SU(4) unification of electroweak interactions

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Electroweak unification is obtained in an SU(4) model with $SU_L(2)\times SU_R(2)\times U(1)$ as its subgroup and $\sin^2\theta_W^0=\frac{1}{4}$ in the symmetry limit. The unification mass scale is $3.3\times 10^4 \geq m_X \geq 6.4\times 10^3$ GeV for $240\leq m_R \leq 6.4\times 10^3$ GeV. Some consequences of the model are discussed.

I. INTRODUCTION

It is well known that the standard $SU_L(2)\times U(1)$ model of electroweak interactions[1] does not provide a true unification of electromagnetic and weak interactions since, because of the U(1) factor, $\sin^2\theta_W$ is not fixed by the model. Another aspect of this model is that parity violation in the weak interaction is treated as an intrinsic property of that interaction. In the left-right-symmetric $SU_L(2)\times SU_R(2)\times U(1)$ model,[2] on the other hand, parity violation is a low-energy phenomena and parity conservation is restored at an energy scale $\gg m_{WL}$. But $\sin^2\theta_W$ is not fixed even in this model. If one wishes to have the latter point of view (i.e., the spontaneous breaking of parity) and a true unification of electroweak interactions, one is naturally led to an SU(4) group [which has $SU_L(2)\times SU_R(2)\times U(1)$ as a subgroup], where the charge operator

$$Q=\frac{1}{2}\left[\lambda_3+\frac{1}{\sqrt{3}}\lambda_8\right]+\frac{2}{\sqrt{6}}\lambda_{15}$$

has eigenvalues 1,0,0,−1. This then naturally accommodates the known leptons per generation in the fundamental representation

$$F_e=\begin{bmatrix} e_R^c \\ \nu_{eR}^c \\ -N_{eR} \\ e_R \end{bmatrix}, \qquad (1)$$

where N_{eR} may or may not be identical with ν_{eR}. The superscript c denotes the charge conjugate of e^- (for e^c). For hadrons, we take integrally charged quarks,[3] which (for the first generation) are arranged in SU(4) multiplets as

$$F_h^r=\begin{bmatrix} d_R^c \\ u_R^c \\ -u_R \\ d_R \end{bmatrix}^r, \quad F_h^{g,b}=\begin{bmatrix} u_L \\ -d_L \\ d_L^c \\ u_L^c \end{bmatrix}^{g,b}, \qquad (2)$$

where r, g, and b refer to the colors. As will be shown, one prediction of the model [as also in an SU(3) formulation considered in Ref. 4, which incorporates $SU_L(2)\times U(1)$ rather than the left-right-symmetric model] is that $\sin^2\theta_W$ in the SU(4)-symmetry limit has the value $\frac{1}{4}$. Since all the physical hadrons at present energies are color singlets, the predictions of the above SU(4) model in the hadron sector are the same as in the left-right-symmetric model. For the lepton sector, additional muon-number-violating processes[4] such as

$$e^-+e^-\to\mu^-+\mu^- ,$$
$$e^-+\mu^+\to e^++\mu^- ,$$

which are higher order in the left-right-symmetric model (i.e., involve two gauge-boson exchanges and mixing of electron- and muon-type neutrinos), can occur through some single gauge-boson exchanges. But they are not likely to cause any trouble with the present observations, since, as will be shown in Sec. III, the gauge bosons (called X and Y) responsible for these processes get sufficiently high masses when the SU(4) symmetry is spontaneously broken. However, they could (we hope) be observed in future experiments at some level, thus testing the model.

The plan of the paper is as follows. In Sec. II, we give the basic Lagrangians both for leptons and hadron sectors and show that $\sin^2\theta_W^0=\frac{1}{4}$ in the SU(4)-symmetry limit. Section III is devoted to details of the spontaneous breaking of SU(4) symmetry. There, we also consider the renormalization-group analysis in order to bring $\sin^2\theta_W^0=\frac{1}{4}$ to its experimental value ≈ 0.22 at present energies. This also provides us the information about the mass scale at which SU(4) is broken. It is found that

$$3.3\times 10^4 \geq m_X \geq 6.4\times 10^3 \text{ GeV}$$

for

$$240\leq m_R \leq 6.4\times 10^3 \text{ GeV} .$$

In Sec. IV, we summarize the conclusions and also discuss the new processes in the leptonic sector mentioned above.

II. BASIC LAGRANGIANS AND $\sin^2\theta_W^0$

Consider any one of the fermion multiplets mentioned in Sec. I:

$$F=\begin{bmatrix}F_1\\F_2\\F_3\\F_4\end{bmatrix}$$

with the charges 1,0,0,−1. Then the SU(4)-invariant Lagrangian is

$$\mathcal{L}_F=-\bar{F}\gamma_\mu\left[\partial_\mu+\frac{g}{2}i\vec{\lambda}\cdot\vec{\mathrm{W}}_\mu\right]F\ . \tag{3}$$

In matrix notation, 15 gauge bosons of SU(4) are

$$W_{ab}=\frac{1}{\sqrt{2}}(\lambda_i W_i)_{ab}$$

with the following identification with the known gauge bosons:

$$W_{12}=W_L^-\ ,\quad W_{21}=W_L^+\ ,\quad \frac{1}{\sqrt{2}}(W_{11}-W_{22})=W_{3L}\ ,$$
$$W_{34}=W_R^-\ ,\quad W_{43}=W_R^+\ ,\quad \frac{1}{\sqrt{2}}(W_{33}-W_{44})=W_{3R}\ ,$$
$$W_{11}+W_{22}=-(W_{33}+W_{44})=B\ ,$$

and new gauge bosons

$$W_{13}=X_1\ ,\quad W_{14}=Y_1\ ,\quad W_{23}=X_2\ ,\quad W_{24}=Y_2$$

together with their charge conjugates. Then, noting that

$$\begin{aligned}\bar{F}^c_{1R}\gamma_\mu F^c_{2R}&=-\bar{F}_{2L}\gamma_\mu F_{1L}\ ,\\ \bar{F}^c_{1L}\gamma_\mu F^c_{2L}&=-\bar{F}_{2R}\gamma_\mu F_{1R}\ ,\end{aligned} \tag{4}$$

we have the following Lagrangians for the leptons and hadrons, respectively:

$$\begin{aligned}\mathcal{L}^e_{\text{int}}=&\frac{ig_2}{\sqrt{2}}(\bar{\nu}_{eL}\gamma_\mu e_{L\mu}W_L^-+\bar{N}_{eR}\gamma_\mu e_{R\mu}W_{R\mu}^-+\text{H.c.})+\frac{ig_2}{2}(-J^e_{0\mu L}W_{3L\mu}-J^e_{0\mu R}W_{3R\mu})\\&+\frac{ig_1}{2}\{[J^e_{0\mu L}+J^e_{0\mu R}-2J^{\text{EM}}_\mu(e)]B_\mu\}\\&+\frac{ig}{\sqrt{2}}(\bar{e}^c_R\gamma_\mu N_{eR}X_{1\mu}+\bar{\nu}^c_{eR}\gamma_\mu N_{eR}X_{2\mu}-\bar{e}^c_R\gamma_\mu e_R Y_{1\mu}-\bar{\nu}^c_{eR}\gamma_\mu e_R Y_{2\mu}+\text{H.c.})\ ,\end{aligned} \tag{5}$$

$$\begin{aligned}\mathcal{L}^h_{\text{int}}=&\frac{i}{\sqrt{2}}g_2\left[\sum_{i=r,g,b}(\bar{u}_{iL}\gamma_\mu d_{iL}W^-_{L\mu}+\bar{u}_{iR}\gamma_\mu d_{iR}W^-_{R\mu})+\text{H.c.}\right]+\frac{i}{2}g_2(-J^h_{0\mu L}W_{3L\mu}-J^h_{0\mu R}W_{3R\mu})\\&+i\frac{g_1}{2}\{[J^h_{0\mu L}+J^h_{0\mu R}-2J^{\text{EM}}_\mu(h)]B_\mu\}\\&+\frac{ig}{\sqrt{2}}\{[(\bar{d}^c_R\gamma_\mu u_R)^r-(\bar{u}_L\gamma_\mu d^c_L)^g-(\bar{u}_L\gamma_\mu d^c_L)^b]X_{1\mu}+[(\bar{u}^c_R\gamma_\mu u_R)^r+(\bar{d}_L\gamma_\mu d^c_L)^g+(\bar{d}_L\gamma_\mu d^c_L)^b]X_{2\mu}\\&\quad-[(\bar{d}^c_R\gamma_\mu d_R)^r+(\bar{u}_L\gamma_\mu u^c_L)^g+(\bar{u}_L\gamma_\mu u^c_L)^b]Y_{1\mu}+[-(\bar{u}^c_R\gamma_\mu d_R)^r+(\bar{d}_L\gamma_\mu u^c_L)^g+(\bar{d}_L\gamma_\mu u^c_L)^b]Y_{2\mu}+\text{H.c.}\}\ ,\end{aligned} \tag{6}$$

where

$$\begin{aligned}J^e_{0\mu L}&=\bar{\nu}_{eL}\gamma_\mu\nu_{eL}-\bar{e}_L\gamma_\mu e_L\ ,\\ J^e_{0\mu R}&=\bar{N}_{eR}\gamma_\mu N_{eR}-\bar{e}_R\gamma_\mu e_R\ ,\\ J^{\text{EM}}_\mu(e)&=-\bar{e}\gamma_\mu e\ ,\end{aligned} \tag{7a}$$

$$\begin{aligned}J^h_{0\mu L}&=\sum_{i=r,g,b}(\bar{u}_{iL}\gamma_\mu u_{iL}-\bar{d}_{iL}\gamma_\mu d_{iL})\ ,\\ J^h_{0\mu R}&=\sum_{i=r,g,b}(\bar{u}_{iR}\gamma_\mu u_{iR}-\bar{d}_{iR}\gamma_\mu d_{iR})\ ,\end{aligned} \tag{7b}$$

$$J^{\text{EM}}_\mu(h)=-(\bar{d}\gamma_\mu d)^r+(\bar{u}\gamma_\mu u)^g+(u\gamma_\mu u)^b\ . \tag{7c}$$

Note the fact from Eqs. (3) and (5) or (6) that in the symmetry limit

$$g_L=g_R\equiv g_2=g\ , \tag{8a}$$

$$g_1=\frac{g}{\sqrt{2}}=\frac{1}{C_1}g\ , \tag{8b}$$

where

$$\tfrac{1}{2}C_1{}^2=\text{Tr}(\tfrac{1}{4}Y^2)=1\ . \tag{8c}$$

We also note the relation

$$\frac{A_\mu}{e}=\frac{W_{3L\mu}}{g_L}+\frac{W_{3R\mu}}{g_R}+\frac{B_\mu}{g_1}\ , \tag{9a}$$

so that

$$\begin{aligned}\frac{1}{e^2}&=\frac{1}{g_L{}^2}+\frac{1}{g_R{}^2}+\frac{1}{g_1{}^2}\\&=\frac{2}{g_2{}^2}+\frac{1}{g_1{}^2}\ .\end{aligned} \tag{9b}$$

In the symmetry limit, this becomes

$$\frac{1}{e^2}=\frac{4}{g^2}\ ,$$

giving in that limit

$$\sin^2\theta_W^0=\frac{e^2}{{g_L}^2}=\tfrac{1}{4}\ . \qquad (10)$$

The above definition of the couplings correspond to the following breaking pattern of SU(4):

$$\mathrm{SU}(4)\underset{m_X}{\longrightarrow}\mathrm{SU}_L(2)\times\mathrm{SU}_R(2)\times\mathrm{U}(1)$$
$$\underset{m_R}{\longrightarrow}\mathrm{SU}_L(2)\times\mathrm{U}'(1)\underset{m_L}{\longrightarrow}\mathrm{U}_{\mathrm{EM}}\ , \qquad (11)$$

where g, g_L, g_R, and g_1 correspond to the groups SU(4), $\mathrm{SU}_L(2)$, $\mathrm{SU}_R(2)$, and U(1).

Note that $J_\mu^{\mathrm{EM}}(h)$ in Eq. (7c) is not a pure color singlet:

$$J_\mu^{\mathrm{EM}}(h)=J_\mu^{\mathrm{EM}}(\text{color singlet})-J_\mu^{\mathrm{EM}}(\text{color octet})\ , \qquad (12a)$$

where

$$J_\mu^{\mathrm{EM}}(\text{color singlet})=\tfrac{2}{3}\sum_i \bar{u}_i\gamma_\mu u_i-\tfrac{1}{3}\sum_i \bar{d}_i\gamma_\mu d_i \qquad (12b)$$

and only the color-singlet part of $J_\mu^{\mathrm{EM}}(h)$ will contribute in a physical process. Note also that the currents coupled with X and Y gauge bosons in Eq. (6) do not contain any color singlets (these terms belong to 3×3 or $\bar{3}\times\bar{3}$ which do not contain color singlets) and as such are not relevant for a physical process since all physical hadrons at present energies are color singlets. Thus $\mathcal{L}_{\mathrm{int}}^h$(color singlet) is identical with that of $\mathrm{SU}_L(2)\times\mathrm{SU}_R(2)\times\mathrm{U}(1)$ with $\sin^2\theta_W^0$ fixed to be $\frac{1}{4}$. Similarly the $\mathcal{L}_{\mathrm{int}}^e$ in Eq. (5), except for the terms coupled with X and Y gauge bosons, is also identical with that of $\mathrm{SU}_L(2)\times\mathrm{SU}_R(2)\times\mathrm{U}(1)$. These extra terms coupled with X and Y bosons for the electron and muon can give rise to processes of the form

$$e^-+e^-\rightarrow Y_1^-\rightarrow\mu^-+\mu^-$$

and

$$e^-+\mu^+\rightarrow e^++\mu^-\ ,$$

which will be discussed in Sec. IV.

In order to see the identification with the Lagrangians of the left-right-symmetric model, consider the diagonalization of the mass matrix for W_{3L}, W_{3R}, and B which is obtained through the equations[5]

$$A=\cos\gamma\frac{W_{3L}+W_{3R}}{2}-\sin\gamma\, B\ ,$$
$$Z_L=-\left[\sin\gamma\frac{W_{3L}+W_{3R}}{\sqrt{2}}+\cos\gamma\, B\right]\sin\beta+\frac{W_{3L}-W_{3R}}{\sqrt{2}}\cos\beta\ ,$$
$$Z_R=\left[\sin\gamma\frac{W_{3L}+W_{3R}}{\sqrt{2}}+\cos\gamma\, B\right]\cos\beta+\frac{W_{3L}-W_{3R}}{\sqrt{2}}\sin\beta\ . \qquad (13)$$

Then from Eq. (9b), we have

$${g_2}^2=\frac{2e^2}{\cos^2\gamma}\ ,\quad {g_1}^2=\frac{e^2}{\sin^2\gamma}\ , \qquad (14a)$$

giving

$$\sin^2\theta_W=\tfrac{1}{2}\cos^2\gamma\ ,\quad \tan\gamma=-\frac{g_2}{\sqrt{2}g_1}\ , \qquad (14b)$$

so that in the symmetry limit [cf. Eq. (8)]

$$\tan\gamma^0=-1\ ,\quad \sin\gamma^0=\frac{-1}{\sqrt{2}}\ ,\quad \cos\gamma^0=\frac{1}{\sqrt{2}}\ . \qquad (14c)$$

Then it is easy to see that A_μ is coupled to J_μ^{EM} with

$$-\frac{g_2}{\sqrt{2}}\cos\gamma=e\ ,\quad g_2\sin\theta_W=e\ . \qquad (14d)$$

For $m_A=0$ and $m_{ZR}\gg m_{ZL}$ (see next section on how one can obtain this and $m_{WR}\gg m_{WL}$), one obtains

$$\frac{\sin\beta}{\sin\gamma}\simeq\frac{1}{\sqrt{2}\cos\theta_W}\ ,\quad \cos\beta=-\frac{1}{\sqrt{2}\cos\theta_W}\ , \qquad (15)$$

giving

$$\mathcal{L}_{\mathrm{int}}^h(\text{color singlet})=\frac{i}{\sqrt{2}}g_2\left[\sum_i(\bar{u}_{iL}\gamma_\mu d_{iL}W_{L\mu}^-+\bar{u}_{iR}\gamma_\mu d_{iR}W_{R\mu}^-)+\text{H.c.}\right]$$
$$+\frac{g_2}{4\cos\theta_W}[2J_{0\mu L}^h-4\sin^2\theta_W J_\mu^{\mathrm{EM}}(\text{singlet})]Z_{L\mu}$$
$$+\frac{g_2}{4\cos\theta_W\cos2\theta_W}[2\sin^2\theta_W J_{0\mu L}^h+2\cos^2\theta_W J_{0\mu R}^h-4\sin^2\theta_W J_\mu^{\mathrm{EM}}(\text{singlet})]Z_{R\mu}\ , \qquad (16)$$

where $J_{0\mu L}^h$, $J_{0\mu R}^h$, and J_μ^{EM}(singlet) are given in Eqs. (7b) and (12b), respectively. The expression for $\mathcal{L}_{\mathrm{int}}^e$ except the terms involving X_1, Y_1, X_2, and Y_2 given in Eq. (5) is the same as above where the currents coupled with $W_{L\mu}^-$ and $W_{R\mu}^-$ are given by the first term in Eq. (5), while $J_{0\mu L}^e$, $J_{0\mu R}^e$, and $J_\mu^{\mathrm{EM}}(e)$ are given in Eq. (7a).

III. SPONTANEOUS BREAKING OF SU(4)

The gauge symmetry SU(4) is spontaneously broken according to Eq. (11).

The first stage of symmetry breaking is accomplished by a 15-plet of Higgs scalars Φ:

$$\langle \Phi \rangle = \frac{v}{2}\,\mathrm{diag}\,(1,1,-1,-1)\,.$$

The second and third stages of symmetry breaking are accomplished by Higgs scalars H and S belonging to the fundamental representation and representation 10 of SU(4), respectively. This is done by giving the vacuum expectation values as follows:

$$\langle H \rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ \lambda \\ \lambda' \\ 0 \end{bmatrix}, \quad \lambda \approx 0, \quad \langle S \rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 0 & 0 & -\kappa \\ 0 & 0 & \kappa' & 0 \\ 0 & \kappa' & 0 & 0 \\ -\kappa & 0 & 0 & 0 \end{bmatrix}.$$

The mass Lagrangian for vector bosons is given by

$$\begin{aligned} \mathscr{L}_{\text{mass}} = & -\tfrac{1}{8} g^2 v^2 (2\bar{X}_1 X_1 + 2\bar{X}_2 X_2 + 2\bar{Y}_1 Y_1 + 2\bar{Y}_2 Y_2) \\ & -\tfrac{1}{8} g^2 \lambda'^2 \left[2W_R^+ W_R^- + W_{3R}{}^2 - 2\frac{1}{\sqrt{2}} W_{3R} B + \tfrac{1}{2} B^2 + 2\bar{X}_1 X_1 + 2\bar{X}_2 X_2 \right] \\ & -\tfrac{1}{8} g^2 [(\kappa^2 + \kappa'^2)(2W_L^+ W_L^- + 2W_R^+ W_R^- + W_{3L}{}^2 + W_{3R}{}^2 - 2W_{3L} W_{3R}) - 2\kappa\kappa'(2W_L^+ W_R^- + 2W_L^- W_R^+) \\ & + \kappa^2 (8\bar{Y}_1 Y_1 + 2\bar{X}_1 X_2 + 2\bar{Y}_2 Y_2) + \kappa'^2 (8\bar{X}_2 X_2 + 2\bar{X}_1 X_1 + 2\bar{Y}_2 Y_2) - 4\kappa\kappa'(\bar{X}_1 Y_2 + X_1 \bar{Y}_2)]\,. \end{aligned} \tag{17}$$

With

$$v^2 \gg \lambda'^2 \gg \kappa^2, \kappa'^2\,,$$

the masses of vector bosons are given by

$$\begin{aligned} m_{X,Y}{}^2 &\approx \tfrac{1}{4} g^2 v^2\,, \\ m_{WR}{}^2 &\approx \tfrac{1}{4} g^2 \lambda'^2\,, \\ m_{WL}{}^2 &\approx \tfrac{1}{4} g^2 (\kappa^2 + \kappa'^2)\,. \end{aligned} \tag{18}$$

For the special case $m_R \approx m_X$, viz. $v \approx \lambda'$, the above masses remain unchanged except that

$$m_X{}^2 \approx \tfrac{1}{4} g^2 (v^2 + \lambda'^2)\,.$$

The mass matrix for neutral vector bosons W_{3L}, W_{3R}, and B can be diagonalized through Eq. (13) to give the masses of physical vector bosons A, Z_L, and Z_R as follows:

$$\begin{aligned} m_A{}^2 &= 0 \quad \text{(photon)}\,, \\ m_{Z_L}{}^2 &\approx (\cos^2\theta_W)^{-1} m_{W_L}{}^2\,, \\ m_{Z_R}{}^2 &\approx (\cos^2\theta_W / \cos 2\theta_W) m_{W_R}{}^2\,. \end{aligned} \tag{19}$$

Only the scalars S are coupled to fermions. The Yukawa coupling of these scalars to fermions are given by

$$-f F_i^T C F_j S_{ij} + \text{H.c.} \tag{20}$$

The fermions get their masses from Eq. (20), when we put the vacuum expectation values

$$\begin{aligned} \langle S_{14} \rangle &= \langle S_{41} \rangle = -\kappa\,, \\ \langle S_{23} \rangle &= \langle S_{32} \rangle = \kappa'\,. \end{aligned}$$

These masses are for the first generation (for example),

$$\begin{aligned} m_e &= f\kappa\,, \\ m_{\nu_e\text{-}N_e} &= +f\kappa'\,, \\ m_u &= m_d = f(\kappa + \kappa')\,. \end{aligned}$$

If there is a left-handed neutrino N_{eL}, this can be put in a singlet representation of SU(4), then we have an additional mass term

$$m_{Ne} = f\lambda'\,.$$

In this case, we have two massive Dirac neutrinos,[6] one of them superheavy and the other one very light.

Finally, we discuss the renormalization of $\sin^2\theta_W$. Using the renormalization-group equations, we get the following formula for $\sin^2\theta_W$ at m_L:

$$\frac{\sin^2\theta_W - \sin^2\theta_W^0}{2\cos^2\theta_W^0} = \alpha \left\{ (\beta_2 - \tan^2\theta_W^0) \ln \frac{m_X}{m_L} + \tan^2\theta_W^0 (\beta' - \beta_2 - \beta_1) \ln \frac{m_X}{m_R} \right\}, \tag{21}$$

where

$$\begin{aligned} \beta_2 &= \frac{1}{4\pi}\left(-\tfrac{22}{3} + \tfrac{2}{3} n_g\right) = -\frac{16}{12\pi}\,, \\ \beta_1 &= \frac{1}{4\pi}\left[\tfrac{4}{3} n_g \sum \tfrac{1}{2} Y^2\right] = \frac{4}{4\pi}\,, \\ \beta' &= \frac{1}{4\pi}\left[\tfrac{4}{3} n_g \sum \tfrac{1}{2} Y'^2\right] = \frac{6}{4\pi}\,. \end{aligned} \tag{22}$$

In the above formulas, we have taken the number of generations $n_g=3$, $\sum \frac{1}{2}Y^2=1$, $\sum \frac{1}{2}Y'^2=\frac{3}{2}$. Using $\sin^2\theta_W^0=\frac{1}{4}$, $\sin^2\theta_W=0.22$, we get for

$$240 \leq m_R \leq 6.4\times 10^3 \text{ GeV} ,$$

the following limits on m_X:

$$3.3\times 10^4 \geq m_X \geq 6.4\times 10^3 \text{ GeV} . \quad (23)$$

IV. CONCLUSIONS

To conclude, we have seen that it is possible to obtain a "true" unification of electroweak interactions with $\sin^2\theta_W^0=\frac{1}{4}$ in an SU(4) group which contains the left-right-symmetric $SU_L(2)\times SU_R(2)\times U(1)$ group. The unification mass scale is given in Eq. (23). It is clear from this equation that the unification scale could be as high as 3.3×10^4 GeV, if the right-handed vector boson is comparatively light, viz. of the order of 240 GeV. In any case, in our unification scheme, W_R cannot have mass greater than 6.4×10^3 GeV.

For the hadron sector with integer-charge quarks, the SU(4) model gives the same predictions as the left-right-symmetric model. For the lepton sector, certain new processes are possible, namely,[4]

$$e^- + e^- \to \mu^- + \mu^- ,$$

$$e^- + \mu^+ \to e^+ + \mu^-$$

with effective Lagrangian [cf. Eq. (5)]

$$\mathcal{L}^{\text{eff}} = C\frac{G_F}{\sqrt{2}}(\bar{\mu}_R^c \gamma_\mu \mu_R)(\bar{e}_R^c \gamma_\mu e_R) ,$$

where the characteristic strength C is given by [cf. Eq. (23)]

$$C \simeq \frac{{m_{W_L}}^2}{{m_{Y_1}}^2} \simeq (6.4\times 10^{-6} \text{ to } 1.5\times 10^{-4}) ,$$

compared with[7] $C\simeq(3\times 10^{-6}$ to $2\times 10^{-5})$ in the conventional theory. The former process has not so far been searched, while the present experimental limit[8] on C for the second process is 5×10^3, very much larger than the above theoretical limits.

Another observable effect could be in the mode[4] (through Y_2 exchange)

$$\mu^- \to e^- + \bar{\nu}_e + \nu_{\bar{\mu}} ,$$

as distinct from the usual mode

$$\mu^- \to e^- + \bar{\nu}_e + \nu_\mu .$$

The former mode would favor a right-handed electron in a polarized-muon decay, but the present limit on the Y_2 mass from such a mode is that $m_{Y_2} > 3m_{W_L}$, much less than that given in Eq. (23).

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