New macroscopic forces?

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The forces mediated by spin-0 bosons are described, along with the existing experimental limits. The mass and couplings of the invisible axion are derived, followed by suggestions for experiments to detect axions via the macroscopic forces they mediate. In particular, novel tests of the T-violating axion monopole-dipole forces are proposed.

MACROSCOPIC FORCES

Very light, weakly coupled bosons are occasionally suggested in the literature, for example, axions,[1] familons,[2] majorons,[3] arions,[4] and spin-1 antigravitons.[5] Such particles must couple very weakly to ordinary matter to have eluded detection thus far. A boson with small enough mass (say, 10^{-5} eV) would have a macroscopic Compton wavelength (say, 2 cm) and would mediate a force on laboratory scales. Even if very weakly coupled at the single-particle level, a macroscopic body with 10^{23} constituents could produce a measurable, coherent light-boson field. In the familiar case of gravity, the dimensionless coupling between two nucleons due to graviton exchange is absurdly small $[(m_{\text{nucleon}}/M_{\text{Planck}})^2 \sim 10^{-38}]$, but two 1-g masses separated by 1 cm experience a measurable force of

$$(6\times 10^{23})^2 (m_N/M_{\text{Pl}})^2 \frac{\hbar c}{(1\ \text{cm})} = 6.7\times 10^{-8}\ \text{dyn}\ .$$

We shall be interested in the possibility of detecting very light spin-0 bosons through the macroscopic forces which they mediate. The possible forces are determined by the allowed couplings; the spin-0 boson must couple to an effectively conserved quantity. There are only two possibilities for couplings to fundamental fermions: the scalar vertex and the pseudoscalar vertex. The scalar and pseudoscalar vertices can be analyzed in momentum space using the Gordon decomposition. For pure spacelike momentum transfer $\vec{q}$, they become

scalar ,

$$g_s\varphi(q)\bar{\psi}(p_f)\psi(p_i) = g_S\varphi(q)\left[\frac{p^\mu p_\mu}{M^2}\bar{\psi}(p_f)\psi(p_i) - i\frac{p_\mu q_\nu}{2M^2}\bar{\psi}(p_f)\sigma^{\mu\nu}\psi(p_i)\right]\ ; \quad (1)$$

pseudoscalar ,

$$g_P\varphi(q)\bar{\psi}(p_f)i\gamma_5\psi(p_i) = g_P\varphi(q)\frac{q^\mu}{2M}\bar{\psi}(p_f)i\gamma_5\gamma_\mu\psi(p_i) = g_P\varphi(q)\frac{\vec{q}}{2M}\cdot[\psi^\dagger(p_f)_i\vec{\Sigma}\psi(p_i)]\ . \quad (2)$$

Here $p_f = p + q/2$ and $p_i = p - q/2$ are the final and initial on-shell momenta and M is the fermion mass. The matrix $\vec{\Sigma}$ is the diagonal spin matrix. In the nonrelativistic limit (small fermion velocity and momentum transfer), the scalar coupling is spin-independent and depends only upon the fermion density $g_S\varphi\rho e^{-i\vec{q}\cdot\vec{r}}$. The pseudoscalar coupling is entirely spin-dependent, however. The virtual boson fields of a fermion in the two cases will thus be "monopole" and "dipole" fields (in the sense of the multiple expansion).

The scalar and pseudoscalar vertices (1) and (2) can appear in one-boson-exchange graphs in three combinations; this allows the existence of three distinct forces. The two-fermion potential can be calculated in the inverse Born approximation,

$$V(r) = \int \frac{d^3q}{(2\pi)^3}\frac{(\text{vertex 1})(\text{vertex 2})e^{i\vec{q}\cdot\vec{r}}}{\vec{q}^{\,2}+m_\varphi^2}\ . \quad (3)$$

The results are (see Fig. 1)

(monopole),[2]

$$V(r) = \frac{-g_S^1 g_S^2 e^{-m_\varphi r}}{4\pi r}\ ; \quad (4)$$

monopole-dipole ,

$$V(r) = (g_S^1 g_P^2)\frac{\hat{\sigma}_2\cdot\hat{r}}{8\pi M_2}\left[\frac{m_\varphi}{r}+\frac{1}{r^2}\right]e^{-m_\varphi r}\ ; \quad (5)$$

(dipole),[2]

$$V(r) = \frac{g_P^1 g_P^2}{16\pi M_1 M_2}\left[(\hat{\sigma}_1\cdot\hat{\sigma}_r)\left[\frac{m_\varphi}{r^2}+\frac{1}{r^3}+\frac{4\pi}{3}\delta^3(r)\right] - (\hat{\sigma}_1\cdot\hat{r})(\hat{\sigma}_2\cdot\hat{r})\left[\frac{m_\varphi^2}{r}+\frac{3m_\varphi}{r^2}+\frac{3}{r^3}\right]\right]e^{-m_\varphi r}\ . \quad (6)$$

Regardless of the assigned parity of the light, spin-0 boson, the (monopole)^2 and (dipole)^2 forces conserve P and T. However, the monopole-dipole force enjoys a unique status amongst possible macroscopic interactions, because $\hat{\sigma}\cdot\hat{r}$ violates P and T and of course macroscopic P and T violation has heretofore not been observed.

A few experimental upper limits exist for the strength of anomalous (monopole)^2 and (dipole)^2 interactions.

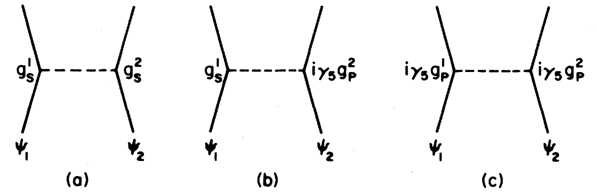

FIG. 1. Graphs for the potentials of Eqs. (4), (5), and (6). (a) (Monopole),[2] (b) monopole-dipole, (c) (dipole).[2]

Spero *et al.*[6] performed a Cavendish experiment to test deviations from the Newtonian $1/r$ potential over the distance range 2 to 5 cm. Their experiment established an upper bound for additional Yukawa-type interactions given by

$$V(r)=-\frac{Gm_1m_2}{r}(1+\alpha e^{-r/\lambda}) ;$$

at their scale of greatest sensitivity $\lambda \sim 3$ cm, α was found to be less than $10^{-3.8}$. Since the dimensionless coupling constant for the gravitational interaction between two nucleons is $(m_p/m_{\rm Pl})^2=10^{-38}$, we see that any anomalous Yukawa coupling at a scale of 3 cm must have a dimensional magnitude of $10^{-41.5}$ or smaller.

The measured g factor of the electron provides a limit on nonelectromagnetic electron spin-spin interactions. Since the experimental findings agree with the predictions of QED to eight digits for experiments using ferromagnets, we get a limit for any nonelectromagnetic spin-spin coupling at a scale of 1 cm of $10^{-8}\times\alpha(\lambda_e/1\text{ cm})^2=10^{-31}$, where λ_e is the electron Compton wavelength and $\alpha=\frac{1}{137}$.

A limit on photon spin-spin tensor interactions is provided by Ramsey, based upon studies of the hydrogen molecule. Ramsey[7] finds that any nonmagnetic interaction must be 4×10^{-4} smaller than that between proton magnetic moments. Extrapolated to a distance of 1 cm, this establishes an upper limit on the dimensionless coupling for an r^{-3} tensor force of 10^{-32}.

Of these various limits, only the anomalous (monopole)^2 interaction limit of $10^{-41.5}$ obtained by Spero *et al.* comes close to testing the range of possible strengths for axion-mediated forces. Furthermore, we know of no obvious experimental limit on the macroscopic P- and T-violating monopole-dipole interaction. Thus, the opportunity is ripe for pushing past known limits and perhaps finding something new. We shall shortly discuss some experiments which may do so.

AXIONS

A particularly well-motivated proposal for a very light spin-0 boson is the axion.[1] It arises in models to explain the smallness of a potentially large P- and T-violating coupling in QCD.

The axion is the quasi-Nambu-Goldstone boson of a spontaneously broken Peccei-Quinn[8] quasisymmetry. If the Peccei-Quinn symmetry were not broken by the 't Hooft vertex associated with fermion emission in instanton fields, the axion would be massless and would couple to quarks only through a T-conserving pseudoscalar vertex:

$$a\frac{m_q}{F}\bar{q}i\gamma_5 q ,$$

where F is the scale of Peccei-Quinn symmetry breaking.

However, a pure Peccei-Quinn transformation changes the phase multiplying the 't Hooft vertex. It is energetically unfavorable to change this phase (which requires energies of the order of the mass of the η'), so the Peccei-Quinn transformation is compensated for by a combined chiral U(1) and chiral SU(N) transformation which leaves the phase invariant and minimizes the energy. Since the quark masses are not zero, these combined (Peccei-Quinn)$\otimes[\text{U}(1)_A]\otimes[\text{SU}(N)_A]$ transformations cost energy, and the axion acquires a small mass. If, in addition, the effective θ parameter $\theta_{\rm eff}$ is not zero, the axion will also couple to the quarks with T-violating scalar vertices.

To see how this all works, consider the quark-mass and T-violating sectors,

$$H_{\rm mass}=m_u\bar{u}_L u_R+m_d\bar{d}_L d_R+\cdots+\text{H.c.} \tag{7a}$$

and

$$H_T=\theta\frac{g^2}{32\pi^2}G\tilde{G} . \tag{7b}$$

Under a Peccei-Quinn transformation,

$$m_q\to m_q e^{-i\eta},\quad q_L\to e^{-i\eta/2}q_L,\quad q_R\to e^{i\eta/2}q_R , \tag{8}$$

the phase of the 't Hooft vertex varies as

$$\arg\left[\prod_q \bar{q}_L q_R\right] ;$$

hence, $e^{i\theta}$ becomes $e^{i(\theta+N\eta)}$, where $N=$ number of quark flavors. Similarly, under chiral U(1),

$$\begin{aligned} q_R &\to e^{i\nu/2}q_R ,\\ q_L &\to e^{-i\nu/2}q_L , \end{aligned} \tag{9}$$

and the 't Hooft vertex changes as $e^{i\theta}\to e^{i(\theta+N\nu)}$. Thus, a combined Peccei-Quinn and chiral U(1) transformation with $\nu=-\eta$ leaves θ invariant.

To calculate the mass of the axion, we imagine performing a Peccei-Quinn transformation; this leaves the quark mass terms unchanged, but changes θ to $\theta+\Delta\theta$. We now undo this change of θ by reabsorbing $\Delta\theta$ into the quark mass sector by the combined chiral SU(N)$\times$U(1) transformation which minimizes the energy. This gives

$$H_{\rm maa}=m_u\bar{u}u\,\cos\Delta\theta_u+m_d\bar{d}d\,\cos\Delta\theta_d+\cdots , \tag{10}$$

subject to the constraint $\Delta\theta_u+\Delta\theta_d+\Delta\theta_s+\cdots=\Delta\theta$. Since the quark bilinears acquire the vacuum expectation value $\langle\bar{u}u\rangle=\langle\bar{d}d\rangle=\cdots=v<0$, the minimum is found to be at

$$\Delta\theta_q=\frac{\prod_{i\neq q} m_i}{\sum_j\prod_{i\neq j} m_j}\Delta\theta . \tag{11}$$

For example, in SU(3),

$$\Delta\theta_u = \frac{m_d m_s}{m_u m_d + m_d m_s + m_s m_u}\Delta\theta \ .$$

By expanding H_{mass} out to second order, we obtain $\frac{1}{2}m_A a^2$, where

$$m_A{}^2 = \frac{-v}{F^2}\left[\frac{m_u m_d m_s}{m_u m_d + m_d m_s + m_s m_u}\right]$$

in SU(3). Here, we have identified $F\Delta\theta = a$ from the properly normalized axion kinetic energy: $\frac{1}{2}F^2(\partial\Delta\theta)^2$; F is the scale at which Peccei-Quinn symmetry is broken. By using $m_s \gg m_u, m_d$ and the pion-mass formula

$$m_\pi{}^2 = \frac{-v}{f_\pi^2}(m_u + m_d) \ ,$$

we obtain

$$m_A{}^2 = \frac{m_\pi{}^2 f_\pi{}^2}{F^2}\frac{m_u m_d}{(m_u+m_d)^2} \ . \tag{12}$$

We can now extract the pseudoscalar couplings for the axion. Labeling $a = F\Delta\theta$ as above and expanding H_{mass} (7a) to first order, we get

$$H_{\text{mass}} = m_u \bar{u}_L u_R(1+i\Delta\theta_u) + m_d \bar{d}_L d_R(1+i\Delta\theta_d) + \cdots + \text{H.c.}$$

$$= \sum_q m_q \bar{q}q + \frac{a}{F}\frac{m_u m_d m_s}{m_u m_d + m_d m_s + m_s m_u} \times (\bar{u}i\gamma_5 U + \bar{d}i\gamma_5 d + \bar{s}i\gamma_5 s) \ .$$

In the limit $m_s \to \infty$, the pseudoscalar couplings are just

$$H_{\text{int}} = \frac{a}{F}\frac{m_u m_d}{m_u + m_d}(\bar{u}i\gamma_5 u + \bar{d}i\gamma_5 d + \bar{s}i\gamma_5 s) \ . \tag{13}$$

The T-violating scalar axion-quark couplings are obtained by absorbing the θ term (7b) into the quark-mass sector while minimizing the energy as before and expanding. From

$$H_{\text{mass}} = m_u \bar{u}u \cos(\theta_u + \Delta\theta_u) + m_d \bar{d}d \cos(\theta_d + \Delta\theta_d) + \cdots \ ,$$

we get

$$H_{\text{int}} = \frac{a}{F}\frac{m_u m_d}{m_u + m_d}\theta(\bar{u}u + \bar{d}d + \bar{s}s) \ . \tag{14}$$

In the calculation leading to (14), we have assumed that the energy is minimized when the coefficient of the 't Hooft vertex is θ, while the quark masses and v are real. In a theory with axions, this θ is, in principle, dynamically determined by minimizing the vacuum energy with respect to the phases of appropriate Higgs fields. This can be done at the effective Lagrangian level by considering a more complete Lagrangian including P- and T-violating four-quark terms induced by the weak interactions, whose coefficients depend on quark masses and thereby effectively on θ. Since both the coefficients and the vacuum matrix elements of these operators are uncertain, due to our ignorance of the mechanism of P and T violation and our lack of computational skill in QCD, respectively, a reliable calculation of this type is impossible. Accordingly, we have treated θ as a phenomenological parameter. It should be remarked that the additional operators will also have nonzero nucleon matrix elements, so the scalar coupling (14) is incomplete, although it is probably a reasonable approximation.

To calculate the effective scalar coupling of the axion to nucleons, we make use of the value of the σ term in current algebra. The effective scalar coupling is

$$g_{aNN} = \left\langle N \left| \frac{\theta}{F}\frac{m_u m_d}{m_u + m_d}(\bar{u}u + \bar{d}d + \bar{s}s) \right| N \right\rangle$$

$$= \left\langle N \left| \frac{\theta}{F}\frac{m_u m_d}{m_u + m_d}(\bar{u}u + \bar{d}d) \right| N \right\rangle$$

$$= \frac{\theta}{F}\frac{2m_u m_d}{(m_u+m_d)^2}\langle N \mid \sigma \mid N \rangle \ , \tag{15}$$

where $\langle N \mid \sigma \mid N \rangle$ is the pion-nucleon σ term, taken to be 60 MeV.

In a wide class of axion models, basically all those deriving from grand unified theories, the axion-electron pseudoscalar vertex is

$$H_{\text{int}} = a\frac{m_e}{F}\bar{e}i\gamma_5 e \ . \tag{16}$$

Several limits exist for the key parameters of axion physics, θ and F. Measurements of the electric dipole moment of the neutron have established[9] that $\theta < 10^{-8}$. The expected value of θ depends upon the model of CP violation chosen. In the conservative Kobayashi-Maskawa model, one of us (FW) has estimated that $\theta \sim 10^{-14}$; other models should give higher values. Stellar energy-loss rates constrain the scale of Peccei-Quinn-symmetry breaking F to be $F > 10^8$ GeV.[10] Finally, cosmological considerations[11] suggest a preferred value of $F \approx 10^{12}$ GeV; in this case, axions would account for the missing mass of the Universe. However, F is probably less than 10^{13} GeV; otherwise, axions would overdominate the Universe.

Before proceeding with our discussion of possible experiments, let us write down the three force laws which will be of practical interest, those involving electron spins and nucleons. In terms of the effective interaction strengths at the scale of the axion Compton wavelength, they are [following Eqs. (4), (5), and (6)]

nucleon [(monopole)^2] ,

$$V_{NN} = -G_{NN}(\lambda_A)\frac{e^{-1}}{4\pi\lambda_A} \ ; \tag{17}$$

nucleon—electron [monopole-dipole] ,

$$V_{Ne} = G_{Ne}(\lambda_A)\frac{\hat{\sigma}\cdot\hat{r}\, e^{-1}}{4\pi\lambda_A} \ ; \tag{18}$$

electron [(dipole)^2] ,

$$V_{ee} = -G_{ee}(\lambda_A)[\tfrac{7}{4}(\hat{\sigma}_1\cdot\hat{r})(\hat{\sigma}_2\cdot\hat{r}) - \tfrac{1}{2}\hat{\sigma}_1\cdot\hat{\sigma}_2]\frac{e^{-1}}{4\pi\lambda_A} . \quad (19)$$

The effective interaction strengths in terms of m_A using (15) and (16) are

$$G_{NN} = \left[\frac{\theta(60\text{ MeV})}{F}\frac{2m_u m_d}{(m_u+m_d)^2}\right]^2 \sim (\theta/F)^2 , \quad (20)$$

$$G_{Ne}(m_A) = \frac{\theta(60\text{ MeV})m_A}{F^2}\frac{2m_u m_d}{(m_u+m_d)^2} \sim \theta/F^3 , \quad (21)$$

$$G_{ee}(m_A) = \left[\frac{m_A}{F}\right]^2 \sim 1/F^4 . \quad (22)$$

For convenience, the axion mass and Compton wavelength are given by

$$m_A = 10^{-5}\text{ eV}\left[\frac{10^{12}\text{ GeV}}{F}\right] , \quad (23)$$

$$\lambda_A = 2\text{ cm}\left[\frac{F}{10^{12}\text{ GeV}}\right] . \quad (24)$$

EXPERIMENTAL POSSIBILITIES

We now discuss techniques for measuring very weak forces and suggest experiments to detect axions. Novel "monopole-dipole" experiments to detect the macroscopic T-violating axion interaction are proposed.

Axion sources are macroscopic collections of nucleons (ordinary masses) or coherent, spin-polarized electron or nucleon systems. We shall be interested in using dense or highly spin-polarized test masses of dimension comparable to the axion Compton wavelength. The dimensionless axion coupling strengths for the three interaction types [Eqs. (20), (21), and (22)] as functions of F and θ are shown in Fig. 2. The gravitational coupling is shown for comparison. Clearly, the axion forces are weak.

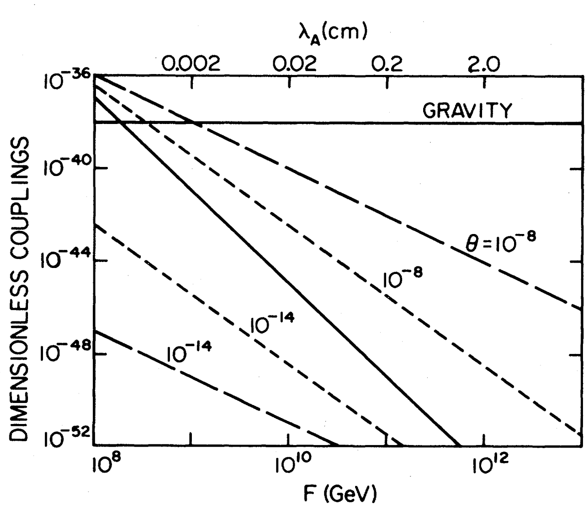

FIG. 2. Dimensionless axion couplings (at scale λ_A) as functions of F and θ. Long dashed lines are $G_{NN}/4\pi$ for $\theta=10^{-8}$ and $\theta=10^{-14}$. Short-dashed lines are $G_{Ne}/4\pi$ for $\theta=10^{-8}$ and $\theta=10^{-14}$. Solid diagonal line is $G_{ee}/4\pi$. Gravitational coupling between two nucleons $(M_N/M_{\rm Pl})^2$ shown for comparison.

For assessing experimental sensitivity, a useful quantity is the acceleration experienced by a nucleon at a distance λ_A [axion Compton wavelength, Eq. (24)], due to a semi-infinite axion source composed of iron. This acceleration is shown in Fig. 3 for the three interaction types along with the gravitational acceleration exerted by an iron slab of thickness λ_A. (Of course, the geometry is important for the spin-dependent forces, but we are concerned only with order-of-magnitude comparisons.) The accelerations and range depend upon F and θ; realistically accessible lengths are 0.2 mm and greater.

The techniques for detecting axions are borrowed from experimental gravity. The experiments are of two types: those which employ broad-band torsion balances to measure Newton's constant or to test the inverse-square law ("Cavendish experiments"), and those proposed for sensitive tests of post-Newtonian gravity which use high-Q monocrystals and torsion balances operated at resonance ("high-Q experiments"). Cavendish experiments represent an existing technology currently capable of probing accelerations as weak as 10^{-12} cm/sec^2. For example, the Eotvos-Dicke equivalence-principle test attained this precision.[12] The cryogenic, high-Q experiments represent a goal to strive for; having sensitivity to accelerations as weak as perhaps 10^{-20} cm/sec^2, they would be limited only by attainable Q's and the corresponding (thermal) Nyquist noise limits.[13] The evolution from the current state-of-the-art Cavendish experiments to the high-Q ideal may be a difficult process, requiring several generations of experiments.

(MONOPOLE)^2 AND (DIPOLE)^2 EXPERIMENTS

Currently, two experiments of the Cavendish type to detect anomalous gravitational (monopole)^2 forces at distances of less than 2 cm are proposed or in progress.[14,15] The sensitivity of these experiments is expected to be 10^{-11} cm/sec^2, and thus probe the range $\vartheta > 10^{-9}$, $F > 10^{10.5}$. Since the (monopole)^2 axion force competes directly with gravity, such experiments are fundamentally limited by metrological precision. For example, the best measurement[16] of G [$(6.6726\pm0.0005)\times10^{-8}$ cm^3 sec^{-2} g^{-1}] is good to only one part in 10^4. Clever geometries, such as Newman's nested cylinders,[14] can be used to nullify the Newtonian force, but without perfect precision. The fine tuning of these experiments will prove a challenge for experimentalists. Perhaps forces 10^8 times weaker than gravity are accessible.

A test for anomalous gravitational spin-spin interactions is currently being built by Graham and Newman.[17] This Cavendish experiment measures the torque between two spin-polarized bodies. By using state-of-the-art superconducting magnetic shielding,[18] Newman expects to achieve sensitivity to anomalous forces 10^{15} times weaker than the magnetic spin-spin interaction. This sensitivity is 7 orders of magnitude better than the limit placed by the measured electron g factor (which agrees with QED to eight digits). Unfortunately, since the axion coupling is 10^{-29} times the photon coupling [$\alpha(m_e/F)^2$, $F=10^{12}$ GeV], it is unlikely that such experiments will be able to

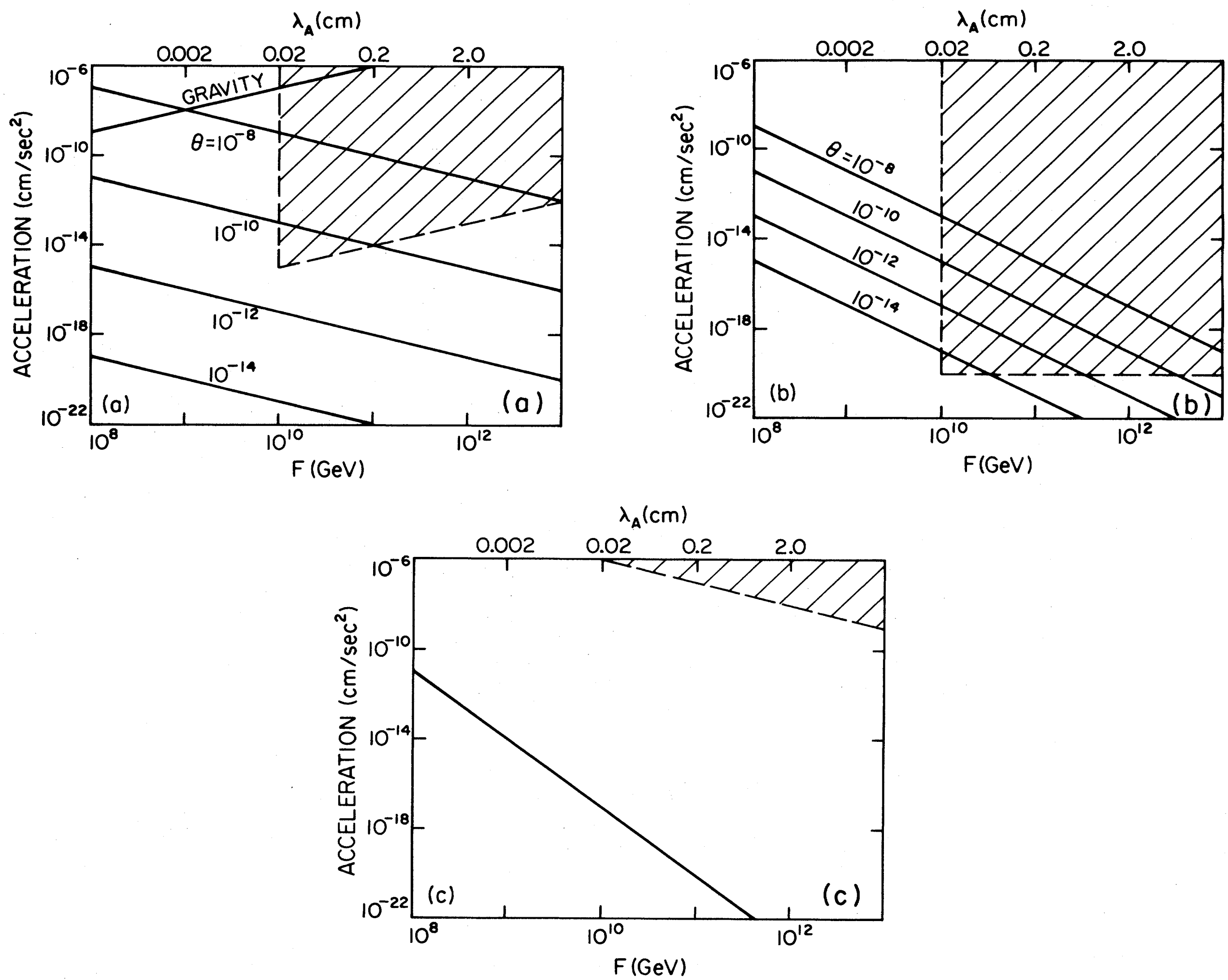

FIG. 3. Order-of-magnitude accelerations expected for axion-mediated forces as functions of F and θ. The accelerations are for an atom of iron at a distance λ_A outside a semi-infinite axion source with the nucleon density or the electron spin density (two free spins per atom) of iron. The geometry dependence for the spin-dependent forces is ignored. The coupling strengths are those of Eqs. (20), (21), and (22) shown in Fig. 2. (a) Nucleon-nucleon (monopole)^2 acceleration, obtained from the formula $a(r)=2\pi\hbar c(G_{NN}/4\pi)\lambda_A\rho_{\rm iron}e^{-r/\lambda_A}/m_N$, where $\rho_{\rm iron}$ is the nucleon density of iron. The competing gravitational acceleration due to a slab of iron of thickness λ_A is shown for comparison. Experimentally accessible region (shaded) is limited by size and metrology (precision with which competing gravitational acceleration can be canceled); it is arbitrarily taken to be $\lambda_A > 0.02$ cm and (acceleration) $> 10^{-8}$ gravitational. (b) Nucleon-electron monopole-dipole acceleration (spin-dependence ignored). Experimentally accessible region (shaded) is limited by size and acceleration sensitivity. We take these to be $\lambda_A > 0.02$ cm and $a > 10^{-20}$ cm/sec^2. (c) Electron-electron (dipole)^2 acceleration (spin-dependent ignored). Experimentally accessible region (shaded) is limited by ability to cancel competing electromagnetic interaction. Even with a magnetic shielding factor of 10^{-15}, the axion spin-spin force is inaccessible.

see the axion spin-spin force.

The above torsion balance experiments are state of the art. What are the present limitations on the performance of torsion balances for measuring weak forces? Borrowing from the experience of Roll, Krotkov, and Dicke,[12] the most serious systematic effects which need to be controlled are (1) temperature variations, (2) changing gravity gradients, and (3) seismic noise. Some possible cures are as follows.[12–15]

(1) Temperature gradients and temperature changes cause a multitude of problems, notably radiometer forces and electrostatic forces due to varying contact potentials. Both radiometer forces and contact potentials can be controlled by the use of clean, uniform surfaces on torsion balance parts (resulting in uniform accommodation coefficients and work functions), operation in high vacuum and at low temperatures, and careful maintenance of temperature stability.

(2) Changing gravity gradients due to moving people and even low-frequency air pressure fluctuations can exert significant torques on torsion balances, unless they are designed with minimized quadrupole and even higher multipole moments.

(3) Seismic noise, whether from natural earth tremors or human activity, can cause erroneous response in the torsional mode via direct torsional motion or nonlinear

mechanical couplings of translational or tilt excitations to the torsional mode. Rotational seismic noise is fortunately almost nonexistent,[19] so the keys to seismic noise reduction are minimization of nonlinearities in mechanical design and effective damping or shielding of translational and tilt excitations. Electronic refrigeration of pendulum and vertical "spring" modes can critically damp translational excitations, while active, antiseismic shielding can be used to insulate against tilt disturbances and torsional excitations. Finally, a quiet location such as a mine is desirable.

Solving such problems will be important for the low-frequency, high-Q experiments to be discussed later.

MONOPOLE-DIPOLE EXPERIMENTS

Monopole-dipole experiments offer distinct advantages over (monopole)^2 and spin-spin experiments. Unlike the (monopole)^2 interaction, the monopole-dipole force can be switched on and off without moving the sources. This allows the use of an alternating axion force which can be distinguished from static gravitational or van der Waals forces. Unlike the spin-spin force, the monopole-dipole force should not be magnetic-shielding limited. A monopole body is in principle magnetically neutral, so magnetic shielding would be necessary only to safeguard against the presence of magnetic impurities. By using "dirty" superconducting shields[20] around both monopole and dipole bodies, the unwanted magnetic couplings could hopefully be reduced to a dc effect. A monopole body made of zone-refined monocrystal could have a uniform impurity distribution of only one part in 10^{12}, with no macroscopic magnetic ordering.

The switchability of the monopole-dipole force lends itself to measurement by high-Q mechanical oscillators. A high Q can buy you two things: a large equilibrium-oscillation amplitude for a given acceleration and small Nyquist noise.

It is the small Nyquist noise which we wish to exploit by using high Q's; for our purposes, the oscillator amplitude built up over time will be limited by the uncertainty in our knowledge of the resonant frequency ω_R and our observation time $\hat{\tau}$, not the intrinsic Q of the system itself. Ideally, the equilibrium amplitude of the oscillator excited with acceleration *a at resonance* ω_R is

$$(\Delta x)_{\max}=\frac{a}{(\omega_R)^2}Q_{\text{intrinsic}}=\frac{1}{4\pi}a\tau_R\tau^* ,$$

where τ^* is the relaxation time. In practice we will not know ω_R precisely and will excite the oscillator slightly off resonance at $\omega=\omega_R+\Delta\omega$. This will induce an amplitude

$$(\Delta x)_{\text{eff}}=\frac{a}{\omega_R{}^2}Q_{\text{eff}} ,$$

where the effective Q is

$$Q_{\text{eff}}=\frac{\omega_R}{2\Delta\omega} .$$

The uncertainty in the frequency depends upon how long we are willing to spend determining the frequency. If we wait for $\hat{\tau}$ seconds, then $\omega_R/\Delta\omega=\hat{\tau}/\tau_R$, so we find

$$(\Delta x)_{\text{eff}}=\frac{a}{2\omega_R{}^2}\frac{\hat{\tau}}{\tau_R}=\frac{1}{8\pi^2}a\hat{\tau}\tau_R . \tag{25}$$

[For our purposes below, we will set both the time we spend determining the oscillator's natural frequency and the time we spend exciting the oscillator at (or near) resonance to be $\hat{\tau}=10^6$ sec or 10 days.] From Eq. (25), we see that sensitivity is increased in proportion to the period. The principal advantage of a high Q is seen in the Nyquist noise formula:

$$a_{\text{Brownian}}\sim\left[\frac{8\pi kT}{m\hat{\tau}\tau_R Q}\right]^{1/2} . \tag{26}$$

Two high-Q devices have been proposed to measure weak forces: the high-frequency monocrystal and the low-frequency torsion balance.[13,21] Here, we propose a monopole-dipole experiment using a sapphire or niobium crystal and existing technology which could detect an anomalous acceleration of better than 10^{-15} cm/sec^2. We then discuss the promising possibility of doing more sensitive low-frequency experiments using torsion balances or other devices. These, however, require good seismic shielding.

A CRYSTAL EXPERIMENT

We envision a monopole-dipole crystal experiment as follows (see Fig. 4). A 10-g, 3-cm-long monocrystal of sapphire or niobium would passively respond to an oscillating field produced by an adjacent, but magnetically shielded, electron-spin system. The crystal would have a resonant frequency in its quadrupole mode of roughly $f=150$ kHz. At $T=0.001$ K its relaxation time is expected to be $\tau^*=10^7$ sec (see below). If one then observes the crystal response to an oscillating axion field applied at

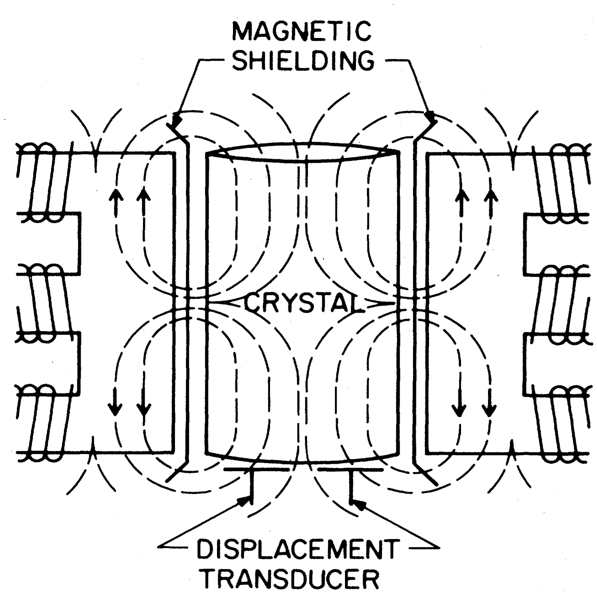

FIG. 4. Sample schematic for a monopole-dipole high-Q crystal experiment. Electron spins and resulting axion field shown.

$\omega_R \pm \Delta\omega$ for $r=10^6$ sec (where $\Delta\omega=2\pi/\hat{\tau}$), one finds a thermal-acceleration noise level of

$$a_{\text{Brownian}} \sim \left[\frac{8kT}{m\hat{\tau}\tau^*}\right]^{1/2} \sim 10^{-16}\ \text{cm/sec}^2\ . \qquad (27)$$

For a displacement sensitivity of $\Delta x = 10^{-17}$ cm using a capacitive transducer[22] or an inductive SQUID (superconducting quantum-interference device) transducer[23] or laser interferometry,[24] an acceleration of

$$a \sim 8\pi^2 f\, \Delta x/\hat{\tau} \sim 10^{-16}\ \text{cm/sec}^2$$

would be detectable. Such an experiment could *unambiguously* detect an anomalous monopole-dipole acceleration as small as 10^{-15} cm/sec^2 or could establish very stringent upper bounds for the existence of such a force. While the axion monopole-dipole force is expected to be slightly weaker than this experiment could detect, no experimental limit exists for a macroscopic T-violating force, so the results of such an experiment would be very significant.

There is good reason to believe that the above proposal is practical. Bagdasorov *et al.* (see Ref. 21) constructed a 15-cm sapphire crystal which oscillated at $f=34$ kHz with an amplitude of 10^{-17} cm in its quadrupole mode with a Q of 5×10^9 at 4.2 K. A 3-cm sapphire cylinder would have a frequency 5 times that of a 15-cm crystal. For compressional modes, the primary internal losses (from thermoelastic dissipation and phonon-phonon interactions) and surface dissipation mechanisms scale as ${Q_{\text{loss}}}^{-1} \propto \omega T$ for sufficiently low temperatures and frequencies.[25] Thus, our 3-cm crystal operating at $T=1$ mK should yield a $Q_{\text{intrinsic}}$ of at least 4×10^{12}.

However, the Q of the Bagdasorov *et al.* crystal may have been limited by their wire-loop suspension system; the intrinsic Q could have been much higher than 5×10^9. Suspension losses scale with the frequency ${Q_{\text{susp}}}^{-1} \propto \omega$ and go down slightly with temperature.[25] The suspension losses may also be amplitude dependent; we would expect the nonlinear, amplitude-dependent losses to be smaller for small amplitudes. If the attainable ${Q_{\text{susp}}}^{-1}$ proves too large for mechanical suspension systems, we propose the use of a Meissner-effect levitation system with either a niobium-coated sapphire crystal or a pure niobium crystal. The attainable Q's should be quite high, because superconducting microwave cavities have demonstrated Q's of 10^{12} at 1 GHz,[25] so the Q's for a 10^5-Hz crystal could be as good as $Q_{\text{susp}}=10^{16}$. (It should be noted that a mysterious ac dissipation mechanism has been found in inductive SQUID readout systems[22,23] which may be due to motion of magnetic fluxoids penetrating superconducting films. It might be possible to cure such an effect by using a pure niobium crystal through which fluxoids could not penetrate and dirty shielding to pin fluxoids.)

LOW-FREQUENCY EXPERIMENTS

The above discussion of the intrinsic, surface, suspension, and superconductor dissipation mechanisms suggests that a low-frequency monopole-dipole experiment would accrue a significant advantage. The relation $\tau^*=Q\tau_R/\pi$ becomes $\tau^*=C{\tau_R}^2$, where C depends upon the materials, the mechanical design, and the temperature. The Brownian acceleration level scales as

$$a_{\text{Brown}} \sim \frac{1}{\tau_R}(8kT/m\hat{\tau}C)^{1/2}\ ,$$

while the measurable accelerations are

$$a_{\text{meas}} \sim \frac{1}{\tau_R} 8\pi^2(\Delta x)_{\text{meas}}/\hat{\tau}\ .$$

The major additional problem which must be contended with at low frequencies is seismic noise. The power spectrum of seismic disturbances generally increases with the period in the range of interest (100 to 10^{-5} Hz).[26,27] Low-frequency experiments must thus be designed to minimize the coupling of seismic vibrations to the oscillator itself. At very low frequencies, say, 10^{-3} to 10^{-4} Hz, it may be necessary to use active antiseismic shielding, which is in principle possible because seismic noise is nonthermal.

Braginsky, Caves, and Thorne (1977) have proposed the use of very-low-frequency torsion balances in tests of post-Newtonian gravity.[13] They desire sensitivity to a signal of 10^{-17} cm/sec^2 and a Brownian acceleration level of 10^{-18} cm/sec^2. They cite state-of-the-art (as of 1977) relaxation times of $\tau^*=10^{10}$ sec and suggest that numbers like $\tau^*=10^{13}$ should be attainable in the near future by going to low temperature and refining existing techniques (see earlier discussion). The experiment they propose would operate at $f=10^{-4}$ Hz and would require active antiseismic shielding of only 3 orders of magnitude or less for the various translational, tilt, or rotational modes. The practical limits on the performance of torsion balances are not really known at present. With a displacement sensitivity of 10^{-14} cm, it is conceivable that one could perform an experiment with a sensitivity of 10^{-20} cm/sec^2 and at a higher frequency such as $f=10^{-2}$ Hz where the seismic-noise problems would be easier to control.

To appreciate the difficulties imposed on such an experiment by seismic noise, let us note that the uncertainty in horizontal or vertical position introduced by earth motion during a 10^6-sec integration time at 10^{-2} Hz is 10^{-7} cm.[26] This is to be compared with a 10^{-14}-cm signal excited in the torsional mode. Active antiseismic shielding would be required to the extent that nonlinear mechanical couplings allow translational noise to leak into the torsional mode.

A very clever variation of the low-frequency resonance technique which greatly reduces the effect of seismic disturbances is available; it is so-called electronic refrigeration technique.[12,14] The torsional mode is critically damped by a velocity-sensitive electronic-feedback system; this effectively erases memory of past unwanted disturbances. Since the damping is accomplished by pumping energy of the system rather than through dissipation, the Brownian noise level is unchanged. Operating, for example, at $\tau_R=10^4$ sec for a time $\hat{\tau}=10^6$ sec, one effectively does 100 separate experiments where the amplitude obtained with each half cycle is now given by the free mass formula

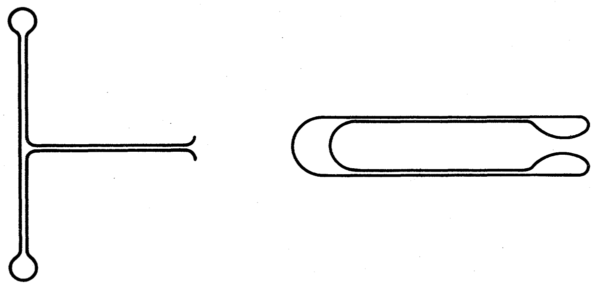

FIG. 5. Hybrid crystal oscillators.

$$\Delta x = \tfrac{1}{2} a (\tau_R/2)^2 \ .$$

This is in contrast to the single measurement of

$$\Delta x = \frac{1}{8\pi^2} a \tau_R \hat{\tau}$$

one would obtain in an equivalent resonance experiment. The electronic-refrigeration technique is more forgiving than the standard resonance technique, because not only are unwanted disturbances "forgotten," but bad data can be judiciously excised. However, slightly greater measurement sensitivity is required. For $a = 10^{-20}$ cm/sec^2, $\tau_R \sim 10^4$ sec, and $\hat{\tau} \sim 10^6$ sec, the resonance technique gives a steady-state amplitude of $\Delta x \sim 10^{-12}$ cm, while the electronic-refrigeration technique requires $\Delta x \sim 10^{-13}$ cm sensitivity.

Several possibilities besides standard torsion balances exist for low-frequency oscillators. One genre of devices is intermediate between crystals and torsion balances; these are the hybrid oscillators cut from monocrystals; examples are tuning forks and dumbbell-like torsion oscillators (Fig. 5). Depending upon how delicately these devices could be machined, they could operate at frequencies as low as $f = 10^2$ or $f = 10^0$ Hz. Hybrid torsion oscillators made from BeCu with frequencies of ~ 1 kHz and Q's of $\sim 10^6$ at temperatures as low as 1 mK have been used in superfluid helium experiments.[28] Furthermore, a torsion crystal oscillator has been proposed to measure the post-Newtonian Faraday effect.[13] Whether Q's higher than 10^6 (as in BeCu) are attainable for carefully polished sapphire is as yet uncertain, because the internal dissipation in noncompressional modes includes defect motion in addition to the thermoelastic and phonon-phonon processes. Dissipation due to defect motion, however, is probably highly amplitude dependence and may be unimportant for the very small amplitudes we seek to measure. Projecting optimistically, a magnetically levitated sapphire tuning fork operating at $T = 0.001$ K with $f = 100$ Hz could have a relaxation time of $\tau^* \sim 10^{13}$ sec. The associated Brownian noise level would be 10^{-19} cm/sec^2.

Another genre of low-frequency devices are Meissner-levitated masses or torsion arms which experience magnetic restoring forces. The restoring force could in principle be tuned to yield an arbitrary natural frequency of, say, $f = 10^0$ to 10^{-2} Hz. Extrapolating from attained Q's of microwave resonators, the Q of such a system could be astronomical, say $Q \sim 10^{19}$ at $f = 1$ Hz. The corresponding Brownian noise limits could be $a_{\rm Brown} = 10^{-22}$ cm/sec^2 for an observation time $\hat{\tau} = 10^6$ sec with an accompanying measurement sensitivity $a_{\rm meas} = 10^{-20}$ cm/sec^2.

There may be unanticipated dissipation effects (e.g., uncontrollable fluxoid motion in superconductors, frequency, and temperature-independent-loss mechanisms, etc.) which could reduce these sensitivities. However, if a measurement sensitivity of 10^{-20} cm/sec^2 were attained, the entire suggested range in θ for the axion monopole-dipole force could be tested. Regardless of whether one believes in axions, we feel that the possibility of detecting a weak P- and T-violating force is important and exciting and that specific ideas for experiments such as those presented here deserve further study.

Before closing, we note that Sikivie has proposed detecting physical axions from the Sun and galactic halo by taking advantage of the Primakov process which couples axions to the electromagnetic field.[29]

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[1]S. Weinberg, Phys. Rev. Lett. **40**, 233 (1978); F. Wilczek, *ibid.* **40**, 279 (1978); M. Dine, W. Fischler, and M. Srednicki, Phys. Lett. **104B**, 199 (1981); M. Shifman, A. Vainshtein, and V. Zakharov, Nucl. Phys. **B166**, 493 (1980); J. Kim, Phys. Rev. Lett. **43**, 103 (1979).

[2]F. Wilczek, Phys. Rev. **49**, 1549 (1982); G. Gelmini, S. Nussinov, and T. Yanagida, Nucl. Phys. **B219**, 31 (1983).

[3]G. Gelmini and M. Roncadelli, Phys. Lett. **99B**, 411 (1981); Y. Chikashige, R. Mohapatra, and R. Peccei, *ibid.* **98B**, 265 (1981).

[4]A. Ansel'm, Pis'ma Zh. Eksp. Teor. Fiz. **35**, 266 (1982) [JETP Lett. **36**, 55 (1982)].

[5]J. Scherk, Phys. Lett. **88B**, 265 (1979); D. Neville, Phys. Rev. D **25**, 573 (1982).

[6]R. Spero *et al.*, Phys. Rev. Lett. **44**, 94 (1980).

[7]N. Ramsey, Physica (Utrecht) **96A**, 285 (1979); see also G. Feinberg and J. Sucher, Phys. Rev. D **20**, 1717 (1979).

[8]R. Peccei and H. Quinn, Phys. Rev. Lett. **38**, 1440 (1977); Phys. Rev. D **16**, 1791 (1977).

[9]R. Crewther *et al.*, Phys. Lett. **88B**, 123 (1979); **91B**, 487(E) (1980); W. B. Dress *et al.*, Phys. Rev. D **15**, 9 (1977).
[10]M. Fukugita, S. Watamura, and M. Yoshimura, Phys. Rev. Lett. **48**, 1522 (1982).
[11]J. Preskill, M. Wise, and F. Wilczek, Phys. Lett. **120B**, 127 (1983); L. Abbott and P. Sikivie, *ibid.* **120B**, 133 (1983); M. Dine and W. Fischler, *ibid.* **120B**, 137 (1983).
[12]P. Roll, R. Krotkov, and R. Dicke, Ann. Phys. (N.Y.) **26**, 442 (1964).
[13]V. Braginsky, C. Caves, and K. Thorne, Phys. Rev. D **15**, 2047 (1977).
[14]R. Newman, NSF Grant Proposal, 1980.
[15]W. Oelfke, in Proceedings of the 2nd International Conference on Precision Measurements and Fundamental Constants, Gaithersburg, Maryland, 1981 (unpublished).
[16]G. Luther and W. Towler, Phys. Rev. Lett. **48**, 121 (1982).
[17]D. Graham and R. Newman, in Proceedings of the Tenth International Conference on Relativity and Gravitation (unpublished).
[18]See, for example, B. Cabrera, Phys. Rev. Lett. **48**, 1378 (1982), or B. Cabrera and F. J. van Kann, Acta Astron. **5**, 125 (1978).
[19]R. Weiss (private communication).
[20]R. Newman (private communication).
[21]V. B. Braginsky and A. B. Manukin, *Measurement of Weak Forces in Physics Experiments* (University of Chicago Press, Chicago, Illinois, 1977).
[22]D. Blair, in *Gravitational Radiation,* 1982 Les Houches Lectures, edited by N. Deruelle and T. Piran (North-Holland, Amsterdam, 1983).
[23]P. Michelson, in *Gravitational Radiation* (Ref. 22).
[24]S. Whitcomb (private communication). Broad-band displacement sensitivities of 10^{-14} cm have been attained. Narrow-band sensitivities of 10^{-17} cm are conceivable.
[25]V. B. Braginsky, V. P. Mitrofanov, and V. I. Panov, *Systems with Small Dissipation* (Isvatizdatel'stvo Nauka, Moskow, 1982) (English translation in preparation, University of Chicago Press, Chicago).
[26]J. Fix, Bull. Seismol. Soc. Am. **62**, 1753 (1972).
[27]J. Berger and J. Levine, J. Geophys. Res. **57**, 1210 (1974).
[28]D. J. Bishop and J. D. Reppy, Phys. Rev. Lett. **40**, 1727 (1978), and references cited therein.
[29]P. Sikivie, Phys. Rev. Lett. **51**, 1415 (1983).