Nonstandard fermion mass matrices and nucleon decay

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We discuss how nonstandard fermion mass matrices (with hierarchical eigenvalues) lead to mixing effects in nucleon decay which can enhance the branching ratios of naively unexpected decay modes, and could suppress the rates for $N \to e^+ +$ nonstrange hadron(s).

Much has been said over the past five years on the subject of the uncertainties surrounding calculations of the proton lifetime in grand unified models (see, for example, Ref. 1). The grand unification scale is fixed in a given grand unified model by the particle content of the theory and by low-energy inputs, including the QCD scale parameter. It is possible to considerably increase the naively expected proton lifetime by the addition of suitable light fermions or scalars, a possibility recently reemphasized in the literature.[2] Another important source of uncertainty is found in the calculation of matrix elements for nucleon decay, e.g., in the widely varying treatments of spin, phase space, recoil effects, etc. This still leaves another major source of uncertainty regarding our expectations for nucleon decay, namely, the possibility of unusual (non-Cabibbo-type) generation mixings in baryon-number-violating vertices, which could lead to *a priori* peculiar final states. All the mixing angles are, in principle, fixed, given a set of fermion mass matrices: it is in fact known[3] [in an SU(5) grand unified model] that if fermion mass matrices arise only from Yukawa couplings to one or more $\underline{5}$'s of Higgs scalars, the mixing angles in the baryon-number-violating gauge-boson vertices are just the Cabibbo-Kobayashi-Maskawa angles of the low-energy charged weak currents (with the possibility of additional *CP*-violating phases). In this case, one would expect nucleons to decay mainly, say, into $e^+ +$ nonstrange channels. However, it should perhaps be recalled here that this kind of grand unified model fails miserably in accounting for light-quark and lepton mass ratios.

Following the lead of Jarlskog,[4] many people have addressed the question of whether the proton could be made completely stable at tree level through inherent mixing effects.[5–8] This does not seem possible in general for reasons alluded to in what follows. Our concern in this paper is to point out by means of examples that, as a consequence of the latitude in the choice of fermion mass matrices, nucleons could conceivably decay into naively unexpected channels with a relatively large branching ratio, and that this is a potentially important source of information on the problem of families and of fermion mass generation.

Our considerations will be restricted to standard, nonsupersymmetric grand unified models. In supersymmetric grand unified models the number of particle species more than doubles, and nucleon decay can proceed through different mechanisms which can give rise to a hierarchy of branching ratios quite dissimilar from the naively expected results of the corresponding standard grand unified models (see, for example, Ref. 9 for a review). The analysis of peculiar mixing effects given in what follows can be extended to supersymmetric grand unified models, taking into account the properties and spectra (especially in the expanded scalar sector) of specific models.

There are very few constraints on the exact form of the fermion mass matrices. One constraint is that the eigenvalues of these matrices give rise to the observed hierarchy of fermion masses. A second constraint is that the mixing matrix appearing in the left-handed charged weak current [the Kobayashi-Maskawa (KM) matrix] $U_{\rm KM}$ has the approximate form (with small θ)

$$U_{\rm KM} \sim \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} ,$$

which we refer to as "Cabibbo-type".[10] There are a large number of possibilities in the choice of fermion mass matrices which satisfy these two requirements.

We write the fermion mass terms (in the group-eigenstate basis) as

$$l_L^\dagger M_l l_R + q^\dagger_{(-1/3)L} M_{-1/3} q_{(-1/3)R} + q^\dagger_{(2/3)L} M_{2/3} q_{(2/3)R} + \text{Hermitian conjugate} ,$$

where each of l_L, l_R, $q_{(-1/3)L}, \ldots$ is a vector in family space and we are using two-component spinor notation. The generic fermion mass term $f_L^\dagger \mathscr{M} f_R$ can be reexpressed in the mass-eigenstate basis as $\tilde{f}_L^\dagger \mathscr{M}^{(D)} \tilde{f}_R$, where $\mathscr{M} = U \mathscr{M}^{(D)} V^\dagger$ is diagonalized by the unitary matrices U and V and the eigenvalues are placed along the diagonal of $\mathscr{M}^{(D)}$ in ascending magnitude. Thus,

$$f_L = U\tilde{f}_L ,$$
$$f_R = V\tilde{f}_R .$$

If $\mathscr{M}$ is Hermitian, then $U=V$, and if $\mathscr{M}$ is symmetric, then $V^\dagger = U^T$. Generally, however, U is the matrix which diagonalizes $\mathscr{M}\mathscr{M}^\dagger$ and V that which diagonalizes $\mathscr{M}^\dagger\mathscr{M}$,

$$U^\dagger \mathscr{M}\mathscr{M}^\dagger U = \mathscr{M}^{(D)2} ,$$
$$V^\dagger \mathscr{M}^\dagger \mathscr{M} V = \mathscr{M}^{(D)2} .$$

In this notation the left-handed charged current is

$$j_\mu^{(L)} \sim q^\dagger_{(2/3)L}\sigma_\mu q_{(-1/3)L}$$
$$= \tilde{q}^\dagger_{(2/3)L} U^\dagger_{2/3} U_{-1/3}\sigma_\mu \tilde{q}_{(-1/3)L} ,$$

and thus, $U_{\rm KM} = U^\dagger_{2/3} U_{-1/3}$.

An example of mass matrices (for three families) that yield a fermion mass hierarchy and give rise to a Cabibbo-type KM matrix is

$$M_l^{(F)} = \begin{bmatrix} 0 & A' & 0 \\ A & B & 0 \\ 0 & 0 & C \end{bmatrix} , \tag{1}$$

$$M_{-1/3}^{(F)} = \begin{bmatrix} 0 & D' & 0 \\ D & E & 0 \\ 0 & 0 & F \end{bmatrix} , \tag{2}$$

$$M_{2/3}^{(F)} = \begin{bmatrix} 0 & G' & 0 \\ G & 0 & H' \\ 0 & H & J \end{bmatrix} , \tag{3}$$

where (A, $A' \ll B \ll C$), (D, $D' \ll E \ll F$), and (G, $G' \ll H$, $H' \ll J$). Matrices of this type (or similar types) have been of interest to model builders because they have few parameters and consequently they can give rise to relationships between fermion masses and mixing angles such as the well known phenomenological relation $\tan\theta_C \simeq (m_d/m_s)^{1/2}$. The problem of achieving the matrices (1)—(3), or ones similar to them, in a technically natural way has been a long-standing industry (see, e.g., the review of Weyers[11]). For these matrices, $U_{\rm KM}$ is of Cabibbo type as a consequence of both $U_{-1/3}$ and $U_{2/3}$ being of Cabibbo type. However, in the general case, all that is required is that $U_{2/3}$ is close to $U_{-1/3}$: $U_{2/3} = U_{-1/3} + \Delta$, where $\Delta = (U_{2/3} - U_{-1/3})$ has small entries. It is worth noting that this does not require $U_{2/3}$ and $U_{-1/3}$ to be of Cabibbo type; however, it requires that if either one is not of Cabibbo type then the same is true of the other. Some examples of this last possibility will be given in what follows.

We will now illustrate some alternative possibilities for the fermion mass matrices by considering a two-family case (for simplicity we will work with real matrices)

$$\mathcal{M} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} .$$

One way to get a hierarchy in the eigenvalues of $\mathcal{M}$ is to have a hierarchy in the magnitudes of the entries of $\mathcal{M}\mathcal{M}^\dagger$ (and consequently $\mathcal{M}^\dagger\mathcal{M}$) as was the case for the 3×3 matrices (1)—(3).

Case 1. A conventional choice might be $a \ll b = c \ll d$. Then $U = V$ is of Cabbibo type.

Case 2. An example of an unconventional choice along these lines might be $b \ll a = d \ll c$. Note that the matrix $\mathcal{M}K$ where

$$K = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

is of the conventional form (case 1) that we just discussed. Thus, here, U is of Cabbibo type and $V = KU$.

Case 3. The choice $c \ll a = d \ll b$ is similar to the preceding case where now, V is of Cabibbo type and $U = KV$. If both $M_{-1/3}$ and $M_{2/3}$ are of this form then (since $K^2 = 1$) $U_{\rm KM}$ is still of Cabibbo type.

We can also get a fermion mass hierarchy and a Cabibbo-type KM matrix without a hierarchy of magnitudes in the entries of $\mathcal{M}$. Thus we have the following.

Case 4. Allow $a \approx b \approx c \approx d$. In fact, consider

$$\mathcal{M} = \mu \begin{bmatrix} 1+\epsilon & 1 \\ 1 & 1 \end{bmatrix} \tag{4}$$

with small ϵ. The eigenvalues of $\mathcal{M}$ are $\approx \epsilon\mu/2$ and $\approx 2\mu + \epsilon\mu/2$, and

$$U = V \simeq \frac{1}{\sqrt{2}} \begin{bmatrix} 1-\epsilon/4 & -(1+\epsilon/4) \\ 1+\epsilon/4 & 1-\epsilon/4 \end{bmatrix} . \tag{5}$$

If both $M_{-1/3}$ and $M_{2/3}$ are of this form (with small parameters $\epsilon_{-1/3}$ and $\epsilon_{2/3}$) we do indeed get a Cabibbo-type KM matrix:[12] to lowest order in $\epsilon_{-1/3}$ and $\epsilon_{2/3}$ [$\delta \equiv (\epsilon_{-1/3} - \epsilon_{2/3})/4$],

$$U_{\rm KM} \simeq \begin{bmatrix} 1-\frac{1}{2}\delta^2 & -\delta \\ \delta & 1-\frac{1}{2}\delta^2 \end{bmatrix} .$$

These alternative forms for the fermion mass matrices are by no means an exhaustive catalog. They simply illustrate the latitude of choice one has, although in a particular grand unified model the possibilities may be limited by the group and Higgs choice.[13] It is easy to generalize the above examples to three or more families and to matrices with complex entries.

Unconventional fermion mass matrices can give rise to mixing matrices in the baryon-number-violating vector-boson vertices that are not of Cabibbo type. Under some circumstances this can Cabibbo-suppress nucleon decay altogether.[5–8,14] However, in a greater variety of circumstances it will allow nucleon decay to proceed, but will give rise to an unusual final-state spectrum. We will now review baryon-number-violating boson vertices and then give some examples of this latter possibility.

Baryon-number-violating interactions involving the light fermions can be induced by the following $\mathrm{SU(3)}\times\mathrm{SU(2)}\times\mathrm{U(1)}$ multiplets of vectors

$$\begin{bmatrix} X \\ Y \end{bmatrix} \sim (\underline{3}, \underline{2}, \tfrac{5}{6}) ,$$

$$\begin{bmatrix} X' \\ Y' \end{bmatrix} \sim (\underline{3}, \underline{2}, -\tfrac{1}{6}) ,$$

and the following multiplets of scalars

$$S \sim (\underline{3}, \underline{1}, \tfrac{1}{3}) , \ (\underline{3}, \underline{3}, \tfrac{1}{3}) , \ (\underline{3}, \underline{1}, \tfrac{4}{3}) ,$$

where the electric charge operator is $Q = T_{3L} - Y$. In all but some very particular cases the final-state spectrum of a nucleon decay via the scalars S depends upon arbitrary Yukawa couplings which are unrelated to the fermion mass matrix. We will therefore not analyze scalar-mediated decays in the following, although given a particular model the analysis can be easily carried out using the interactions written down in Ref. 15. We note in passing that, due to this arbitrariness in the scalar couplings, the final states expected in scalar-mediated nucleon decay can be quite different from what would be expected in, say, the minimal SU(5) model.

The interactions of the vector multiplets with the fermions are as follows[15] ($a,b,\ldots,$ are family indices; $\alpha,\beta,\gamma,\ldots,$ are color indices; χ and η indicate the possible repetition of the various baryon-number-violating multiplets):

$$g_{\chi,ab}[(\nu_{La})^{\dagger}\sigma_{\mu}\sigma_{2}(q_{Rb\alpha}^{(-1/3)})^{*}X_{\chi\alpha}^{\mu} + (l_{La})^{\dagger}\sigma_{\mu}\sigma_{2}(q_{Rb\alpha}^{(-1/3)})^{*}Y_{\chi\alpha}^{\mu}]+\text{H.c.} \,, \tag{6}$$

$$h_{\chi,ab}[-\epsilon_{\alpha\beta\gamma}q_{Ra\alpha}^{(2/3)T}\sigma_{2}\sigma_{\mu}q_{Lb\beta}^{(-1/3)}X_{\chi\gamma}^{\mu} + \epsilon_{\alpha\beta\gamma}q_{Ra\alpha}^{(2/3)T}\sigma_{2}\sigma_{\mu}q_{Lb\beta}^{(2/3)}Y_{\chi\gamma}^{\mu}]+\text{H.c.} \,, \tag{7}$$

$$j_{\chi,ab}[(q_{La\alpha}^{(2/3)})^{\dagger}\sigma_{\mu}\sigma_{2}(l_{Rb})^{*}X_{\chi\alpha}^{\mu} + (q_{La\alpha}^{(-1/3)})^{\dagger}\sigma_{\mu}\sigma_{2}(l_{Rb})^{*}Y_{\chi\alpha}^{\mu}]+\text{H.c.} \,, \tag{8}$$

$$g'_{\eta,ab}[(\nu_{La})^{\dagger}\sigma_{\mu}\sigma_{2}(q_{Rb\alpha}^{(2/3)})^{*}X'^{\mu}_{\eta\alpha} + (l_{La})^{\dagger}\sigma_{\mu}\sigma_{2}(q_{Rb\alpha}^{(2/3)})^{*}Y'^{\mu}_{\eta\alpha}]+\text{H.c.} \,, \tag{9}$$

$$h'_{\eta,ab}[-\epsilon_{\alpha\beta\gamma}q_{Ra\alpha}^{(-1/3)T}\sigma_{2}\sigma_{\mu}q_{Lb\beta}^{(-1/3)}X'^{\mu}_{\eta\gamma} + \epsilon_{\alpha\beta\gamma}q_{Ra\alpha}^{(-1/3)T}\sigma_{2}\sigma_{\mu}q_{Lb\beta}^{(2/3)}Y'^{\mu}_{\eta\gamma}]+\text{H.c.} \tag{10}$$

We specialize to the case

$$g_{\chi,ab}\sim h_{\chi,ab}\sim j_{\chi,ab}\sim g'_{\eta,ab}\sim h'_{\eta,ab}\sim \delta_{ab} \,,$$

which is characteristic of models in which no more than one family is contained in any one irreducible representation of the group which contains the baryon-number-violating vector bosons.[16] In this case the mixing matrices that appear in the couplings (6)—(10), when we go over to the fermion mass-eigenstate basis, are, respectively,

$$(V_{-1/3}^{*},\ U_{l}^{\dagger}V_{-1/3}^{*}) \,, \tag{11}$$

$$(V_{2/3}^{T}U_{2/3}U_{\text{KM}},\ V_{2/3}^{T}U_{2/3}) \,, \tag{12}$$

$$(U_{2/3}^{\dagger}V_{l}^{*},\ U_{\text{KM}}^{\dagger}U_{2/3}^{\dagger}V_{l}^{*}) \,, \tag{13}$$

$$(V_{2/3}^{*},\ U_{l}^{\dagger}V_{2/3}^{*}) \,, \tag{14}$$

$$(V_{-1/3}^{T}U_{2/3}U_{\text{KM}},\ V_{-1/3}^{T}U_{2/3}) \,. \tag{15}$$

In this notation, for example, (11) is taken to mean that (6) becomes

$$g_{\chi}[\nu_{L}^{\dagger}\sigma_{\mu}\sigma_{2}V_{-1/3}^{*}(\tilde{q}_{R\alpha}^{(-1/3)})^{*}X_{\chi\alpha}^{\mu} + \tilde{l}_{L}^{\dagger}\sigma_{\mu}\sigma_{2}U_{l}^{\dagger}V_{-1/3}^{*}(\tilde{q}_{R\alpha}^{(-1/3)})^{*}Y_{\chi\alpha}^{\mu}]$$

in the fermion mass-eigenstate basis.[17]

To illustrate the effects of some unusual choices for fermion mass matrices on nucleon decay, we will first focus on decays mediated by the (X,Y) multiplet of vectors [as, for example, in a grand unified model based on SU(5)]. We concentrate on giving examples where there are no substantial suppression factors that impede the decay of the nucleon; however, some situations in which the latter does occur will be mentioned.

We start by discussing possibilities for two fermion families. As a basis for comparison in the following assume M_l, $M_{-1/3}$, and $M_{2/3}$ to have the generic forms discussed in case 1 above. Then, since all of the unitary matrices $U_{2/3}$, $V_{2/3}$, $U_l,\ldots,$ are of Cabibbo type, nucleon decay amplitudes will have the standard final-state preferences, e.g., $p\to e^{+}+$nonstrange, $p\to\mu^{+}+$strange, $p\to\bar{\nu}+$nonstrange, $p\to\bar{\nu}+$strange, but very little of either $p\to e^{+}+$strange or $p\to\mu^{+}+$nonstrange, and similarly for the neutron. As a contrast to this, assume now that M_l, $M_{-1/3}$, and $M_{2/3}$ all have the forms discussed in case 2 (case 3 is similar). Then U_l, $U_{-1/3}$, and $U_{2/3}$ are of Cabibbo-type and Eqs. (11)—(13)) become, respectively (using the fact that U_l, $U_{-1/3}$, and $U_{2/3}$ are real for real mass matrices),

$$(KU_{-1/3},\ U_{l}^{T}KU_{-1/3}) \,,$$

$$(U_{2/3}^{T}KU_{2/3}U_{\text{KM}},\ U_{2/3}^{T}KU_{2/3}) \,,$$

$$(U_{2/3}^{T}KU_{l},\ U_{\text{KM}}^{T}U_{2/3}^{T}KU_{l}) \,.$$

In this circumstance nucleon decay through s-channel exchange is Cabibbo-suppressed. However, not all t-channel-exchange decays are suppressed. One easily verifies that the following hierarchies obtain for proton and bound-neutron decay branching ratios on the basis of flavor-mixing effects in matrix elements at the quark level [in decreasing order of suppression by powers of small parameters, $O(1)\gg O(\epsilon)\gg O(\epsilon^{2})$]:

$$\begin{aligned} B(p\to\mu^{+}+\text{strange}) &\sim B(p\to\bar{\nu}+\text{strange}) \\ &\gg B(p\to\bar{\nu}+\text{nonstrange})\sim B(p\to\mu^{+}+\text{nonstrange}) \\ &\sim B(p\to e^{+}+\text{strange}) \\ &\gg B(p\to e^{+}+\text{nonstrange}) \,, \end{aligned}$$

and

$$\begin{aligned} B(n\to\bar{\nu}+\text{strange}) &\gg B(n\to\bar{\nu}+\text{nonstrange})\sim B(n\to\mu^{+}+\text{nonstrange}) \\ &\gg B(n\to e^{+}+\text{nonstrange}) \,. \end{aligned}$$

Note that as always, decays of bound neutrons into a charged antilepton and a strange particle are forbidden by charge conservation.

Another circumstance occurs if we assume, for example, that M_l is of the standard form as in case 1 and $M_{-1/3}$ and $M_{2/3}$ have the form given in case 2. Equations (11)—(13) then take the form, respectively,

$$(KU_{-1/3},\ U_l^T KU_{-1/3})\ ,$$
$$(U_{2/3}^T KU_{2/3}U_{\mathrm{KM}},\ U_{2/3}^T KU_{2/3})\ ,$$
$$(U_{2/3}^T U_l,\ U_{\mathrm{KM}}^T U_{2/3}^T U_l)\ .$$

As in the preceding example, only t-channel exchanges are important. The resulting hierarchies of decay modes are then

$$B(p\to e^+ +\text{strange})\sim B(p\to\bar{\nu}+\text{strange})$$
$$\gg B(p\to\bar{\nu}+\text{nonstrange})\sim B(p\to e^+ +\text{nonstrange})$$
$$\sim B(p\to\mu^+ +\text{strange})$$
$$\sim B(p\to\mu^+ +\text{nonstrange})\ ,$$

and

$$B(n\to\bar{\nu}+\text{strange})\gg B(n\to\bar{\nu}+\text{nonstrange})\sim B(n\to e^+ +\text{nonstrange})$$
$$\sim B(n\to\mu^+ +\text{nonstrange})\ .$$

If all of M_l, $M_{-1/3}$, and $M_{2/3}$ have the form given in case 4 then all of the relevant matrices in (11)—(15) are of Cabibbo type. The final states in nucleon decay are then the conventional ones. However, if say, $M_{-1/3}$ and $M_{2/3}$ have the form given in case 4 and M_l is of conventional form, then (11)—(13) become

$$(U_{-1/3},\ U_l^T U_{-1/3})\ , \tag{16}$$
$$(U_{\mathrm{KM}},\ 1)\ , \tag{17}$$
$$(U_{2/3}^T U_l,\ U_{\mathrm{KM}}^T U_{2/3}^T U_l)\ . \tag{18}$$

The matrices in (16) and (18) all have entries of order unity. The consequence of this is that the decays $p\to e^+ +$nonstrange, $p\to e^+ +$strange, $p\to\mu^+ +$nonstrange, $p\to\mu^+ +$strange, $p\to\bar{\nu}+$strange, and $p\to\bar{\nu}$ +nonstrange all have amplitudes of comparable order. For the neutron, $n\to e^+ +$nonstrange, $n\to\mu^+ +$nonstrange, $n\to\bar{\nu}+$strange, and $n\to\bar{\nu}+$nonstrange have comparable amplitude.

An interesting case occurs for decays mediated by the (X',Y') multiplet of vectors when $M_{-1/3}$ and $M_{2/3}$ are of the standard form (case 1) and M_l has the form given in case 3 ($V=KU$ is of Cabibbo type). Equations (14) and (15) are then

$$(U_{2/3},\ V_l^T KU_{2/3})\ ,$$
$$(1,\ U_{\mathrm{KM}}^T)\ .$$

X' (Y') mediated decays always involve final-state antineutrinos (charged antileptons). In this final example, the decay branching ratios follow the ordering, for protons,

$$B(p\to\mu^+ +\text{nonstrange})\sim B(p\to\bar{\nu}+\text{nonstrange})$$
$$\gg B(p\to e^+ +\text{nonstrange})\sim B(p\to\mu^+ +\text{strange})$$
$$\gg B(p\to e^+ +\text{strange})\ ,$$

and for bound neutrons,

$$B(n\to\bar{\nu}+\text{nonstrange})\sim B(n\to\mu^+ +\text{nonstrange})$$
$$\gg B(n\to e^+ +\text{nonstrange})\ .$$

The preceding examples show (in the 2×2 case) how unusual final states in nucleon decay might be traced to the structure of the fermion mass matrices. When the number of families is increased the possible forms for the fermion mass matrices become legion. It is worth noting that (for three or more families) the straightforward generalizations of the preceding examples, for decays mediated by the (X,Y) bosons (except for those based on case 4) give rise to Cabibbo-suppressed nucleon decay (this corresponds to an explicit realization of the comment made in Ref. 14): Such a generalization of our first nonstandard example (to three families) might be $M_l=M_l^{(F)}K'$, $M_{-1/3}=M_{-1/3}^{(F)}K'$, and $M_{2/3}=M_{2/3}^{(F)}K'$, where $M_l^{(F)}$, $M_{-1/3}^{(F)}$, and $M_{2/3}^{(F)}$ are the matrices exhibited in Eqs. (1)—(3), and K'=antidiagonal (1,1,1). An example that was not considered above, that in which M_l is antidiagonal (case 2 or case 3) and $M_{2/3}$ and $M_{-1/3}$ are of the standard form, gives conventional nucleon decay final states. We also note that for *all* of these examples (save those based on case 4) nucleon decay mediated by an (X',Y') multiplet is Cabibbo-suppressed.

How then can we get unsuppressed unusual final states in nucleon decay given three or more families of fermions? One way is to have some of the mass matrices with elements all of the same order (as in the 2×2 example above based on case 4). Another way is to have the 2×2 submatrix (for the first two generations) have unconventional entries such as those discussed in the examples above, and to have conventional values for the remaining elements of the mass matrices [the remaining entries might be, for example, ones of magnitude comparable to the corresponding ones in Eqs. (1)–(3)]. These are just two relatively simple possibilities. Clearly, there may be many more complicated (and even uglier) examples.

We hope we have demonstrated that given our relative ignorance of the fermion mass matrices, nucleon decay could conceivably proceed unsuppressed into unusual final states even in the context of conventional grand unified models. It is interesting that in many of the examples we have considered, this occurs as a consequence of a qualitative difference in the form of the lepton versus the quark mass matrices. The implementation of any of the examples discussed above in the framework of a specific grand unified model requires an extension of the Higgs sector beyond the usual minimal choices. This, in any case, may be required to cope with the well known difficulties in reproducing the light-fermion mass ratios.

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[8] D. Altschuler, P. Eckert, and T. Schucker, Phys. Lett. **119B**, 351 (1982).

[9] S. Rudaz, in *Proceedings of the Third Workshop on Grand Unification, Chapel Hill, North Carolina, 1982,* edited by P. H. Frampton, S. L. Glashow, and H. van Dam (Birkhauser, Boston, 1982), p. 191.

[10] We refer to the upper left-hand 2×2 as Cabibbo-type as well. More generally we also refer to the 2×2 matrix by this name if the diagonal entries are close to 1 in magnitude (and thus, the off-diagonal entries are small).

[11] J. Weyers, in *Proceedings of the 1979 Cargèse Summer Institute,* edited by M. Levy, J. L. Basdevant, J. Weyers, R. Gastmans, and M. Jacob (Plenum, New York, 1980), p. 515.

[12] Note that we have $\tan\theta_C \simeq m_d/m_s$ in this case. This is a consequence of having only one free parameter in (4). By letting the off-diagonal elements in (4) vary away from 1, the relation $\tan\theta_C \simeq (m_d/m_s)^{1/2}$ can be reproduced.

[13] Arbitrary mass matrices M_l, $M_{-1/3}$, and $M_{2/3}$ can be achieved in an SU(5) model with at least one $\underline{5}_H$ and one $\underline{45}_H$ of Higgs. In SO(10) models with Higgs only in the symmetric part of $(\underline{16}\times\underline{16})_S=\underline{10}+\underline{126}$, none of the above cases except for case 4 can occur. Of course one can include Higgs in the antisymmetric representation $(\underline{16}\times\underline{16})_A=\underline{120}$. All of these comments are well known.

[14] A number of the papers in Refs. 5–8 discuss attempts at making the nucleon decay rate *vanish* (at tree level) by means of such mixings. While generally this cannot be achieved (the relevant exact algebraic equalities among mixing angles cannot be satisfied subject to the constraints of the KM matrix), it may still be possible to have nucleon decay be Cabibbo-suppressed.

[15] D. V. Nanopoulos and S. Weinberg, Phys. Rev. D **20**, 2484 (1979).

[16] Grand unified models in which more than one family is contained in an irreducible representation [such as SO$(4n+2)$ models for $n>2$ and some E_6, E_7, and E_8 models] may contain baryon-number-violating vector bosons that also induce transitions between families even in the absence of mixings arising from the fermion mass matrices. If such vectors are the lowest-mass baryon-number-violating ones, they may give rise to unusual final states in nucleon decay.

[17] The mixing matrices appearing in the vertices involving neutrinos are irrelevant if the final-state neutrino is not observed. For the same reason we have ignored the possibility of mixings originating in the neutral mass matrix. All of these mixings will be irrelevant unless there are neutrinos with masses $\gtrsim 1$ GeV.