where we can immediately identify from (B1)

$$\varphi=\tfrac{1}{2}(n_1+n_2+m)=\tfrac{1}{2}(n-1)\,,$$
$$M=\tfrac{1}{2}(n_2-n_1+m)\,,\quad M'=\tfrac{1}{2}(n_1-n_2+m) \tag{B4}$$

or, in our case, with $m=0$,

$$\varphi=\tfrac{1}{2}(n-1)\,,$$
$$M=-M'=\tfrac{1}{2}(n_2-n_1)\,. \tag{B4$'$}$$

Then the matrix element in (3.6) is given by

$$D=\langle n_1n_20|e^{i(\pi+\theta)L_2}|n_1n_20\rangle$$
$$=\langle\varphi,M|e^{i(\pi+\theta)L_2{}^a}|\varphi,M\rangle\otimes\langle\varphi,-M|e^{i(\pi+\theta)L_2{}^b}|\varphi,-M\rangle\,.$$

Now each factor is a rotation matrix element with "spin" φ. Hence

$$D=D_{M,M'}{}^{\varphi}(\theta+\pi)D_{-M,-M'}{}^{\varphi}(\theta+\pi) \tag{B5}$$

or, in terms of the hypergeometric functions,

$$D=F(-\varphi-M,\,1+\varphi-M,\,1,\,\sin^2[\tfrac{1}{2}(\theta+\pi)])$$
$$\times F(-\varphi+M,\,1+\varphi+M,\,1,\,\sin^2[\tfrac{1}{2}(\theta+\pi)])\,. \tag{B6}$$

Finally, we continue this expression to our continuum values,

$$\varphi=\tfrac{1}{2}(n-1)\,,$$
$$M=\tfrac{1}{2}(n+1)\,,$$

and obtain

$$D=F(-n,\,0,\,1,\,\sin^2(\tfrac{1}{2}\theta+\tfrac{1}{2}\pi))$$
$$\times F(1,\,1+n,\,1,\,\sin^2(\tfrac{1}{2}\theta+\tfrac{1}{2}\pi))$$
$$=[1-\sin^2(\tfrac{1}{2}\theta+\tfrac{1}{2}\pi)]^{-1-n}=[\cos^2(\tfrac{1}{2}\theta+\tfrac{1}{2}\pi)]^{-n-1}$$
$$=[\sin^2(\tfrac{1}{2}\theta)]^{-n-1}\,. \tag{B7}$$

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Scalar Mesons, Hadron Masses, and Approximate Scale Invariance*

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The possible role of a scalar-meson nonet in determining hadron masses by means of generalized Goldberger-Treiman formulas is studied. Existing data on scalar-pseudoscalar meson couplings are analyzed, assuming SU_3-invariant couplings for the unmixed mesons. The d/f ratio for the coupling of octet scalars to the baryon octet is predicted to be the same as in the Gell-Mann–Okubo mass formula. The strength of the transition vacuum → scalar meson (induced by the energy-momentum tensor $\theta_{\mu\nu}$) is measured by the constant F_σ. Denoting the SU_3 singlet scalar meson by σ_0 and the octet η-like scalar by σ_8, we find F_{σ_0} to be much larger than F_{σ_8}, provided that the σ_0BB coupling is not considerably smaller than the σ_8BB coupling. Then F_{σ_0} is comparable to the usual pion decay constant F_π. The pseudoscalar octet dispersion relation is not scalar dominated. This result is also suggested by analysis of partial conservation of axial-vector current for the scalar-pseudoscalar system, and consideration of the relation to the underlying scale-invariant limit. Implications for the underlying dynamics are discussed.

I. INTRODUCTION

THE existence and properties of scalar mesons have been the subject[1] of much theoretical speculation.[1,2] The difficulty of obtaining unequivocal experimental information has prevented a decisive clarification of the situation. It is also possible that some of the resonances in question are too broad to be described as particles. However, it seems reasonable to assume the existence of a nonet (with average mass around 1 GeV) of 0^+ mesons. Recent studies[3–6] of broken scale invariance have attributed a more fundamental significance to the scalar mesons, i.e., that they should dominate matrix elements of the trace of the energy-momentum tensor $\theta_{\mu\nu}$ and thereby determine the masses of the hadrons. Variants of this idea have been expressed previously.[7–9] This paper is primarily concerned with this question.

In Sec. II the couplings of the scalar and pseudoscalar nonets are analyzed, assuming that SU_3 symmetry is maintained in the vertices except for mixing effects. This view is not universally maintained; for

* Supported in part by the National Science Foundation.
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[4] P. Carruthers, Phys. Rev. D **2**, 2265 (1970).
[5] S. P. de Alwis and P. J. O'Donnell, Phys. Rev. D **2**, 1023 (1970).
[6] G. Mack, Nucl. Phys. **85**, 499 (1968).
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[9] G. B. West, Phys. Rev. **183**, 1496 (1969).

example, Gilman and Harari have suggested[10] that some of the mesons in question belong to simple representations of chiral $SU_2 \times SU_2$ at infinite momentum. The experimental situation with regard to candidates for the scalar nonet is still sufficiently obscure that one cannot say whether the masses are compatible with a standard nonet mass relation. We take the $\epsilon(700)$ and $\epsilon'(1060)$ (the latter called $\eta_0{}^+$ in the Particle Data Group compilation[11]) to be the 0^+ isoscalars. For the $I=1$ scalar meson we have three candidates: the narrow $\delta(962)$, the $\pi_N(980)$, and the $\pi_N(1016)$. We shall tentatively regard these as manifestations of the same state and shall use the parameters of the $\pi_N(980)$ in our analysis.[12,13] The status of the κ meson is even more obscure. Although the $\kappa(725)$ seems to be discredited,[11] there are several $K\pi$ bumps between 1.1 and 1.2 GeV and also peripheral phase-shift analyses indicating a $K\pi$ resonance around 1 GeV.[14] According to the standard mixing scheme, m_κ has to be less than $m_{\epsilon'}$; our subsequent analysis suggests $m_\kappa \cong 1$ GeV and a scalar mixing angle of about 23°. Should the κ mass violate the restriction $m_\kappa < m_{\epsilon'}$, one would have to abandon SU_3 symmetry plus mixing or introduce more particles into the mixing scheme. In particular, it is interesting to consider the possibility of a third scalar, isoscalar meson as suggested by several authors.[4,15,16] We give a brief analysis of three-particle mixing.

In Sec. III the conjectured mass formulas, based on dominating unsubtracted dispersion relations for $\theta^\mu{}_\mu$ by scalar mesons, are studied. In Ref. 4 it was noted that these assumptions lead to difficulties for the pseudoscalar octet if only two (nonet) scalars are present. Thus it is also possible that the breaking of scale invariance is sufficiently severe that at least one of the dispersion relations needs a subtraction; otherwise more scalar mesons are required. On the basis of partial conservation of axial-vector current (PCAC) relations, Crewther and Gell-Mann have suggested[17] that the pseudoscalar octet masses calculated assuming scalar dominance are subject to substantial corrections. This point of view is adopted in the present work. Analysis of the baryon octet masses leads to a determination of the ratio $F_{\sigma_8} g_{\sigma_8 BB} / F_{\sigma_0} g_{\sigma_0 BB}$, where σ_0, σ_8 are (unmixed) scalars and F_{σ_i} is analogous to the usual Goldberger-Treiman constant F_π.[18] We also predict the d/f ratio of the octet scalar-baryon octet couplings and discuss how one can determine the scalar nonet couplings and thereby check the assumptions of the theory. In order to disentangle F_{σ_0} from F_{σ_8}, we need to know g_8/g_0, or, equivalently, $g_{\epsilon' NN}/g_{\epsilon NN}$ and the mixing angle. As an example, if ϵ' is made of strange quarks and does not couple to the nucleon, a "canonical" mixing angle $\theta = \tan^{-1}(1/\sqrt{2})$ leads to $F_{\sigma_8}/F_{\sigma_0} = 0.19$. In this case F_{σ_0} is about 120 MeV, comparable to $F_\pi = 95$ MeV. F_{σ_0} is much larger than F_{σ_8} if $g_{\sigma_8 BB}$ is comparable with $g_{\sigma_0 BB}$.

In Sec. IV the relevance of PCAC to scale breaking is studied, in particular, the matrix element $\langle \pi | \mathfrak{F}_\mu{}^5 | \sigma \rangle$, which leads to the relation $F_\pi G_{\sigma\pi\pi} = (m_\pi{}^2 - m_\sigma{}^2) F_1(0)$. The form factor $F_1(0)$ would be unity in the limit of chiral symmetry, but is around $\frac{1}{2}$ according to our best value of $G_{\sigma\pi\pi}$. This relation suggests that $G_{\sigma\pi\pi}$ does not vanish in the limit of chiral symmetry (unless this limit coincides with that of scale invariance, in which case the dilaton mass vanishes).

The form factors F_1 and F_2 appearing in the matrix element $\langle \pi | \mathfrak{F}_\mu{}^5 | \sigma \rangle$ are evaluated in terms of π and A_1 meson parameters. The assumption that the divergence vanishes for $t \to \infty$ determines $F_1(\infty)$. We suggest that $F_1(\infty)$ is zero, in which case the $A_1 \epsilon \pi$ coupling constant is determined. $\Gamma_{A_1 \to \epsilon\pi}$ is predicted to be 30 MeV. Consistency of this determination of $g_{A\epsilon\pi}$ with that of a hard-pion calculation gives the interesting relation $F_\pi = m_\epsilon{}^2 / \gamma_\rho m_A$ in good agreement with experiment. In the approximation $m_\epsilon = m_\rho$ this is in essence the KSRF relation.[19,20]

In Sec. V we discuss our results in the light of various dynamical and structural assumptions about the underlying theory. Throughout the paper we tacitly assume that when interactions that break scale invariance are turned off, scale "invariance" is realized by the existence of a massless Goldstone boson. Only in this way can we justify the mass formulas based on scalar dominance of matrix elements of $\theta^\mu{}_\mu$.

In Appendix A we consider the role of ϵ exchange in πN scattering and find that $g_{\epsilon NN} \approx g_{\epsilon\pi\pi}$. Appendix B estimates the ratio $F_{\sigma_0}/F_{\sigma_8}$ from the baryon-decuplet masses. The result (0.15) is in reasonable agreement with the baryon-octet determination.

II. COUPLINGS OF SCALAR AND PSEUDOSCALAR NONETS

We consider the SU_3-invariant trilinear couplings which can be formed from a nonet of scalar fields σ_0, σ_i and a nonet of pseudoscalar fields ϕ_0, ϕ_i $(i=1,2,\ldots,8)$.

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[11] Particle Data Group, Rev. Mod. Phys. **42**, 87 (1970); **39**, 1 (1967). Appendix A of the latter paper analyzes $\kappa(725)$.

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[13] M. A. Abolins, R. Graven, G. A. Smith, L. H. Smith, A. B. Wicklund, R. L. Lander, and D. E. Pellet, Phys. Rev. Letters **25**, 469 (1970).

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[15] H. A. Kastrup, Nucl. Phys. **B15**, 179 (1970).

[16] L. N. Chang and P. G. O. Freund, Ann. Phys. (N. Y.) **61**, 182 (1970).

[17] R. Crewther and M. Gell-Mann (private communication).

[18] We define F_π and F_σ by the following formulas: $\langle 0 | \mathfrak{F}_{\mu j}{}^5 | \pi_k(p) \rangle (2\omega)^{1/2} = i p_\mu F_\pi \delta_{jk}$; $\langle 0 | \theta_{\mu\nu} | \sigma(p) \rangle (2\omega)^{1/2} = \frac{1}{3}(g_{\mu\nu} p^2 - p_\mu p_\nu) F_\sigma$. We use a metric $g_{\mu\nu}$ such that $x^2 = x_0{}^2 - \mathbf{x}^2$. Numerically F_π is about 95 MeV and the PCAC relation is $\partial^\mu \mathfrak{F}_{\mu i}{}^5 = F_\pi m_\pi{}^2 \pi_i$.

[19] K. Kawarabayashi and M. Suzuki, Phys. Rev. Letters **16**, 255 (1966).

[20] Riazuddin and Fayyazuddin, Phys. Rev. **147**, 1071 (1966).

These are conveniently written as

$$\mathfrak{L}_{SPP}=g_0\sigma_0\phi_0^2+g_1\sigma_0\phi_i^2+\sqrt{3}g_2d_{ijk}\sigma_i\phi_j\phi_k+g_3\phi_0\sigma_i\phi_i\,, \tag{2.1}$$

$$\mathfrak{L}_{SSS}=h_0\sigma_0^3+h_1\sigma_0\sigma_i^2+\sqrt{3}h_2d_{ijk}\sigma_i\sigma_j\sigma_k\,. \tag{2.2}$$

SU_3-symmetry breaking is taken into account to the extent that σ_0-σ_8 and ϕ_0-ϕ_8 mixing occurs with mixing angles θ and ϕ:

$$\begin{aligned}\epsilon&=\sigma_0\cos\theta-\sigma_8\sin\theta\,,\\ \epsilon'&=\sigma_0\sin\theta+\sigma_8\cos\theta\,,\end{aligned} \tag{2.3}$$

$$\begin{aligned}\eta&=\phi_8\cos\phi-\phi_0\sin\phi\,,\\ \eta'&=\phi_8\sin\phi+\phi_0\cos\phi\,.\end{aligned} \tag{2.4}$$

The notation adopted above was chosen for convenience.

It is useful to rewrite Eqs. (2.1) and (2.2) in terms of the constituent isospin couplings. This yields the following effective Lagrangians:

$$\begin{aligned}\mathfrak{L}_{SPP}=&g_{\epsilon'\eta'\eta'}\epsilon'(\eta')^2+g_{\epsilon'\eta\eta}\epsilon'\eta^2+g_{\epsilon'\eta\eta'}\epsilon'\eta\eta'+g_{\epsilon\eta'\eta'}\epsilon(\eta')^2\\ &+g_{\epsilon\eta\eta}\epsilon\eta^2+g_{\epsilon\eta\eta'}\epsilon\eta\eta'+g_{\pi_N\eta'\pi}\eta'\boldsymbol{\pi}_N\cdot\boldsymbol{\pi}+g_{\pi_N\eta\pi}\eta\boldsymbol{\pi}_N\cdot\boldsymbol{\pi}+g_{\pi_NKK}\boldsymbol{\pi}_N\cdot\bar{K}\boldsymbol{\tau}K+g_{\pi K\kappa}(\boldsymbol{\pi}\cdot\bar{K}\boldsymbol{\tau}\kappa+\text{H.c.})\\ &+g_{K\kappa\eta'}(\eta'\bar{K}\kappa+\text{H.C.})+g_{K\kappa\eta}(\eta\bar{K}\kappa+\text{H.c.})+g_{\epsilon\pi\pi}\epsilon\pi^2+g_{\epsilon'\pi\pi}\epsilon'\pi^2+g_{\epsilon KK}\bar{K}K+g_{\epsilon'KK}\epsilon'\bar{K}K\,,\end{aligned} \tag{2.5}$$

$$\mathfrak{L}_{SSS}=g_{\epsilon'\epsilon'\epsilon'}(\epsilon')^3+g_{\epsilon'\epsilon'\epsilon}(\epsilon')^2\epsilon+g_{\epsilon'\epsilon\epsilon}\epsilon'\epsilon^2+g_{\epsilon\epsilon\epsilon}\epsilon^3+g_{\epsilon'\pi_N\pi_N}\epsilon'\pi_N^2+g_{\epsilon'\kappa\kappa}\epsilon'\bar{\kappa}\kappa+g_{\epsilon\pi_N\pi_N}\epsilon\pi_N^2+g_{\epsilon\kappa\kappa}\epsilon\bar{\kappa}\kappa+g_{\pi_N\kappa\kappa}\boldsymbol{\pi}_N\bar{\kappa}\boldsymbol{\tau}\kappa\,. \tag{2.6}$$

The coupling constants are given by the following expressions:

$$\begin{aligned}
g_{\epsilon\eta'\eta'}&=g_0\cos^2\phi\cos\theta+\sin^2\phi\,(g_1\cos\theta+g_2\sin\theta)-g_3\sin\theta\cos\phi\sin\phi\,,\\
g_{\epsilon\eta\eta}&=g_0\sin^2\phi\cos\theta+\cos^2\phi\,(g_1\cos\theta+g_2\sin\theta)+g_3\sin\theta\cos\phi\sin\phi\,,\\
g_{\epsilon\eta\eta'}&=\sin2\phi\,(-g_0\cos\theta+g_1\cos\theta+g_2\sin\theta)-g_3\sin\theta\cos2\phi\,,\\
g_{\epsilon'\eta\eta'}&=\sin2\phi\,(-g_0\sin\theta+g_1\sin\theta-g_2\cos\theta)+g_3\cos\theta\cos2\phi\,,\\
g_{\epsilon'\eta'\eta'}&=g_0\cos^2\phi\sin\theta+\sin^2\phi\,(g_1\sin\theta-g_2\cos\theta)+g_3\cos\theta\cos\phi\sin\phi\,,\\
g_{\epsilon'\eta\eta}&=g_0\sin^2\phi\sin\theta+\cos^2\phi\,(g_1\sin\theta-g_2\cos\theta)-g_3\cos\theta\cos\phi\sin\phi\,,\\
g_{\pi_N\eta'\pi}&=2g_2\sin\phi+g_3\cos\phi,\quad g_{\pi_N\eta\pi}=2g_2\cos\phi-g_3\sin\phi,\quad g_{\pi_NKK}=\sqrt{3}g_2\,,\\
g_{\kappa K\pi}&=\sqrt{3}g_2,\quad g_{\kappa K\eta'}=-g_2\sin\phi+g_3\cos\phi,\quad g_{\kappa K\eta}=-g_2\cos\phi-g_3\sin\phi\,,\\
g_{\epsilon'\pi\pi}&=g_1\sin\theta+g_2\cos\theta,\quad g_{\epsilon'KK}=2g_1\sin\theta-g_2\cos\theta\,,\\
g_{\epsilon\pi\pi}&=g_1\cos\theta-g_2\sin\theta,\quad g_{\epsilon KK}=2g_1\cos\theta+g_2\sin\theta\,,\\
g_{\epsilon\epsilon\epsilon}&=h_0\cos^3\theta+h_1\cos\theta\sin^2\theta+h_2\sin^3\theta\,,\\
g_{\epsilon'\epsilon'\epsilon'}&=h_0\sin^3\theta+h_1\sin\theta\cos^2\theta-h_2\cos^3\theta\,,\\
g_{\epsilon'\epsilon\epsilon}&=3h_0\cos^2\theta\sin\theta+h_1(1-3\cos^2\theta)\sin\theta-3h_2\sin^2\theta\cos\theta\,,\\
g_{\epsilon'\epsilon'\epsilon}&=3h_0\sin^2\theta\cos\theta+h_1(1-3\sin^2\theta)\cos\theta+3h_2\cos^2\theta\sin\theta\,,\\
g_{\epsilon\pi_N\pi_N}&=h_1\cos\theta-3h_2\sin\theta,\quad g_{\epsilon'\pi_N\pi_N}=h_1\sin\theta+3h_2\cos\theta\,,\\
g_{\epsilon\kappa\kappa}&=2h_1\cos\theta+3h_2\sin\theta,\quad g_{\epsilon'\kappa\kappa}=2h_1\sin\theta-3h_2\cos\theta,\quad g_{\pi_N\kappa\kappa}=3\sqrt{3}h_2\,.
\end{aligned} \tag{2.7}$$

We have expressed 25 coupling constants in terms of seven constants g_i, h_i and two mixing angles θ, ϕ. Unfortunately it is very difficult to obtain experimental information about most of these constants. The success of the Gell-Mann–Okubo mass formula for the pseudoscalar octet indicates[1] that the η-η' mixing angle ϕ is small, say, 8°. Therefore to analyze the properties of the scalars we shall simplify the relations (2.7) by setting $\cos^2\phi=1$, $\sin^2\phi=0$.

The most direct information bearing on the couplings comes from the decays of the (presumed) scalar nonet into pseudoscalars. Elementary calculations give the following decay widths:

$$\begin{aligned}
\Gamma_{\epsilon\to\pi\pi}&=\frac{g_{\epsilon\pi\pi}^2}{4\pi}\frac{3p_\pi}{m_\epsilon^2}, & \Gamma_{\epsilon'\to\pi\pi}&=\frac{g_{\epsilon'\pi\pi}^2}{4\pi}\frac{3p_\pi}{m_{\epsilon'}^2},\\
\Gamma_{\epsilon'\to K\bar{K}}&=\frac{g_{\epsilon'KK}^2}{4\pi}\frac{p_K}{m_{\epsilon'}^2}, & \Gamma_{\pi_N\to\pi\eta}&=\frac{g_{\pi_N\pi\eta}^2}{4\pi}\frac{p_\pi}{2m_{\pi_N}^2},\\
\Gamma_{\kappa\to K\pi}&=\frac{g_{\kappa K\pi}^2}{4\pi}\frac{3p_\pi}{2m_\kappa^2}, & \Gamma_{\kappa\to K\eta}&=\frac{g_{\kappa K\eta}^2}{4\pi}\frac{p_K}{2m_\kappa^2}.
\end{aligned} \tag{2.8}$$

Since the coupling constants are dimensional, it is convenient to extract a characteristic mass m_0, $g_{SPP}=m_0\bar{g}_{SPP}$, and to parametrize the coupling strength by the dimensionless quantity

$$\gamma_{SPP}\equiv g_{SPP}^2/4\pi m_0^2. \tag{2.9}$$

We choose m_0 to be 1 GeV.

In order to obtain some information about the coupling strengths, we assume the following parameters. Further clarification of the experimental situation could lead to drastic revision of these values. For ϵ we assume mass 700 MeV and width 320 MeV. The decay modes of ϵ' (1060) into $\pi\pi$, $K\bar{K}$ are imprecisely known ($\Gamma_{\text{tot}}=80$ MeV, $\Gamma_{\pi\pi}<65$ MeV, $\Gamma_{K\bar{K}}>35$ MeV) and we use $\Gamma_{2\pi}=\frac{1}{2}\Gamma_{K\bar{K}}=27$ MeV to get a rough estimate. We assume the decay width of $\pi_N(980)$ into $\pi\eta$ to be 50 MeV. (The κ is controversial, so we do not use it as input.) Using these parameters, we find

$$\begin{aligned}&\gamma_{\epsilon\pi\pi}=0.17,\ \gamma_{\epsilon'\pi\pi}=0.020,\\&\gamma_{\epsilon'KK}=0.32,\quad \gamma_{\pi_N\pi\eta}=0.31.\end{aligned} \tag{2.10}$$

The strengths of the singlet-octet-octet coupling ($\gamma_1\equiv g_1^2/4\pi m_0^2$) and the octet-octet-octet coupling ($\gamma_2\equiv g_2^2/4\pi m_0^2$) are found, using the relations

$$\gamma_{\pi_N\pi\eta}=4\gamma_2,\quad \gamma_{\epsilon\pi\pi}+\gamma_{\epsilon'\pi\pi}=\gamma_1+\gamma_2, \tag{2.11}$$

to be comparable:

$$\gamma_1=0.112,\quad \gamma_2=0.078. \tag{2.12}$$

Having chosen θ positive, the relative sign of g_2 and g_1 has to be negative since the ratio

$$r_1\equiv\frac{\gamma_{\epsilon'\pi\pi}}{\gamma_{\epsilon'KK}}=\frac{1}{4}\left(\frac{\tan\theta+g_2/g_1}{\tan\theta-g_2/2g_1}\right)^2 \tag{2.13}$$

is much less than unity (0.062). Our value $g_2=-0.83g_1$ may be compared with the $SU_3\times SU_3$-symmetry relation $g_2=-\sqrt{2}g_1$. When $\gamma_{\epsilon'\pi\pi}$ vanishes, a canonical mixing angle goes with the chiral symmetry relation $g_2/g_1=-\sqrt{2}$. Solving (2.13) gives two solutions: $\theta=22°40'$ and $\theta=63°$. The latter may be rejected by using the prediction

$$r_2\equiv\frac{\gamma_{\epsilon'KK}}{\gamma_{\epsilon\pi\pi}}=4\left[\frac{\tan\theta-(g_2/2g_1)}{1-(g_2/g_1)\tan\theta}\right]^2. \tag{2.14}$$

According to (2.10) this ratio is 1.89, while the values 22°40′ and 63° found from Eq. (2.13) predict $r_2=1.52$ and 3.24, respectively. Both (2.13) and (2.14) are compatible with $\theta\cong 23°$. Canonical mixing can be obtained only if we have overestimated $\Gamma_{\epsilon\to 2\pi}$ and underestimated $\Gamma_{\pi_N\to\pi\eta}$ by substantial amounts. From the foregoing parameters, we predict $m_\kappa=1$ GeV and $\Gamma_{\kappa\to K\pi}=140$ MeV.

Next suppose there is a third isoscalar meson S which mixes with σ_0, σ_8. (An explicit model of this sort was considered in Ref. 4.) Since the mixed fields ϵ_i ($i=1, 2, 3$) may be chosen real, the transformation from the unmixed basis is exactly the same as the rotation of a vector

$$\epsilon_i=[\exp(-i\theta\hat{n}\cdot\mathbf{t})]_{ij}\epsilon_j^{(0)},$$

where $\boldsymbol{\varepsilon}^0=(\sigma_0,\sigma_8,S)$ and $(t_i)_{jk}=-i\epsilon_{ijk}$. The axis of rotation is identified by two mixing angles ξ_1 and ξ_2,

$$\hat{n}=(\sin\xi_1\cos\xi_2,\ \sin\xi_1\sin\xi_2,\ \cos\xi_1).$$

The case of σ_0-σ_8 mixing corresponds to a rotation around the $z(s)$ axis. In detail, the general mixing transformation is

$$\begin{pmatrix}\epsilon_1\\\epsilon_2\\\epsilon_3\end{pmatrix}=\begin{pmatrix}\cos\theta+n_1^2(1-\sin\theta) & -n_3\sin\theta+n_1n_2(1-\cos\theta) & n_2\sin\theta+n_1n_3(1-\cos\theta)\\ n_3\sin\theta+n_1n_2(1-\cos\theta) & \cos\theta+n_2^2(1-\cos\theta) & -n_1\sin\theta+n_2n_3(1-\cos\theta)\\ -n_2\sin\theta+n_1n_3(1-\cos\theta) & n_1\sin\theta+n_2n_3(1-\cos\theta) & \cos\theta+n_3^2(1-\cos\theta)\end{pmatrix}\begin{pmatrix}\sigma_0\\\sigma_8\\S\end{pmatrix}. \tag{2.15}$$

The angles can be related to the matrix elements m_{ij}^2 of the unmixed basis in a familiar way.

III. HADRON MASSES AND TRACE OF ENERGY-MOMENTUM TENSOR

The energy-momentum tensor $\theta_{\mu\nu}$ occupies a key position in elementary particle physics because of its familiar role in the construction of the Poincaré generators. In addition, the dilatation and special conformal operators are expressible as simple moments of $\theta_{\mu\nu}$ in renormalizable field theories. In this manner the violation of scale invariance and conformal invariance is directly related to the trace $\theta=\theta_\mu{}^\mu$ by the local relations $\partial^\mu D_\mu=\theta$ and $\partial^\mu K_{\mu\nu}=2x_\nu\theta$, where D_μ is the dilatation current and $K_{\mu\nu}$ the conformal current. A review of the current status of the subject is given in Ref. 21. It is interesting to explore the consequences of assuming that the violation of scale invariance is gentle in the sense that the single-particle matrix elements of θ vanish as the momentum transfer $t\to\infty$ and that the dispersion relation is dominated by a set of scalar mesons. These assumptions lead to mass formulas analogous to the usual Goldberger-Treiman relations when one recalls that the matrix elements of θ coincide with those of the energy density θ_{00} for single-particle rest states. In this section we analyze more closely the formulas derived in Ref. 4.

The main questions of interest are as follows. (1) Is it reasonable to set $\theta(\infty)$ equal to zero? Only then can the masses be simply expressed in terms of scalar-meson parameters. (2) How many scalar mesons are there? (3) Do the scalar mesons dominate the dispersion relation, even when $\theta(\infty)=0$?

[21] P. Carruthers, Phys. Repts. (to be published).

The assumption that $\theta \to 0$ as $t \to \infty$ is indeed reasonable, although not true in simple perturbation-theory models. However, such models ignore the key feature of hadron physics, the large multiparticle amplitudes (which through unitarity tend to suppress a given matrix element). Broken SU_3 requires at least two scalars if the subsequent mass formulas are correct. With regard to question (3), it seems likely that the scalar-dominance hypothesis is reasonable for baryons but possibly wrong for the pseudoscalar octet. Therefore we first derive the various parameters of the theory from baryon data. In Sec. IV we take up the question of symmetry breaking for the pseudoscalars.

For each scalar meson σ, we define a constant F_σ (∞ mass) by

$$\langle 0|\theta_{\mu\nu}|\sigma(p)\rangle(2\omega)^{1/2}=\tfrac{1}{3}F_\sigma(g_{\mu\nu}p^2-p_\mu p_\nu) \quad (3.1)$$

so that $\langle 0|\theta_\mu{}^\mu|\sigma\rangle(2\omega)^{1/2}$ is $m_\sigma{}^2F_\sigma$. For spin-zero mesons m, we define a σmm coupling constant by

$$G_{\sigma mm}=(4\omega_\sigma\omega_m)^{1/2}\langle\sigma|j_m{}^\dagger|m\rangle \quad (3.2)$$

with all particles on-shell. $j_m=(m^2+\partial^2)\phi_m$ is the meson current, so that $G_{\sigma mm}$ will in general differ from the constants $g_{\sigma mm}$ by a constant depending on the number of times the field ϕ_m occurs in the effective couplings (2.1) and (2.2). Writing $\theta(t)=(4\omega\omega')^{1/2}\langle p'|\theta(0)|p\rangle$, we find

$$\theta(t)=\sum_\sigma \frac{m_\sigma{}^2F_\sigma G_{\sigma mm}}{m_\sigma{}^2-t}+\text{background}. \quad (3.3)$$

Dropping the background term and setting $t=0$ gives

$$2m^2=\sum_\sigma F_\sigma G_{\sigma mm}, \quad (3.4)$$

and a similar analysis for spin-$\frac{1}{2}$ baryons gives

$$M_B=\sum_\sigma F_\sigma g_{\sigma BB}. \quad (3.5)$$

Since $G_{\sigma mm}$ has dimension (mass) while $g_{\sigma BB}$ is dimensionless, a further analysis is required to establish that the spinless mesons obey quadratic mass formulas. We shall return to this question in Sec. V.

For the baryon octet, Eq. (3.5) has the unmixed form

$$M_B=F_{\sigma_0}g_{\sigma_0BB}+F_{\sigma_8}g_{\sigma_8BB}. \quad (3.6)$$

The first term gives the average baryon mass and the second term gives the SU_3 breaking. g_{σ_0BB} is independent of g, while the g_{σ_8BB} are given[22] by

$$\begin{aligned} g_{\sigma_8NN}&=[(3-4\alpha)/\sqrt{3}]g_8,\\ g_{\sigma_8\Xi\Xi}&=-[(3-2\alpha)/\sqrt{3}]g_8,\\ g_{\sigma_8\Lambda\Lambda}&=-(2\alpha/\sqrt{3})g_8,\\ g_{\sigma_8\Sigma\Sigma}&=(2\alpha/\sqrt{3})g_8, \end{aligned} \quad (3.7)$$

where g_8 is the π_NNN coupling constant and α is defined by the effective scalar octet-baryon octet coupling

$$\mathcal{L}_{SBB}=2g_8\bar{B}_i[\alpha d_{ijk}+i(1-\alpha)f_{ijk}]B_kS_j. \quad (3.8)$$

We note that the Gell-Mann–Okubo mass formula is automatically satisfied. Combining (3.6) with (3.7), we find ($g_0\equiv g_{\sigma_0BB}$)

$$\begin{aligned} m_\Sigma-m_\Lambda&=(4\alpha/\sqrt{3})F_{\sigma_8}g_8,\\ m_\Sigma+m_\Lambda&=2F_{\sigma_0}g_0. \end{aligned} \quad (3.9)$$

The mixing parameter α is easily estimated using the observed masses of the baryon octet. One finds

$$\frac{m_\Sigma-m_\Lambda}{m_\Xi-m_N}=-\frac{2}{3}\frac{\alpha}{1-\alpha}=0.132. \quad (3.10)$$

Equation (3.10) gives a d/f ratio of -0.198, or $\alpha=-0.25$. Now Eq. (3.9) gives

$$F_{\sigma_8}g_8/F_{\sigma_0}g_0=-0.117. \quad (3.11)$$

In order to evaluate $F_{\sigma_8}/F_{\sigma_0}$, one needs information about the coupling constants g_8 and g_0.

We note that g_8 is the π_NNN coupling constant defined by $\mathcal{L}=g_8\boldsymbol{\pi}_N\cdot\bar{N}\boldsymbol{\tau}N$. Knowledge of the d/f ratio then allows one to predict the pure octet scalar (π_N and κ) couplings to the baryon octet. The ϵ, ϵ' couplings require in addition a knowledge of both g_0 and the mixing angle.

The mass formula predicts the d/f ratio for the coupling of the unmixed scalar octet to the baryon octet. This can be checked by an eventual determination of some of the π_NBB and κBB couplings. The mixing angle θ is perhaps most easily determined from the scalar decay strengths as in Sec. II. The coupling constant g_0 could be obtained from $g_{\epsilon NN}{}^2+g_{\epsilon'NN}{}^2=g_{\sigma_8NN}{}^2+g_{\sigma_0NN}{}^2$ provided $g_{\epsilon NN}$ and $g_{\epsilon'NN}$ can be found.

In Sec. II we found that the $\epsilon'\pi\pi$ coupling is suppressed, as in the picture wherein ϵ' is composed of two strange quarks. It is reasonable and illustrative to see what this model predicts for scalar-baryon couplings, since $g_{\epsilon'NN}=0$ allows some predictions. Supposing the mixing to be canonical (even though our rough estimates of Sec. II gave $\theta=23°$) gives $g_{\sigma_8NN}/g_{\sigma_0NN}=-1/\sqrt{2}$, or $g_0/g_8=-2(3-4\alpha)/\sqrt{3}=-1.64$, and

$$F_{\sigma_8}/F_{\sigma_0}=0.19. \quad (3.12)$$

In Appendix B the baryon decuplet is shown to give the value 0.15. This prediction is quite reasonable in view of the approximate SU_3 invariance of the vacuum. A reliable determination of the ratio $F_{\sigma_8}/F_{\sigma_0}$ would provide an important clue to low-energy hadron dynamics.

Eliminating $F_{\sigma_8}g_{\sigma_8NN}$ from Eq. (3.6) using Eqs. (3.7) and (3.11) gives

$$\begin{aligned} F_{\sigma_0}g_0&=1.15\ m_N=1.08\ \text{GeV},\\ F_{\sigma_8}g_8&=-0.126\ \text{GeV}, \end{aligned} \quad (3.13)$$

[22] P. Carruthers, *Introduction to Unitary Symmetry* (Interscience, New York, 1966), p. 118.

Given the corrections of the nonet assumption, an experimental determination of g_0 and g_8 will determine F_{σ_0} and F_{σ_8}. It is interesting to explore the consequences of the quark-motivated relation $g_{\epsilon' NN}=0$, since we have some information on $g_{\epsilon NN}$ from πN and NN scattering (see Appendix A): $g_{\sigma_0}=(\sqrt{\frac{2}{3}})g_{\epsilon NN}\approx 9$ for $g_{\epsilon NN}^2/4\pi\approx 10$ and, hence,

$$F_{\sigma_0}\cong 120 \text{ MeV}, \quad F_{\sigma_8}\cong 23 \text{ MeV}. \tag{3.14}$$

It is interesting and suggestive that F_{σ_0} is roughly equal to F_π (=95 MeV).

A similar analysis of the mass formula (3.4) for the pseudoscalar octet leads to difficulties; in particular, that $F_{\sigma_0}<F_{\sigma_8}$. In detail, the unsubtracted mass formulas for the pseudoscalar and scalar nonets are

$$\begin{aligned} m_\pi^2&=F_\epsilon g_{\epsilon\pi\pi}+F_{\epsilon'}g_{\epsilon'\pi\pi},\\ m_\eta^2&=F_\epsilon g_{\epsilon\eta\eta}+F_{\epsilon'}g_{\epsilon'\eta\eta},\\ 2m_K^2&=F_\epsilon g_{\epsilon KK}+F_{\epsilon'}g_{\epsilon' KK},\\ m_{\eta'}^2&=F_\epsilon g_{\epsilon\eta'\eta'}+F_{\epsilon'}g_{\epsilon'\eta'\eta'},\\ m_{\pi N}^2&=F_\epsilon g_{\epsilon\pi_N\pi_N}+F_{\epsilon'}g_{\epsilon'\pi_N\pi_N},\\ m_\epsilon^2&=3F_\epsilon g_{\epsilon\epsilon\epsilon}+F_{\epsilon'}g_{\epsilon'\epsilon\epsilon},\\ 2m_\kappa^2&=F_\epsilon g_{\epsilon\kappa\kappa}+F_{\epsilon'}g_{\epsilon'\kappa\kappa},\\ m_{\epsilon'}^2&=F_\epsilon g_{\epsilon\epsilon'\epsilon'}+3F_{\epsilon'}g_{\epsilon'\epsilon'\epsilon'}. \end{aligned} \tag{3.15}$$

Suppose we set $m_\pi^2=0$, which corresponds to the energy density having $SU_2\times SU_2$ symmetry. Eliminating F_ϵ then gives for the pseudoscalar mesons (ignoring η-η' mixing)

$$\begin{aligned} m_\eta^2&=2g_1g_2F_{\epsilon'}/g_{\epsilon\pi\pi},\\ 2m_K^2&=3g_1g_2F_{\epsilon'}/g_{\epsilon\pi\pi},\\ m^2_{\eta'}&=-g_0g_2F_{\epsilon'}/g_{\epsilon\pi\pi}. \end{aligned} \tag{3.16}$$

The ratio $m_\eta^2/m_K^2=\frac{4}{3}$ is exactly what is expected from the Gell-Mann–Okubo mass formula. (Our mixing assumptions are essentially equivalent to using lowest-order perturbation theory.) It is to be noted that terms involving g_1^2 and g_2^2 cancel in arriving at (3.7). We also note that the coupling constant g_0 is given by

$$g_0=-2g_1m_{\eta'}^2/m_\eta^2. \tag{3.17}$$

This evaluation of g_0 may be compared with g_1,g_2 found in Sec. II:

$$g_0:g_1:g_2=-6.1:1:-0.83. \tag{3.18}$$

It is interesting to note that if particles σ_i, ϕ_i $(i=0,\ldots,8)$ belonged to a $(3,\bar{3})+(\bar{3},3)$ representation, the $SU_3\times SU_3$-*invariant* coupling would give the ratios $-2:1:-\sqrt{2}$ in place of (3.18). The result (3.18) seems to show a remnant of $SU_3\times SU_3$ symmetry.

Although relations (3.15) are true in some models,[23] they lead to unreasonable numerical results when applied to the real world, as already noticed in Ref. 4. Rewriting the pion mass formula in the unmixed basis gives $m_\pi^2=F_{\sigma_0}g_1+F_{\sigma_8}g_2$, where g_1 and g_2 are $\sim$1 GeV. Neglecting m_π then gives roughly $F_{\sigma_8}/F_{\sigma_0}\cong -g_1/g_2=1.2$. This result, exact in the $SU_3\times SU_3$ limit, contradicts the approximate SU_3 invariance of the vacuum[24] as well as the rough calculation leading to (3.12).

In Ref. 4 it was suggested that the above discrepancy be resolved by the introduction of another scalar meson. Here we follow a more conservative approach in which the scalar-dominated mass formulas are abandoned for the pseudoscalar octet. This attitude is useful since one can make many more predictions if the extra meson does not exist.

IV. PCAC AND BREAKING OF SCALE INVARIANCE

The success of PCAC is generally regarded as due to the smallness of m_π relative to other hadron masses. In the limit of a conserved axial-vector current ($m_\pi=0$), one has the *exact* formulas

$$\begin{aligned} F_\pi g_{\pi NN}&=-MF_1^{NN}(0),\\ F_\pi G_{\sigma\pi\pi}&=-m_\sigma^2F_1^{\pi\sigma}(0), \end{aligned} \tag{4.1}$$

where $F_1^{NN}(0)=G_A/G_V=1.18$ and the form factor $F_1^{\pi\sigma}(0)$, studied in detail below, is of order unity. According to (4.1), $G_{\sigma\pi\pi}$ remains large in the limit of $SU_3\times SU_3$ symmetry, unless that limit also corresponds to the scale-invariant limit, in which case σ is the massless dilaton. The relevance of the second PCAC relation in Eq. (4.1) has been pointed out by Crewther and Gell-Mann.[17]

The PCDC (partial conservation of dilatation current) hypothesis does not have such a reasonable foundation since the lightest candidate for the "massive" dilaton has $m_\epsilon^2\approx 0.5$ GeV². In the limit of exact scale invariance we would have, analogous to (4.1),

$$F_\sigma G_{\sigma mm}=2m^2, \quad F_\sigma g_{\sigma MM}=M, \tag{4.2}$$

for spin-0 and spin-$\frac{1}{2}$ particles. (Here we have assumed only one dilaton, as seems dynamically most reasonable.) Apart from mixing effects, it is reasonable to suppose that F_σ is not changed appreciably as explicit scale-invariance-breaking interactions are turned on. However, Eq. (4.1) indicates that $G_{\sigma\pi\pi}$ changes drastically; although other contributions now occur [see Eq. (3.4) or (3.15)], it seems unlikely that they will compensate. [Neglecting mixing gives $2m_\pi^2\approx -(F_\sigma/F_\pi)m_\sigma^2F_1^{\pi\pi}(0)\approx O(m_\sigma^2)$.]

Hence it seems that PCDC must be abandoned for the pseudoscalar octet. The mass formulas for baryons and heavy mesons ($\sim$1 GeV or greater) may be trustworthy, although not with the same accuracy as pion PCAC relations. If ϵ(700) is the main remnant of the dilaton in the real world, we can organize the low-lying

[23] In a purely mesonic model in which scale breaking is due to a linear term $\sigma_0+c\sigma_8$ alone, Eqs. (3.15) are satisfied. The 18 meson fields σ_i, ϕ_i are put in the representation $(3,\bar{3})+(\bar{3},3)$ and interact via quartic couplings in the scale-invariant limit.

[24] M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. **175**, 2195 (1968).

particles according to the following pattern: (a) $m^2<m_\epsilon^2$ (pseudoscalar octet), (b) $m^2\approx m_\epsilon^2$ (ρ meson), (c) $m^2>m_\epsilon^2$ (baryons, scalar octet, pseudoscalar singlet). It is a moot point whether even the mass relations in category (c) are reliable. Further work is needed to learn how to calculate scale-invariance-breaking corrections. Nevertheless, the picture is consistent if we assume the validity of the scalar-dominated mass formulas for category (c). For particles in category (b) corrections are expected to be substantial but not so drastic as for the pseudoscalar octet.

It is interesting to combine the CAC and CDC relations for the nucleon to obtain

$$|F_\sigma/F_\pi|=g_{\pi NN}/G_Ag_{\sigma NN}\approx 1 \tag{4.3}$$

using the physical values of the couplings. This calculation ignores mixing but supports the analysis of Sec. III, which found $F_{\sigma_0}\approx F_\pi$.

In order to study the implications of PCAC for SPP couplings, we define form factors F_1 and F_2 by the relation

$$\begin{aligned}(4\omega\omega')^{1/2}\langle P_i(p')|\mathfrak{F}_{\mu j}{}^5|S_k(p)\rangle\\ =-i[(p'+p)_\mu F_{1ijk}(t)+(p'-p)_\mu F_{2ijk}(t)],\end{aligned} \tag{4.4}$$

where $\mathfrak{F}_{\mu j}{}^5$ is the usual octet axial-vector current while i and k label pseudoscalar and scalar states p and S. We write the PCAC assumption in the form $\partial^\mu\mathfrak{F}_{\mu i}{}^5=F_im_i{}^2\phi_i$ and obtain from (4.4) the relation

$$F_jG_{P_iP_jS_k}(0)=[m^2(P_i)-m^2(S_k)]F_{1ijk}(0). \tag{4.5}$$

$G_{p_ip_jS_k}(0)$ is not quite the physical coupling constant[25]; for pions the correction is expected to be small.

Thus far the indices i, j, and k are only labels with no group-theoretical significance. However, the normalization has been chosen so that for free nonet fields (σ_i,ϕ_i) in $(3,\bar{3})+(\bar{3},3)$, for which $\mathfrak{F}_{\mu i}{}^5=d_{ijk}(\phi_j\partial_\mu\sigma_k-\sigma_j\partial_\mu\phi_k)$, the form factor F_1 is unity. The sign is chosen to conform to the usual convention[26] $[F_i{}^5,\sigma_j]=-id_{ijk}\phi_k$, etc.

If the extrapolation from $t=0$ to $m_p{}^2$ is ignored in the coupling constant, we can use (4.5) to estimate $F_1(0)$. For the π and K, we then find

$$\begin{aligned}F_1{}^{\pi\epsilon}(0)&=-\frac{2F_\pi g_{\epsilon\pi\pi}}{m_\epsilon{}^2-m_\pi{}^2},\\ F_1{}^{K\epsilon}(0)&=\frac{F_Kg_{\epsilon KK}}{m_\epsilon{}^2-m_K{}^2},\end{aligned} \tag{4.6}$$

and similar relations for ϵ'. We can use the analysis of Sec. II to evaluate the coupling constants, finding for the absolute values

$$\begin{aligned}F_1{}^{\pi\epsilon}(0)=0.52,\quad F_1{}^{\pi\epsilon'}(0)=0.08,\\ F_1{}^{K\epsilon}(0)=0.43,\quad F_1{}^{K\epsilon'}(0)=0.20.\end{aligned} \tag{4.7}$$

It will be noted that as one goes away from the $SU_3\times SU_3$ limit ($m_\pi=m_K=0$), $F_1{}^{K\epsilon}(0)$ is much more sensitive to the purely kinematical corrections than is $F_1{}^{\pi\epsilon}(0)$. The evaluation of $F_1{}^{\pi\epsilon}(0)$ is clearly more reliable.

The contribution of ϵ and ϵ' to the Adler sum rule[27] for π-π scattering may be written as

$$1=|F_1{}^{\pi\epsilon}(0)|^2+|F_1{}^{\pi\epsilon'}(0)|^2+\cdots, \tag{4.8}$$

showing that ϵ and ϵ' contribute 0.26 and 0.01 of the total, respectively, according to this evaluation. This is somewhat smaller than found in Ref. 27, and could indicate a broader ϵ, or another scalar.

We now consider the dispersion theory of the form factors F_1 and F_2. (This is simple because the $\pi\sigma$ channel is dominated by π and A_1.) This allows one to estimate $F_1(0)-F_1(m_\sigma{}^2)$ in terms of parameters of the A_1 meson. In the annihilation channel we write

$$\begin{aligned}\langle 0|\mathfrak{F}_{\mu i}{}^5|\pi_i(\bar{p}')\sigma(p)\text{in}\rangle(4\omega\bar{\omega}')^{1/2}\\ =-i[(-\bar{p}'+p)_\mu F_1-(\bar{p}'+p)_\mu F_2].\end{aligned} \tag{4.9}$$

The discontinuity of this amplitude is given by

$$\begin{aligned}-i[(-\bar{p}'+p)_\mu\,\mathrm{Im}F_1-(\bar{p}'+p)_\mu\,\mathrm{Im}F_2]\\ =\tfrac{1}{2}(2\pi)^4\sum_n\delta(p+\bar{p}'-p_n)\\ \times\langle 0|\mathfrak{F}_{\mu i}{}^5|n\rangle\langle n|j_{\pi i}|\sigma\rangle(2\omega)^{1/2}.\end{aligned} \tag{4.10}$$

The pion contribution is

$$\begin{aligned}\mathrm{Im}F_1{}^\pi&=0,\\ \mathrm{Im}F_2{}^\pi&=\pi\delta(t-m_\pi{}^2)F_\pi G_{\sigma\pi\pi}.\end{aligned} \tag{4.11}$$

In order to give the A_1 contribution, we make the definitions

$$\begin{aligned}\mathfrak{L}_{A_1\sigma\pi}=g_{A\sigma\pi}\sigma\mathbf{A}^\mu\cdot\partial_\mu\boldsymbol{\pi},\\ \langle 0|\mathfrak{F}_{\mu i}{}^5|A_j(p,\lambda)\rangle(2E_A)^{1/2}=g_Ae_\mu(p,\lambda)\delta_{ij},\end{aligned} \tag{4.12}$$

where e_μ is the spin-one wave function. Now Eq. (4.10) gives

$$\begin{aligned}\mathrm{Im}F_1{}^A&=\pi\delta(t-m_A{}^2)\tfrac{1}{2}g_Ag_{A\sigma\pi},\\ \mathrm{Im}F_2{}^A&=\pi\delta(t-m_A{}^2)g_Ag_{A\sigma\pi}(m_\sigma{}^2-m_\pi{}^2)/2m_A{}^2.\end{aligned} \tag{4.13}$$

The π and A_1 pole contribution to the form factors is exhibited by the equations

$$F_1(t)=\frac{\frac{1}{2}g_Ag_{A\sigma\pi}}{m_A{}^2-t}+F_1(\infty), \tag{4.14}$$

$$F_2(t)=\frac{F_\pi G_{\sigma\pi\pi}}{m_\pi{}^2-t}+\frac{g_Ag_{A\sigma\pi}}{m_A{}^2-t}\left(\frac{m_\sigma{}^2-m_\pi{}^2}{2m_A{}^2}\right). \tag{4.15}$$

One conventionally assumes that F_1 requires a subtraction but that F_2 does not.

In order to agree with PCAC, one also assumes that the form factor of $\partial^\mu\mathfrak{F}_{\mu i}{}^5$, $(m_\pi{}^2-m_\sigma{}^2)F_1+tF_2\equiv H(t)$,

[25] G is defined as in Eq. (3.2).

[26] M. Gell-Mann, Physics 1, 63 (1964).

[27] S. Adler, Phys. Rev. 140, B736 (1965).

obeys an unsubtracted dispersion relation dominated by the pion pole. Requiring that $H(\infty)$ vanish then determines $F_1(\infty)$:

$$F_1(\infty)=\frac{F_\pi G_{\sigma\pi\pi}}{m_\pi^2-m_\sigma^2}-\frac{g_A g_{A\sigma\pi}}{2m_A^2}. \quad (4.16)$$

Putting this result back into Eq. (4.14) then gives

$$F_1(0)=\frac{F_\pi G_{\sigma\pi\pi}}{m_\pi^2-m_\sigma^2}, \quad (4.17)$$

in agreement with "operator" PCAC used in deriving (4.5), apart from the difference of $G_{\sigma\pi\pi}(0)$ from $G_{\sigma\pi\pi}$.

The form factors (4.14) and (4.15) *automatically* satisfy the soft-π theorem

$$F_1(m_\sigma^2)=F_2(m_\sigma^2), \quad (4.18)$$

which follows in the usual way by extrapolating $p_\pi\to 0$. The matrix element $\langle\pi|\mathfrak{F}_\mu^5|\sigma\rangle$ vanishes in this limit, leaving (4.18). Equations (4.14)–(4.16) combine to give $F_1(m_\sigma^2)-F_2(m_\sigma^2)=g_A g_{A\sigma\pi}(m_\pi^2/2m_A^2)$, which vanishes as $m_\pi\to 0$ and is in any case numerically small.

We now investigate the hypothesis that $F_1(\infty)$ vanish, i.e., that the A_1 dominate the form factor F_1 completely. In the case of the isovector current the analogous assumption leads to universal ρ coupling. In the present case we do not have a conserved current but we can still make predictions. The first prediction is a relation between the $\sigma\pi\pi$ and $A\sigma\pi$ coupling constants:

$$g_A g_{A\sigma\pi}=2m_A^2 F_\pi G_{\sigma\pi\pi}/(m_\pi^2-m_\sigma^2). \quad (4.19)$$

This allows F_1 and F_2 to be written in the form

$$F_1(t)=\frac{m_A^2}{m_A^2-t}\,\frac{F_\pi G_{\sigma\pi\pi}}{m_\pi^2-m_\sigma^2}, \quad (4.20)$$

$$F_2(t)=\frac{m_A^2-m_\pi^2}{m_A^2-t}\,\frac{F_\pi G_{\sigma\pi\pi}}{m_\pi^2-t}. \quad (4.21)$$

From this we learn that

$$F_1(m_\epsilon^2)/F_1(0)=m_A^2/(m_A^2-m_\epsilon^2)\approx 2.$$

Further, $\lambda(t)\equiv F_1(t)/F_2(t)$ explicitly obeys $\lambda(0)\cong -m_\pi^2/m_\sigma^2\cong 0$ and $\lambda(m_\sigma^2)\approx 1+m_\pi^2/m_A^2\approx 1$. If we use Weinberg's second sum rule[28] $g_A=g_\rho=m_\rho^2/\gamma_\rho$ $(\gamma_\rho^2/4\pi\approx 2)$ and neglect m_π compared with m_σ, we get $(G_{\sigma\pi\pi}=2g_{\sigma\pi\pi})$

$$g_{A\sigma\pi}=-8\gamma_\rho g_{\sigma\pi\pi}F_\pi/m_\sigma^2. \quad (4.22)$$

Applying (4.22) to the $\epsilon(700)$, we obtain $g_{A\epsilon\pi}^2/4\pi=10$, and the width

$$\Gamma_{A_1\to\epsilon\pi}=\frac{g_{A\epsilon\pi}^2}{24\pi}\,\frac{p^3}{m_A^2} \quad (4.23)$$

is 30 MeV, using the $\epsilon\pi\pi$ coupling of Eq. (2.10). This value seems to be a reasonable fraction of the total width (80 MeV).

Conversely, if the experimental value of $\Gamma_{A_1\to\epsilon\pi}$ is 30 MeV, the A_1 pole contribution to F_1 agrees with the PCAC determination, which would support our conjecture that $F_1(\infty)=0$. A similar observation has been made by Nieh with regard to the nucleon form factor.[29] It is tempting to assume the generality of the principle of π, A_1 dominance of the axial-vector current (unsubtracted) form factors of *all* particles. The PCAC condition will then lead to "universal" predictions of A_1 couplings.

It is interesting to compare the result (4.22) with the "hard-pion" prediction of Golowich,[30]

$$g_{A\epsilon\pi}=-8g_{\epsilon\pi\pi}/m_A. \quad (4.24)$$

Numerically this formula gives $g_{A\epsilon\pi}^2/4\pi=9.3$, in close agreement with the prediction of Eq. (4.22). Consistency of (4.22) and (4.24) gives the relation

$$F_\pi=m_\epsilon^2/\gamma_\rho m_A. \quad (4.25)$$

Numerically, Eq. (4.25) predicts $F_\pi=91$ MeV, in surprisingly good agreement with experiment. In the approximation $m_\epsilon=m_\rho$, $m_A=\sqrt{2}m_\rho$, Eq. (4.25) reduces to the KSRF relation.[19,20] The foregoing should not be regarded as a derivation of that relation but rather an indication of the consistency of the prediction (4.22) with hard-pion approximations.

V. SUMMARY AND DISCUSSION

The main conclusions of the preceding sections are as follows. (1) The fragmentary data on scalar mesons (decaying into pseudoscalars) are consistent with a mixed nonet $(\theta\approx 23°)$ and SU_3-invariant couplings provided the κ mass is around 1 GeV. (2) The validity of the mass formula (3.5) for the baryon octet predicts the d/f ratio for the coupling constants of the unmixed scalar octet to baryons, providing a test of the basic assumptions of approximate scale invariance. (3) The universal constant F_{σ_8} is much less than F_{σ_0}, provided that σ_0 and σ_8 couple to baryons with comparable strength. F_{σ_0} is comparable to F_π, suggesting a similar origin for the two constants. (4) PCAC relations show that $G_{\epsilon\pi\pi}$ is very sensitive to the dilaton mass and that the assumption of scalar dominance of the $\theta^\mu{}_\mu$ dispersion relation is wrong for the pseudoscalar octet unless there is yet another scalar meson. Other parameters, like F_π, F_σ, $g_{\pi NN}$, etc., may be close to their physical values in the scale-invariant limit. (5) As an incidental result we are able to predict the $A_1\epsilon\pi$ coupling constant.

The foregoing results have important bearing on the structure of the energy density with regard to scale and chiral transformations. An important question is

[28] S. Weinberg, Phys. Rev. Letters 18, 507 (1967).

[29] H. T. Nieh, Phys. Rev. 164, 1780 (1968).

[30] E. Golowich (private communication).

whether the limit of $SU_3\times SU_3$ symmetry coincides with that of scale invariance. Thus far it has been difficult to give model-independent judgments on this problem. We follow Gell-Mann[3] in writing $\theta_{00}=\bar{\theta}_{00}+\delta+u$, where $\bar{\theta}_{00}$ is chiral and scale invariant, δ violates scale invariance but is $SU_3\times SU_3$ invariant, and u breaks scale invariance *and* chiral symmetry. If δ vanishes, not only $m_\pi\to 0$ as $u\to 0$, but also the dilaton mass vanishes so that $G_{\sigma\pi\pi}\to 0$ in this limit. This would make the success of soft-meson theorems difficult to understand and violate the popular belief that the $SU_3\times SU_3$ world is "nearby" the real world.[24,31,32] (However, we can have approximate $SU_2\times SU_2$ symmetry and $m_\sigma\neq 0$ with $\delta=0$.) That δ must have a c-number part follows[17] from the models of Refs. 21, 31, and 32 in which u is assumed to have the form u_0+cu_8, where the u_i are scalar components in $(3,\bar{3})+(\bar{3},3)$ and $c\approx -1.25$. From this work, one concludes that the vacuum is approximately SU_3 invariant and that $\langle u_0\rangle\neq 0$, $\langle u_8\rangle\cong 0$. Since $\langle\theta^\mu{}_\mu\rangle$ vanishes, $\langle\delta\rangle$ cannot vanish except for the (non-scale-invariance-breaking) dimension $l_u=-4$. Other evidence bearing on the existence of δ was discussed in Refs. 3 and 4.

In effective Lagrangian models[4,33] it is easy to understand why SU_3 symmetry is good for masses and vertices (apart from mixing corrections) provided that the operator δ exists. In purely mesonic models, the scale-invariance-breaking operator δ occurs naturally as a trilinear coupling (of canonical dimension -3)

$$\delta=-g\,\det M+\text{H.c.}, \tag{5.1}$$

where $M=\sigma+i\phi$ is the usual sum of nonet matrices. $SU_3\times SU_3$ symmetry is broken by a linear term $f_0(\sigma_0+c\sigma_8)=-u$. The interaction (5.1) leads to instability of the normal vacuum and to a new ground state with $\langle\sigma_0\rangle\neq 0$, $\langle\sigma_0\rangle/\langle\sigma_8\rangle\gg 1$, so that SU_3 symmetry results. The masses characterizing σ are of order 1 GeV. Thus *corrections* due to μ are of order u/δ and are small.

The details of the foregoing model are unreliable but the dimensional argument is general; in this view there are two dimensional parameters, one of which fixes the scale. The small parameter u/δ gives the breaking of SU_3 symmetry. It is not presently clear whether u and δ are independent.

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APPENDIX A

The $\epsilon\pi\pi$ coupling constant also enters into πN scattering in a significant way. Defining the effective ϵNN coupling by $\mathcal{L}_{\epsilon NN}=g_{\epsilon NN}\epsilon\bar{N}N$, the contribution of ϵ exchange to the standard invariant πN amplitude is

$$A=A^{(+)}=\frac{2g_{\epsilon NN}g_{\epsilon\pi\pi}}{m_\epsilon^2-t}, \qquad B=0. \tag{A1}$$

The contributions to the partial-wave amplitudes are found to be[34] ($j=l\pm\frac{1}{2}$)

$$f_{l\pm}=\frac{g_{\epsilon NN}g_{\epsilon\pi\pi}}{4\pi m_\pi}\left(\frac{m_\pi}{2k^2W}\right)[(E+M)Q_l(y) -(E-M)Q_{l\pm 1}(y)], \tag{A2}$$

where W is the total center-of-mass energy, E is the nucleon energy, and k is the center-of-mass momentum; y is $1+m_\epsilon^2/2k^2$. At reasonably low energies this depends on l, not j. Comparison with the numerical results of Hamilton and collaborators[35] for $l=1$ gives $g_{\epsilon NN}g_{\epsilon\pi\pi}/(4\pi m_\pi)\approx 12$. Since $g_{\epsilon\pi\pi}^2/4\pi m_\pi^2\approx 10$, we expect $g_{\epsilon NN}^2/4\pi\approx g_{\epsilon\pi\pi}^2/(4\pi m_\pi^2)\approx 10$. This value is entirely compatible with results from nucleon-nucleon single-particle-exchange models.

APPENDIX B: BARYON DECUPLET AND CONSTANTS F_{σ_i}

In Sec. III we used information on the baryon octet, supplemented by quark-model considerations, to estimate the ratio $F_{\sigma_0}/F_{\sigma_8}$. A similar analysis of the baryon decuplet gives an independent estimate of this important ratio. The decuplet masses may be written in a form[30] analogous to (3.6),

$$\begin{aligned} M_\Delta&=F_{\sigma_0}g_0'+F_{\sigma_8}g_8',\\ M_{Y_1^*}&=F_{\sigma_0}g_0',\\ M_{\Xi^*}&=F_{\sigma_0}g_0'-F_{\sigma_8}g_8',\\ M_\Omega&=F_{\sigma_0}g_0'-2F_{\sigma_8}g_8'. \end{aligned} \tag{B1}$$

The equal-spacing rule is evident in (B1). Numerically one has $F_0g_0'=1385$ MeV, $F_8g_0'=-149$ MeV. Again, the naive quark model with canonical mixing predicts $g_{\epsilon'\Delta\Delta}=0$, or $g_8'/g_0'=-1/\sqrt{2}$, which gives

$$F_{\sigma_8}/F_{\sigma_0}=0.15. \tag{B2}$$

This result is in reasonable agreement with the value 0.19 obtained from the baryon octet.

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