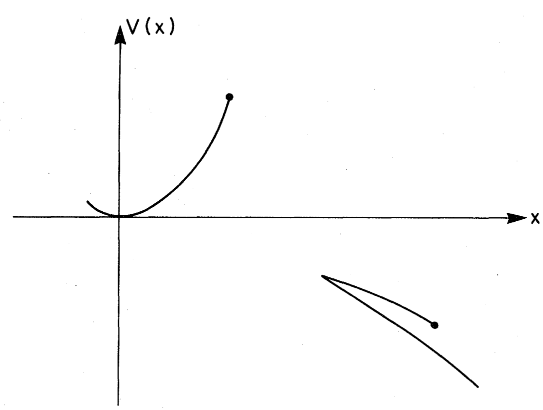

FIG. 2. The one-loop effective potential in the modified minimal-subtraction scheme. Units are arbitrary.

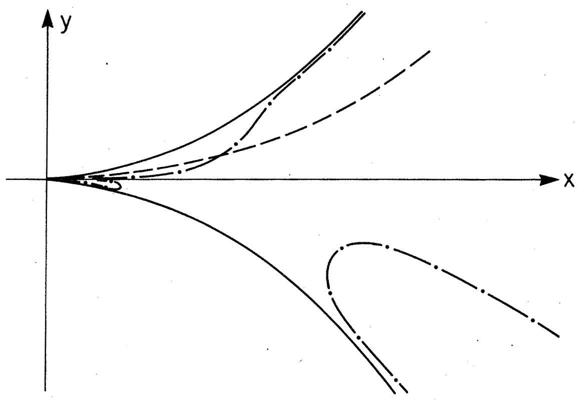

FIG. 3. The trajectory $y(x)$ in the tree-level (dashed) and two-loop (dashed-dotted) approximations in the modified minimal-subtraction scheme.

third branch which is very close to the origin of the x-y plane. This third branch is shown in Fig. 4. The two-loop effective potential is real everywhere on the x axis but is triple valued in places. In particular, the effective potential is triple valued at points infinitesimally close to the origin $x=0$.

The pathological behavior exhibited by the effective potential in Eqs. (3.8)—(3.10) is due to the use of an inappropriate renormalization scheme (minimal subtraction). This was pointed out by Amati and Chou[10] in reference to the model of Zanon.[7] To see this, consider the coefficient of f^*f in the effective *action*. To one loop it is given by

$$-\left.\frac{\partial^2 V_{\rm eff}}{\partial f^*\partial f}\right|_{f=f^*=0} = 1-\left[\frac{\lambda^2}{16\pi^2}\right]\frac{1}{4}\left[2\ln\left[\frac{m'^2}{\mu^2}\right]+3\right], \quad (3.25)$$

and the two-loop contribution is

$$-\left[\frac{\lambda^2}{16\pi^2}\right]^2\frac{1}{4}\left[2\ln^2\left[\frac{m'^2}{\mu^2}\right]+5\ln\frac{m'^2}{\mu^2}-2\right]. \quad (3.26)$$

The coefficient of $a^*\Box a$ in the effective action must be given by (3.25)—(3.26). This is because $a^*\Box a$ and f^*f both arise from the $\phi^\dagger\phi$ term in the Lagrangian (2.6). From (3.25) it can be seen that for large enough m' ($m'=\lambda a$) the coefficient of $a^*\Box a$ becomes negative. Hence, the kinetic energy term of the physical field, a, has the wrong sign for large a. The value of a for which (3.25) changes sign roughly corresponds to the value of x where the one-loop effective potential becomes complex (see Fig. 2).

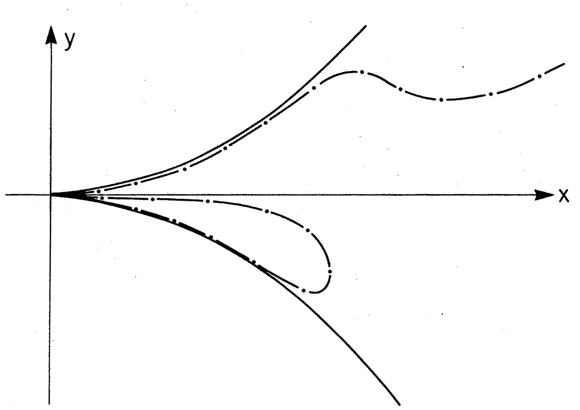

FIG. 4. The two-loop trajectory $y(x)$ in the modified minimal-subtraction scheme for $x \ll 1$.

When the two-loop contribution (3.26) is added the coefficient of $a^*\Box a$ becomes negative for small values of a as well as for large values. This is because of the logarithm-squared term in (3.26). The appearance of ghosts at small a is reflected by the triple valuedness of the effective potential at small x (see Fig. 4).

It can now be seen that the decreased number of renormalization constants can cause complications in the calculation of the effective potential. If the coupling constant and wave-function renormalizations were independent (as they are in $\lambda\phi^4$ theories), then the coefficient of $a^*\Box a$ changing signs would not be a problem. The independent wave-function renormalization could be adjusted to keep (3.25)—(3.26) positive.

In supersymmetric models all of the above must be taken into account in the choice of the renormalization prescription. This is done in Sec. IV.

IV. A NO-GHOST SCHEME

The renormalization must be done in such a way as to keep the kinetic terms in the effective action positive and to remove the divergent terms. The easiest way to do this is to impose the renormalization condition[2,14,17]

$$\left.\frac{\partial^2 V_{\rm eff}}{\partial f^*\partial f}\right|_{f=f^*=0} = -1 . \quad (4.1)$$

In this case

$$Z_1 = -\left[\frac{\lambda^2}{16\pi^2}\right]\frac{1}{4}\left[\frac{4}{\epsilon}-2\gamma-2\ln\frac{m'^2}{\mu^2}\right] \quad (4.2)$$

and

$$Z_2 = -\left[\frac{\lambda^2}{16\pi^2}\right]^2\left[\frac{1}{\epsilon^2}-\left[\tfrac{1}{2}+\gamma+\ln\frac{m'^2}{\mu^2}\right]\frac{1}{\epsilon} + \tfrac{1}{2}(\gamma^2-3\gamma+4)\right] \quad (4.3)$$

(see Appendix C).

The one-loop running coupling can be determined from (2.5) and (4.2):

$$\lambda^2(m'^2) = \frac{\lambda^2(\mu^2)}{1-\frac{3}{2}[\lambda^2(\mu^2)/16\pi^2]\ln(m'^2/\mu^2)} . \quad (4.4)$$

$\lambda^2(\mu^2)$ is the coupling measured when $m'=m+\lambda a=\mu$ and $\lambda^2(m'^2)$ is the coupling at any other value of m'. This dependence of λ on a was discussed in Ref. 10 and noticed for small a in Refs. 4 and 9. From (4.4) it can be seen that the model is not asymptotically free. As a increases the coupling λ increases and there will be a point

($\lambda^2=4\pi$ or $B=\frac{1}{4}$) beyond which the perturbation expansion does not make sense.

The two-loop β function is discussed in Appendix C [see (C11)] and gives the following equation for the coupling λ:[18]

$$\mu\frac{\partial\lambda}{\partial\mu}=\lambda\left[\frac{3}{2}\left[\frac{\lambda^2}{16\pi^2}\right]-\frac{3}{2}\left[\frac{\lambda^2}{16\pi^2}\right]^2\right] . \qquad (4.5)$$

We solved this equation numerically and found that the two-loop coupling, λ, reaches 4π at a slightly larger value of the field a than does the one-loop coupling.

In the minimal-subtraction scheme the coupling becomes large at values of x corresponding to complex effective potential for one loop (see Fig. 2) and to the large-x triple-valued effective potential for two loops (Fig. 3). The triple valuedness of the two-loop effective potential at small x (see Fig. 4) is due to the logarithm-squared terms which appear in the inappropriate minimal-subtraction scheme [for example, in (3.26)].

Using the scheme (4.1) the problems encountered in Sec. III are circumvented. The effective potential is

$$V_{\text{eff}}=C(\widetilde{V}^{(0)}+\widetilde{V}^{(1)}+\widetilde{V}^{(2)}+\cdots) , \qquad (4.6)$$

where C is defined in (3.7) and

$$\widetilde{V}^{(0)}=-y^2+yx^2-\xi(m)y , \qquad (4.7)$$

$$\widetilde{V}^{(1)}=B[(x^2+y^2)^2\ln(x^2+y)+(x^2-y)^2\ln(x^2-y)-2x^4\ln x^2-(3+2\ln x^2)y^2] , \qquad (4.8)$$

$$\begin{aligned}\widetilde{V}^{(2)}=2B^2\{&[(x^2+y)\ln(x^2+y)+(x^2-y)\ln(x^2-y)-2x^2\ln x^2]^2-8x^4\ln^2x^2+2(x^2+y)^2\ln^2(x^2+y)\\&+2(x^2-y)^2\ln^2(x^2-y)+2x^2[(x^2+y)\ln^2(x^2+y)+(x^2-y)\ln^2(x^2-y)]\\&-4(3-\gamma+\ln x^2)[(x^2+y)^2\ln(x^2+y)+(x^2-y)^2\ln(x^2-y)-2x^4\ln x^2\\&-\tfrac{1}{2}y(x^2+y)\ln(x^2+y)+\tfrac{1}{2}y(x^2-y)\ln(x^2-y)]+4[(4-2\gamma)+(1-\gamma)\ln x^2]y^2\} . \qquad (4.9)\end{aligned}$$

B, x, y, and $\xi(m)$ are defined in (3.11)–(3.14).

First consider the one-loop effective potential

$$\widetilde{V}(x,y)=\widetilde{V}^{(0)}(x,y)+\widetilde{V}^{(1)}(x,y) .$$

The function

$$\widetilde{V}(x,y)-y\frac{\partial}{\partial y}\widetilde{V}(x,y)=y^2+B\left[(x^4-y^2)\ln\left[\frac{x^4-y^2}{x^4}\right]+y^2\right] \qquad (4.10)$$

is non-negative. Thus along the trajectory $y(x)$, which is the solution of

$$\frac{\partial\widetilde{V}(x,y)}{\partial y}=0 , \qquad (4.11)$$

$\widetilde{V}(x,y(x))$ must be non-negative.

Substituting

$$y(x)=\alpha x^2 \qquad (4.12)$$

into (4.11) with $m=0$ the equation

$$(1-2\alpha)+2B[(1+\alpha)\ln(1+\alpha)-(1-\alpha)\ln(1-\alpha)-2\alpha]=0 \qquad (4.13)$$

is obtained. This is a transcendental equation for α which may be solved numerically. For $B=0$ the solution is the tree-level $\alpha=\frac{1}{2}$. As B increases α decreases monotonically to $\alpha=0.487$ at $B=\frac{1}{4}$.

Substitute (4.12) into $\widetilde{V}(x,y)$ to get

$$\widetilde{V}(x,y(x))=x^4\delta(B) , \qquad (4.14)$$

where

$$\delta(B)=(-\alpha^2+\alpha)+B[(1+\alpha)^2\ln(1+\alpha)+(1-\alpha)^2\ln(1-\alpha)-3\alpha^2] , \qquad (4.15)$$

with α in (4.15) determined by Eq. (4.13). For $B=0$, $\delta=\frac{1}{4}$. As B increases δ decreases monotonically to $\delta=0.247$ at $B=\frac{1}{4}$.

Thus the bizarre behavior of the one-loop trajectory and effective potential in the minimal-subtraction scheme is replaced by boring behavior when the condition (4.1) is used. The one-loop effective potential is proportional to x^4. It is even well-behaved for values of x where the perturbation expansion has no meaning.

When the two-loop contribution (4.9) is included, analysis similar to that discussed in (4.10) through (4.15) does not work. Numerical computations, however, show that both the trajectory and the effective potential stay very close to the one-loop case described above. In particular, nothing unusual happens for small values of x. The two-loop $\widetilde{V}(x)$ is a well-behaved non-negative function of x which vanishes only at $x=0$. This is true up to a B-dependent (and therefore λ-dependent) point, x_B, beyond which $\widetilde{V}(x)$ is multivalued and negative along one of its branches. It is possible that this behavior is a result of the approximation made in evaluating the integrals in Eq. (2.36) (see Appendix C). In any case no significance can be attached to this behavior since x_B is well beyond the point where perturbation theory is valid. To illustrate this let x_p be where λ becomes 4π (B becomes $\frac{1}{4}$) using the two-loop running coupling defined by Eq. (4.5). For $B=0.2$, $x_p=1.17$ and $x_B\simeq 89$. For $B=0.15$, $x_p=1.48$ and $x_B\simeq 226$. As B decreases x_B increases very rapidly and $(x_p/x_B)\to 0$ as $B\to 0$.

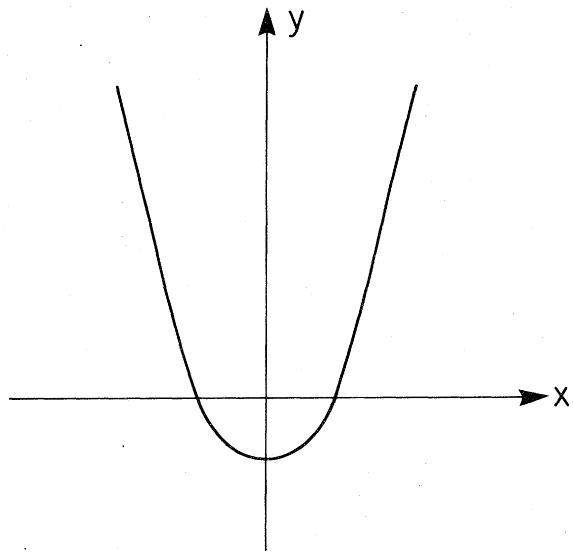

FIG. 5. The tree-level trajectory for $m \neq 0$.

The two-loop effective potential calculated with the renormalization condition (4.1) is qualitatively the same as the tree-level potential. It is quantitatively very close to the tree-level potential for values of the field where the perturbative expansion is good.

All of the above results apply equally to the $m \neq 0$ case. There are, however, a few subtleties involved in the massive case. The tree-level trajectory,

$$y^{(0)}(x)=\tfrac{1}{2}(x^2-1) \tag{4.16}$$

and potential

$$V^{(0)}(x)=\tfrac{1}{4}(x^2-1)^2 \tag{4.17}$$

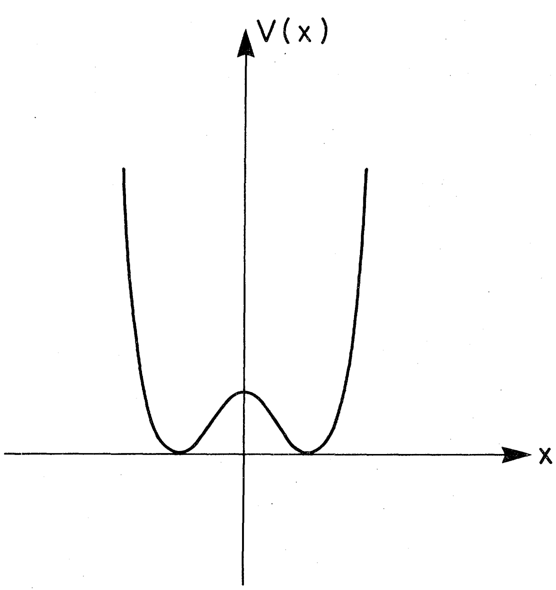

FIG. 6. The tree-level potential for $m \neq 0$.

are shown in Figs. 5 and 6. Both of these approach the $m=0$ results for large x. This continues at higher order. The minima in $V^{(0)}(x)$ at $x=\pm 1$ [$a=0$, $(-2m/\lambda)$] are also present at higher order. The differences at one-loop and two-loop occur between these minima (i.e., for $-1<x<1$). The tree-level trajectory (see Fig. 5) crosses the parabola $y=-x^2$. The one-loop trajectory runs into this parabola so there is a region around $x=0$ where the one-loop effective potential is complex. The two-loop trajectory approaches the $y=-x^2$ parabola asymptotically as x decreases and passes through the origin. The two-loop effective potential is everywhere real and non-negative but has a third minimum at $x=0$. No physical significance can be placed on any of these features since the perturbative expansion for the effective potential between the minima of a double-well potential (such as in Fig. 6) is known to be untrustworthy.[19]

V. SUMMARY AND CONCLUSIONS

We have shown that the vacuum of the Wess-Zumino model is stable under quantum fluctuations. Supersymmetry is not broken by radiative corrections to the effective potential. In order to show this it was necessary to use a renormalization procedure which preserved the positivity of the kinetic terms in the effective action. If a modified minimal-subtraction scheme is used instead, then the effective potential displays pathological behavior such as multivaluedness, complexity, and negativity. Care must be taken in the choice of renormalization prescription because the Wess-Zumino model possesses only one independent renormalization constant. A bad choice of renormalization scheme cannot be compensated by adjusting a second independent renormalization constant.

ACKNOWLEDGMENTS

We would like to thank Glenn Starkman for assistance with the numerical computations. One of us (K.S.V.) expresses appreciation to the TRIUMF theory group for their hospitality.

APPENDIX A: ONE-LOOP EFFECTIVE POTENTIAL

In this appendix the one-loop contribution to the effective potential is discussed to order $\hbar^2$. $V^{(1)}$ is given in (2.27) as

$$V^{(1)}=\frac{\hbar}{2}\int\frac{d^4p}{(2\pi)^4}\ln\left[\frac{(Z^2p^2+|m'|^2)^2-Z^2\lambda^2|f|^2}{(Z^2p^2+|m'|^2)^2}\right]. \tag{A1}$$

Dimensional regularization will be used (although other regularization techniques, such as Pauli-Villars, give the same result), so re-express (A1) as

$$V^{(1)}=\frac{\hbar}{2}\frac{1}{Z^n}\int\frac{d^np}{(2\pi)^n}\ln\left[1-\frac{\lambda^2|f'|^2}{(p^2+|m'|^2)^2}\right], \tag{A2}$$

where the change of variables $p \to p/Z$ has been made and where

$$|f'| \equiv Z|f| \ . \tag{A3}$$

Performing the integration in (A2) and expanding Z as

$$Z = 1\ \hbar Z_1 + \cdots \ , \tag{A4}$$

$V^{(1)}$ becomes

$$V^{(1)} = \hbar V_1^{(1)} + \hbar^2 V_2^{(1)} + \cdots \ , \tag{A5}$$

where

$$V_1^{(1)} = \frac{1}{4}\left[\frac{1}{16\pi^2}\right]\left[-\left[\frac{4}{\epsilon}+3-2\gamma\right]\lambda^2 f^* f + m_1{}^4 \ln\left[\frac{m_1{}^2}{\mu^2}\right] + m_2{}^4 \ln\left[\frac{m_2{}^2}{\mu^2}\right] - 2m'^4 \ln\left[\frac{m'^2}{\mu^2}\right]\right] \tag{A6}$$

and

$$V_2^{(1)} = \frac{1}{4}\left[\frac{1}{16\pi^2}\right] Z_1 \left\{ 2\left[\frac{4}{\epsilon}+2-2\gamma\right]\lambda^2 f^* f - 4\left[m_1{}^4 \ln\left[\frac{m_1{}^2}{\mu^2}\right] + m_2{}^4 \ln\left[\frac{m_2{}^2}{\mu^2}\right] - 2m'^4 \ln\left[\frac{m'^2}{\mu^2}\right] - \tfrac{1}{2}(\lambda f) m_1{}^2 \ln\left[\frac{m_1{}^2}{\mu^2}\right] + \tfrac{1}{2}(\lambda f) m_2{}^2 \ln\left[\frac{m_2{}^2}{\mu^2}\right]\right]\right\} . \tag{A7}$$

In (A6) and (A7) μ is the renormalization point, γ is Euler's constant, $\epsilon = 4 - n$,

$$m_1{}^2 = m'^2 + \lambda f \ , \tag{A8}$$

and

$$m_2{}^2 = m' - \lambda f \ . \tag{A9}$$

The $\hbar^2$ part of the one-loop contribution to the effective potential, $V_2^{(1)}$, will be needed to cancel $(1/\epsilon)\times$ logarithm terms in the two-loop contribution.

APPENDIX B: EVALUATION OF THE TWO-LOOP INTEGRALS

Two types of integrals appear in (2.36):

$$I_A(a,b) = \int \frac{d^4 p_1}{(2\pi)^4}\frac{d^4 p_2}{(2\pi)^4} D_1(a) D_2(b) \ , \tag{B1}$$

$$I_B(a,a,b) = \int \frac{d^4 p_1}{(2\pi)^4}\frac{d^4 p_2}{(2\pi)^4}\frac{d^4 p_3}{(2\pi)^4}\delta^{(4)}(p_1+p_2+p_3)(2\pi)^4 \times D_1(a) D_2(a) D_3(b) \ , \tag{B2}$$

where

$$D_i(a) = \frac{1}{p_i{}^2 + a^2} \ . \tag{B3}$$

The integral $I_A(a,b)$ is straightforward:

$$I_1(a,b) = \frac{a^2 b^2}{(16\pi^2)}\left\{\frac{4}{\epsilon^2} + \frac{2}{\epsilon}\left[2(1-\gamma) - \left[\ln\frac{a^2}{\mu^2} + \ln\frac{b^2}{\mu^2}\right]\right] + \left[(3-4\gamma+\gamma^2) - 2(1-\gamma)\left[\ln\frac{a^2}{\mu^2}+\ln\frac{b^2}{\mu^2}\right] + \frac{1}{2}\left[\ln\frac{a^2}{\mu^2}+\ln\frac{b^2}{\mu^2}\right]^2\right]\right\} . \tag{B4}$$

Factors of $\ln(4\pi)$ and order ϵ terms in the expansion of $\Gamma(\epsilon)$ are ignored since they either cancel or may be absorbed into renormalization counterterms.

The integral I_B is more complicated. Use the identities

$$1 = \frac{1}{2n}\sum_{i,j}\left[\frac{\partial p_{1i}}{\partial p_{1j}} + \frac{\partial p_{2i}}{\partial p_{2j}}\right]\delta_i^j \ , \tag{B5}$$

$$1 = \frac{1}{n}\sum_{i,j}\frac{\partial p_{1i}}{\partial p_{1j}}\delta_i^j \ , \tag{B6}$$

$$1 = \frac{1}{n}\sum_{i,j}\frac{\partial p_{2i}}{\partial p_{2j}}\delta_i^j \ , \tag{B7}$$

once each, integrate by parts and rearrange to obtain

$$I_B = \frac{2}{(n-3)(n-4)}[\,2a^4 I_1 + 4a^2 b^2 I_2 + (2a^2+b^2) I_3\,] \ , \tag{B8}$$

where

$$I_1 = \int (dp_1)(dp_2)\Delta_1(b)\Delta_2{}^2(a)\Delta_{1+2}{}^2(a) \ , \tag{B9}$$

$$I_2 = \int (dp_1)(dp_2)\Delta_1{}^2(b)\Delta_2(a)\Delta_{1+2}{}^2(a) \ , \tag{B10}$$

$$I_3 = \int (dp_1)(dp_2)(p_1{}^2 + p_1\cdot p_2)\Delta_1{}^2(b)\Delta_2(a)\Delta_{1+2}{}^2(a) \ , \tag{B11}$$

with

$$(dp_i)\equiv\frac{d^n p_i}{(2\pi)^n} \tag{B12}$$

and

$$\Delta_{1+2}(a)\equiv\frac{1}{(p_1+p_2)^2+a^2}\ . \tag{B13}$$

The integrals (B9)–(B11) can be evaluated using dimensional regularization. In terms of $\epsilon\equiv4-n$, (B8) becomes

$$I_B=\frac{-(a^2)^{-\epsilon}}{(16\pi^2)^2}(4\pi)^\epsilon\frac{\Gamma(\epsilon)}{1-\epsilon}\times\left[4a^2I_a+4b^2I_b+(2a^2+b^2)\frac{4-\epsilon}{\epsilon}I_c\right]\ , \tag{B14}$$

where

$$I_a=\int_0^1 dx\,dy\,y[x(1-x)y]^{\epsilon/2}\times[y+Ax(1-x)(1-y)]^{-(1+\epsilon)}\ , \tag{B15}$$

$$I_b=\int_0^1 dx\,dy(1-y)[x(1-x)y]^{\epsilon/2}\times[y+Ax(1-x)(1-y)]^{-(1+\epsilon)}\ , \tag{B16}$$

$$I_c=\int_0^1 dx\,dy(1-y)[x(1-x)y]^{\epsilon/2}\times[y+Ax(1-x)(1-y)]^{-\epsilon}\ , \tag{B17}$$

with

$$A\equiv\frac{b^2}{a^2}\ . \tag{B18}$$

Expand (B14) in powers of ϵ. The following identities are useful:

$$\int_0^1 dx\frac{1-Ax^2}{[1-Ax(1-x)]^2}=1\ , \tag{B19}$$

$$(2-\tfrac{1}{2}A)\int_0^1 dx\frac{1}{1-Ax(1-x)}+(4-A)\int_0^1 dx\frac{\ln x}{[1-Ax(1-x)]^2}-2\int_0^1 dx\frac{\ln x}{1-Ax(1-x)}=0\ . \tag{B20}$$

Expand the left-hand side of (B20) in a power series to prove the last identity. Using (B19) and (B20), I_B becomes

$$I_B(a,a,b)=\frac{-1}{(16\pi^2)^2}\left\{\frac{2}{\epsilon^2}(2a^2+b^2)+\frac{1}{\epsilon}\left[(3-2\gamma)(2a^2+b^2)-2\left[2a^2\ln\frac{a^2}{\mu^2}+b^2\ln\frac{b^2}{\mu^2}\right]\right]+\left[\left[2a^2\ln^2\frac{a^2}{\mu^2}+b^2\ln^2\frac{b^2}{\mu^2}\right]-(3-2\gamma)\left[2a^2\ln\frac{a^2}{\mu^2}+b^2\ln\frac{b^2}{\mu^2}\right]+(3-3\gamma)(2a^2+b^2)\right]\right\}+a^2J(A)\ , \tag{B21}$$

where $J(A)$ is a function of $A=(b^2)/(a^2)$ which cannot be expressed in terms of elementary functions. $J(A)$ is given by

$$J(A)=\frac{-1}{(16\pi^2)^2}\{A\ln A-A\ln^2A+4[I_{a_1}(A)+AI_{b_1}(A)+(2+A)I_{c_2}(A)]-(2+A)I_{c_1}(A)\}\ , \tag{B22}$$

where I_{a_1}, etc., are defined by expanding I_a, I_b, and I_c in powers of ϵ:

$$I_a=I_{a0}+\epsilon I_{a_1}+\cdots\ , \tag{B23}$$

$$I_b=I_{b0}+\epsilon I_{b_1}+\cdots\ , \tag{B24}$$

$$I_c=I_{c0}+\epsilon I_{c_1}+\epsilon^2 I_{c_2}+\cdots\ . \tag{B25}$$

Note that if $a=b$ then $J(A)=J(1)$ is just a constant.

APPENDIX C: RENORMALIZATION AND β FUNCTION

Divide $V^{(2)}$ as follows:

$$V^{(2)}=V_a^{(2)}+V_b^{(2)}+V_c^{(2)}\ , \tag{C1}$$

where $V_a^{(2)}$ is the part of $V^{(2)}$ which involves integrals of the type I_A [Eq. (B1)] and where $V_b^{(2)}+V_c^{(2)}$ is the part of $V^{(2)}$ which involves integrals of the type I_B [Eq. (B2)]. $V_c^{(2)}$ contains all the $J(A)$ terms [see (B21)] and $V_b^{(2)}$ contains the rest.

Substituting (B4) into (2.36), $V_a^{(2)}$ is seen to be

$$V_a^{(2)}=\frac{\hbar^2\lambda^2}{(16\pi^2)^2}\frac{1}{8}\left[m_1{}^2\ln\frac{m_1{}^2}{\mu^2}+m_2{}^2\ln\frac{m_2{}^2}{\mu^2}-2m'^2\ln\frac{m'^2}{\mu^2}\right]^2\ . \tag{C2}$$

Note that all pole terms in $V_a^{(2)}$ have canceled.

Substituting (B21) with $J(A)=0$ into (2.36) $V_b^{(2)}$ is

$$V_b^{(2)}=\frac{\lambda^2\hbar^2}{(16\pi^2)^2}\left\{\frac{1}{\epsilon}(\lambda^2f^2)+\frac{1}{\epsilon}[(\tfrac{3}{2}-\gamma)\lambda^2f^2-(m_1{}^4\ln m_1{}^2+m_2{}^4\ln m_2{}^2-2m'^4\ln m'^2-\tfrac{1}{2}m_1{}^2\lambda f\ln m_1{}^2+\tfrac{1}{2}m_2{}^2\lambda f\ln m_2{}^2)]\right.$$
$$+\tfrac{1}{8}[-8m'^4\ln^2 m'^2+2m_1{}^4\ln^2 m_1{}^2+2m_2{}^4\ln^2 m_2{}^2+2m'^4(m_1{}^2\ln^2 m_1{}^2+m_2{}^2\ln^2 m_2{}^2)]$$
$$\left.-(\tfrac{3}{2}-\gamma)[m_1{}^4\ln m_1{}^2+m_2{}^4\ln m_2{}^2-2m'^4\ln m'^2-\tfrac{1}{2}m_1{}^2\lambda f\ln m_1{}^2+\tfrac{1}{2}m_2{}^2\lambda f\ln m_2{}^2]\right\}. \tag{C3}$$

Note that $V_a^{(2)}$ and $V_b^{(2)}$ are invariant under $f\to -f$ (i.e., $m_1\leftrightarrow m_2$). This will not be true of $V_c^{(2)}$, which will be discussed later in this appendix.

All the pole terms in $V^{(2)}$ are in $V_b^{(2)}$. Considering (2.12) with Z expanded as in (2.39), $V_1^{(1)}$ and $V_2^{(1)}$ as given in (A6) and (A7), and $V_b^{(2)}$ in (C3) it can be seen that if

$$Z_1=-\left[\frac{\lambda^2}{16\pi^2}\right]\frac{1}{\epsilon}+(\text{finite}) \tag{C4}$$

and

$$Z_2=\left[\frac{\lambda^2}{16\pi^2}\right]^2\left[\frac{1}{\epsilon^2}+\left[\frac{3-2\gamma}{2}\right]\frac{1}{\epsilon}\right]+\frac{\lambda^2}{16\pi^2}Z_1\left[\frac{2}{\epsilon}+1-\gamma\right]+(\text{finite})\,, \tag{C5}$$

then all pole terms in the effective potential cancel. In particular the $(1/\epsilon)\times$ logarithm terms cancel with Z_1 defined as in (C4).

The finite parts of (C4) and (C5) are determined by the renormalization prescription. In order to compute the two-loop β function, and for reference, the values of Z_1 and Z_2 in the modified minimal-subtraction scheme are

$$Z_1=-\frac{1}{4}\left[\frac{\lambda^2}{16\pi^2}\right]\left[\frac{4}{\epsilon}+3-2\gamma\right], \tag{C6}$$

$$Z_2=-\left[\frac{\lambda^2}{16\pi^2}\right]^2\left[\frac{1}{\epsilon^2}+\frac{(1-\gamma)}{\epsilon}+\text{constant}\right], \tag{C7}$$

and in the scheme of Eq. (4.1) they are

$$Z_1=-\frac{\lambda^2}{16\pi^2}\frac{1}{4}\left[\frac{4}{\epsilon}-2\gamma-2\ln\frac{m'^2}{\mu^2}\right], \tag{C8}$$

$$Z_2=-\left[\frac{\lambda^2}{16\pi^2}\right]^2\left[\frac{1}{\epsilon^2}-\left[\tfrac{1}{2}+\gamma+\ln\frac{m'^2}{\mu^2}\right]\frac{1}{\epsilon}+\text{constant}\right]. \tag{C9}$$

The β function is given by (for a discussion of the calculation of two-loop β functions using dimensional regularization see Ref. 20)

$$\beta(\lambda)=\mu\frac{\partial\lambda}{\partial\mu}=\frac{-\epsilon/2}{(\partial/\partial\lambda)[\ln(\lambda Z^{-3/2})]}\,. \tag{C10}$$

Either scheme gives the same result:[18]

$$\beta(\lambda)=\lambda\left[\frac{3}{2}\left[\frac{\lambda^2}{16\pi^2}\right]-\frac{3}{2}\left[\frac{\lambda^2}{16\pi^2}\right]^2+\cdots\right]. \tag{C11}$$

The remaining finite part of the effective potential is obtained by substituting $J(A)$ [Eq. (B22)], where $A=(b^2/a^2)$, from $I_B(a,a,b)$ into (2.36) to get

$$V_c^{(2)}=\frac{\lambda^2\hbar^2}{(16\pi^2)^2}(\tfrac{1}{8})m'^4\left[3(1+z)J(1)+(1-z)J\left[1+\frac{2z}{1-z}\right]-8J(1+z)+2(1+z)J(1+z)+2(1-z)J(1-z)\right], \tag{C12}$$

where

$$z\equiv\frac{\lambda f}{m'^2}\,. \tag{C13}$$

In the notation of Eqs. (3.11)—(3.14)

$$z=\frac{y}{x^2} \tag{C14}$$

and

$$\tilde{V}_c^{(2)}=2B^2x^4\left[3(1+z)J(1)+(1-z)J\left[1+\frac{2z}{1-z}\right]-8J(1+z)+2(1+z)J(1+z)+2(1-z)J(1-z)\right]. \tag{C15}$$

Expand in powers of z (note that $-1<z<+1$ if $V^{(2)}$ is to be real) to get

$$\tilde{V}_c^{(2)}=2B^2x^4\{[2J(1)-6J'(1)]z+[4J'(1)]z^2+[2J''(1)]z^3+\cdots\}\,. \tag{C16}$$

Now $V^{(2)}$ in the form (2.33) can be expanded in a power series in λf. If this is done it is seen that the first term in the series is order $(\lambda f)^2$. The first term in the power series for $V_a^{(2)}$ and $V_b^{(2)}$ is also order $(\lambda f)^2$, so the coefficient of the term linear in z in (C16) must vanish. By the same technique it can be shown that the coefficient of z^3 does not vanish. Thus, $V_c^{(2)}$ is not symmetric in f.

Note that

$$\frac{\partial \widetilde{V}_c^{(2)}}{\partial y}=2B^2x^2\{[8J'(1)]z+[6J''(1)]z^2+\cdots\}\ . \tag{C17}$$

Along a parabola $y=\alpha x^2$, α a constant, this becomes

$$\left.\frac{\partial \widetilde{V}_c^{(2)}}{\partial y}\right|_{y=\alpha x^2}=2B^2x^2\gamma(\alpha)\ , \tag{C18}$$

where $\gamma(\alpha)$ is a number depending on α. Note that, because of $\ln x^2$ terms, $V_a^{(2)}$ and $V_b^{(2)}$ are not so simple on these parabolas. In the discussion of the two-loop trajectory $y(x)$ using the renormalization scheme (4.1) the contribution of (C18) was ignored. It was found that $y(x)$ lies very close to the parabola $y=(\frac{1}{2})x^2$. The effect of neglecting the contribution from (C18) will be to shift the trajectory a small amount from its true value. Thus approximating $V^{(2)}$ by ignoring $V_c^{(2)}$ will have a small quantitative but no qualitative effect on the results discussed in this paper. It is also straightforward to show (although the functional analysis is somewhat involved) that terms of the form (C18) cannot remove the triple valuedness of $y(x)$ and $V_{\text{eff}}(x)$ near $x=0$ in the modified minimal-subtraction scheme (see Fig. 4).

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