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Static properties of baryons in the SU(3) Skyrme model

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We study the SU(3)×SU(3) Skyrme model with explicit chiral- and flavor-symmetry-breaking terms. We evaluate the SU(3)-symmetric meson-baryon coupling-constant ratio α, SU(3) mass breaking in the octet and decuplet, and the $\Delta I=1$ part of the electromagnetic mass splitting in baryons. The theoretical numbers are in reasonable agreement with the experimental values.

I. INTRODUCTION

The idea that baryons might appear as solitons of a nonlinear σ model has been receiving a great deal of attention recently.[1] Several static properties of the nucleons (magnetic moment, g_A, charge radii, etc.) have been studied with reasonable agreement with experiment.[2] We present here a calculation based on an SU(3) generalization of these calculations. In particular, we calculate the ratio α $[=D/(D+F)]$ of the meson-baryon coupling constants in SU(3). Further assuming an SU(3) mass breaking in the meson sector we derive SU(3) mass breaking for baryons in the octet as well as the decuplet. Lastly, we calculate the electromagnetic mass difference among the isomultiplets using the $(K^+\text{-}K^0)$ mass difference as input.

One starts from the Lagrangian density

$$\mathscr{L}_{\rm KE}(U)=\frac{F_\pi{}^2}{16}\mathrm{Tr}[(\partial_\mu U)^\dagger\partial^\mu U]+\frac{1}{32e^2}\mathrm{Tr}[\partial_\mu UU^\dagger,\partial_\nu UU^\dagger]^2\ , \tag{1}$$

where

$$U=\exp\left(\frac{2i}{F_\pi}\lambda\phi\right)\ ,$$

with ϕ as the pseudoscalar octet. The classical Skyrme-soliton solution is given by

$$U_0(x)=\begin{pmatrix}\exp[iF(r)\vec{\tau}\cdot\hat{x}] & 0\\ 0 & 1\end{pmatrix}\ , \tag{2}$$

where $F(r)=\pi$ at $r=0$ and $F(r)=0$ at $r=\infty$. The topological number associated with such a solution has been identified with the conventional baryon number. The quantization of this Lagrangian in the baryon sector is done through the use of collective coordinates:

$$U=g(t)U_0g^{-1}(t)\ , \tag{3}$$

where $g(t)$ is an element of SU(3) expressed in the fundamental representation.

The baryon wave functions in the SU(3) space are given by the SU(3) generalizations of the D functions, $D^{(n)}_{\alpha\beta}(g)$, where n denotes the SU(3) representation,[3] α denotes the three quantum numbers (I,I_3,Y), and β determines the spin quantum number of the baryon state. In particular, for the spin-$\frac{1}{2}$ octet the wave funtions are $D^{(8)}_{I,I_3,Y;1/2,m,1}$ with $m=-\frac{1}{2}$ for spin-up and $m=+\frac{1}{2}$ for spin-down states. Similarly, for the decuplet the wave functions are given by $D^{(10)}_{I,I_3,Y;3/2,m,1}$ with $m=-\frac{3}{2},\ -\frac{1}{2},\ +\frac{1}{2}$, and $+\frac{3}{2}$ describing the states with spin $\frac{3}{2},\ \frac{1}{2},\ -\frac{1}{2}$, and $-\frac{3}{2}$, respectively. Given the wave functions it is straightforward to calculate the matrix elements of the relevant operators.

II. DETERMINATION OF α

In order to compute the effective coupling constants between the baryons states and the pseudoscalar mesons, it is convenient to start from the expression

$$I_\gamma(B',B)=\int d^3x\,\vec{q}\cdot\vec{x}(B'|\mathrm{Tr}[\lambda_\gamma(U-U^\dagger)]|B)\ . \tag{4}$$

Using Eqs. (3) and (2), on performing the angular integrations, we obtain

$$\begin{aligned}I_\gamma(B'B)&=iJq_j(B'|D^{(8)}_{j\gamma}(g^{-1})|B)\\&=iJq_j(B'|D^{*(8)}_{\gamma j}(g)|B)\ ,\end{aligned} \tag{5}$$

where

$$J=\frac{16}{3}\pi\int_0^\infty r^3dr\,\sin F(r)\ . \tag{6}$$

We have used the relation

$$g^{-1}\lambda_\gamma g=\lambda_\beta D^{(8)}_{\beta\gamma}(g^{-1})\ .$$

Now,

$$(B';\beta'\alpha'|D^*_{\gamma j}(g)|B,\beta\alpha)=\int d\mu(g)D^{*(8)}_{\beta'\alpha'}(g)D^{*(8)}_{\gamma j}(g)D^{(8)}_{\beta\alpha}(g)\ , \tag{7}$$

where the D functions are normalized:

$$\int d\mu(g)D^{(8)*}_{\beta'\alpha'}(g)D^{(8)}_{\beta\alpha}(g)=\delta_{\beta\beta'}\delta_{\alpha\alpha'}\ .$$

To evaluate the above we use the Clebsch-Gordan series

$$D^{(8)}_{\beta'\alpha'}(g)D^{(8)}_{\gamma j}(g)=\begin{pmatrix}8 & 8 & 8_1\\ \beta' & \gamma & \beta\end{pmatrix}\begin{pmatrix}8 & 8 & 8_1\\ \alpha' & j & \alpha\end{pmatrix}D^{(8)}_{\beta\alpha}+\begin{pmatrix}8 & 8 & 8_2\\ \beta' & \gamma & \beta\end{pmatrix}\begin{pmatrix}8 & 8 & 8_2\\ \alpha' & j & \alpha\end{pmatrix}D^{(8)}_{\beta\alpha}+\cdots\ , \tag{8}$$

where 8_1 (8_2) refers to the symmetric (antisymmetric) combination. ($\cdots$ refers to the elements of the series belonging to $\underline{1}$, $\underline{10}$, $\overline{\underline{10}}$, and $\underline{27}$.) Choosing $j=3$ ($I=1$, $I_3=0$, $Y=0$) and $B'=B=$ neutron state, we obtain

$$I_{\pi^0}(n,n)=\tfrac{7}{30}J\vec{\sigma}\cdot\vec{q}\ . \tag{9}$$

This may be interpreted as the π^0nn coupling if we consider the leading term of the expansion in $(U-U^\dagger)$. This is equivalent to dropping higher-order terms in the $1/N$ expansion, since they involve higher-loop corrections. Similarly, we can also evaluate $I_{\eta^0}(n,n)$ ($j=8$) and obtain

$$I_{\eta^0}(n,n)=-\frac{1}{10\sqrt{3}}J\vec{\sigma}\cdot\vec{q}\ .$$

This determines the SU(3)-symmetric coupling-constant ratio α $[=D/(D+F)]$ through

$$\frac{g_{\eta^0nn}}{g_{\pi^0nn}}=\frac{4\alpha-3}{\sqrt{3}}=-\frac{\sqrt{3}}{7}\ ,$$

leading to $\alpha=\frac{9}{14}$, which compares well with the experimental number[4] $\alpha=0.65\pm0.03$. This result, we may observe, is identical with what one obtains in the strong-coupling theory.[5]

III. BARYON MASS DIFFERENCES

We introduce an explicit SU(3)-symmetry breaking in the meson sector. This will induce a definite symmetry breaking in the soliton sector as well. The symmetry-breaking term that incorporates the meson masses is

$$\Delta\mathscr{L}=\frac{M_\pi^2+2m_K^2}{48}F_\pi^2\mathrm{Tr}[(U+U^\dagger)-6]-\frac{m_K^2-m_\pi^2}{8\sqrt{3}}F_\pi^2\mathrm{Tr}[\lambda_8(U+U^\dagger)]\ .$$

This implies an SU(3)-mass-breaking Hamiltonian

$$\Delta H^{(8)}=\int d^3x\frac{m_K^2-m_\pi^2}{8\sqrt{3}}F_\pi^2\mathrm{Tr}[\lambda_8(U+U^\dagger)]\ .$$

We may now evaluate SU(3) breaking in the baryon sector by computing the matrix element of $\Delta H^{(8)}$ between the corresponding baryon states. Using the wave functions for the baryons and performing the angular integrations we have

$$(B\alpha\beta|\Delta H^{(8)}|B\alpha\beta)=-\Delta m\int d\mu(g)D^{(n)*}_{\alpha\beta}(g)D^{(8)*}_{88}(g)D^{(n)}_{\alpha\beta}(g)\ ,$$

where[6]

$$\Delta m=\frac{2\pi}{3}(m_K^2-m_\pi^2)F_\pi^2\int_0^\infty(1-\cos F)r^2dr\simeq 388\ \mathrm{MeV}\ .$$

The SU(3)-mass-breaking contribution to the baryon octet is

$$(N|\Delta H^{(8)}|N)=-\frac{3\Delta m}{10},\quad(\Lambda|\Delta H^{(8)}|\Lambda)=-\frac{\Delta m}{10}\ ,$$

$$(\Sigma|\Delta H^{(8)}|\Sigma)=\frac{\Delta m}{10},\quad(\Xi|\Delta H^{(8)}|\Xi)=\frac{2\Delta m}{10}\ ,$$

for the decuplet is

$$(\Delta|\Delta H^{(8)}|\Delta)=-\frac{\Delta m}{8},\quad(Y^*|\Delta H^{(8)}|Y^*)=0\ ,$$

$$(\Xi^*|\Delta H^{(8)}|\Xi^*)=\frac{\Delta m}{8},\quad(\Omega^-|\Delta H^{(8)}|\Omega^-)=\frac{\Delta m}{4}\ .$$

This unfortunately cannot explain the mass differences completely even though the Gell-Mann–Okubo sum rule is reproduced. However, it has been argued[7] that there is a need to supplement it with an extra term proportional to hypercharge which contributes to the mass difference as well. The analysis using both terms has been done and it leads to $\Delta m\simeq 390$ MeV, which is in agreement with our predictions.

IV. ELECTROMAGNETIC MASS DIFFERENCES ($\Delta I=1$ PART)

The $\Delta I=1$ electromagnetic mass splitting in the meson sector is introduced by means of the extra term

$$\Delta H^{(3)}=-\frac{m_{K^+}{}^2-m_{K^0}{}^2}{16}F_\pi^2\int d^3x\,\mathrm{Tr}[\lambda_3(U+U^\dagger)]\ .$$

This gives rise to $\Delta I=1$ mass differences, which in particular are responsible for the neutron-proton mass difference. Evaluating the matrix element $(B|\Delta H^{(3)}|B)$ gives us the $\Delta I=1$ part of the mass difference. For the octet we have

$$(p|\Delta H^{(3)}|p)=-\frac{\mu}{10},\quad(n|\Delta H^{(3)}|n)=\frac{\mu}{10}\ ,$$

$$(\Sigma^+|\Delta H^{(3)}|\Sigma^+)=-\frac{\mu}{2},\quad(\Sigma^0|\Delta H^{(3)}|\Sigma^0)=0\ ,$$

$$(\Sigma^-|\Delta H^{(3)}|\Sigma^-)=\frac{\mu}{2}\ ,$$

$$(\Xi^0|\Delta H^{(3)}|\Xi^0)=-\tfrac{2}{5}\mu,\quad(\Xi^-|\Delta H^{(3)}|\Xi^-)=\tfrac{2}{5}\mu\ ,$$

where

$$\mu=\tfrac{2}{3}\pi(m_{K^0}{}^2-m_{K^+}{}^2)F_\pi^2\int_0^\infty(1-\cos F)r^2dr\simeq 3.3\ \mathrm{MeV}\ .$$

The mass splittings are $m_n-m_p=0.6$ MeV (1.3), $m_{\Sigma^-}-m_{\Sigma^+}=3.3$ MeV (8), $m_{\Xi^-}-m_{\Xi^0}=2.4$ MeV (6.4). (The numbers in parentheses are the experimental values.) Certainly all the signs are correct. An introduction of $\Delta I=2$ part in terms of $m_{\pi^+}-m_{\pi^0}$ mass difference could similarly give the $\Delta I=2$ part of the Σ mass differences.

We conclude our discussion by observing that the static properties of the baryons can be obtained using the properties of the meson sector as input. The SU(3) generalization we have attempted here appears sufficiently encouraging that other dynamical properties of the model should be considered seriously.

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[1]T. H. R. Skyrme, Proc. R. Soc. London, Ser. A **260**, 127 (1961); A. P. Balachandran *et al.*, Phys. Rev. Lett. **49**, 1124 (1982); Phys. Rev. D **27**, 1369 (1983); E. Witten, Nucl. Phys. **B223**, 422 (1983); **B223**, 433 (1983).

[2]G. S. Adkins, C. R. Nappi, and E. Witten, Nucl. Phys. **B228**, 552 (1983).

[3]A. P. Balachandran, lectures at IIT, Kanpur, 1983 (unpublished); Sanjay Jain and Spenta Wadia, Tata Institute for Fundamental Research, Bombay report (unpublished).

[4]R. E. Marshak, Riazuddin, and C. P. Ryan, *Theory of Weak Interactions in Particle Physics* (Wiley Interscience, New York, 1969).

[5]We thank Professor V. Singh for pointing this out. The relationship between the Skyrme model and static strong-coupling theory has been noted by J. L. Gervais and B. Sakita [Phys. Rev. Lett. **52**, 87 (1984)].

[6]The integral

$$\int_0^\infty (1-\cos F)r^2 dr = \frac{49\ \mathrm{MeV}}{m_\pi{}^2 F_\pi{}^2} ,$$

evaluated by G. S. Adkins and C. R. Nappi [Nucl. Phys. **B233**, 109 (1984)].

[7]E. Guadagnini, Nucl. Phys. **B236**, 35 (1984).