First-order phase transitions in a U(1)-lattice-gauge—Higgs theory

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A U(1) lattice-gauge theory coupled to a radially variable scalar (Higgs) field in the fundamental representation is studied using Monte Carlo techniques. We present the phase diagram of the theory at zero temperature on a 4^4 Euclidean lattice. First-order Higgs-Coulomb transitions, which suggest the realization of the Coleman-Weinberg mechanism, and first-order Higgs-confinement transitions are observed.

I. INTRODUCTION

It is well established that lattice regularization is a powerful method to study nonperturbative effects in gauge theories. Among various lattice-gauge theories, those coupled to scalar fields have been studied as a prototype of the Weinberg-Salam theory, or as an effective factory to study gauge-fermion systems. The phase diagram of these theories was first studied analytically by several authors,[1] who, for simplicity, froze out the radial mode of the Higgs field. Some authors[2–4] then made Monte Carlo studies of these radially frozen models and observed their rich phase structures. None of them, however, found first-order phase transitions in the weak-gauge-coupling region so it remained an open question if the Coleman-Weinberg mechanism[5] works on a lattice or not.

In previous papers[6] Munehisa and the author studied a Z_2 gauge-scalar system on a lattice without fixing the magnitudes of the scalar field. We pointed out that there exist some phase transitions driven by the radial degrees of freedom of the scalar field and that these degrees of freedom might be essential to construct more realistic models. Recently several authors[7–9] investigated Z_N, SU(2), and SU(3) lattice-gauge—Higgs theories taking radial fluctuations into account. Their results are somewhat confusing; some of them indicate effects of the radial degrees of freedom while others do not. None of them established the Coleman-Weinberg mechanism.

In this paper we present results from a Monte Carlo study of a U(1)-lattice-gauge theory which is coupled to a radially variable scalar field in the fundamental representation. We will see that this theory undergoes first-order phase transitions in the weak-gauge-coupling region when the self-coupling constant of the scalar field is small enough. This suggests that the spontaneous symmetry breaking of the vacuum discussed by Coleman and Weinberg is realized in our model. We also see that radial degrees of freedom of the scalar field cause first-order phase transitions in the strong-gauge-coupling region.

This paper is organized as follows. In Sec. II we define the model and discuss some limiting cases. We also describe the methods of Monte Carlo simulations briefly. Details of our methods are in Ref. 6. The results of the simulations are presented in Sec. III. Finally, Sec. IV is devoted to discussions.

II. THE MODEL AND THE METHODS

We introduce a link variable $U_{i,\mu}$ located on a link from one site i to a neighboring site $i+\hat{\mu}$, where $\hat{\mu}$ denotes the unit vector in the μ direction, on a four-dimensional hypercubic lattice. We also introduce a site variable ϕ_i sitting on site i, which is written as

$$\phi_i = R_i V_i \ , \tag{2.1}$$

where R_i denotes $|\phi_i|$. Both $U_{i,\mu}$ and V_i are elements of the gauge group U(1) while R_i is a real, continuous number which ranges from 0 to $+\infty$.

The action of the model is

$$S = S_G + S_H \ , \tag{2.2}$$

where

$$S_G = \beta_g \sum_{\square} [1 - \tfrac{1}{2}(U_{i,\mu} U_{i+\hat{\mu},\nu} U^\dagger_{i+\hat{\nu},\mu} U^\dagger_{i,\nu} + \text{H.c.})] \ , \tag{2.3}$$

with i, $i+\hat{\mu}$, $i+\hat{\mu}+\hat{\nu}$, and $i+\hat{\nu}$ circulating around the plaquette $\square$, and

$$S_H = \sum_i [\lambda(\phi_i \phi_i^\dagger)^2 + m^2 \phi_i \phi_i^\dagger] + \sum_{i,\mu} [2\phi_i\phi_i^\dagger - (\phi_i U_{i,\mu} \phi^\dagger_{i+\hat{\mu}} + \text{H.c.})] \ . \tag{2.4}$$

The sum $\sum_\square$ in (2.3) runs over all the plaquettes while the sums $\sum_i$ and $\sum_{i,\mu}$ in (2.4) are over all sites and links, respectively.

In the naive continuum limit, the action (2.2) reduces to the conventional Euclidean gauge-scalar action:

$$S \underset{a\to 0}{\to} \int d^4x [\tfrac{1}{4}\mathscr{F}_{\mu\nu}\mathscr{F}^{\mu\nu} + |D_\mu \phi|^2 + m^2\phi\phi^\dagger + \lambda(\phi\phi^\dagger)^2] \ , \tag{2.5}$$

where

$$\mathscr{F}_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{2.6}$$

and a denotes the lattice spacing. The partition function is defined by

$$Z = \sum_{\{U_{i,\mu}\}\{\phi_i\}} e^{-S} \ . \tag{2.7}$$

When $m^2 = \infty$ the model reduces to the pure U(1) lattice-gauge theory, which is known to undergo a second-order phase transition at $\beta_g = 1.01$,[10] since any configuration having $\phi_i \neq 0$ gives no contribution to the partition function in this case. For $\beta_g = \infty$, on the other hand, only those configurations which are equivalent to the total gauge-ordering give a nonvanishing contribution to Z. Then we obtain a theory of a self-interacting scalar field. Hereafter we refer to this theory as the pure scalar theory. Finally, when $\lambda \to \infty$, the magnitude of the scalar field is frozen to the value $(-m^2/2\lambda)^{1/2}$ for negative m^2 and zero for non-negative m^2. Thus our model reduces to the radially frozen model studied by several authors.[3]

Let us describe our method for Monte Carlo simulations. Each gauge variable and each site variable on a lattice are updated, by means of the Metropolis method,[11] once in one sweep. We measure the average plaquette $\langle S_\Box \rangle$ and the average squared length of the scalar field $\langle |\phi|^2 \rangle$, defined by

$$\langle S_\Box \rangle = -\frac{1}{6}\frac{\partial F}{\partial \beta_g} \tag{2.8}$$

and

$$\langle |\phi|^2 \rangle = -\frac{\partial F}{\partial m^2} , \tag{2.9}$$

respectively. Here F denotes the free energy of the system:

$$F = \frac{1}{N_{\text{site}}} \ln Z , \tag{2.10}$$

N_{site} being the number of lattice sites. We investigate behaviors of $\langle S_\Box \rangle$ ($\langle |\phi|^2 \rangle$) for fixed values of m^2 (βg) and λ by increasing or decreasing values of β_g (m^2) by small amounts. A few hundred sweeps are performed first in such a simulation to get a proper initial configuration. We also measure $\langle |\phi|^2 \rangle$ as a function of the number of sweeps, which we call hereafter time evolution of $\langle |\phi|^2 \rangle$. The maximum value of R_i in (2.1), which is denoted by $R_{\max}$, is set at a finite value in each simulation. We chose the values of $R_{\max}$ large enough so that the effect of the finite upper bound should be negligible.

Most simulations are performed on a 4^4 lattice with periodic boundary conditions for both the gauge and the scalar variables. Some calculations are also done on a 6^4 lattice, which show no significant difference except that values of $\langle S_\Box \rangle$ or $\langle |\phi|^2 \rangle$ fluctuate less on the larger lattice.

III. THE RESULTS

In this section we present the results of the simulations for several values of λ. First we discuss the results at $\lambda = 0.8$. Figure 1 summarizes the phase structure of the theory. Our results suggest that the second-order phase transition of the pure U(1) gauge theory, which corresponds to the $m^2 = \infty$ case, survives in the region $m^2 \gtrsim -1.5$. In Fig. 2 we plot $\langle S_\Box \rangle$ at $m^2 = -1.0$ as a function of β_g. The result indicates that the system undergoes the second-order phase transition at $\beta_g = 1.00 \pm 0.05$.

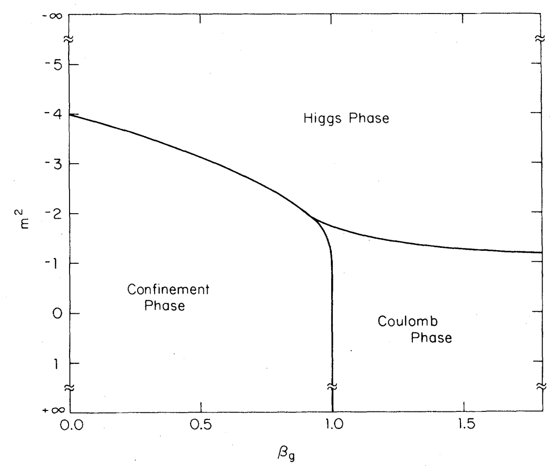

FIG. 1. Phase diagram of the theory at $\lambda = 0.8$

Another line of transitions is observed to extend from $\beta_g = 0.0$ to $\beta_g \lesssim 1.0$. Figure 3(a) shows $\langle |\phi|^2 \rangle$ versus m^2 at $\beta_g = 0.0$, in which we see a clear discontinuity of $\langle |\phi|^2 \rangle$ together with the hysteresis effect. In Figure 3(b) we plot time evolution of $\langle |\phi|^2 \rangle$ at $\beta_g = 0.0$ and $m^2 = -3.95$. The result shows that two states having different values of $\langle |\phi|^2 \rangle$ coexist there, which indicates the occurrence of a first-order phase transition. From these results we conclude that the radial degrees of freedom do cause the first-order Higgs-confinement transition. (Note that the gauge-group degrees of freedom are completely irrelevant at $\beta_g = 0.0$.) Singular behaviors similar to Fig. 3(a) are also observed when $\beta_g \lesssim 1.0$.

In the region where $\beta_g \gtrsim 1.0$, which seems to be most interesting from the theoretical point of view, we find that the system shows another kind of hysteresis effect. In Fig. 4 we plot $\langle |\phi|^2 \rangle$ at $\beta_g = 1.5$ as a function of m^2. We see that the singularity of the transition is strong enough to suggest the occurrence of the first-order transition. It is hard, however, to confirm the order of the transition by measuring the time evolution of $\langle |\phi|^2 \rangle$ because

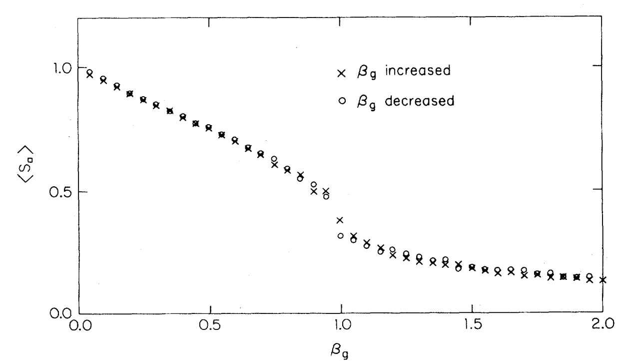

FIG. 2. $\langle S_\Box \rangle$ vs β_g at $\lambda = 0.8$ and $m^2 = -1.0$. At each value of β_g 100 sweeps are performed and the last 50 configurations are used to calculate $\langle S_\Box \rangle$.

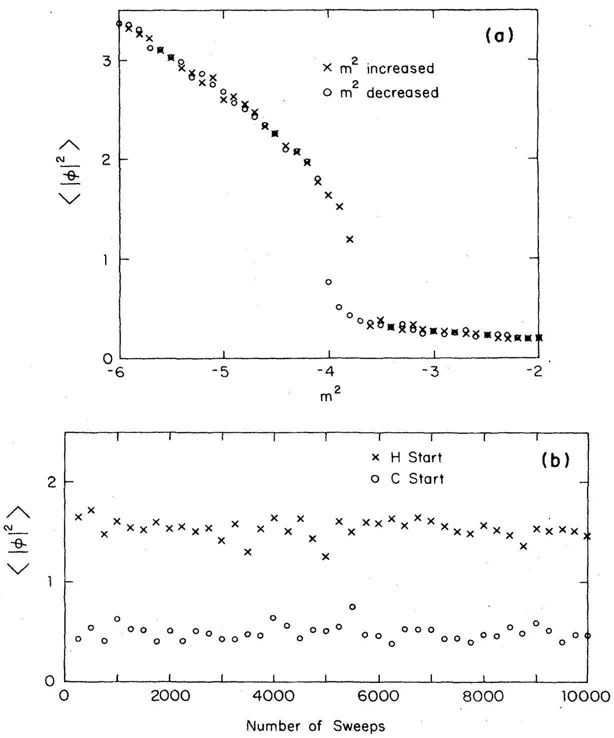

FIG. 3. (a) $\langle |\phi|^2 \rangle$ vs m^2 at $\lambda=0.8$ and $\beta_g=0.0$. At each value of m^2 400 sweeps are performed and the last 100 configurations are used to calculate $\langle |\phi|^2 \rangle$. (b) Time evolutions of $\langle |\phi|^2 \rangle$ at $\lambda=0.8$, $\beta_g=0.0$, and $m^2=-3.95$. "*H* start" means that the system starts from the configuration in which $R_i=R_{\max}$ and $V_i=1$ for all sites *i*. "*C* start" means, on the other hand, that the system starts from a configuration where every V_i is chosen randomly while all $R_i=0.0$. In both starts we set $U_{i,\mu}=1$ for all links.

the expected discontinuity is not large enough compared with the fluctuations on a 4^4 lattice.

We, therefore, looked for the first-order phase transitions at large β_g by going into the region of smaller λ,

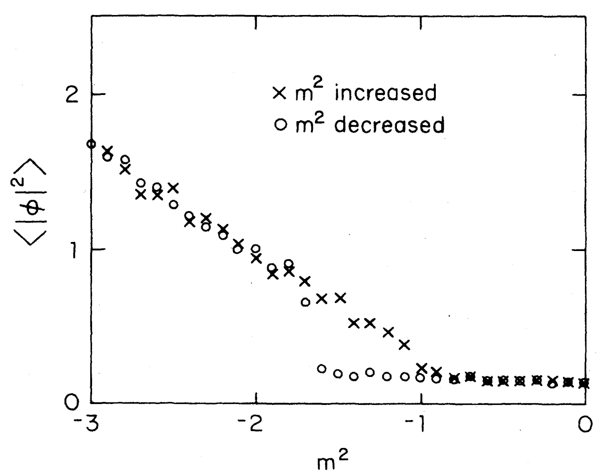

FIG. 4. $\langle |\phi|^2 \rangle$ vs m^2 at $\lambda=0.8$ and $\beta_g=1.5$. At each value of m^2 400 sweeps are performed and the last 100 configurations are used to calculate $\langle |\phi|^2 \rangle$.

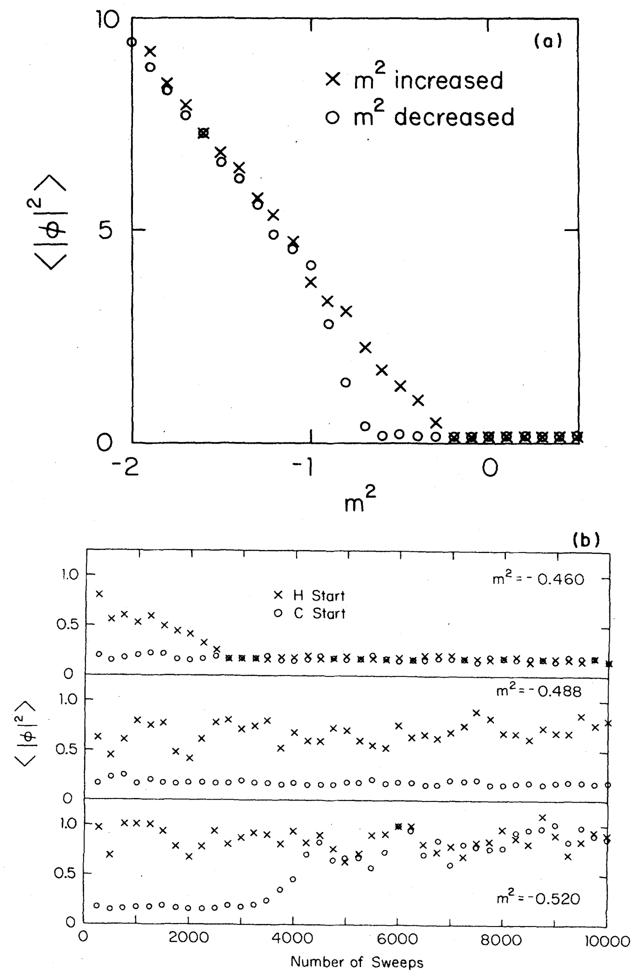

FIG. 5. (a) $\langle |\phi|^2 \rangle$ vs m^2 at $\lambda=0.1$ and $\beta_g=1.5$. At each value of m^2 800 sweeps are performed and the last 100 configurations are used to calculate $\langle |\phi|^2 \rangle$. (b) Time evolutions of $\langle |\phi|^2 \rangle$ at $\lambda=0.1$, $\beta_g=1.5$, and $m^2=-0.460$, -0.488, and -0.520. Initial configurations are the same as those in Fig. 3(b).

where the discontinuity is expected to be larger. Figure 5 shows several results at $\lambda=0.1$. In Fig. 5(a) we plot $\langle |\phi|^2 \rangle$ at $\beta_g=1.5$ versus m^2, in which we see much greater discontinuity compared with Fig. 4(b). Figure 5(b) gives plots of time evolutions of $\langle |\phi|^2 \rangle$. Although there is considerable fluctuation in the data because of the small lattice size, we can see that two metastable states persist at $m^2=-0.488$. The gap of $\langle |\phi|^2 \rangle$ between these two states seems to hardly depend on the values of β_g; gaps calculated from the last 5000 configurations of 10 000 sweeps are 0.57 ± 0.12 at $\beta_g=1.2$, 0.54 ± 0.10 at $\beta_g=1.5$, and 0.60 ± 0.12 at $\beta_g=1.8$. Thus, we come to a very important conclusion that the radial degrees of freedom change the nature of the transition in the weak-gauge-coupling region.

Let us comment on the λ dependence of the transitions. In the region where $\beta_g \lesssim 1.0$ our data show that the singularity weakens rapidly as λ grows and finally disappears. Therefore the Higgs and the confinement phases are

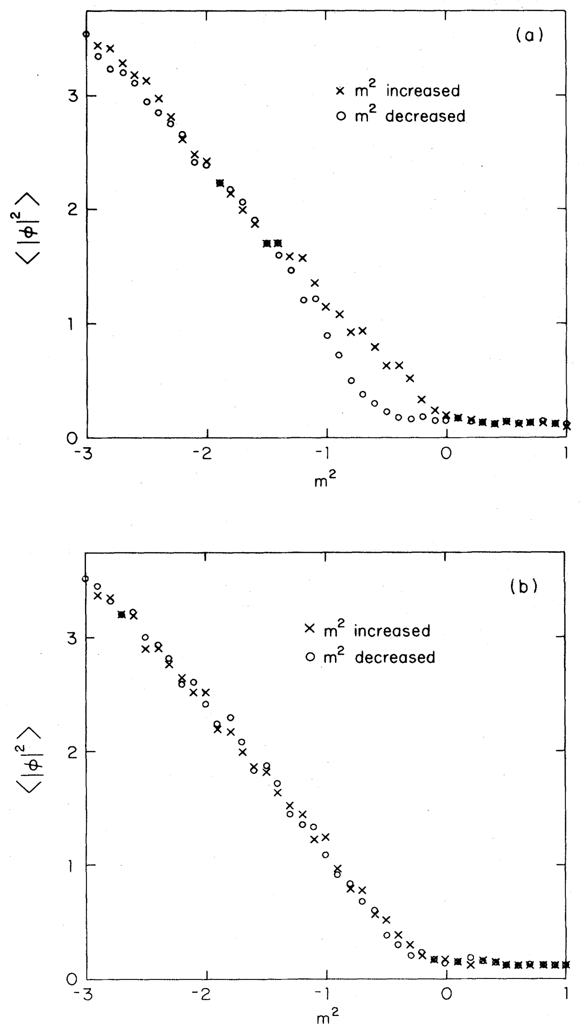

FIG. 6. (a) $\langle |\phi|^2 \rangle$ of the pure scalar theory vs m^2 at $\lambda=0.4$. At each value of m^2 100 sweeps are performed and the last 50 configurations are used to calculate $\langle |\phi|^2 \rangle$. (b) Same as (a) except that 400 sweeps are done at each value of m^2 and the last 100 configurations are used to calculate $\langle |\phi|^2 \rangle$.

analytically connected in the region where λ is large. As for the Higgs-Coulomb transition, on the other hand, the singularity seems to survive at large λ, although the hysteresis effect is observed only for small values of λ.

Finally, we present the results for the pure scalar theory, which corresponds to the $\beta_g=\infty$ case. Figure 6 shows the behavior of $\langle |\phi|^2 \rangle$ at $\lambda=0.4$ as a function of m^2. We see that a weak hysteresis effect observed in Fig. 6(a), where 100 sweeps are done at each value of m^2, vanishes in Fig. 6(b) with 400 sweeps at each point. This singularity becomes strong as λ decreases, but we did not see any signal of first-order phase transition in the region we investigated: $\lambda \geq 0.1$. Thus, all the results indicate that this theory has higher-order phase transitions.

IV. DISCUSSIONS

In the previous sections we discussed the phase structure of the U(1)-lattice-gauge—Higgs theory with radial degrees of freedom. Our results for large λ are in good agreement with those of the radially frozen model.[3] When λ is small, on the other hand, the theory undergoes the first-order Higgs-confinement transitions in the region where $\beta_g \lesssim 1.0$. We also see that the first-order Higgs-Coulomb transitions, which suggest the realization of the Coleman-Weinberg mechanism on a lattice, take place for $\beta_g \gtrsim 1.0$ provided λ is small enough.

Some of our results stated above are in good agreement with those given by Gerdt, Ilchev, and Mitrjushkin.[7] They studied the Z_N lattice-gauge—Higgs model with radial degrees of freedom for the cases $\beta_g=0.0$ and ∞. For large values of N, Z_N is expected to be a good approximation to U(1). We found that our results at $\beta_g=0.0$ are quantitatively consistent with theirs for $N=200$. We, however, did not observe any first-order phase transitions they found at $\beta_g=\infty$.

Quite interesting is whether the Coleman-Weinberg mechanism works in non-Abelian lattice-gauge—Higgs theories. Discussion by Coleman and Weinberg[5] indicates qualitatively similar results to the Abelian case. However, only higher-order phase transitions have been observed in Monte Carlo studies of the radially frozen non-Abelian models.[4] Recently Gupta and Heller[8] investigated the SU(3) adjoint Higgs model allowing for radial fluctuations. Their results suggest the existence of the first-order phase transitions in the region where λ is small. Kühnelt, Lang, and Vones,[9] who studied the SU(2) fundamental Higgs model, found, on the other hand, that the critical line in the model's phase diagram is second order everywhere. More studies would be necessary to this problem before we draw a definitive conclusion.

It is also interesting to study the first-order transitions of gauge-Higgs models in three dimensions. This problem has been discussed by people who study transitions in superconductors and in liquid crystals.[12,13] Halperin, Lubensky, and Ma[12] argued that some transitions in these matters should be weakly first order. The estimated size of the transition is too small in superconductors to be experimentally detected, but is sizable in liquid crystals. So far, however, no such transitions are observed in the experiments. Some recent theoretical work[13] pointed out, therefore, the existence of the gauge-Higgs models which escape the mechanism suggested in Ref. 12. Since the radial mode of the Higgs field had been excluded in these discussions, Monte Carlo study of our model in three dimensions would be worthwhile in order to get a better understanding in this field.

Phase structure of a lattice-gauge—Higgs model at finite temperature is another attractive subject. As is well known, it has much to do with cosmological studies. For example, the so-called inflationary scenario for the early universe is based on the existence of a symmetry-restoring first-order phase transition of such a model. At present our knowledge of the model's phase structure is not enough. There is a conjecture by Banks and Rabinovici[1] which says that the Abelian fundamental Higgs model does not undergo a symmetry-restoring phase transition at finite temperature. They also suggested that, if the Higgs charge is a multiple of the basic unit, two of the three phases observed at zero temperature disappear above some critical point. Recently Karsch, Seiler, and Stamatescu[14] made a Monte Carlo study of the SU(2) adjoint Higgs model at finite temperature. They established a line of deconfinement transitions, but found no unambiguous signal for the symmetry-restoring transition. As they pointed out in their paper, one should consider the effect of the radial degrees of freedom in the Higgs action to get a definitive view of the phase structure.

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