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New approach to left-right-symmetry breaking in unified gauge theories

D. Chang, R. N. Mohapatra, and M. K. Parida*
Department of Physics and Astronomy, University of Maryland, College Park, Maryland 20742
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We propose a new approach to the left-right-symmetric models of weak interactions where parity- and $SU(2)_R$-breaking scales are decoupled from each other. This changes the spectrum of Higgs bosons, which in turn affects the evolution of various gauge coupling constants with energy. This has profound implications for mass hierarchies in partial unification models based on $SU(2)_L \times SU(2)_R \times SU(4)_C$ and grand unified SO(10) models. We find several interesting SO(10)-breaking chains with and without an intermediate $U(1)_R$ symmetry for which both $M_{W_R^\pm}$ and M_{Z_R} masses are in the range of 1—20 TeV. We also find a symmetry-breaking pattern for which the $SU(4)_C$-breaking scale is in the range of 10^5 GeV. These patterns lead to observable $\Delta B=2$ as well as other right-handed-current effects at low energies.

I. INTRODUCTION

Left-right-symmetric models[1,2] based on the gauge group $G_{LR}=SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ have been the focus of a great deal of attention as a possible new symmetry of weak interactions beyond the standard $SU(2)_L \times U(1)_Y$ electroweak model. A central question in these models is the scale of parity violation and the strength of new interactions involving the right-handed currents. This new scale is conventionally associated with the breaking of the $SU(2)_R \times U(1)_{B-L}$ gauge symmetry down to $U(1)_Y$. In these models, the breaking of discrete parity symmetry and local $SU(2)_R$ symmetry occurs simultaneously.

Extensive analyses of the weak-interaction phenomenology have been carried out in the left-right-symmetric models. Since in the left-right-symmetric phase of the theory, the two SU(2) gauge couplings are equal ($g_{2L}=g_{2R}$), if the mass of the right-handed gauge boson, M_{W_R}, is not much higher than that of the left-handed gauge boson, M_{W_L}, then at $\mu \simeq M_{W_L}$, we have $g_{2L} \simeq g_{2R}$, which must be assumed in the phenomenological analysis of the model. It is, however, sometimes useful to assume that, even though the ratio (M_{W_L}/M_{W_R}) is not much smaller than 1, $\delta \equiv g_{2L}/g_{2R}$ is very different[3] from 1. It would, therefore, be of great interest to know if this latter class of models can arise out of a theory which is strictly left-right symmetric above a certain energy.

Another question of considerable practical importance is whether low-M_{W_R} theories can be consistently embedded in grand unified theories such as SO(10) or SU(16), etc. This has been studied by Rizzo and Senjanović,[4] who pointed out that with the conventional breaking pattern of $SU(2)_R \times U(1)_{B-L} \times P$ down to $U(1)_Y$, a low-mass W_R requires $\sin^2\theta_W \simeq 0.27$. Furthermore, they argued that neutral-current data available at the time allowed for such a large $\sin^2\theta_W$. A similar situation arises in other grand unified theories,[5] such as SU(16). However, the discovery of W and Z bosons at the CERN $\bar{p}p$ collider[6] has ruled out such large $\sin^2\theta_W$ values and implies $\sin^2\theta_W \simeq 0.23$—0.24. This would imply $M_{W_R} \geq 10^{10}$ GeV for SO(10) grand unification with conventionally assumed breaking patterns; for other unified models, the situation is similar. Does this then imply that a low-mass right-handed gauge boson is incompatible with simple grand unified models? An affirmative answer to this question would have serious experimental implications, such as rendering unobservable $\Delta B=2$ transitions such as n-$\bar{n}$ oscillations, $\Delta L=2$ transitions [$(\beta\beta)_{0\nu}$ decay], etc., within the framework of grand unified models with the minimal-fine-tuning hypothesis.[7]

In the present paper we propose a new approach to left-right-symmetric models which has bearing on all the above questions and makes the right-handed scale M_{W_R} more accessible without conflicting with the attractive hypothesis of grand unification. Our new proposal, which has been briefly reported earlier,[8] is to decouple the breaking scales of discrete spacetime symmetry, the parity from that of the local $SU(2)_R$ symmetry, giving rise to the new scale M_P, different from the scale of the right-handed gauge boson mass M_{W_R}. The original symmetry $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times P$, where P denotes the parity symmetry, is first broken down to $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ at a mass scale M_P. This manifests itself in different values of g_{2L} and g_{2R} as well as a different spectrum of Higgs-boson masses and *not* in a nonzero $W_R^\pm$ mass as in the left-right models discussed to date. The $SU(2)_R$ gauge symmetry is subsequently broken down at a mass scale $M_{W_R} \ll M_P$. The asymmetry in the Higgs-boson mass spectrum changes the behavior of coupling constants also when the model is embedded in the partial-unification schemes or unified theories and changes the value of M_{W_R} allowed by the constraints of grand unification. It is this decoupling of parity- and $SU(2)_R$-gauge-symmetry breaking and its impact on the low-energy predictions of partially unified and grand unified models that we study in this paper. Common to all the scenarios investigated in this paper is the feature that between M_P and M_{W_R}, only the right-handed Higgs boson (in our case triplets) contribute to the β functions and not the left-handed ones. This has a profound effect on the mass hierarchies.

We examine two classes of models in detail: (i) the partial-unification model $SU(2)_L \times SU(2)_R \times SU(4)_C \times P$ and (ii) the SO(10) model, in their various symmetry-breaking chains. We have isolated several symmetry-breaking chains in the SO(10) model where either the M_C (the partial-unification scale) or M_{W_R} or both are low enough to lead to observable matter-antimatter mixing as well as to observable $\Delta L = 2$ and *CP*-violating effects at low energies. We find a specially interesting chain where SO(10), parity, and $SU(4)_C$ break down at one scale to $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C$ and which can lead to a value of M_{W_R} and $M_{Z_R} \approx 20$ TeV for $\sin^2\theta_W \simeq 0.245$ to 0.25. In our opinion, this ought to provide new motivations to search for signatures of low-mass $B-L$ symmetry breaking such as $n-\bar{n}$ oscillation, double-β decay, and the like. Of course, our result makes the models with $g_{2L} \neq g_{2R}$ and arbitrarily low M_{W_R} quite compatible with exact left-right symmetry.

This paper is organized as follows: in Sec. II, we illustrate how to implement our idea in the case of the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ model; Sec. III is devoted to explaining the notation used in the paper; in Sec. IV, we begin studying the impact of our idea on intermediate mass scales in the case of the $SU(2)_L \times SU(2)_R \times SU(4)_C \times P$ model; in Sec. V, we discuss embedding of our idea in grand unified models and the multiplets needed to implement our idea in those models; in Sec. VI, we examine various symmetry-breaking chains in the SO(10) models and the associated mass scales; Sec. VII is devoted to concluding remarks. In an Appendix, we note how we can spontaneously break *CP* symmetry along with parity without breaking any gauge symmetry and discuss the embedding of parity symmetry in the SO(10) model.

II. ILLUSTRATION OF THE BASIC IDEA WITH A SIMPLE EXAMPLE

In this section, we illustrate explicit realization of our idea using the Higgs potential for an $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times P$ model. The fermions in this model are represented, as before, in a left-right-symmetric manner. To implement the symmetry-breaking pattern envisaged by us, i.e.,

$$SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times P \underset{M_P}{\longrightarrow} SU(2)_L \times SU(2)_R \times U(1)_{B-L} \underset{M_W}{\longrightarrow} SU(2)_L \times U(1)Y , \quad (2.1)$$

we choose the following Higgs multiplets with the group transformation properties,

$$\Delta_L(3,1,2), \quad \Delta_R(1,3,2), \quad \phi(2,2,0), \quad \eta(1,1,0) , \quad (2.2)$$

possessing the following transformation properties under parity,

$$\Delta_L \leftrightarrow \Delta_R, \quad \phi \leftrightarrow \phi^\dagger, \quad \eta \leftrightarrow -\eta . \quad (2.3)$$

The most general Higgs potential involving these fields can be written as follows:

$$V = V_\Delta + V_\phi + V_\eta + V_{\eta\Delta} + V_{\Delta\phi} + V_{\eta\phi} , \quad (2.4)$$

where

$$V_\Delta = \mu_\Delta{}^2[\mathrm{Tr}(\Delta_L^\dagger \Delta_L) + \mathrm{Tr}(\Delta_R^\dagger \Delta_R)] + \text{quartic terms} , \quad (2.5)$$

$$V_\eta = -\mu_\eta{}^2\eta^2 + \lambda_1\eta^4 , \quad (2.6)$$

$$V_{\eta\Delta} = M\eta(\Delta_L^\dagger \Delta_L - \Delta_R^\dagger \Delta_R) + \lambda_2\eta^2(\Delta_L^\dagger \Delta_L + \Delta_R^\dagger \Delta_R) , \quad (2.7)$$

$$V_{\eta\phi} = \lambda_2\eta^2 \,\mathrm{Tr}(\phi^\dagger\phi) + \lambda_2^1\eta^2(\mathrm{Det}\phi + \mathrm{Det}\phi^\dagger) . \quad (2.8)$$

Here V_ϕ and $V_{\eta\phi}$ are chosen in the most general manner so as to lead to $\langle \phi \rangle \neq 0$ that breaks $SU(2)_L \times U(1)_Y$ symmetry. It is clear from Eqs. (2.6) and (2.7) that for $\mu_\eta{}^2 > 0$, the minimum of the potential occurs at

$$\langle \eta \rangle \equiv M_P = \frac{\mu_\eta}{(2\lambda_1)^{1/2}} . \quad (2.9)$$

Equations (2.5) and (2.7) then imply

$$\mu_{\Delta_R}{}^2 = \mu_\Delta{}^2 - M\langle \eta \rangle + \lambda_2\langle \eta \rangle^2$$

and (2.10)

$$\mu_{\Delta_L} = \mu_\Delta{}^2 + M\langle \eta \rangle + \lambda_2\langle \eta \rangle^2 .$$

At this stage, we impose the minimal-fine-tuning condition in Eq. (2.10) such that $\mu_{\Delta_R}{}^2 < 0$ and $|\mu_{\Delta_R}{}^2| \ll \langle \eta \rangle^2$, this leads to a minimum of the potential where $\langle \Delta_R \rangle = V_R \neq 0$. Thus, we note that the $SU(2)_R$-breaking scale is induced by the parity-breaking scale. We also note that the masses of the components of Δ_L are of order $\langle \eta \rangle$. Thus, the spectrum of Higgs bosons exhibits the left-right asymmetry even though $SU(2)_R$ symmetry is unbroken. This is the first known mechanism which makes the left-handed triplets much heavier than their right-handed counterparts, consistent with minimal fine-tuning.

We now discuss the impact of this parity breaking on the gauge couplings. Since the original Lagrangian has discrete parity invariance, in the absence of radiative corrections, $g_{2L}^{(0)} = g_{2R}^{(0)}$. But once we include radiative corrections, the effective gauge couplings will become different. A typical set of graphs that lead to this difference in the gauge coupling constant are those in Fig. 1. The two graphs induce corrections to $g_{2L,R}$ with opposite sign. It is clear that $g_{2L} - g_{2R}$ is finite and of order g^3. Leading contributions to these corrections can be summed up by using renormalization-group equations, as is done below. To start with, at a mass scale $\mu > M_P$, the complete Lagrangian is invariant under $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times P$ and possesses $L \leftrightarrow R$ discrete symmetry. At this stage the $SU(2)_L$ coupling constant $g_{2L}(\mu)$ and the $SU(2)_R$ coupling constant $g_{2R}(\mu)$ are equal, i.e.,

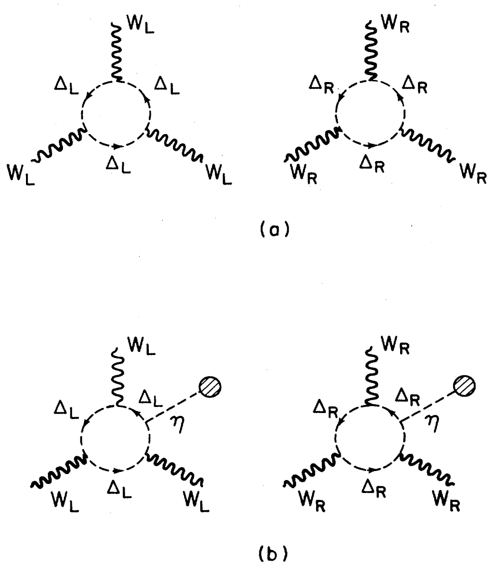

FIG. 1. One-loop radiative corrections that contribute to the difference between g_L and g_R.

$$\delta(\mu)=g_{2L}(\mu)/g_{2R}(\mu)=1,\ \ \mu>M_P\ . \tag{2.11}$$

But below M_P, when the parity-odd singlet η at first acquires a nonzero vacuum expectation value, the parity symmetry is spontaneously broken without breaking the gauged $SU(2)_R$ symmetry.

To describe the effect of parity breaking below this point, we consider the ratio $\delta(\mu)$ for $\mu<M_P$. We note that, due to the asymmetric spectrum of the Higgs-boson masses, only the right-handed Higgs multiplets contribute for $v_R<\mu<M_P$ and we obtain

$$\delta^2(M_R)=1+\frac{2\alpha(M_R)N_\Delta}{6\pi\sin^2\theta_W(M_R)}\ln\frac{M_P}{M_R}\ , \tag{2.12}$$

where N_Δ stands for the number of triplet Higgs bosons. We emphasize that even though the gauge couplings g_{2L} and g_{2R} are different and the theory is left-right asymmetric, the gauge symmetry $SU(2)_L\times SU(2)_R\times U(1)_{B-L}$ is unbroken. Note that in conventional methods of breaking left-right asymmetry the ratio $\delta^2(M_R)=1$. In most of the physical applications we will need the ratio $\delta(M_W)$ given by

$$\delta^2(M_W)=1+\frac{\alpha(M_W)}{2\pi\sin^2\theta_W(M_W)}\times\left[\left[\frac{22}{3}-\frac{4}{3}N_G-\frac{N_\phi}{6}\right]\ln\frac{M_R}{M_W}+\frac{2}{3}N_\Delta\ln\frac{M_P}{M_R}\right]\ , \tag{2.13}$$

where extrapolation of g_{2L} has been taken into account. In Eq. (2.13) N_ϕ stands for the number of complex doublets mentioned in Eq. (2.2). It is clear that the variations of $\delta(M_R)$ and $\delta(M_W)$ from unity depends on mass scales M_P and M_{W_R} and the number of doublets and triplets.

As an immediate application of our idea, we consider the bounds on M_{W_R} implied by the K_L-K_S mass-difference analysis.[9] The bound $M_{W_R}\geq 1.6$ TeV is derived on the assumption that $g_{2L}=g_{2R}$. But if the parity symmetry is broken at $M_P\gg M_{W_R}$, the above bound becomes $M_{W_R}\geq[1.6/\delta(M_W)]$ TeV and since $\delta>1$, the bound becomes weaker. In fact, this may enable us to accommodate an M_{W_R} in the hundreds-of-GeV region.

III. NOTATION AND CONVENTIONS

In the rest of this paper we use the following notations for various groups:

$$\begin{aligned}
&G_{224}\equiv SU(2)_L\times SU(2)_R\times SU(4)_C\ ,\\
&G_{224P}\equiv SU(2)_L\times SU(2)_R\times SU(4)_C\times P\ ,\\
&G_{2213}\equiv SU(2)_L\times SU(2)_R\times U(1)_{B-L}\times SU(3)_C\ ,\\
&G_{2213P}\equiv SU(2)_L\times SU(2)_R\times U(1)_{B-L}\times SU(3)_C\times P\ ,\\
&G_{2113}\equiv SU(2)_L\times U(1)_R\times U(1)_{B-L}\times SU(3)_C\ ,\\
&G_{213}\equiv SU(2)_L\times U(1)_Y\times SU(3)_C\ ,\\
&G_{13}\equiv U(1)_{EM}\times SU(3)_C\ ,\\
&G_{221}\equiv SU(2)_L\times SU(2)_R\times U(1)_{B-L}\ ,\\
&G_{221P}\equiv SU(2)_L\times SU(2)_R\times U(1)_{B-L}\times P\ ,
\end{aligned} \tag{3.1}$$

where P denotes the discrete $L\leftrightarrow R$ symmetry. The symbols g_{2L}, g_{2R}, g_{1R}, g_Y, g_3, g_{BL}, and g_4 denote coupling constants for the unitary groups $SU(2)_L$, $SU(2)_R$, $U(1)_R$, $U(1)_Y$, $SU(3)_C$, $U(1)_{B-L}$, and $SU(4)_C$, respectively.

In our calculations for mass hierarchies we make use of the evolution equation for coupling constants due to Georgi, Quinn, and Weinberg,[10]

$$\frac{1}{g_N{}^2(\mu)}=\frac{1}{g_N{}^2(M)}+\frac{A_N}{8\pi^2}\ln\frac{M}{\mu}\ , \tag{3.2}$$

where, for the $SU(N)$ group with $N\geq 2$,

$$A_N=\frac{-11N}{3}+\frac{4}{3}N_G+T(R_S)\ . \tag{3.3}$$

In Eq. (3.3) the first, second, and third terms denote the contributions of the gauge bosons, fermions, and Higgs scalars, respectively, and N_G denotes the number of generations of fermions. For any U(1) group, the first term in Eq. (3.3) is zero.

We will use the following notations for the value of $\sin^2\theta_W$ at different mass scales:

$$\begin{aligned}
&x_W(M_W)=\sin^2\theta_W(M_W)=e^2(M_W)/g_{2L}{}^2(M_W)\ ,\\
&x_W(M_R)=\sin^2\theta_W(M_R)=\frac{e^2(M_R)}{g_{2L}{}^2(M_R)}\ ,
\end{aligned} \tag{3.4}$$

where M_W (M_R) denotes the left- (right-) handed W-boson mass. Whenever the mass of the right-handed neutral boson Z_R is different from that of the right-handed charged gauge bosons $W_R^\pm$, it will be denoted by M_{R^0}.

Unless specified parenthetically, the quantities x_W, α, and α_s stand for their values at M_W,

$$\alpha=\frac{e^2(M_W)}{4\pi}, \quad \alpha_s=\frac{g_3{}^2(M_W)}{4\pi} . \tag{3.5}$$

The asymmetry parameter $\delta(M_R)$, which is a measure of left-right asymmetry at the $W_R^{\pm}$-boson mass and the parameter $\delta(M_W)$ which enters into the charged- and neutral-current phenomenology are defined as

$$\delta^2(M_R)=g_{2L}{}^2(M_R)/g_{2R}{}^2(M_R) ,$$
$$\delta^2(M_w)=g_{2L}{}^2(M_W)/g_{2R}{}^2(M_W) . \tag{3.6}$$

But when $SU(2)_R$ breaks in two steps, with $U(1)_R$ as an intermediate symmetry, the mass M_{R^+} of W_R turns out to be different from the mass M_R of Z_R. In this case the ratio that is of interest in neutral-current phenomenology and the mass matrix of neutral Z bosons is

$$\delta_N{}^2=g_{2L}{}^2(M_W)/g_{1R}{}^2(M_{R^0}) , \tag{3.7}$$

whereas the corresponding quantity of interest for K_L-K_S mass difference, electric dipole moment of the neutron, neutrinoless double-β decay, and other charged-current parameters, etc., at low energies is $\delta^2(M_W)$ of Eq. (3.6).

IV. PARITY BREAKING AND THE SCALE OF PARTIAL-UNIFICATION SYMMETRY $SU(2)_L \times SU(2)_R \times SU(4)_C$

In this section, we consider the implication of separating parity- and $SU(2)_R$-breaking scales on the partial-unification mass at which $SU(2)_L \times SU(2)_R \times SU(4)_C$ symmetry emerges. In the conventional treatment of this group, $SU(2)_R$ and parity symmetry are broken at the same scale. As a result constraints of $\sin^2\theta_W$ and α_s lead to a unification scale $M_C \sim 10^{11}-10^{12}$ GeV. One implication of such a high unification scale is the complete suppression of all effects associated with right-handed currents as well as quark-lepton unification such as n-$\bar{n}$ oscillation, $K_L^0 \to \mu \bar{e}$, etc. It is, therefore, important to study if by separating the parity breaking from the $SU(2)_R$ scale, M_C can be lowered. As we see below, it is indeed the case, although it is not low enough to be highly visible.

In contrast with grand unification schemes, the partial-unification scheme[11,12] based on the gauge group G_{224P} involves two coupling constants at the unification mass, $M_U=M_P$:

$$g_{2L}(M_P)=g_{2R}(M_P) ,$$
$$g_3(M_P)=g_4(M_P)=(\tfrac{2}{3})^{1/2}g_{BL} . \tag{4.1}$$

As a result, we obtain only one relation involving $\sin^2\theta_w$, α_s and the mass scales. We will consider three intermediate symmetry-breaking chains between $G_{224P} \to G_{213}$. All three can be obtained from the following general symmetry-breaking chain by equating various mass scales,

$$G_{224P} \xrightarrow[M_P]{} G_{224} \xrightarrow[M_C]{} G_{2213} \xrightarrow[M_{R^+}]{} G_{2113} \xrightarrow[M_{R^0}]{} G_{213} \xrightarrow[M_W]{} G_{13} . \tag{4.2}$$

Using the evolution equations for the various coupling constants in the standard manner, we obtain

$$x_W(M_W)=\frac{1}{2}-\frac{1}{3}\frac{\alpha(M_W)}{\alpha_s(M_W)}-\frac{\alpha(M_W)}{4\pi}\left[(\tfrac{44}{3}+\tfrac{5}{3}T_Y-T_{2L})\ln\frac{M_{R^0}}{M_W}+(\tfrac{44}{3}+\tfrac{2}{3}T_{BL}+T_{1R}-T_{2L})\ln\frac{M_{R^+}}{M_{R^0}}\right.$$
$$\left.+(\tfrac{22}{3}+\tfrac{2}{3}T_{BL}+T_{2R}-T_{2L})\ln\frac{M_C}{M_{R^+}}+(T_{2R}T_{2L})\ln\frac{M_P}{M_C}\right] , \tag{4.3}$$

where the T's are the Higgs-boson contributions to the various β functions defined in Sec. III in the appropriate mass range. Similarly, using the evolution equations for g_{2L} and g_{2R}, we find

$$\delta^2(M_W)=1+\frac{\alpha(M_W)}{2\pi x_W(M_W)}\left[(T_{2R}-T_{2L})\ln\frac{M_p}{M_C}+(T_{2R}-T_{2L})\ln\frac{M_C}{M_{R^+}}+(\tfrac{10}{3}-T_{2L})\ln\frac{M_{R^+}}{M_W}\right] , \tag{4.4}$$

$$\delta^2(M_R)=\delta^2(M_W)-\frac{\alpha(M_W)}{2\pi x_W(M_W)}\left[\frac{10}{3}-T_{2L}\right]\ln\frac{M_R}{M_W} . \tag{4.5}$$

In deriving these expressions, we have assumed three generations of light fermions below M_W. For N_G generations in the formulas (4.4) and (4.5), $\frac{10}{3}$ gets replaced by $(22-4N_G)/3$.

Let us now look at the various special cases and obtain the values of the intermediate mass scales in each case.

Case (i). We consider the case, where there is only one intermediate symmetry G_{224}, which can be implemented by the following choice of Higgs multiplets:[8,13]

$$\eta(1,1,1), \quad \Delta_L(3,1,10)+\Delta_R(1,3,10), \quad \phi(2,2,1) . \tag{4.6}$$

Here the quantities in parentheses denote the transformation property of each multiplet under G_{224}. This leads to the following symmetry-breaking pattern:

$$G_{224P} \underset{M_P=\langle\eta\rangle\neq 0}{\longrightarrow} G_{224} \underset{M_C=g\langle\Delta_R\rangle\neq 0}{\longrightarrow} G_{213} .$$

We note that the triplet-Higgs-boson masses become asymmetric with $\Delta_L \equiv (3,1,10)$ being at a mass scale of order M_P, whereas $\Delta_R \equiv (1,3,10)$ has a mass of order M_C. Taking this asymmetry into account we find the following expression for $\sin^2\theta_W$ ($\equiv x_W$) by substituting $M_c = M_{R^+} = M_{R^0}$ in Eq. (4.3):

$$x_W(M_W) = \frac{1}{2} - \frac{1}{3}\frac{\alpha(M_W)}{\alpha_s(M_W)} - \frac{\alpha(M_W)}{4\pi}\left[\frac{44}{3}\ln\frac{M_R}{M_W} + \frac{20}{3}\ln\frac{M_P}{M_R}\right], \tag{4.7}$$

$$\delta^2(M_W) = 1 + \frac{\alpha(M_W)}{2\pi x_W(M_W)}\left[\frac{20}{3}\ln\frac{M_P}{M_R} + \frac{19}{6}\ln\frac{M_R}{M_W}\right], \tag{4.8}$$

$$\delta^2(M_R) = 1 + \frac{\alpha(M_R)}{2\pi x_W(M_R)}\left[\frac{20}{3}\ln\frac{M_P}{M_R}\right]. \tag{4.9}$$

In Eq. (4.9), $x_W(M_R)$ can be evaluated with the help of Eq. (4.7), putting $M_W = M_R$ on both sides.

The implication of Eq. (4.7) for M_R and M_P is given in Table I. We find that for $\sin^2\theta_W \simeq 0.24$, the lowest-allowed value is $M_R \geq 10^8$ GeV for $M_P \leq 10^{19}$ GeV. This is considerably better than the previous results. Unfortunately, however, its implications for low-energy processes such as n-$\bar{n}$ oscillation or $K \to \mu\bar{e}$ are not so hopeful for detection in the near future. For instance, assuming Higgs-boson masses are 10 times smaller than the scale M_C (i.e., $M_{\Delta qq} \simeq 10^7$ GeV), the maximum value of the six-quark $\Delta B = 2$ amplitude is[7,13]

$$A_{\Delta B=2} \simeq 10^{-35}\ (\text{GeV})^{-5} . \tag{4.10}$$

After the quark-wave-function effects are taken into account, this would lead to an n-$\bar{n}$ mixing time $\tau_{n\text{-}\bar{n}} \simeq 10^6-10^7$ yr. Similarly, we would obtain for the $K \to \mu e$ branching ratio

$$\frac{B(K \to \mu e^-)}{B(K \to \text{all})} \simeq 10^{-24} . \tag{4.11}$$

Case (ii). In this case, we include the Higgs multiplets transforming as $(3,1,15) \equiv D_L$ and $(1,3,15) \equiv D_R$ in addition to the ones already discussed and consider the following chain of symmetry breaking.

$$G_{224P} \underset{M_P}{\overset{\langle\eta\rangle\neq 0}{\longrightarrow}} G_{224} \underset{M_C=M_{R^+}}{\overset{\langle D_R\rangle\neq 0}{\longrightarrow}} G_{2113} \underset{M_{R^0}}{\overset{\langle\Delta_R^0\rangle\neq 0}{\longrightarrow}} G_{123} , \tag{4.12}$$

where Δ_R^0 is the neutral component of $\Delta_R(1,3,10)$.

In this case, the effect of discrete parity breaking (i.e., $\langle\eta\rangle \neq 0$) is to elevate both Δ_L and D_L masses to the scale M_P, thereby enhancing the asymmetric contributions from the Higgs bosons to the gauge couplings. The equations for the various parameters in this case can be obtained from Eq. (4.3) through (4.5) by setting $M_C = M_{R^+}$ and we obtain

$$x_W(M_W) = \frac{1}{2} - \frac{1}{3}\frac{\alpha(M_W)}{\alpha_s(M_W)} - \frac{\alpha(M_W)}{4\pi}\left[\frac{35}{3}\ln\frac{M_P}{M_{R^+}} + \frac{46}{3}\ln\frac{M_{R^+}}{M_{R^0}} + \frac{44}{3}\ln\frac{M_{R^0}}{M_W}\right], \tag{4.13}$$

$$\delta^2(M_W) = 1 + \frac{\alpha(M_W)}{2\pi x_W(M_W)}\left[\frac{35}{3}\ln\frac{M_P}{M_{R^+}} + \frac{19}{6}\ln\frac{M_{R^+}}{M_W}\right]. \tag{4.14}$$

The values of $x_W(M_R)$ and $\delta^2(M_R)$ can be obtained from Eqs. (4.13) and (4.14) by choosing $M_W = M_{R^0} = M_{R^+}$. The value of the U(1) gauge coupling constant at M_W can be obtained by defining a function $\delta_N(M_W) \equiv g_{2L}/g_{1R}$:

$$\delta_N{}^2(M_W) = 1 + \frac{\alpha(M_W)}{2\pi x_W(M_W)}\left[\frac{35}{3}\ln\frac{M_P}{M_{R^+}} + \frac{23}{3}\ln\frac{M_{R^+}}{M_{R^0}} + \frac{19}{6}\ln\frac{M_{R^0}}{M_W}\right]. \tag{4.15}$$

This parameter is needed in the analysis of neutral-current data in $SU(2)_L \times U(1)_{B-L} \times U(1)_R$-type models. The allowed mass scales for this model are given in Table II. We note that for $\sin^2\theta_W \simeq 0.23-0.24$ and $\alpha_s = 0.10-0.11$, $M_C \equiv M_{R^+}$ turns out to be 10^5-10^6 GeV, for $M_{R^0} \simeq$ TeV and $M_P \simeq 10^{14}-10^{15}$ GeV. This model has interesting implications. Since the diquark Higgs bosons in the (1,3,10) representation acquire masses of order $\sim M_C$, this leads to

TABLE I. Mass scales M_P and M_R and values of the asymmetry parameter for the symmetry-breaking chain in case (i) of the partial-unification group.

$\alpha_s(M_W)$	$x_W(M_W)$	M_P (GeV)	$M_R = M_C$ (GeV)	$\delta^2(M_W)$	$\delta^2(M_R)$
0.10	0.24	10^{19}	2×10^8	2.1	1.7
	0.24	10^{16}	6×10^{10}	1.75	1.31
	0.23	10^{19}	1.6×10^9	2.1	1.6

TABLE II. Intermediate mass scales for case (ii) of the partial-unification scheme with an intermediate $U(1)_{I_{3R}}$.

$\alpha_s(M_W)$	$x_W(M_W)$	M_P (GeV)	$M_{R^+}=M_C$ (GeV)	M_{R^0} (GeV)	$\delta^2(M_W)$	$\delta_N{}^2(M_W)$
0.1	0.24	3×10^{14}	10^7	3×10^2	2.2	2.5
	0.24	6×10^{14}	10^6	10^3	2.4	2.5
	0.24	10^{15}	10^5	10^3	2.5	2.6
	0.23	10^{15}	10^7	3×10^2	2.4	2.6
	0.23	3×10^{15}	10^6	10^3	2.5	2.7
	0.23	5×10^{15}	10^5	10^3	2.7	2.8

observable n-$\bar{n}$ oscillation times,[7,13] $\tau_{n\text{-}\bar{n}}\simeq10^7$ sec. This also can give rise to hydrogen-antihydrogen[7,14] oscillation with oscillation times $\tau_{\text{H-}\bar{\text{H}}}\simeq10^{12}$–$10^{13}$ yr. We also wish to point out that, since $M_{R^+}\simeq10^5$–10^6 GeV, lepton-number-violating processes such as $(\beta\beta)_{0\nu}$ decay, $\mu^-\to e^-\gamma$, etc., that involve the exchange of right-handed charged W_R bosons will be highly suppressed.[15,16] On the other hand, neutrino masses are inversely proportional to $\langle\Delta_R^0\rangle$ (Ref. 17) which is of order of 1 TeV. Thus, we would expect $m_{\nu_e}\simeq1$ eV, $m_{\nu_\mu}\simeq40$ keV, and $m_{\nu_\tau}\simeq15$–20 MeV.

In this case, $B(K_L^0\to\mu\bar{e})$ will be of order 10^{-9}–10^{-13}, which is within the observable range of proposed experiment.[18]

We also wish to point out here that, in order to account for the realistic quark-lepton mass spectrum, this model needs to be extended by including a Higgs multiplet $\xi(2,2,15)$, whose color neutral component acquires a vacuum expectation value of order $\sim M_W/g$. It has been recently pointed out[12] that this can lead to $(B+L)$-conserving proton decay modes such as $p\to e^-\pi^+\pi^+$ and $p\to ll\bar{l}+$ mesons, etc. In our case, we estimate the amplitude for this process to be (within the framework of the minimal-fine-tuning hypothesis)

$$A_{p\to e^-\pi^+\pi^+}\simeq\frac{h_D{}^2h_M\lambda'V_R}{M_C{}^6}\,. \tag{4.16}$$

For $h_D\simeq h_M\simeq\lambda'\simeq10^{-1}$, we obtain $A_{p\to e^-\pi^+\pi^+}\simeq10^{-32}$ GeV^{-5}, which could be observable. Note that we have chosen "natural" mass scales for all Higgs multiplets.

Case (iii). In this case, in addition to the Higgs bosons in case (ii), we include a Higgs multiplet Σ belonging to the (1,1,15) representation under $SU(2)_L\times SU(2)_R\times SU(4)_C$. In this case, we realize the most general symmetry-breaking pattern allowed for this model, i.e.,

$$G_{224P}\underset{M_P}{\overset{\langle\eta\rangle\neq0}{\longrightarrow}}G_{224}\underset{M_C}{\overset{\langle\Sigma\rangle\neq0}{\longrightarrow}}G_{2213}\underset{M_{R^0}}{\overset{\langle D_R\rangle\neq0}{\longrightarrow}}G_{2113}\underset{M_{R^0}}{\overset{\langle\Delta_R^0\rangle\neq0}{\longrightarrow}}G_{213}\,.$$

In order to write down the expression for x_W, we allow the Higgs bosons to acquire their natural mass scales consistent with the survival hypothesis.[7] In that case, Higgs bosons contribute in various mass ranges as follows:

$M_W<\mu<M_{R^0}$: one $SU(2)_L$ doublet of $\phi(2,2,1)$;

$M_{R^0}<\mu<M_{R^+}$: one doublet of ϕ, Δ_R^0 ;

$M_{R^+}<\mu<M_C$: $\phi(2,2,1)$, Δ_R, (1,3,1) of D_R ;

$M_C<\mu<M_P$: ϕ, Δ_R, D_R, Σ .

This leads to the following equations:

$$x_W(M_W)=1-\frac{1}{3}\frac{\alpha(M_W)}{\alpha_s(M_W)}-\frac{\alpha}{4\pi}\left[\frac{35}{3}\ln\frac{M_P}{M_C}+\frac{28}{3}\ln\frac{M_C}{M_{R^+}}+\frac{46}{3}\ln\frac{M_{R^+}}{M_{R^0}}+\frac{44}{3}\ln\frac{M_{R^0}}{M_W}\right], \tag{4.17}$$

$$\delta^2(M_W)=1+\frac{\alpha}{2\pi x_W}\left[\frac{35}{3}\ln\frac{M_P}{M_C}+\ln\frac{M_C}{M_{R^+}}+\frac{19}{6}\ln\frac{M_{R^+}}{M_W}\right], \tag{4.18}$$

$$\delta_N{}^2(M_W)=1+\frac{\alpha}{2\pi x_W}\left[\frac{35}{3}\ln\frac{M_P}{M_C}+\ln\frac{M_C}{M_{R^+}}+\frac{23}{3}\ln\frac{M_{R^+}}{M_{R^0}}+\frac{19}{6}\ln\frac{M_{R^0}}{M_W}\right]. \tag{4.19}$$

Allowed solutions for the various mass scales are given in Table III. We find that with $\sin^2\theta_W=0.22$–0.23 and $\alpha_s(M_W)=0.1$–0.11, it is possible to have a value for $M_{W_R^\pm}$ and M_{Z_R} in the 300 GeV to 1 TeV range. The M_C turns out to be 10^5–10^{10} GeV depending on the values of M_P. Thus one may, in some cases, have observable matter-antimatter oscillations.

The light right-handed charged gauge boson permitted in this scenario would lead to a variety of low-energy

TABLE III. Mass scales and asymmetry parameters for case (iii) of the partial-unification scheme.

$\alpha_s(M_W)$	$x_W(M_W)$	M_P (GeV)	M_C (GeV)	M_{R^+} (GeV)	M_{R^0} (GeV)	$\delta^2(M_W)$	$\delta_N{}^2(M_W)$
	0.24	10^{16}	10^5	10^3	3×10^2	2.6	2.6
0.1	0.24	2.5×10^{16}	10^6	10^3	10^3	2.5	2.5
	0.24	10^{17}	10^{10}	10^3	3×10^2	2.1	2.1
	0.23	5×10^{16}	10^5	10^3	3×10^2	2.8	2.8
	0.23	10^{17}	10^6	10^3	10^3	2.7	2.7
	0.23	5×10^{17}	10^{10}	10^3	3×10^2	2.2	2.3

$\Delta L\neq0$ processes such as $\mu^-\to e^-\gamma$, $(\beta\beta)_{0\nu}$-decay processes to be in the detectable range.[15,16] The rare kaon-decay process $K_L^0\to\mu e^-$ will also be observable for the cases where $M_C\simeq10^5$ GeV. Similarly, including a (2,2,15) multiplet in this case will also lead to observable $(B+L)$-conserving proton decay models.[12]

V. LEFT-RIGHT-SYMMETRIC GRAND UNIFICATION AND BREAKING OF DISCRETE PARITY SYMMETRY

In this section, we discuss possible embedding of our idea in the framework of various left-right-symmetric grand unified models. There are two different classes of grand unified models that are left-right symmetric: (i) grand unified groups[19,20] SO(10) and E$_6$, and (ii) maximal unification groups[21] SU(16) and $[\mathrm{SU}(2N)]^4$.

The fundamental difference between these two classes of models is in the chiral structure of the fermionic sector. The models of class II, in order to be anomaly free need mirror fermions and therefore, without additional discrete symmetries will lead to fermion masses which are superheavy ($m_f\sim M_u$). The models of type I do not have this feature. Second, besides the D-parity-breaking method, as discussed in this paper for SO(10), being applicable for SU(16) and $[\mathrm{SU}(8)]^2$ types of theories, there is also a different way which can ascribe asymmetry in the coupling constants, g_{2L} and g_{2R}, as mentioned below:[22]

$$\mathrm{SU}(16)\underset{M_U}{\to}\mathrm{SU}(8)_L\times\mathrm{SU}(8)_R\ ,$$
$$\underset{M_P}{\to}\mathrm{SU}(8)_L\times\mathrm{SU}(2)_R\times\mathrm{SU}(4)_R^C\ ,$$
$$\underset{M'}{\to}\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R\times\mathrm{SU}(4)_{L+R}^C\quad(g_{2L}\neq g_{2R})\ .$$

In order to discuss the left-right asymmetry in models of type I, we note that there is an element of the SO(10) group denoted by D [and since E$_6\to$SO(10)$\times$U(1), this is true also for the E_6 group], where[23] $D=-\Sigma_{67}\Sigma_{23}$, that behaves almost like the parity operator. It takes, for instance q_L to q_L^C. In general, it cannot be identified with the parity or charge-conjugation operator; however, under the special circumstance when all couplings in the Lagrangian are real, it becomes the same as the parity operator P, which takes $q_L\to q_R$. To avoid confusion, we will call the mass scale associated with breaking of D parity, M_P. Since we do not want to break the gauge symmetry when we break D parity, we have to look for an SO(10) Higgs multiplet, which contains an $\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R\times\mathrm{SU}(3)_C\times\mathrm{U}(1)_{B-L}$ singlet which is odd under D parity. An irreducible representation of this kind is the totally antisymmetric fourth-rank SO(10) tensor, which is 210-dimensional. Under $\mathrm{SU}(2)_L\times\mathrm{SU}(2)_L\times\mathrm{SU}(4)$:

$$\{210\}=(1,1,1)+(2,2,20)+(3,1,15)+(1,3,15)+(2,2,6)+(1,1,15)\ . \tag{5.1}$$

In component notation,[24] if we identify $\alpha,\beta=1,\ldots,6$ as SU(4) color indices and $a,b,\ldots=7,\ldots,10$ as the $\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R$ indices, then the D-parity-odd singlet is $\eta\equiv\phi_{78910}$. In all our discussions of SO(10) breaking, we will use 210-dimensional Higgs field to break parity without breaking the gauge symmetry. Also, we wish to note that the 45-dimensional Higgs boson contains an $\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R\times\mathrm{U}(1)_{B-L}\times\mathrm{SU}(3)_C$-singlet piece which is odd under parity. The relevant components are ϕ_{12}, ϕ_{34}, and ϕ_{67} components of $\phi_{\mu\nu}$ where it is antisymmetric under μ,ν. Thus, breaking SO(10) down to G_{2213} by using a 45-dimensional Higgs multiplet also breaks the D parity.

For completeness, we note that the corresponding multiplet in E_6 which serves the same purpose is 650-dimensional. Under $E_6\supset\mathrm{SO}(10)\times\mathrm{U}(1)$, we get

$$\{650\}=(1,0)+(10,6)+(10,-6)+(16,-3)+(\overline{16},+3)+(45,0)+(54,0)+(144,-3)+(\overline{144},3)+(210,0)\ .$$

This will break the E_6 group down to $\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R\times\mathrm{SU}(4)_C\times\mathrm{U}(1)$. In fact, under $E_6\supset\mathrm{SU}(3)_L\times\mathrm{SU}(3)_R\times\mathrm{SU}(3)_C$, the same 650-dimensional representation contains two singlets (1,1,1), one of which is odd under parity. This multiplet can therefore be used to split the $\mathrm{SU}(3)_R$ or $\mathrm{SU}(3)_L$ coupling constant from the other two SU(3) coupling constants. This will then lead to a picture of E_6 grand unification which is distinct from the ones previously discussed.[20] This will be the subject of a separate publication.

VI. SPONTANEOUS DESCENT OF LEFT-RIGHT ASYMMETRY FROM SO(10) GRAND UNIFIED THEORY

In this section, we will study the various SO(10)-symmetry-breaking patterns embedding our idea and their implications for intermediate mass scales. In particular, we will isolate scenarios, where the $\mathrm{SU}(4)_C$ scale and/or the $\mathrm{SU}(2)_R$-breaking scale can be low enough to lead to physically interesting low-energy effects. Our discussion in this section will parallel the discussion of Sec. IV. In fact, the Higgs multiplets that will be employed will be straightforward extensions of those discussed in Sec. IV. The parity-odd singlet field will be embedded in the 210-dimensional representation. We will discuss the following four cases separately.

Case (i). This is the embedding of case (i) of Sec. IV into the SO(10) grand unification group.[19] This is implemented by the following set of Higgs mesons: {210}, {126}, and {10}, i.e.,

$$\mathrm{SO}(10) \underset{\{210\}}{\overset{M_U \equiv M_P}{\longrightarrow}} G_{224} \underset{\{126\}}{\overset{M_C = M_R}{\longrightarrow}} G_{123} \underset{\{10\}}{\overset{M_W}{\longrightarrow}} G_{13} \; .$$

In this case, the parity (D-parity) symmetry is broken at the grand unification scale by $\langle \phi_{78910} \rangle \neq 0$. This fact has implications for cosmology of the early universe which concern the question of formation of strings.[25] We comment on this briefly in Sec. VII. The rest of the symmetry breaking has been widely discussed in the literature[7] and therefore, we do not repeat it here. We wish, however, to note the various Higgs representations that contribute in various mass regions of evolution of the coupling constant:

$$M_W < \mu < M_R: \; \mathrm{SU}(2)_L \text{doublets of } \{10\} \; ,$$

$$M_R < \mu < M_P: \; (2,2,1) \text{ of } \{10\}, \; (1,3,\overline{10}) \text{ of } \{126\} \; .$$

This mass hierarchy follows within the framework of the minimal-fine-tuning hypothesis, as we noted in Sec. II in the case of $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{B-L} \times P$ theory.

Including their effect, one can obtain expressions for x_W and $\alpha(M_W)/\alpha_s(M_W)$:

$$x_W(M_W) = \frac{3}{8} - \frac{3\alpha(M_W)}{16\pi} \left[\frac{32}{9} \ln \frac{M_P}{M_R} + \frac{109}{9} \ln \frac{M_R}{M_W} \right] , \tag{6.1}$$

$$\frac{\alpha(M_W)}{\alpha_s(M_W)} = \frac{3}{8} - \frac{3\alpha(M_W)}{16\pi} \left[16 \ln \frac{M_P}{M_R} + \frac{67}{3} \ln \frac{M_R}{M_W} \right] . \tag{6.2}$$

The expressions for $\delta^2(M_W)$ and $\delta^2(M_R)$ are the same as in case (i) of Sec. IV [see Eqs. (4.8) and (4.9)]. Table IV lists the allowed mass scales and the $\delta^2(M_W)$ for the various cases. The lowest value for $M_R \simeq M_C$ turns out to be constrained by Eq. (6.2) strongly for $\alpha_s \simeq 0.1$, we find $M_C \simeq M_R \simeq 10^{11}$ GeV for a unification mass around the Planck mass. In this case we simply point out the interesting coincidence that the value of $M_R \simeq 10^{11}$ GeV is of the same order as the scales of supersymmetry breaking in supergravity or models with geometric hierarchy.[26] Other than this matter of theoretical interest, this case is practically devoid of any implications for low-energy physics. Even proton decay[12] is highly suppressed.

Case (ii). Let us now consider the effect of introducing an intermediate symmetry in the symmetry-breaking chain just discussed, i.e.,

$$\mathrm{SO}(10) \underset{\{210\}}{\overset{M_U = M_P}{\longrightarrow}} G_{224} \underset{\{45\}}{\overset{M_C}{\longrightarrow}} G_{2213} \underset{\{126\}}{\overset{M_R}{\longrightarrow}} G_{213} \; .$$

The left-right asymmetry of the Higgs spectrum remains in this case the same as in the previous case; however, above M_C, a new Higgs multiplet transforming as (1,1,15) under G_{224} contributes. The equations for x_W and α_s then become

$$x_W(M_W) = \frac{3}{8} - \frac{3\alpha}{16\pi} \left[\frac{109}{9} \ln \frac{M_R}{M_W} + \frac{19}{3} \ln \frac{M_C}{M_R} + 4 \ln \frac{M_U}{M_C} \right] , \tag{6.3a}$$

$$\frac{\alpha(M_W)}{\alpha_s(M_W)} = \frac{3}{8} - \frac{3\alpha}{16\pi} \left[\frac{67}{3} \ln \frac{M_R}{M_W} + 17 \ln \frac{M_C}{M_R} + \frac{44}{3} \ln \frac{M_U}{M_C} \right] . \tag{6.3b}$$

The implications of these equations for the mass scales are summarized in Table V. The equations for the asymmetry parameters are the same as in case (i). We see from Table V that there is one solution $M_U \simeq M_C \simeq 4 \times 10^{18}$ GeV, for which $M_{W_R} \approx M_{Z_R} \approx 20$ TeV for $x_W \simeq 0.25$. In fact, the most economical Higgs system for this case is {45}, {126}, and {10} where the (1,1,15) part of {45} can break $\mathrm{SO}(10) \rightarrow \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{B-L} \times \mathrm{SU}(3)$. This case leads to a right-handed W_R-boson mass which is in the accessible range of the proposed Superconducting Super Collider.

Case (iii). Implications for low-energy physics become more interesting[15–17,27] if more intermediate steps are introduced as was done in Sec. IV, cases (ii) and (iii). We consider the following symmetry-breaking chain:

TABLE IV. Mass scales and asymmetry parameter for case (i) of the SO(10) grand unified scheme.

$\alpha_s(M_W)$	$x_W(M_W)$	$M_U \equiv M_P$ (GeV)	M_R (GeV)	$\delta^2(M_W)$
0.10	0.247	10^{16}	10^{10}	1.75
0.12	0.244	6×10^{16}	10^{10}	1.8
0.10	0.24	4×10^{15}	10^{11}	1.71

TABLE V. Mass scales for the symmetry-breaking chain $\mathrm{SO}(10) \underset{M_U}{\rightarrow} \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{SU}(4)_C \underset{M_C}{\rightarrow} \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{B-L} \times \mathrm{SU}(3)_C \underset{M_R}{\rightarrow} G_{213}$.

x_W	α_s	$M_U \equiv M_P$ (GeV)	M_C (GeV)	M_R (GeV)
0.246	0.11	10^{17}	10^{12}	10^{10}
0.245	0.11	10^{18}	10^{12}	10^{7}
0.246	0.12	3×10^{18}	10^{11}	10^{7}
0.24	0.1	3×10^{16}	3×10^{16}	10^{8}
0.245	0.1	3×10^{16}	7×10^{16}	7×10^{6}
0.25	0.1	10^{17}	10^{17}	3.3×10^{5}
0.24	0.12	3.3×10^{17}	3.3×10^{17}	7×10^{6}
0.245	0.12	10^{18}	10^{18}	4×10^{6}
0.25	0.12	1.5×10^{18}	1.5×10^{18}	2×10^{4}

$$\mathrm{SO}(10) \underset{\{210\}}{\overset{M_U}{\longrightarrow}} G_{2213P} \underset{\{210\}}{\overset{M_P}{\longrightarrow}} G_{2213} \underset{\{210\}}{\overset{M_P}{\longrightarrow}} G_{2113} \underset{\{126\}}{\overset{M_{R^0}}{\longrightarrow}} G_{213} .$$

In this case, we have decoupled M_p from both M_U and M_{R^+}. The Higgs multiplets contributing at different stages are as follows:

$M_W < \eta < M_{R^0}$: SU(2)L doublet of {10} ;

$M_{R^0} < \eta < M_{R^+}$: SU(2)$_L$ doublet of {10} and Δ_R^0 component of $(1,3,\overline{10})$ of {126};

$M_{R^+} < \mu < M_P$: $(2,2,1)_0$ of {10} , $(1,3,1)_{+2}$ of {126} , $(1,3,1)_0$ of {210} ;

$M_P < \mu < M_U$: $(2,2,1)_0$ of {10} , $(1,3,1)_{+2} + (3,1,1)_{+2}$ of {126} .

This leads to the following equations for $x_W(M_W)$ and $\alpha(M_W)/\alpha_s(M_W)$:

$$x_W(M_W) = \frac{3}{8} - \frac{3\alpha}{16\pi}\left[6\ln\frac{M_U}{M_P} + \frac{20}{3}\ln\frac{M_P}{M_{R^+}} + \frac{115}{9}\ln\frac{M_{R^+}}{M_{R^0}} + \frac{109}{9}\ln\frac{M_{R^0}}{M_W}\right], \tag{6.4}$$

$$\frac{\alpha(M_W)}{\alpha_s(M_W)} = \frac{3}{8} - \frac{3\alpha}{16\pi}\left[\frac{58}{3}\ln\frac{M_U}{M_P} + \frac{52}{3}\ln\frac{M_P}{M_{R^+}} + 23\ln\frac{M_{R^+}}{M_{R^0}} + \frac{67}{3}\ln\frac{M_{R^0}}{M_W}\right], \tag{6.5}$$

$$\delta^2(M_W) = 1 + \frac{\alpha}{2\pi x_W}\left[\ln\frac{M_P}{M_R} + \frac{19}{6}\ln\frac{M_R}{M_W}\right]. \tag{6.6}$$

From Eq. (6.5), $\delta^2(M_{R^+})$ can be calculated by setting $M_R = M_W$ and evaluating α and x_W at M_R. The formula for $x_W(M_R)$ is given by

$$x_W(M_R) = \frac{3}{8} - \frac{3\alpha(M_R)}{16\pi}\left[6\ln\frac{M_U}{M_P} + \frac{20}{3}\ln\frac{M_P}{M_R}\right]. \tag{6.7}$$

Also, we obtain the parameter

$$\delta_N{}^2(M_W) = 1 + \frac{\alpha}{2\pi x_W}\left[\ln\frac{M_P}{M_R} + \frac{23}{3}\ln\frac{M_{R^+}}{M_{R^0}} + \frac{19}{6}\ln\frac{M_{R^0}}{M_W}\right]. \tag{6.8}$$

The mass scales and asymmetry parameters for this case are listed in Table VI. It is clear that low-mass right-handed bosons $W_R^{\pm}$ and Z_R anywhere in the mass range 1–5 TeV are allowed by the model, for $\alpha_s(M_W) \simeq 0.12$ and $\sin^2\theta_W \simeq 0.24$. The result, we believe, is very striking. We have not done any unnecessary fine-tuning of our masses to obtain it. Note that in the conventional treatment of SO(10) models, such a low value for M_{W_R} and M_{Z_R} would require very large $\sin^2\theta_W$ ($\simeq 0.27$), which is incompatible with present experiments. The main reason for our success in accommodating a low M_{W_R} in the SO(10) model is the idea of decoupling parity- and SU(2)$_R$-breaking scales. This ought to provide new incentive to continue the search for right-handed current effects at low energies.[15–17,27]

Case (iv). We repeat the above procedure for another symmetry-breaking chain:

$$\mathrm{SO}(10) \underset{\{210\}}{\overset{M_u}{\longrightarrow}} G_{224} \underset{\{45\}}{\overset{M_C}{\longrightarrow}} G_{2213} \underset{\{210\}}{\overset{M_{R^+}}{\longrightarrow}} G_{2113} \underset{\{126\}}{\overset{M_{R^0}}{\longrightarrow}} G_{213} .$$

The relevant equations in this case are

$$x_W(M_W) = \frac{3}{8} - \frac{3\alpha}{16\pi}\left[\frac{31}{3}\ln\frac{M_U}{M_C} + \frac{20}{3}\ln\frac{M_C}{M_{R^+}} + \frac{115}{9}\ln\frac{M_{R^+}}{M_{R^0}} + \frac{109}{9}\ln\frac{M_{R^0}}{M_W}\right], \tag{6.9}$$

$$\frac{\alpha(M_W)}{\alpha_s(M_W)} = \frac{3}{8} - \frac{3\alpha(M_W)}{16\pi}\left[\frac{47}{3}\ln\frac{M_U}{M_C} + \frac{52}{3}\ln\frac{M_C}{M_{R^+}} + 23\ln\frac{M_{R^+}}{M_{R^0}} + \frac{67}{3}\ln\frac{M_{R^0}}{M_W}\right], \tag{6.10}$$

TABLE VI. Mass scales and asymmetry parameters for case (iii) of the SO(10) grand unified sheme.

$\alpha_s(M_W)$	$x_W(M_W)$	$M_U = M_P$ (GeV)	M_{R^+} (GeV)	M_{R_0} (GeV)	$\delta^2(M_W)$	$\delta_N{}^2(M_W)$
	0.25	2×10^{18}	10^3	5×10^2	1.21	1.36
0.12	0.244	10^{18}	5×10^3	10^3	1.23	1.38
	0.243	10^{18}	10^4	5×10^{12}	1.24	1.41

$$\delta^2(M_W)=1+\frac{\alpha(M_W)}{2\pi x_W(M_W)}\left[\frac{35}{3}\ln\frac{M_U}{M_C}+\ln\frac{M_C}{M_{R^+}}+19\ln\frac{M_R}{M_W}\right]. \tag{6.11}$$

$\delta^2(M_R)$ is obtained by replacing M_W by M_{R^+} in Eq. (6.11). Finally,

$$\delta_N{}^2(M_W)=1+\frac{\alpha}{2\pi x_W}\left[\frac{35}{3}\ln\frac{M_U}{M_C}+\ln\frac{M_C}{M_{R^+}}+\frac{23}{3}\ln\frac{M_{R^+}}{M_{R^0}}+\frac{19}{6}\ln\frac{M_{R^0}}{M_W}\right], \tag{6.12}$$

$$x_W(M_R)=\frac{3}{8}-\frac{3\alpha(M_R)}{16\pi}\left[\frac{31}{3}\ln\frac{M_U}{M_C}+\frac{20}{3}\ln\frac{M_C}{M_{R^+}}\right]. \tag{6.13}$$

The results for mass scales and asymmetry parameters for this case are given in Table VII. We see that, in this case also, W_R and Z_R can have low masses anywhere between 400 GeV to 5 TeV (with the constraint that $M_{R^+}>M_{R^0}$) for allowed values of x_W and α_s. The M_C scale corresponding to $SU(4)_C$ breaking is, however, very high in this case. Therefore, matter-antimatter oscillations are suppressed. In case (v), we offer a chain of symmetry breaking with a lower value of M_C.

Case (v). Let us consider the following symmetry-breaking scenario:

$$SO(10)\underset{\{54\}}{\overset{M_U}{\longrightarrow}}G_{224P}\underset{\{210\}}{\overset{M_P}{\longrightarrow}}G_{224}\underset{\{210\}}{\overset{M_C=M_{R^+}}{\longrightarrow}}G_{2113}\underset{\{126\}}{\overset{M_{R^0}}{\longrightarrow}}G_{213}\ .$$

The relevant Higgs multiplets contributing at various stages are noted below:

$M_W<\mu<M_{R^0}$: $SU(2)_L$ doublet of $\{10\}$;

$M_{R^0}<\mu<M_{R^+}$: $SU(2)_L$ doublet of $\{10\}$, Δ_R^0 of $\{126\}$;

$M_{R^+}<\mu<M_P$: $(2,2,1)$ of $\{10\}$, $\Delta_R(1,3,\overline{10})$ of $\{126\}$, $D_R(1,3,15)$ of $\{210\}$;

$M_P<\mu<M_U$: $(2,2,1)$ of $\{10\}$, $\Delta_L(3,1,10)+\Delta_R(1,3,\overline{10})$ of $\{126\}$, $D_L(3,1,15)+D_R(1,3,15)$ of $\{210\}$.

The expressions for various parameters such as x_W, $\alpha(M_W)$, $\alpha_s(M_W)$, etc., are given in this case by

$$x_W(M_W)=\frac{3}{8}-\frac{3\alpha}{16\pi}\left[\frac{56}{9}\ln\frac{M_U}{M_P}+\frac{89}{9}\ln\frac{M_P}{M_{R^+}}+\frac{115}{9}\ln\frac{M_{R^+}}{M_{R^0}}+\frac{109}{9}\ln\frac{M_{R^0}}{M_W}\right], \tag{6.14}$$

$$\frac{\alpha(M_W)}{\alpha_s(M_W)}=\frac{3}{8}-\frac{3\alpha(M_W)}{16\pi}\left[\frac{56}{3}\ln\frac{M_U}{M_P}+17\ln\frac{M_P}{M_{R^+}}+23\ln\frac{M_{R^+}}{M_{R^0}}+\frac{67}{3}\ln\frac{M_{R^0}}{M_W}\right], \tag{6.15}$$

$$\delta^2(M_W)=1+\frac{\alpha}{2\pi x_W}\left[\frac{35}{3}\ln\frac{M_P}{M_{R^+}}+\frac{19}{6}\ln\frac{M_{R^+}}{M_W}\right], \tag{6.16}$$

$$\delta_N{}^2(M_W)=1+\frac{\alpha}{2p x_W}\left[\frac{35}{3}\ln\frac{M_P}{M_{R^+}}+\frac{23}{3}\ln\frac{M_{R^+}}{M_{R^0}}+\frac{19}{6}\ln\frac{M_{R^0}}{M_W}\right], \tag{6.17}$$

and

$$\delta^2(M_{R^+})=\delta^2(M_W\to M_{R^+})\ .$$

In Table VIII we give the results for this case. We observe that for $x_W=0.22$–0.24 and $\alpha_s(M_W)=0.1$ to 0.11, we find the neutral right-handed boson to have low mass $M_Z\simeq 400$ GeV with $M_{R^+}=M_C\simeq 10^5$–10^6 GeV, for $M_P\simeq 10^{12}$–10^{13} GeV and $M_U\simeq 10^{16}$ GeV. This case is also phenomenologically quite interesting; since using the extended survival hypothesis,[7] we find the diquark Higgs boson responsible for n-$\bar{n}$ and H-$\bar{\text{H}}$ oscillation to have mass around 10^5 GeV. Now, in the manner similar to that discussed in Sec. IV, it leads to observable[28] $\tau_{n\text{-}\bar{n}}\simeq 10^8$–$10^9$ sec and a hydrogen-antihydrogen mixing time $\simeq 10^{12}$ –10^{13} yr. The latter converts to a measurable double-proton-decay ($pp\to e^+e^+$) lifetime of about

TABLE VII. Mass scales and asymmetry parameters in case (iv) of the SO(10) grand unified scheme.

$\alpha_s(M_W)$	$x_W(M_W)$	$M_U=M_P$ (GeV)	M_c (GeV)	M_{R^+} (GeV)	M_{R^0} (GeV)	$\delta^2(M_W)$	$\delta_N{}^2(M_W)$
0.10	0.227	1.5×10^{18}	10^{12}	10^3	5×10^2	2.05	2.1
0.11	0.235	3×10^{18}	10^{13}	10^3	5×10^2	1.95	2
0.10	0.217	2.5×10^{18}	10^{10}	10^3	5×10^2	2.4	2.48

TABLE VIII. Intermediate mass scales and asymmetry parameters for case (v) of the SO(10) scheme.

$\alpha_s(M_W)$	$x_W(M_W)$	M_U (GeV)	M_P (GeV)	$M_{R^+} \equiv M_C$ (GeV)	M_{R^0}	$\delta^2(M_W)$	$\delta_N{}^2(M_W)$
0.10	0.232	10^{17}	3×10^{15}	10^5	5×10^2	2.63	2.76
0.10	0.227	5×10^{16}	3×10^{15}	10^6	5×10^2	2.56	2.75
0.11	0.226	3×10^{17}	10^{16}	10^5	5×10^2	2.75	2.88
0.11	0.225	10^{17}	5×10^{15}	10^6	5×10^2	2.61	2.8

10^{32}–10^{33} yr.[7,14] Also, we obtain[28] $B(K_L^0\to\mu\bar{e})\simeq10^{-9}-10^{-13}$ as in case (iii) of Sec. IV. The neutrino masses owe their origin to $\langle\{126\}\rangle$ which is less than 1 TeV; as a result a spectrum of neutrino masses such as $m_{\nu_e}\simeq1$ eV, $m_{\nu_\mu}\simeq40$ keV, and $m_{\nu_\tau}\simeq20$ MeV may emerge.[17] However, charged-W_R-mediated processes such as $(\beta\beta)_{0\nu}$ decay, etc., will be suppressed.[15–17]

VII. CONCLUSION

In this section, we summarize our results. We have investigated the new possibility of breaking discrete parity symmetry and $SU(2)_R$ gauge symmetry at different mass scales. After discussing various ways of implementing this hypothesis in the context of $SU(2)_L\times SU(2)_R\times U(1)_{B-L}\times P$ and SO(10) models, we study its physical implications for the intermediate mass scales. We find the new result that, in this approach to left-right-symmetric models, the SO(10) model can accommodate both a low-mass scale for parity restoration ($M_R\sim1$ TeV) and a low mass for $SU(4)_C$-color breaking[29] ($M_C\sim10^5$ GeV) in different symmetry-breaking chains. Both these results are of great phenomenological significance since they lead to observable $\Delta B=2$ and $\Delta L=2$ processes at low energies. This way of symmetry breaking also has interesting cosmological implications of avoiding string formation in the early universe in SO(10) models. It was noted by Kibble, Lazarides, and Shafi[25] that if SO(10) breaks down to $SU(2)_L\times SU(2)_R\times SU(4)\times D$, string appears in the early universe for temperature $T<M_U/g$. After the $SU(2)_R\times D\times SU(4)_C$ breaks down to $U(1)_Y$ at $T_c\simeq M_R/g$, domain walls that are bounded by these strings appear, dominating the energy density of the Universe contradicting the standard cosmological picture. In contrast, in our approach, when the SO(10) group breaks down to $SU(2)_L\times SU(2)_R\times SU(4)_C$, the discrete D symmetry is also broken preventing string formation, thus avoiding the above cosmological problem altogether.[30]

Finally, we note another area of cosmology where our idea may be useful. Kuzmin and Shaposhnikov[31] have noted that the existence of the D-parity operator as a part of the SO(10) group implies that to generate baryons, one must break it at a scale of about 10^{14} GeV. In the context of earlier SO(10) models where D parity and $SU(2)_R$ symmetry were broken at the same scale, this would have implied $M_{W_R}>10^{12}$ GeV. Within our framework, since they are decoupled, we do not expect any conflict between baryon generation and low-mass W_R bosons. This point will be the subject of a forthcoming publication.[32]

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APPENDIX: SIMULTANEOUS BREAKDOWN OF *P* AND *CP* SYMMETRIES

In this Appendix, we wish to point out that singlet field η used to break parity can also be used to break both parity and CP symmetry starting with a theory that respects both these symmetries prior to spontaneous symmetry breaking. In order to achieve this, we have to define the operations of CP and P on the fields ϕ, Δ, and η:

$$P:\ \phi\to\phi^+\ ,\quad \eta\to-\eta\ ,\quad \Delta_L\to\Delta_R\ , \tag{A1}$$

$$CP:\ \phi\to\phi^+\ ,\quad \eta\to-\eta\ ,\quad \Delta_L\to\Delta_R^+\ . \tag{A2}$$

The most general gauge-invariant Higgs potential invariant under these symmetries can be written as

$$V=V_\Delta+V_\phi+V_\eta+V_{\eta\Delta}+V_{\Delta\phi}+V_{\eta\phi}\ . \tag{A3}$$

V_Δ, V_ϕ, V_η, $V_{\eta\Delta}$ remain the same as in Sec. II. $V_{\Delta\phi}$ which was not specified before will change such that all coupling constants become real to respect CP invariance prior to spontaneous breakdown. $V_{\eta\phi}$, however, changes in an essential manner:

$$V_{\eta\phi}=\lambda_2\eta^2\mathrm{Tr}\phi^\dagger\phi+\lambda_2'\eta^2(\mathrm{Det}\phi+\mathrm{Det}\phi^\dagger)+i\lambda_3M\eta(\mathrm{Det}\phi-\mathrm{Det}\phi^\dagger)\ . \tag{A4}$$

Note that if η was chosen CP even, this term would be absent. It is now clear that if parity symmetry is broken, i.e., $\langle\eta\rangle\neq0$, this breaks CP symmetry and will introduce a phase into $\langle\phi\rangle$ for all values of parameters in $V_\phi+V_{\Delta\phi}+V_{\eta\phi}$.

It is possible to argue that the CP-violating phase in $\langle\phi\rangle$ is inversely proportional to the parity-breaking scale M_P. To see this, note that in the limit of $M_P\gg M_{W_R}$, M_{W_L}, the η field completely decouples from the low-energy Lagrangian, which then becomes completely CP-conserving. In this case, $\langle\phi\rangle=$ real. Hence, the result is proven.

To see how this idea can be embedded in the SO(10) grand unified theory, we note that the operator D that transforms a left-handed particle (say, e_L^-) to its parity partner which is a left-handed antiparticle (e.g., e_L^+) is given by

$$D=-\Sigma_{67}\Sigma_{23}\ , \tag{A5}$$

where $\Sigma_{\mu\nu}$ is the antisymmetric generator of SO(10). We note that under this "D" conjugation, the color and electrically neutral members of the 10-dimensional Higgs boson H_μ (i.e., H_9, H_{10}) are even, whereas those of the 120-dimensional Higgs boson $\Delta_{\mu\nu\lambda}$ (i.e., Δ_{789} or Δ_{7810}) are odd. Therefore, the following coupling will be the SO(10) analog of the last term in (A4), i.e.,

$$\mathscr{L}'=\lambda_3 M\phi_{\mu\nu\lambda\sigma}\Delta_{\mu\nu\lambda}H_\sigma\ . \tag{A6}$$

Since $\langle\phi_{78910}\rangle$ breaks parity or D conjugation, $\mathscr{L}'$ will also break charge conjugation. This will give rise to a new mechanism for introducing CP violations into gauge theories.

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*On leave from Sambalpur University, Orissa, India.

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