Constraints on the right-W-boson mass in nonmanifest left-right gauge theories

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We obtain a model-independent lower bound on M_R of 300 GeV for a general class of $SU(2)_L \times SU(2)_R \times U(1)_Y$ left-right theories *without* any assumptions about the structure of the Cabibbo-Kobayashi-Maskawa matrices or the right-handed neutrino mass. Our result is also independent of Higgs-boson, dispersive, and W_L-W_R mixing contributions, and it holds for a range of bag constants.

I. INTRODUCTION

With the recent discovery of the W and Z bosons,[1] it is of interest to place constraints on the presence of right-handed currents and the masses of the right-handed bosons. There have been numerous papers which derive limits on the above masses by making specific assumptions of the models,[2] such as equivalent left and right Cabibbo-Kobayashi-Maskawa (CKM) matrices[3–5] ($V^L = V^R$) or a light right-handed neutrino,[6,7] and the resulting bounds are highly dependent on these assumptions.[8,9] There also have been recent papers which employ a light right-handed boson.[10,11]

The major contribution of this paper is to establish a bound on the right-handed W-boson mass of $M_R \geq 300$ GeV independent of (1) the structure of the leptonic sector, (2) any restrictions or implied symmetries on the right CKM-matrix parameters such as a manifest ($V^L = V^R$) or pseudomanifest symmetry ($V^L = V^{R*}$), (3) the existence or nature of Higgs-boson contributions, (4) dispersive contributions from intermediate particle states, (5) W_L-W_R mixing contributions, and (6) any value of the bag constant B in the customary range[12–15] ($\sim 0.3 \leq |B| \leq 2.5$) or similar QCD contributions. Clearly our limits on M_R are not always as restrictive as those models which make specific assumptions about the right-hand sector, but our work serves to legitimize the hypothesis that the right-handed boson is substantially heavier than the left-handed boson, and, barring a highly contrived cancellation, we prove this result in Sec. VII for the general class of left-right theories discussed in Sec. III. While there has been much work performed on left-right theories with the hypothesis $M_R \gg M_L$ (Ref. 16), the structure of the right-handed sector still remained arbitrary enough such that the $A(K^0 \to \bar{K}^0)$ amplitude and the other experimental data (K decays and muon decays) permitted a light right-handed W boson ($M_R \simeq M_L$) for a wide range of models.[10,11] Only by parametrizing the right-handed sector in a general manner and then imposing experimental constraints from *both* the kaon mass difference and the b-quark lifetime were we able to obtain such a universal limit on M_R without any assumptions about the form of V^R, or the leptonic structure. To our knowledge, no similar limit currently exists.

Our approach will be first to review previous limits on M_R (Sec. II) and examine what conditions are assumed in obtaining each limit, and then to formally parametrize a general class of left-right models (Sec. III) without assuming manifest symmetry. Next, we impose the experimental constraints from the kaon mass difference ΔM_K (Sec. IV) and then from the b decay (Sec. VI), and prove that these constraints are incompatible for any model in our general class of theories unless $M_R \geq 300$ GeV. We will show this for a specific case in Sec. VI, and then prove our results in general in Sec. VII. Finally, we consider other sources which might affect our calculation and demonstrate that such contributions *do not* change our conclusions.

II. REVIEW OF LIMITS

First, we briefly analyze some previous constraints on M_R to review what values are allowed and specifically what assumptions are made.[17] Bég, Budny, Mohapatra, and Sirlin[6] utilized muon decay data to require the right-handed charged current to be less than 13% of the left-handed charged current. In a symmetric ($g_L = g_R$) model with a light right-handed neutrino, this measurement implies a limit of $M_R \geq 230$ GeV so that the right-handed current is adequately suppressed. However, if the right-handed neutrino ν_R is a heavy Majorana neutrino, the right-handed process is suppressed for *any* value of M_R and we have no constraint. More stringent limits using recent data from similar experiments[7] require $M_R \geq 380$–450 GeV, again with the above assumptions.

Donoghue and Holstein[18] analyzed the nonleptonic $\Delta S = 1$ weak decays and concluded that $M_R \geq 300$ GeV assuming the left- and right-handed mixings to be similar, as they would be in the case of an approximate manifest left-right symmetry ($\theta_i^L \simeq \theta_i^R$).

Beall, Bander, and Soni[3] used the $K_L - K_S$ mass difference to determine that $M_R \geq 1600$ GeV independent of the right-handed neutrino mass, but this constraint is dependent on the assumption of *manifest* left-right symmetry, i.e., $V^L = V^R$. Mohapatra, Senjanović, and Tran[4] extended this analysis by including contributions from the third generation of quarks and Higgs bosons, and conclude that for sufficiently light Higgs bosons ($M_H \simeq 100$ GeV to 1 TeV) the additional terms can yield a contribution large enough to cancel the W-boson contribution and that no absolute, model independent limit can be placed on M_R from the $K_L - K_S$ mass difference and $K \to 2\pi$ decays. We will discuss such cancellations in Sec. V and show how we avoid the uncertainties of the Higgs contribution in Sec. VIII.

The applicability of all the above constraints are restricted by the inherent assumptions necessary to derive each limit. Our goal is to derive a limit on M_R which is independent of the leptonic sector (i.e., the mass of the right-handed neutrino) and manifest left-right symmetry, (i.e., our constraints should hold for $V_L \neq V_R$). These are the two assumptions which plague the previous limits on M_R. Furthermore, we will demonstrate that our limit is essentially independent of Higgs-boson contributions, dispersive contributions, left-right mixing, and QCD corrections.

III. FORMALISM

The class of models we will examine exhibit a zeroth-order symmetry of $SU(2)_L \times SU(2)_R \times U(1)_Y$.[19] Our results apply to any model where the quarks are placed in left- and right-handed doublets with no more than three generations. In particular, all our results are independent of the structure of the leptonic sector. For the left- and right-handed quark fields we use the standard representation[8,10] with $T_L = \frac{1}{2}$, $T_R = 0$, and $Y = 1$ for the left quarks Ψ_L, and $T_L = 0$, $T_R = \frac{1}{2}$, and $Y = 1$ for the right quarks Ψ_R. We choose a symmetric theory ($g_L = g_R$) for simplicity. Note that for $M_R / M_L \simeq 1$ and $m_i / M_L \simeq 0$ it is only the quantity (M_R / g_R) which is constrained and then the above choice can be made without any loss of generality.

To understand what freedom we have in defining the most general quark-mixing matrices V^L and V^R, let us investigate the origin of these matrices in the theory. The quarks obtain mass through a coupling to a complex Higgs field Φ. The term which will yield the quark masses is of the Yukawa form: $L_{\text{mass}} = g \bar{\Psi}_L \Phi \Psi_R$. We can reduce the Higgs field Φ by an invariant decomposition into $\Phi = \Phi_+ + \Phi_-$ with

$$\Phi_{\pm} = \tfrac{1}{2}(\Phi \pm S \Phi^* S^{-1})$$

and S such that $S^{-1} U S = U^*$ for any U in SU(2). Assuming different coupling constants for $\Phi_{\pm}$, we have

$$L_{\text{mass}} = \sum_i \sum_j g_{ij}^{+} \bar{\Psi}_L^i \Phi_+ \Psi_R^j + g_{ij}^{-} \bar{\Psi}_L^i \Phi_- \Psi_R^j + \text{H.c.} \quad (3.1)$$

Note that the absence of any horizontal symmetry permits g_{ij} to be an arbitrary matrix. To obtain massive quarks, we break the symmetry and take the vacuum expectation value of the Higgs fields as

$$\langle \Phi \rangle = \begin{bmatrix} u_1 & 0 \\ 0 & u_2 \end{bmatrix} \neq 0 . \quad (3.2)$$

To ensure β-decay universality and to satisfy present limits on $W_L - W_R$ mixing, we must set either u_1 or u_2 equal to zero. To be definite, we choose $u_2 = 0$. The mass matrices we desire enter L_{mass} in the form

$$L_{\text{mass}} = \bar{\Psi}_L^i M_{ij} \Psi_R^j + \bar{\Psi}_R^i M_{ij}^* \Psi_L^j . \quad (3.3)$$

We define matrices A and B such that $g_{ij}^{\pm} = A_{ij} \pm B_{ij}$. Then $u_1 A_{ij}$ is the mass matrix of the upper quarks, and $u_1^* B_{ij}$ is the mass matrix of the lower quarks. We follow the usual convention and take the mass eigenstates and gauge eigenstates of the upper quarks to be identical, i.e., A_{ij} is diagonal. Since the V matrices can be chosen to be unitary, then

$$B = \tfrac{1}{2}(g^+ - g^-) = \frac{1}{u_1^*} V^L \begin{bmatrix} m_d & & \\ & m_s & \\ & & m_b \end{bmatrix} V^{R\dagger} . \quad (3.4)$$

The point is that we can determine both V^L and V^R (and the ratios of the quark masses) from a knowledge of the coupling constants of the theory. To select a parametrization for a set of six-quark Kobayashi-Maskawa (KM) matrices, we take V^L equal to the standard KM form.[20] V^R clearly can be taken to be of the form

$$V^R = \begin{bmatrix} c_1 e^{i\phi_a} & s_1 c_3 e^{i\phi_b} & s_1 s_3 e^{i\phi_c} \\ -s_1 c_2 e^{i\phi_d} & (c_1 c_2 c_3 + s_2 s_3 e^{i\delta}) e^{i\phi_f} & (c_1 c_2 s_3 - s_2 c_3 e^{i\delta}) e^{i\phi_h} \\ -s_1 s_2 e^{i\phi_e} & (c_1 s_2 c_3 - c_2 s_3 e^{i\delta}) e^{i\phi_g} & (c_1 s_2 s_3 + c_2 c_3 e^{i\delta}) e^{i\phi_j} \end{bmatrix} , \quad (3.5)$$

the standard KM form with arbitrary phases ϕ_i added. Unitarity requires $\phi_a - \phi_b = \phi_d - \phi_f = \phi_e - \phi_g$ and $\phi_a - \phi_c = \phi_d - \phi_h = \phi_e - \phi_j$ which implies the most general V^R is a function of three angles and six phases. These are the CKM matrices we will use in our analysis. Specifically, $V^L \neq V^R$.

IV. KAON-MASS CALCULATION AND CONSTRAINTS

To investigate what constraints we must impose on our matrix parameters, we approximate the transition amplitude $\langle \bar{K}^0 | H_W | K^0 \rangle$ by the box-diagram amplitude $A(K^0 \to \bar{K}^0)$, and compare our results with the experimental value for $\Delta M_K = 2\,\text{Re}(\langle \bar{K}^0 | H_W | K^0 \rangle)$. We compute the box-diagram amplitude by evaluating Fig. 1, where we must sum over all combinations of L and R bosons. That is,

$$A(K^0 \to \bar{K}^0) = A_{LL} + A_{RR} + A_{LR} + A_{RL} .$$

In the limit where the external quark momenta are assumed to be negligible compared to the loop momenta, we have for the LL and LR contributions

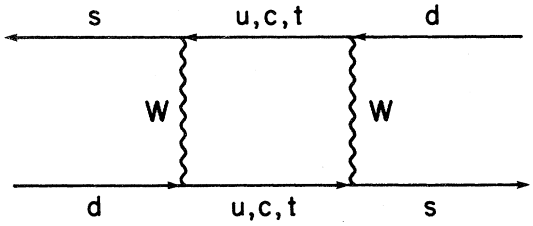

FIG. 1. Box diagram for $A(K^0 \to \bar{K}^0)$. Note that W must be summed over all combinations of left and right helicities. The crossed diagram is also to be included.

$$A_{LR}^{LL} = \begin{Bmatrix} 1 \\ 4 \end{Bmatrix} \beta \frac{-G^2}{4\pi^2} B M_{LR}^{LL} \left[\sum_{i,j} \lambda_i \lambda'_j [ij] \begin{Bmatrix} M_L{}^2 \\ m_i m_j \end{Bmatrix} \right]_{LR}^{LL}, \tag{4.1}$$

where we define

$$M_{LR}^{LL} = \left\langle \bar{K}^0 \middle| \left[\bar{\Psi}_s^\alpha \frac{(1-\gamma_5)}{2} \Psi_d^\alpha \right] \middle| 0 \right\rangle \times \left\langle 0 \middle| \left[\bar{\Psi}_s^\beta \frac{(1\mp\gamma_5)}{2} \Psi_d^\beta \right] \middle| K^0 \right\rangle \tag{4.2}$$

with α,β summed over color, and $i,j = u,c,t$. Also, $\lambda_i^{LR} = V_{id}^L V_{is}^{R*}$, $\lambda_i^{LR'} = V_{is}^{L*} V_{id}^R$ with appropriate definitions for the LL terms, and

$$[ij] = \frac{-\beta^2 \ln(\beta)}{(1-\beta)(\beta-\epsilon_i)(\beta-\epsilon_j)} + \frac{\beta}{(\epsilon_i-\epsilon_j)} \left[\frac{\epsilon_i^n \ln(\epsilon_i)}{(1-\epsilon_i)(\beta-\epsilon_i)} - \frac{\epsilon_j^n \ln(\epsilon_j)}{(1-\epsilon_j)(\beta-\epsilon_j)} \right] \tag{4.3}$$

with $n=2$, $\beta=1$, and $\epsilon_i = (m_i/M_L)^2$ for the A_{LL} term, and $n=1$, $\beta = (M_L/M_R)^2$, and $\epsilon_i = (m_i/M_R)^2$ for the A_{LR} term. It can easily be shown that $A_{LR} = A_{RL}$ since $M_{LR} = M_{RL}$, $\lambda^{LR} = \lambda^{RL'}$, and $[ij] = [ji]$. The A_{RR} term has the same form as the A_{LL}, with appropriate subscripts changed. B is an adjustable constant characteristic of the quark content of the K^0 and $\bar{K}^0$ states which has a range (in the literature)[12–15] of $\sim 0.3 \leq |B| \leq 2.5$. In the vacuum insertion approximation $B = 1.0$. In the MIT bag model, $|B| = 0.4$ for an M_{LL} or M_{RR} matrix element,[12] and $|B| = \sim 0.7$ for a left-right matrix element (M_{LR} or M_{RL}).[13] A value of $|B| = 0.33$ has been determined from PCAC (partial conservation of axial-vector current) using hyperon data.[14] We will examine the B dependence of $A(K^0 \to \bar{K}^0)$ in Sec. VI, and show that the particular value of B has little effect on the limits of M_R. We will also examine other QCD contributions in Sec. VIII and show that these do not change our results. For simplicity, we choose $B = 1.0$ in the following calculations.

Using approximate values for the left CKM-matrix parameters, we find the A_{LL} contribution yields $\sim 70\%$ of the experimental ΔM_K result. Assuming no contrived cancellations, we expect the $\mathrm{Re}(2A_{LR} + A_{RR})$ contribution to ΔM_K to be of the order of magnitude of, or less than, $\Delta M_K/2$. Since A_{RR} can be easily suppressed by choice of angles, our task is to suppress $2A_{LR}$ such that $\mathrm{Re}(2A_{LR}) \leq (\Delta M_K/2)$. Because the expression for $2A_{LR}$ [Eq. (4.1)] contains an extra factor of $2\times 4 = 8$ relative to A_{LL} and $M_{LR} \simeq 7.7 M_{LL}$, this chore is nontrivial.

Since ΔM_K only depends on the real part of the box diagram, we could adjust all the phase angles such that $\mathrm{Re}(2A_{LR}) = 0$, but this would necessarily imply a large $\mathrm{Im}(2A_{LR})$ which would lead to a large CP-violation amplitude in contradiction to experiment. The very small experimentally observed value for the CP-violation parameter of $\mathrm{Re}(\epsilon) \simeq 1.6 \times 10^{-3}$ (Ref. 21) will, in general, lead to a much more restrictive set of constraints on our parameters than the conditions from the ΔM_K data if we do not also suppress the imaginary part of $A(K^0 \to \bar{K}^0)$. Having eliminated the possibility of suppressing $2A_{LR}$ by complex phases, we examine suppression by Cabibbo mixing. In fact, careful analysis[10] shows there are two general forms of V^R which will adequately suppress $2A_{LR}$, given that we also choose an acceptable V^L matrix. For V^R we have

$$V^R = \begin{bmatrix} c_1 e^{i\phi_a} & \sim 0 & -s_1 e^{i\phi_c} \\ s_1 e^{i\phi_d} & \sim 0 & c_1 e^{i\phi_h} \\ \sim 0 & e^{i(\delta+\phi_g)} & \sim 0 \end{bmatrix} \tag{4.4}$$

and the same structure with columns 1 and 2 interchanged. Note that unitarity requires $\phi_a + \phi_h = \phi_c + \phi_d$, and the zeros mean the matrix element is $\lesssim 10^{-3}$. The only restrictions we need impose on the corresponding V^L are that $|V_{td}^L|$ or $|V_{ts}^L|$ be $\lesssim 10^{-3}$, respectively. These values for V^L are acceptable since the only present constraints we have come from unitarity. We conclude that this choice of CKM matrices alone is sufficient to suppress the $2A_{LR}$ contribution to ΔM_K for a light right-handed W boson ($M_R \simeq M_L$), and hence it represents a solution which is consistent with (only) the ΔM_K data. Also note that this form of the V^R matrix immediately implies that $|V_{ub}^R|^2 + |V_{cb}^R|^2 \simeq 1$. We will show this condition to be inconsistent with the b-decay data, first for the above specific cases and then for the general class of models.

V. PHILOSOPHY

To obtain the general forms of the matrices discussed above and the implied limits in M_R, we exploited the fact that the contribution to ΔM_K from the $2A_{LR}$ was too large by at least an order of magnitude. This is the key which allows us to obtain a definite prediction in a theory with so many free parameters. The above forms of the CKM matrices are the only ones which will adequately suppress the $2A_{LR}$ contribution for a *range* of angles and phases. Apart from the possibility of a fortuitous cancellation, we have been unable to find a simple rationale for a set of phases and angles which achieves the necessary suppression. For example, the choice of manifest symmetry ($\theta_i^L = \theta_i^R$) as in Beall *et al.*[3] actually leads to a more stringent limit on M_R (i.e., $M_R \geq 1600$ GeV). Furthermore, the additional constraint from the experimental value of the CP-violation parameter ϵ, forces us to suppress *both* the Im and Re parts of $2A_{LR}$, and thus would require very specific choices of both angles and phases.

Our choice of CKM matrices is in fact a solution set which categorizes the parameters into two classes, those angles which are less than $\sim 10^{-3}$ and those which are unrestricted. This differs from a cancellation mechanism where the parameters must be equal to particular values (not simply near zero) within one part in $\sim 10^3$. *A priori,* the probability for such particular values of both angles and phases is extremely small, and all that follows must be understood in this context.

VI. b-DECAY DATA

The above analysis demonstrates that there exists a class of V^R matrices which allow us to satisfy experimental constraints on ΔM_K, ϵ, ζ, ρ and virtually all other data while maintaining a light right-handed W boson ($M_R \simeq M_L$). In fact, if it were not for the recent results on the b-quark lifetime, we could construct an acceptable model with a light M_R (Ref. 2). We will now demonstrate that it is the b-decay data which force us to place a lower bound on M_R. To satisfy the ΔM_K constraints, the right-handed CKM matrix must be one of the two forms mentioned previously in Eq. (4.4); in either case, $|V_{ub}^R|^2+|V_{cb}^R|^2 \simeq 1$. This is in violation of the b-quark decay results for a light M_R (i.e., less than ~ 320 GeV).

The most recent estimates of the b-quark lifetime are $\tau_b=(1.6\pm0.4\pm0.3)\times10^{-12}$ sec (MAC, Ref. 22) and $\tau_b=(1.2^{+0.45}_{-0.36}\pm0.30)\times10^{-12}$ sec (Mark II, Ref. 23). The relatively long lifetime of the b quark implies a weak coupling to the u and c quarks ($|V_{ub}^R|$ and $|V_{cb}^R|$ small) in both the left and right sectors for a light W_R. To obtain a conservative bound on $|V_{ub}^R|^2+|V_{cb}^R|^2$, we will perform our calculations with $\tau_b \geq 1\times10^{-12}$ sec. Our goal is to compute Γ_R^b, the b-decay rate through the right-handed channel, and require this to be less than the total b-decay rate for the left and right channels combined. This procedure is valid since the left-right interference term is small. To avoid any dependence on the leptonic sector, we compute the Γ_R^b for b decay from the right-handed hadronic channels assuming a hadronic branching ratio of 75%.[22,24] This will avoid assumptions about the lepton masses and couplings as was necessary in the Bég *et al.*[6] and Carr *et al.*[7] limits. This calculation will parallel the work of previous limits derived for the left sector.[25] We ignore nonspectator contributions as these are estimated to be of order of one percent or less,[24] and if included, they would in fact strengthen our bounds. We have[26,24]

$$\Gamma_R^b=\beta^2\Gamma_0^b\left[\frac{1}{0.75}\right]V_{\rm had} \tag{6.1}$$

with

$$\begin{aligned}V_{\rm had}\simeq{}&(1.6)|V_{cb}^R|^2(|V_{ud}^R|^2+|V_{us}^R|^2)\\&+(0.25)|V_{cb}^R|^2(|V_{cd}^R|^2+|V_{cs}^R|^2)\\&+(3.9)|V_{ub}^R|^2(|V_{ud}^R|^2+|V_{us}^R|^2)\\&+(1.4)|V_{ub}^R|^2(|V_{cd}^R|^2+|V_{cs}^R|^2)\,,\end{aligned} \tag{6.2}$$

$\beta=(M_L/M_R)^2$, and

$$\Gamma_0^b=G^2m_b^5/192\pi^3\,.$$

There is a nonleptonic enhancement factor of $(2f_+{}^2+f_-{}^2)/3$ from the QCD corrections due to the gluon exchanges. This factor is estimated to be ~ 1.3 (Refs. 24 and 26), and has been incorporated into the phase-space coefficients in Eq. (6.2). Using the unitarity of V, we can express the above as

$$\begin{aligned}V_{\rm had}\simeq{}&-3.9|V_{ub}^R|^4-0.25|V_{cb}^R|^4\\&-3.0|V_{ub}^R|^2|V_{cb}^R|^2\\&+5.3|V_{ub}^R|^2+1.85|V_{cb}^R|^2\,.\end{aligned} \tag{6.3}$$

From $\tau_b=1/\Gamma^b\geq 1$ psec, we have $(0.75\Gamma^b/\Gamma_0^b)\lesssim 1/140$, and thus we require $\beta^2V_{\rm had}\lesssim 1/140$ with $m_b=5.0$ GeV. To compare these constraints with the ΔM_K constraints, we look for a bound on $|V_{ub}^R|^2+|V_{cb}^R|^2$. To summarize, we find

$$\frac{(0.004)}{\beta^2}\geq|V_{ub}^R|^2+|V_{cb}^R|^2\geq 0\,, \tag{6.4}$$

where this holds for M_R up to ~ 300 GeV. The above analysis is independent of the structure of the leptonic sector. Clearly the constraint $|V_{ub}^R|^2+|V_{cb}^R|^2\simeq 1$ from the kaon mass difference and the above constraint from the b-decay data are in contradiction for values of β near unity.

Figure 2 illustrates the M_R dependence of the contradictory constraints from the b-decay and ΔM_K data. As M_R becomes heavy, the right decay channel is naturally suppressed and the b-decay constraints vanish for $M_R\to 320$ GeV as $M_R{}^4$. Since the A_{LR} amplitude goes as

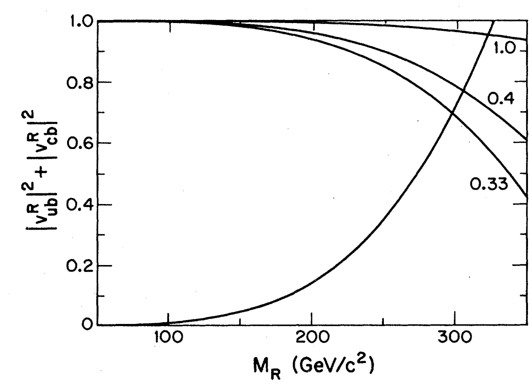

FIG. 2. Constraints on $|V_{ub}^R|^2+|V_{cb}^R|^2$ as a function of M_R. The rising curve represents the limit from b decay with the allowed region being below the curve. The upper three curves show the constraints from the kaon mass difference for the bag constant B equal to 1.0, 0.4, and 0.33 as noted on the curves. The allowed region here is above the curves.

$\beta=(M_L/M_R)^2$, these constraints vanish more slowly than the b-decay constraints, i.e., as $M_R{}^2$. The curves plotted represent *conservative* bounds for the solution space of these specific models described in Sec. IV. Note that since the bag constant B affects only the $2A_{LR}$ amplitude, and then only in a linear manner, the resulting limit on M_R is relatively insensitive to B in the customary range[12–15] of $\sim 0.3 \le |B| \le 2.5$. From Fig. 2, the reader can easily see how additional contributions or constraints will affect the M_R limits. The structure of the curves suggest it is very difficult to obtain a solution consistent with the ΔM_K and b-decay constraints for $M_R < 300$ GeV. While these curves hold rigorously only for the special cases discussed in Sec. IV, we find that the nature of the ΔM_K and b-decay constraints are similar for *all* cases. In light of this knowledge, we now choose $M_R \simeq 300$ GeV and prove our conclusions in general.

VII. GENERAL ANALYSIS

Using Eq. (4.1) for A_{LR} and requiring that $\Delta M_K \ge 2\,\mathrm{Re}(A)$, as discussed in Sec. IV, we can use the ratio $2\,\mathrm{Re}(2A_{LR})/\Delta M_K$ to express the relative contribution of the $2A_{LR}$ part in terms of a few "significant" quantities. Using the experimental value for ΔM_K, we find for $M_R \simeq 300$ GeV that the above constraint becomes

$$\frac{2\,\mathrm{Re}(2A_{LR})}{\Delta M_K} \le 45\left[\sum_{i,j} \lambda_i \lambda'_j m_i m_j [ij]\right]_{LR} \tag{7.1}$$

with $i,j=u,c,t$. The right-hand side is a sum of six independent quantities. The restriction we impose is that each A_{LR}^{ij} term be such that its contribution to the total amplitude is less than or equal to the experimentally determined value. That is,

$$(\lambda_i \lambda'_j m_i m_j [ij])_{LR} \le \tfrac{1}{45} \tag{7.2}$$

for each ij combination. Although not impossible, we find it implausible that six *independent* terms would conspire to create a cancellation to one part in 45, especially without any relation among the quark masses. We will take the phases of the V^R elements to be small and deal only with the magnitudes for the purposes of this calculation. If the V^R terms did have significant imaginary parts, we would necessarily obtain large imaginary contributions which would violate experimental CP-violation measurements, and would hence imply even stronger constraints on M_R as discussed in Sec. IV.

Imposing the constraint (7.2) on the A_{LR}^{cc} contribution, we find for $m_c \simeq 1.5$ GeV, $\lambda_c \lambda'_c \le \frac{1}{700}$, or $V_{cd}^R V_{cs}^{R*} \le \frac{1}{150}$. The b-decay data require $|V_{cb}^R|^2 \le \sim 0.8$, and hence unitarity yields the constraint $|V_{cd}^R|^2 + |V_{cs}^R|^2 \ge \sim 0.2$. Consequently, we find two solution sets for the choices of matrix elements: case A, $|V_{cs}^R| \gtrsim 0.44$ and $|V_{cd}^R| \lesssim 0.01$, and case B, $|V_{cs}^R| \lesssim 0.01$ and $|V_{cd}^R| \gtrsim 0.44$.

We examine case A first. From b decay we have the limit $|V_{tb}^R|^2 \gtrsim 0.2$, and hence if $|V_{cd}^R| \simeq 0.01$, then $|V_{ud}^R|^2 \gtrsim 0.2$. Thus, we have forced $|V_{ud}^R| \gtrsim 0.44$, and $|V_{cs}^R| \gtrsim 0.44$, and the resulting V^R matrix is of the form

$$V^R \simeq \begin{bmatrix} >0.44 & * & * \\ <0.01 & >0.44 & <0.90 \\ <0.90 & * & * \end{bmatrix} . \tag{7.3}$$

But now the A_{LR}^{uc} cross term is too large by a factor

$$45(m_u m_c [uc] \lambda'_u \lambda_c) \simeq 45 \times (3.4) \times V_{us}^{L*} V_{cd}^L V_{ud}^R V_{cs}^{R*} \gtrsim 1.6 . \tag{7.4}$$

Although one might argue that this value is marginally acceptable, as we lower M_R this contribution quickly grows out of proportion. Furthermore, we must add in all other contributions A^{ij} summing over all left-right combinations, and hence we conclude that $M_R \gtrsim 300$ GeV for case A. Case B is identical, except the above contribution will pick up a factor $[\cos(\theta_1^L)]^2$ instead of $[\sin(\theta_1^L)]^2$, and hence the A_{LR}^{uc} contribution will be too large by $1.6 \times 19 = \sim 30$.

Thus, neglecting contrived cancellations, we conclude that in the general case it is impossible for M_R to be less than ~ 300 GeV and still satisfactorily conform to the ΔM_K and b-decay data. We have made some approximations by working only with the magnitudes of the CKM-matrix elements, and by requiring each A_{LR}^{ij} piece to satisfy Eq. (7.2) individually. We have already argued against cancellations, but we have also checked our assumptions and approximations numerically and found them to be valid. In a search of $\sim 10^4$ points in the parameter space, we found no solutions which complied with our constraints from the ΔM_K and b-decay data for M_R values below 300 GeV. The point of this exercise was not to attempt to characterize the solution space of the problem for a light W_R, but instead to independently demonstrate that any solution space for a $M_R < 300$ GeV must rely on sensitive cancellations, and thus be exceedingly small. Our tests would indicate that such a solution space, if it exists, must be no greater than one part in 10^4 relative to the volume of the parameter space. Hence, we can claim with confidence that 300 GeV is a very reasonable model-independent lower bound for M_R.

VIII. OTHER CONTRIBUTIONS

Having already argued against cancellations as a mechanism to resolve the difficulty of such a large $2A_{LR}$ amplitude, we should, in principle, ignore the independent contributions from other sources for the purpose of determining limits on the right-handed boson mass. However, a few remarks regarding such contributions may be in order.

Mohapatra *et al.*[4] and various other authors[27] have examined possible Higgs contributions for values of M_H from 100 GeV to 1 TeV. For a heavy Higgs boson, such terms are suppressed compared to the W-boson channels by the factor $(M_W/M_H)^2$, and can be ignored relative to $2A_{LR}$ for the purposes of our calculation. For a lighter Higgs boson, the contributions to A_{LR} still can be of the order of the experimental value or more, and hence it is not impossible for these contributions to cancel our large $2A_{LR}$ contribution. As discussed in Sec. V, we find such a solution to be highly contrived and not of general benefit in the understanding of the physical processes. However, once the $2A_{LR}$ term is suppressed, the Higgs-boson terms clearly can be important in an accurate calculation of $A(K^0 \rightarrow \bar{K}^0)$. Additionally, the same philosophy we used to constrain the W_R mass has been used to impose limits on the Higgs-boson mass by requiring the Higgs-boson contribution to the $A(K^0 \rightarrow \bar{K}^0)$ amplitude to be on the order of the experimental value.[27]

Hochberg and Sachs[28] and others[29] have computed the low-energy dispersive contributions for 2π, 3π, and one-particle intermediate states and determined that the sum of these terms is comparable to the measured ΔM_K value in magnitude. Wolfenstein[30] and various other authors[31] have highly restricted the W_L-W_R mixing angle to be $\zeta \leq 0.0055$. Since contributions to $A(K^0 \rightarrow \bar{K}^0)$ from W_L-W_R mixing are suppressed by a factor $\tan^2(\zeta)$, this limit makes them essentially ignorable in our calculations. The dispersion and L-R mixing contributions to $A(K^0 \rightarrow \bar{K}^0)$, which are comparable in size to, or less than, the experimental value, are thus too small by at least an order of magnitude to cancel the dominant $2A_{LR}$ contribution to ΔM_K for a light W_R. If, by some unknown means, the above contributions were large enough to possibly cancel the $2A_{LR}$ term, such a cancellation would be highly sensitive to the tuning of the parameters as we discussed in the Higgs-boson case. Consequently, such a solution is very unlikely and not really useful as an aid to understanding the process of generating $A(K^0 \rightarrow \bar{K}^0)$.

The QCD corrections due to the short-distance effects of the strong interactions have been computed for the box diagram in the context of the standard model by Gilman and Wise,[32] and this analysis has been extended to include right-handed currents by Bigi and Frére.[33] The result is that the right-handed currents are enhanced by a factor of ~ 3 to 8. Although this result will clearly strengthen our analysis of the ΔM_K constraints (see Fig. 2), this has no effect on the b-decay constraints and hence our limit of $M_R \geq 300$ GeV will essentially be unchanged.

It is clear that a realistic calculation of $A(K^0 \rightarrow \bar{K}^0)$ and ΔM_K must carefully include Higgs-boson, dispersive, and W_L-W_R mixing contributions. Our point is that we must *first* bring the $2A_{LR}$ term down to an appropriate size, and hence we obtain our limit on M_R *before* we must seriously consider the above terms.

IX. CONCLUSIONS

We have established the result $M_R \geq 300$ GeV independent of (1) the structure of the leptonic sector, (2) a particular choice of CKM matrices, (3) Higgs-boson contributions, (4) dispersive contributions, (5) W_L-W_R mixing contributions, and (6) the standard range of bag constants and other QCD corrections. Aside from the very remote possibility of a contrived cancellation, this limit applies generally to the class of models discussed.

Experimentally, this result greatly reduces the likelihood of finding M_R below 300 GeV, and such a limit will probably keep M_R out of reach for the near future. For the theorist, this limit *requires* a mechanism for making M_R significantly larger than M_L in the general class of models discussed. If the only Higgs field is the Φ previously introduced, then $W_L \simeq W_R$. Depending upon the choice of the leptonic sector, there may be additional corrections to the boson masses. One common mechanism for making W_R heavy is to include additional Higgs fields as has already been done in manifest left-right-symmetric theories[8] and in grand unified theories.[16]

In light of our conclusions, we believe that the combined constraints from the ΔM_K measurements and the b-quark lifetime are sufficient to eliminate the possibility that a light W_R could exist in a six-quark model, and hence we are forced to an increase in the mass of the W_R boson and the complexity of the theory.

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