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f-Dominance of Gravity

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A Lagrangian theory is formulated describing the intrinsic mixing of the graviton with a massive 2^+ f meson which interacts universally with hadrons through the stress tensor. The theory is developed as an analog of the well-known ρ-γ model of hadron electrodynamics, and in particular a field-current identity is exhibited which equates the massive 2^+ meson with the hadronic energy-momentum tensor. An Einstein-type Lagrangian is used for both spin-2 particles, and general covariance is preserved throughout. The nonlinear coupling of the hadrons to the f meson leads, within the framework of nonpolynomial field theories, to a universal cutoff for strong-interaction physics.

I. INTRODUCTION

NATURE appears prodigal with respect to two fundamental forces, electromagnetic and gravitational, in the following sense. The photon—a neutral 1^- massless particle—is supposed to be the mediator of the electromagnetic force; but there appear to be other neutral 1^- particles that play this role as well. In hadronic physics there are the ρ, ω, ϕ particles, and in leptonic physics there are 1^- states of positronium.

The mixing of γ with the ρ-ω-ϕ complex (hereafter generically called the ρ^0) has been formulated[1] in an elegant manner (the so-called formalism of the field-current identity) which attempts to stress that hadronic electrodynamics can, to a good approximation, be separated from lepton electrodynamics. Indeed, the physical content of this theory is that the photon interacts directly with leptons, but only indirectly with hadrons via a simple ρ^0-γ mixing. A natural consequence of the formalism is the identification, in the field-theoretic sense, of the ρ^0 meson with the hadronic electromagnetic current. The model has a number of successes to its credit, in particular the correlation of photon and ρ-ω-ϕ total and differential cross sections. Among the failures the most prominent is the inability to take into account consistently the individual polarization states of the photon and ρ^0 meson, presumably due to the difficulty of covariantly separating the polarization states of a massless γ and a massive ρ-ω-ϕ.

It is an attractive hypothesis that the Einstein graviton g and some mixture of the known, massive, strongly interacting, spin-2 particles may present, in the field-current identity sense, a complete analogy of this ρ^0 photon scheme. In such a theory the graviton would interact directly with leptons, but only indirectly with hadronic matter, and in the field-current identity the role of the current would be played by the energy-momentum tensor.

It is well known[2] that the existence of a conserved stress tensor which can act as a source of the spin-2 particles necessitates the adoption of an Einstein-type system of field equations. For this reason, as well as natural elegance, we use the usual Einstein graviton Lagrangian together with an identical one for the f meson. The crucial step in the theory is the construction of an f-g mixing term which provides one of the spin-2 fields with a mass while maintaining general covariance.

The plan of the paper is as follows. In Sec. II the essentials of the ρ-γ mixing are summarized in a somewhat simplified form so as to bring out those aspects which have an analog in the f-g theory. In Sec. III we quickly review the usual Einstein generally covariant theory of gravity, paying particular attention to the somewhat knotty problem of the definition of energy-momentum tensors in general relativity. The f-g mixing is then introduced and the existence of a massive state and an associated field-current identity is made manifest.

Finally, in the conclusion, we speculate on some of the consequences of the theory, from both the general-relativistic and the field-theoretic points of view.

II. PHOTON AND ρ-MESON MIXING

We shall discuss the essentials of the photon–ρ^0-meson mixing phenomena so as to motivate the analogous graviton–f-meson mixing proposed in Sec. III. By the ρ^0 meson is meant the neutral component of the ρ-ω-ϕ complex with the same quantum numbers as the photon A_μ. The $SU(3)$ symmetry aspects of the ρ^0 coupling are not essential to the points we wish to stress.

The ρ^0-γ mixing, with the associated field-current identity, may be illustrated in its simplest form using the Lagrangian

$$\mathcal{L}=\mathcal{L}(\rho)+\mathcal{L}(A)+\mathcal{L}_{A\rho}\,, \tag{2.1}$$

where

$$\mathcal{L}(\rho)=-\tfrac{1}{4}\rho_{\mu\nu}{}^0\rho_{\mu\nu}{}^0-\rho_\mu{}^0J_\mu{}^{\mathrm{had}}\,, \tag{2.2}$$

$$\mathcal{L}(A)=-\tfrac{1}{4}A_{\mu\nu}A_{\mu\nu}-A_\mu J_\mu{}^{\mathrm{lep}}\,, \tag{2.3}$$

$$\mathcal{L}_{A\rho}=\tfrac{1}{2}m^2(\rho_\mu{}^0-A_\mu)^2\,, \tag{2.4}$$

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[1] T. D. Lee, N. M. Kroll, and B. Zumino, Phys. Rev. **157**, 1376 (1967).

[2] S. N. Gupta, Phys. Rev. **96**, 1683 (1954). The argument rests on the fact that gravity couples universally to all fields including itself. Thus the stress tensor which is to serve as the source of the spin-2 particle must contain contributions from this particle itself. This fact, together with the need for conservation of the stress tensor, leads almost uniquely to Einstein's generally covariant equations.

with $\rho_{\mu\nu}{}^0 \equiv \partial_\mu \rho_\nu{}^0 - \partial_\nu \rho_\mu{}^0$ and $A_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$. The coupling constants associated with ρ and A have been omitted for the sake of clarity. They may easily be supplied at the end of the manipulations. The leptonic current $J_\mu{}^{\text{lep}}$ includes contributions from electrons, muons, and W mesons, while the hadronic current $J_\mu{}^{\text{had}}$ contains the charged ρ mesons together with all other hadrons.[3]

The physical content of Eqs. (2.2)-(2.4) is that a photon couples directly to leptons but only indirectly to hadrons via the ρ-γ vertex exhibited[4] in Eq. (2.4).

From this Lagrangian we obtain the equations of motion:

$$\partial_\nu \rho_{\mu\nu} = J_\mu{}^{\text{had}} - m^2(\rho_\mu{}^0 - A_\mu)\,, \tag{2.5}$$

$$\partial_\nu A_{\mu\nu} = J_\mu{}^{\text{lep}} + m^2(\rho_\mu{}^0 - A_\mu)\,, \tag{2.6}$$

which, when added together, imply the conservation of the total current

$$\partial_\mu (J_\mu{}^{\text{had}} + J_\mu{}^{\text{lep}}) = 0\,. \tag{2.7}$$

One now *defines* a new hadronic current

$$\mathcal{J}_\mu(\rho^0) = J_\mu{}^{\text{had}} - \partial_\nu \rho_{\mu\nu} \tag{2.8}$$

which is conserved if and only if $J_\mu{}^{\text{had}}$ is individually conserved. This will happen if the $W^\pm$ mesons are decoupled from hadrons,[5] so that no charge passes directly from leptonic to hadronic matter; that is, all lepton-hadron interactions are mediated by the neutral A or ρ^0.

At this point it is conventional to define $\rho_\mu{}^0 - A_\mu$ to be the physical ρ^0 field $\tilde{\rho}^0$ leading to the equations

$$\text{(I)} \quad m^2 \tilde{\rho}_\mu{}^0 = \mathcal{J}_\mu(\tilde{\rho} + A)\,, \tag{2.9}$$

$$\text{(II)} \quad \partial_\nu A_{\mu\nu} = J_\mu{}^{\text{lep}} + m^2 \tilde{\rho}_\mu{}^0\,. \tag{2.10}$$

The first of these equations (I) is known as the field-current identity, while the second (II) is the equation of motion of the photon field. It is important to observe that in spite of the appearance of a m^2 term on the right-hand side of Eq. (2.6), the theory does in fact contain a zero-bare-mass state. This is easily seen if we write the Lagrangian of Eq. (2.1) in the form

$$\mathcal{L} = -\tfrac{1}{4}\rho_{\mu\nu}{}^0 \rho_{\mu\nu}{}^0 - \tfrac{1}{4} A_{\mu\nu} A_{\mu\nu} + \frac{m^2}{2(e^2+g^2)} (g\rho_\mu{}^0 - eA_\mu)^2 - g\rho_\mu{}^0 J_\mu{}^{\text{had}} - eA_\mu J_\mu{}^{\text{lep}}\,, \tag{2.11}$$

where the ρ-meson hadronic coupling constant g and photon electromagnetic coupling constant e have now been correctly inserted. The diagonalized fields are

$$\tilde{\rho}_\mu{}^0 = \frac{1}{(e^2+g^2)^{1/2}} (g\rho_\mu{}^0 - eA_\mu)\,, \tag{2.12}$$

$$\tilde{A}_\mu = \frac{1}{(e^2+g^2)^{1/2}} (e\rho_\mu{}^0 + gA_\mu)\,, \tag{2.13}$$

in terms of which Eq. (2.11) becomes

$$\mathcal{L} = -\tfrac{1}{4}\tilde{\rho}_{\mu\nu}{}^0 \tilde{\rho}_{\mu\nu}{}^0 - \tfrac{1}{4}\tilde{A}_{\mu\nu}\tilde{A}_{\mu\nu} + \tfrac{1}{2} m^2 \tilde{\rho}_\mu{}^0 \tilde{\rho}_\mu{}^0 - \frac{g}{(e^2+g^2)^{1/2}} (g\tilde{\rho}_\mu{}^0 + e A_\mu) J_\mu{}^{\text{had}} - \frac{e}{(e^2+g^2)^{1/2}} (-e\tilde{\rho}_\mu{}^0 + g\tilde{A}_\mu) J_\mu{}^{\text{lep}}\,. \tag{2.14}$$

III. GRAVITON AND f MESON MIXING

In this section we discuss the mixing of the graviton g with the f meson (by which is meant the appropriate combination of f^0, $f^{0\prime}$, $A_2{}^0$, and any other massive spin-2 mesons). The underlying physical idea, in strict analogy with Sec. II, is that gravity should couple directly to leptonic matter but only indirectly to hadronic matter through an f-g mixing. We shall start by summarizing the usual Einstein theory.

The Einstein action integral[6] for pure gravity is

$$S_g = \frac{1}{\kappa_g{}^2} \int (-g)^{-1/2} R(g) d\Omega\,, \tag{3.1}$$

where κ_g is the weak gravitational constant ($\kappa_g = 2.2 \times 10^{-22}\, m_e{}^{-1}$) ($m_e$ being the electron mass) and $d\Omega$ indicates the volume element. The curvature tensor $R(g)$ is defined as

$$R = g^{\mu\nu} R_{\mu\nu}\,,$$

where the Ricci tensor $R_{\mu\nu}$ is a contraction of the curvature tensor $R^\beta{}_{\mu\alpha\nu}$, with

$$R^\beta{}_{\mu\alpha\nu} = \Gamma^\beta{}_{\mu\alpha,\nu} - \Gamma^\beta{}_{\mu\nu,\alpha} + \Gamma^\beta{}_{\lambda\nu}\Gamma^\lambda{}_{\mu\alpha} - \Gamma_{\lambda\alpha}{}^\beta \Gamma^\lambda{}_{\mu\nu} \tag{3.2}$$

and $R_{\mu\nu} = R^\alpha{}_{\mu\alpha\nu}$. We shall be assuming a Riemannian geometry[7] so the connection Γ is simply the Christoffel symbol and may be expressed in terms of the metric $g^{\mu\nu}$ as

$$\Gamma^\alpha{}_{\mu\nu} = \tfrac{1}{2} g^{\alpha\beta} [(g^{-1})_{\beta\mu,\nu} + (g^{-1})_{\beta\nu,\mu} - (g^{-1})_{\mu\nu,\beta}]\,.$$

We wish to emphasize that from a field-theoretic point of view there is only one independent field $g^{\mu\nu}$. The other entity $(g^{-1})_{\mu\nu}$, which is normally written as $g_{\mu\nu}$, must be regarded as a derived quantity. Specifically we have

$$(g^{-1})_{\mu\nu} = (1/6g)\epsilon_{\mu\alpha\beta\gamma}\epsilon_{\nu\delta\rho\lambda} g^{\alpha\delta} g^{\beta\rho} g^{\gamma\lambda}\,, \tag{3.3}$$

[3] We know that the photon does not possess direct strong couplings and so its free Lagrangian belongs to the lepton class. For the W meson, however, at the present stage of our knowledge, it is not really clear whether $-\frac{1}{4}W_{\mu\nu}W_{\mu\nu} + \frac{1}{2}m^2 W_\mu W_\mu$ should be placed in the hadronic or leptonic Lagrangians. The choice does not affect the field-current identity argument.

[4] The combination $(\rho^0 - A)$ is chosen rather than $(\rho^0 + A^0)$ so as to preserve invariance under an electromagnetic gauge-group transformation when

$$\rho_\mu{}^0 \to \rho_\mu{}^0 + \partial_\mu \theta(x),\ A_\mu \to A_\mu + \partial_\mu \theta(x),\ \psi \to e^{iq\theta}\psi,\ \bar{\psi} \to \bar{\psi} e^{-iq\theta}.$$

[5] If we had assigned the W mesons to the hadronic rather than leptonic currents, the required decoupling would be from the leptons.

[6] A very good modern text is J. L. Anderson, *Principles of Relativity Physics* (Academic, London, 1967).

[7] In particular this implies that the covariant derivative of the metric tensor vanishes identically.

where g means the determinant of the contravariant tensor $g^{\mu\nu}$. To emphasize this dependence we shall frequently write the covariant tensor $g_{\mu\nu}$ as $(g^{-1})_{\mu\nu}$. It follows at once from Eq. (3.3) that $(g^{-1})_{\mu\nu}$ is indeed the inverse matrix of $g^{\mu\nu}$ satisfying

$$g^{\mu\alpha}(g^{-1})_{\alpha\nu}=\delta_\nu{}^\mu .$$

This point has great relevance when the techniques[8] for handling nonpolynomial Lagrangians are applied to our theory.

Concerning notation, a comma written in a tensor subscript indicates an ordinary derivative, while a semicolon indicates a covariant derivative.

In the presence of matter fields, the action integral becomes

$$S=\int\left[(-g^{-1/2})\frac{R(g)}{\kappa_g{}^2}+\mathcal{L}_m\right]d\Omega . \tag{3.4}$$

Under a variation of $g^{\mu\nu}$ (which vanishes on the integration boundary) the symmetric energy-momentum tensor $T_{\mu\nu}$ of the matter Lagrangian density $\mathcal{L}_m$ is defined by

$$\delta\int\mathcal{L}_m d\Omega=\frac{1}{2}\int[T_{\mu\nu}(-g)^{-1/2}\delta g^{\mu\nu}]d\Omega . \tag{3.5}$$

Setting the variation of the total action of Eq. (3.4) equal to zero leads to the fundamental field equations

$$\text{(II}'\text{)}\quad G_{\mu\nu}\equiv R_{\mu\nu}-\tfrac{1}{2}g_{\mu\nu}R=-\tfrac{1}{2}\kappa_g{}^2T_{\mu\nu} , \tag{3.6}$$

where the left-hand side arises from the variation of the curvature tensor. The Einstein tensor $G_{\mu\nu}$ has an identically vanishing covariant divergence[9]

$$G_\mu{}^\nu{}_{;\nu}\equiv 0 , \tag{3.7}$$

which implies in particular that

$$T_\mu{}^\nu{}_{;\nu}=0 . \tag{3.8}$$

One of the classic (and still unsolved) problems of general relativity is the construction of some geometric entity which can serve to describe the energy-momentum content of the combined system of gravitational and matter fields. Such objects are of great interest to us as they form the analog of the currents of Sec. II.

One possible construct is due to Einstein himself. First remove the second-derivative terms from the gravitational Lagrangian of (3.1). For example, one may use the action

$$S_g'=\frac{1}{\kappa_g{}^2}\int\left\{(-g)^{-1/2}R(g)-\left[(-g^{-1/2})g_{\mu\nu,\alpha}\frac{\partial R(g)}{\partial g_{\mu\nu,\alpha,\beta}}\right]_{,\beta}\right\}d\Omega \tag{3.9}$$

in which the integrand differs from that of (3.1) by a four-divergence. Now compute the canonical energy-momentum "tensor" from the gravitational Lagrangian $\mathcal{L}'$ in Eq. (3.9) defined as usual by

$$(-g)^{-1/2}\tau_\mu{}^\nu=\frac{\partial\mathcal{L}'}{\partial g_{\alpha\beta,\nu}}g_{\alpha\beta,\mu}-\delta_\mu{}^\nu\mathcal{L}' . \tag{3.10}$$

Using the vanishing of the covariant divergence of the matter field tensor $T_\mu{}^\nu$ [Eq. (3.8)] and defining

$$\theta_\mu{}^\nu=(-g)^{-1/2}(\tau_\mu{}^\nu+T_\mu{}^\nu) , \tag{3.11}$$

it may be shown that

$$\theta_\mu{}^\nu{}_{,\nu}=0 . \tag{3.12}$$

This vanishing of the ordinary divergence is a first requirement of any energy-momentum tensor and led Einstein to choose the definition of (3.11) for the total energy-momentum complex.[10]

One important property, first demonstrated by Freud,[11] of the pseudotensor $\theta_\mu{}^\nu$ is that it may be written as a four-divergence. That is,

$$\theta_\mu{}^\nu=\tfrac{1}{2}\psi_\mu{}^{\nu\alpha}{}_{,\alpha}=(-g)^{-1/2}[-\tfrac{1}{2}\tau_\mu{}^\nu+(1/\kappa^2)G_\mu{}^\nu] , \tag{3.13}$$

where the so-called superpotential $\psi_\mu{}^{\nu\alpha}$ is antisymmetric in the upper two indices and is given explicitly as

$$(-g)^{-1/2}\psi_\mu{}^{\nu\alpha}=(1/\kappa^2)g_{\mu\beta}[g^{-1}(g^{\nu\beta}g^{\alpha\lambda}-g^{\alpha\beta}g^{\nu\lambda})]_{,\lambda} . \tag{3.14}$$

If a complex with two upper indices is required, one might define $\theta^{\mu\nu}=g^{\mu\alpha}\theta_\alpha{}^\nu$. This object, however, does not possess the desirable property of symmetry between its indices. A symmetric complex can be defined as

$$\theta^{\mu\nu}=(-g)^{-1}(T^{\mu\nu}+t^{\mu\nu})\equiv\psi'^{\mu\nu\alpha}{}_{,\alpha} \tag{3.15}$$

with

$$t^{\mu\nu}=g^{\mu\alpha}\tau_\alpha{}^\nu-\tfrac{1}{2}g[(-g)^{-1/2}g^{\mu\alpha}]_{,\beta}\psi_\alpha{}^{\nu\beta} \tag{3.16}$$

and

$$\psi'^{\mu\nu\alpha}=\tfrac{1}{2}(-g)^{-1/2}g^{\mu\beta}\psi_\beta{}^{\nu\alpha} . \tag{3.17}$$

This allows an angular momentum complex to be constructed.

There are very many other possible choices for an energy-momentum complex, none of which is a true tensor under the general coordinate group, and all differing from each other by a four-divergence. For a given set of global boundary conditions on an integration region on the space-time manifold, it may be possible to limit this arbitrariness. Good discussions of this problem may be found in Refs. 6 and 12.

[8] See, for example, Abdus Salam and J. Strathdee, Phys. Rev. D 1, 3296 (1970), and references contained therein.

[9] This is a direct consequence of the contracted Bianchi identities.

[10] The use of the word tensor is misleading since although $T_\mu{}^\nu$ is a genuine mixed tensor, $\tau_\mu{}^\nu$ most certainly is not, as follows at once from its definition in (3.10). It has the correct transformation properties under the Lorentz group but in general not under arbitrary coordinate changes. This has been in the past a cause of great consternation in the general-relativity literature. It may not cause such concern to a particle physicist. The object $\tau_\mu{}^\nu$ is frequently known as a pseudotensor.

[11] P. von Freud, Ann. Math. 40, 417 (1939).

[12] A. Trautman, in *Brandeis Lecture Notes 1964* (Prentice Hall, Englewood Cliffs, N. J., 1965), Vol. I.

We now come to the main part of the paper, which concerns the introduction of a "strong gravity," massive spin-2 particle into the theory.

We shall hypothesize that the pure f-meson part of the Lagrangian has the same form as that of (3.1). Thus we write

$$S_f=\frac{1}{\kappa_f^2}\int(-f)^{-1/2}R(f)d\Omega, \tag{3.18}$$

where κ_f is the coupling constant of the strongly interacting f meson and is roughly equal to the inverse of its mass. All geometric quantities in (3.18) are to be regarded as having their usual definitions in terms of $f^{\mu\nu}$ as the metric tensor. The essential prescription now is that the hadronic matter Lagrangian is to be formed using $f^{\mu\nu}$ as a metric tensor while for the leptonic one must use $g^{\mu\nu}$. Thus we have as the combined Lagrangian[13]

$$\mathcal{L}=\mathcal{L}_f+\mathcal{L}_g, \tag{3.19}$$

where

$$\mathcal{L}_f=(1/\kappa_f^2)(-f)^{-1/2}R(f)+\mathcal{L}(\text{hadrons}, f), \tag{3.20}$$

$$\mathcal{L}_g=(1/\kappa_g^2)(-g)^{-1/2}R(g)+\mathcal{L}(\text{leptons}, g). \tag{3.21}$$

So far the theory simply says that the universe consists of two noncommunicating[14] worlds—the hadronic and the leptonic. The crucial step is the introduction of a mixing term $\mathcal{L}_{fg}$ which causes these two worlds to interact. This term must be chosen so that one of the rank-2 tensor fields (or more precisely some combination of both of them) describes a massive particle.[15]

The simplest mixing term that we can think of is given by a straightforward "covariantization" of the usual mass term for a spin-2 field,

$$\mathcal{L}_{\text{mass}}=\tfrac{1}{4}M^2(F^{\alpha\beta}F^{\alpha\beta}-F^{\alpha\alpha}F^{\beta\beta}), \tag{3.22}$$

whose form is determined by requiring that $F^{\alpha\beta}=F^{\beta\alpha}$ describe a pure spin-2 system.[16] Now in order that the Lagrangians of Eqs. (3.20) and (3.21) make sense, the fields $f^{\mu\nu}$, $g^{\mu\nu}$ viewed as 4×4 matrices must be invertible.[17] In particular, we require that they have nonvanishing vacuum expectation values and then normalize them in such a way that we can write

$$\begin{aligned} f^{\mu\nu}&=\eta^{\mu\nu}+\kappa_f F^{\mu\nu}, \\ g^{\mu\nu}&=\eta^{\mu\nu}+\kappa_g h^{\mu\nu}, \end{aligned} \tag{3.23}$$

where $\eta^{\mu\nu}$ denotes the usual Minkowski metric, diag $(1,-1,-1,-1)$. In order to make expression (3.22) into a scalar density it will be sufficient to make the replacement

$$F^{\alpha\beta}\to(1/\kappa_f)(f^{\alpha\beta}-g^{\alpha\beta})$$

and make contractions relative to $(g^{-1})_{\alpha\beta}$ [or $(f^{-1})_{\alpha\beta}$]. In this way one finds the mixing term

$$\begin{aligned} \mathcal{L}_{fg}&=\frac{M^2}{4\kappa_f^2}(-\det f)^{-1/2}(f^{\alpha\beta}-g^{\alpha\beta})(f^{\kappa\lambda}-g^{\kappa\lambda}) \\ &\qquad\times(g_{\alpha\kappa}{}^{-1}g_{\beta\lambda}{}^{-1}-g_{\alpha\beta}{}^{-1}g_{\kappa\lambda}{}^{-1}) \\ &=\frac{M^2}{4\kappa_f^2}(-\det f)^{-1/2}[f^{\alpha\beta}g_{\alpha\beta}{}^{-1}f^{\beta\gamma}g_{\gamma\alpha}{}^{-1} \\ &\qquad-(f^{\alpha\beta}g_{\alpha\beta}{}^{-1})^2+6f^{\alpha\beta}g_{\alpha\beta}{}^{-1}-12]. \end{aligned} \tag{3.24}$$

One can easily verify that to zeroth order in κ_f and κ_g this expression coincides with (3.22). Different mixing terms with this property can be obtained by using g^{-1} and f^{-1} in different ways to make the contractions. Also one could use $(-\det g)^{-1/2}$ in place of $(-\det f)^{-1/2}$. Another sort of mixing term, one which employs cosmological terms, is discussed in the Appendix.

Consider now the equations of motion. Variation of $f^{\mu\nu}$ and $g^{\mu\nu}$ yields the respective equations

$$\frac{G_{\mu\nu}(f)}{\kappa_f^2(-f)^{1/2}}+\frac{T_{\mu\nu}(\text{hadrons}, f)}{2(-f)^{1/2}}+\frac{\partial\mathcal{L}_{fg}}{\partial f^{\mu\nu}}=0, \tag{3.25}$$

$$\frac{G_{\mu\nu}(g)}{\kappa_g^2(-g)^{1/2}}+\frac{T_{\mu\nu}(\text{leptons}, g)}{2(-f)^{1/2}}+\frac{\partial\mathcal{L}_{fg}}{\partial g^{\mu\nu}}=0, \tag{3.26}$$

where $T_{\mu\nu}$(hadrons, f) is associated with $\mathcal{L}$(hadron, f) in Eq. (3.20) and does not include a contribution from $\mathcal{L}_{fg}$. Likewise for $T_{\mu\nu}$(leptons, g). The contributions of the mixing term are given explicitly by

$$\begin{aligned} \frac{\partial\mathcal{L}_{fg}}{\partial f^{\mu\nu}}&=\left(-\tfrac{1}{2}\delta_\mu{}^\alpha\mathcal{L}_{fg}+\frac{M^2}{2\kappa_f}\mathfrak{F}_\mu{}^\alpha\right)f_{\alpha\nu}{}^{-1}, \\ \frac{\partial\mathcal{L}_{fg}}{\partial g^{\mu\nu}}&=-\frac{M^2}{2\kappa_f}\mathfrak{F}_\mu{}^\alpha g_{\alpha\nu}{}^{-1}, \end{aligned} \tag{3.27}$$

where $\mathfrak{F}_\mu{}^\alpha$ denotes the combination

$$\begin{aligned} \mathfrak{F}_\mu{}^\alpha=\frac{1}{\kappa_f}(-\det f)^{-1/2}[&(g^{-1}fg^{-1}f)_\mu{}^\alpha \\ &-(g^{-1}f)_\mu{}^\alpha(g^{-1}{}_{\kappa\lambda}f^{\kappa\lambda}-3)], \end{aligned} \tag{3.28}$$

which reduces to $\eta^{\alpha\nu}(F_{\mu\nu}-\eta_{\mu\nu}\eta^{\beta\gamma}F_{\beta\gamma})$ in zeroth order in κ_f and κ_g, and therefore can be viewed as an interpolating field for the massive spin-2 particle.

[13] In $\mathcal{L}$(hadrons, f), for example, we shall use the Christoffel symbol $\Gamma_{\mu\nu}{}^\alpha(f)=\frac{1}{2}f^{\alpha\beta}[(f^{-1})_{\beta\mu,\nu}+(f^{-1})_{\beta\nu,\mu}-(f^{-1})_{\mu\nu,\beta}]$ while in $\mathcal{L}$(leptons, g) we shall use $\Gamma_{\mu\nu}{}^\alpha(g)=\frac{1}{2}g^{\alpha\beta}[(g^{-1})_{\beta\mu,\nu}+(g^{-1})_{\beta\nu,\mu}-(g^{-1})_{\mu\nu,\beta}]$.

[14] This is in the absence of electromagnetic and weak interactions. See Ref. 3 for remarks concerning the hadronic or leptonic nature of photons and W mesons.

[15] We must emphasize that the resulting theory does not contain two independent physical *metric* tensors, although it does, of course, include two rank-2 tensors. The *real* metric tensor of space-time (whatever that may mean) is presumably the rank-2 tensor field which corresponds to the massless combination of f and g.

[16] This form was derived by W. Pauli and M. Fierz, Proc. Roy. Soc. (London) **A173**, 211 (1939), by requiring that the spin-1 and spin-0 components of the tensor field should not propagate. In order to construct a generally covariant mass term out of this expression one needs an independent field $g_{\mu\nu}{}^{-1}$ to replace the Minkowskian contractions; i.e., one could not use such an approach to make massive gravitons if there were not also massless ones present.

[17] Cf. the discussion following Eq. (3.2).

The equations of motion (3.25) and (3.26) can be put into the suggestive form

$$\psi_{\mu}{}^{\nu\alpha}{}_{,\alpha}(f)+\left\{\frac{1}{(-f)^{1/2}}[\tau_{\mu}{}^{\nu}(f)+T_{\mu}{}^{\nu}(\text{hadrons},f)]-\delta_{\mu}{}^{\nu}\mathfrak{L}_{fg}\right\}-\frac{M^2}{2\kappa_f}\mathfrak{F}_{\mu}{}^{\nu}=0, \quad (3.29)$$

$$\psi_{\mu}{}^{\nu\alpha}{}_{,\alpha}(g)+\frac{1}{(-g)^{1/2}}[\tau_{\mu}{}^{\nu}(g)+T_{\mu}{}^{\nu}(\text{leptons},g)]+\frac{M^2}{2\kappa_f}\mathfrak{F}_{\mu}{}^{\nu}=0, \quad (3.30)$$

where ψ and τ denote the expressions defined by Eqs. (3.13) and (3.14). In (3.29) the expression

$$[1/(-f)^{1/2}][\tau_{\mu}{}^{\nu}(f)+T_{\mu}{}^{\nu}(\text{hadrons}, f)]$$

is the Einstein complex associated with the hadronic Lagrangian (3.20). On the other hand, the quantity $-\delta_{\nu}{}^{\mu}\mathfrak{L}_{fg}$ is simply the contribution of the mixing term to the total canonical energy-momentum complex. Therefore let us define

$$\theta'_{\mu}{}^{\nu}(\text{hadrons}, f)=[1/(-f)^{1/2}][\tau_{\mu}{}^{\nu}(f)+T_{\mu}{}^{\nu}(\text{hadrons}, f)]-\delta_{\mu}{}^{\nu}\mathfrak{L}_{fg}, \quad (3.31)$$

in terms of which (3.29) reads

$$\psi_{\mu}{}^{\nu\alpha}{}_{,\alpha}(f)=\theta_{\mu}{}'^{\nu}(\text{hadrons}, f)-(M^2/2\kappa_f)\mathfrak{F}_{\mu}{}^{\nu}. \quad (3.32)$$

On comparing this formula with Eq. (2.5) one sees a term-by-term correspondence. Thus $\partial_\alpha\psi_{\mu}{}^{\nu\alpha}$ corresponds to $\partial_\nu\rho_{\nu\mu}$, $\theta'_{\mu}{}^{\nu}$ corresponds to the current $J_{\mu}{}^{\text{had}}$, and $\mathfrak{F}_{\mu}{}^{\nu}$ corresponds to the massive field $\tilde{\rho}_\mu$. Finally, by analogy with (2.8), one should define the hadronic tensor current

$$\Theta_{\mu}{}^{\nu}(\text{hadrons}, f)=\theta'_{\mu}{}^{\nu}(\text{hadrons}, f)+\partial_\alpha\psi_{\mu}{}^{\nu\alpha}, \quad (3.33)$$

in which case Eq. (3.32) takes the form of a field-current identity,

$$(M^2/2\kappa_f)\mathfrak{F}_{\nu}{}^{\mu}=\Theta_{\nu}{}^{\mu}(\text{hadrons}, f). \quad (3.34)$$

This formula, together with the gravitational equation of motion (3.30), demonstrates the similarities between the ρ^0-γ and f-g mixing theories.[18] One slight difference is the following. If, in the ρ-γ model, the $W^\pm$ mesons are decoupled from the hadrons, then the hadronic and leptonic currents are individually conserved. A similar decoupling—of the $W^\pm$ and electromagnetic interactions—in the f-g model will not ensure $\partial_\nu\Theta_{\mu}{}^{\nu}\times(\text{hadrons}, f)=0$. This is because part of this stress tensor is contributed by the mixing term itself which contains the weak graviton g explicitly. To secure such a conservation equation we would have to make the non-generally covariant substitution $\kappa_g=0$ in the mixing term, thereby decoupling the hadronic and leptonic worlds gravitationally as well.

We end this section with the remark that if half-integer spin fields are present in the matter Lagrangian, then the well-known vierbein formalism[19] for the gravitational fields must be introduced. There are no consequences of this, apart from a slightly increased algebraic complexity, and we shall not give the details here.

IV. CONCLUSIONS

The present theory can be surveyed from at least three distinct points of view: (a) that of a particle physicist, (b) that of a general relativist, or (c) that of a cosmologist.

(a) From a particle physicist's point of view this is basically a theory of strong interactions which employs Einstein's famous equation for describing the f meson's universal coupling to the hadronic stress tensor. The field stress-tensor identity could, at a date in the far future, provide a means of correlating graviton f scattering data just as the well-known field-current identity does for photon ρ scattering. Immediately, however, the major testable statement of the theory would be the universality of the f meson's coupling and its stress-tensor form. To check this, it is important to state explicitly if our f meson can be identified with any of the known massive spin-2^+ objects. These are the f^0 (1260 MeV decaying predominantly into two pions), $f^{0\prime}$ (1514 MeV decaying into $K\bar{K}$), and $A_2{}^0$ (1300 MeV decaying predominantly into $\rho+\pi$).

To decide on this, note that the strong tensor transforms for $SU(3)$ as a mixture of a singlet, an octet, and possibly a 27-plet, with the singlet predominating. Identifying, as a first approximation, our f with the singlet mixture of f^0 and $f^{0\prime}$ (in an ideal mixing scheme), a preliminary investigation based on decay-rate data and exchange degeneracy of f^0 and $f^{0\prime}$ and ω and ϕ does not seem to lead to any inconsistency with the hypothesis that f couplings may indeed be proportional to the strong stress tensor.[20] Thus on present evidence it could well be identified with a mixture of the known 2^+ objects, though nothing rules out the more aesthetic possibility[21] that the f of this paper is a new object lying on the Pomeranchuk trajectory which, in view of recent data[22] assigning to this trajectory a slope lying between $0.3<\alpha_P<0.5$, would possess a mass between 1400 and 1700 MeV. The universal coupling of the Pomeranchukon to hadronic matter would then be mirrored in the universal coupling of its spin-2 recurrence to the strong stress tensor.

Notwithstanding the title of this paper, we must confess the immediate incentive we had for using an

[18] The possibility of dominating the matrix elements of the hadronic energy-momentum tensor has been considered before in P. G. O. Freund, Phys. Letters **2**, 136 (1962); R. Delbourgo, Abdus Salam, and J. Strathdee, Nuovo Cimento **49**, 593 (1967).

[19] V. Fock and D. Ivanenko, Compt. Rend. **188**, 1470 (1929).

[20] We are indebted to Professor B. Renner, Professor R. Capps, and Professor P. Rotelli for informative discussions on this point.

[21] P. G. O. Freund, Ref. 18.

[22] G. Beznogikh *et al.*, Phys. Letters **30B**, 274 (1969).

Einstein-type equation for strong f^0 gravity was the search for a universal nonpolynomial feature in strong-interaction physics. From recently developed techniques in field theory we know that for such Lagrangians the conventional ultraviolet infinities are automatically suppressed, the inbuilt ultraviolet cutoff being proportional to the inverse of the (universal) length in the theory. For Einstein's gravity theory—and for lepton physics—it was shown in a recent paper[23] that this inbuilt cutoff comes at around $(\kappa_g)^{-1}\approx 10^{19}$ BeV. For the strong gravity in its present formulation this would come at around $(\kappa_f)^{-1}\approx$ a few BeV. Most theoretical work in strong-interaction physics heuristically employs such a cutoff; the present theory would provide a more rigorous formulation of this.

(b) Consider now the implications of the theory for general relativity in its metrical aspects. The theory works with two second-rank tensors. The first question one may ask is: Which of the two tensors approximates to the "actual" metric tensor on space-time? In regions far removed from hadronic concentrations of matter, clearly the old tensor g predominates. Inside hadrons, however, the situation may perhaps better be described using the f tensor. The geodesics associated with the f metric may provide a semiclassical description of paths of "particles" inside hadronic matter. Likewise one may be tempted to speculate with Wheeler[24] on whether "feons"—the analog of "geons"—may not be the elementary stuff of hadron physics. Also of interest would be the relationship of hadrons to "black holes" in a strong f-gravity field.

(c) The most exciting implications of the present theory may, however, be cosmological. Could f-mediated gravity be *repulsive* for short distances and what implications may this have for the problem of collapse? At the very least, the gravitational law of force (for a particle of mass M) may be expected to be modified exhibiting roughly an $M^{2/3}$ dependence for *nonstatic* high-frequency graviton interactions rather than a linear M dependence. This would be in analogy with the results of the ρ-dominance model of hadron electrodynamics where photonic high-frequency interactions with a large nucleus of charge Z are expected to show a surface dependence[25] (as a consequence of the conversion of the photon to the ρ meson, followed by a short-range surface—rather than volume—absorption of the ρ meson), giving effects proportional to $Z^{2/3}$ rather than Z.

ACKNOWLEDGMENTS

We wish to acknowledge a stimulating conversation with B. Zumino, who informed us of his work with J. Wess along closely related lines.

[23] R. Delbourgo, C. J. Isham, Abdus Salam, and J. Strathdee, Phys. Rev. (to be published).

[24] J. Wheeler, *Geometrodynamics* (Academic, London, 1962).

[25] We are indebted to J. S. Bell for a discussion of this.

APPENDIX

There is at present no criterion (other than that of simplicity) which could serve to limit one's choice of the f-g mixing term. The one exhibited in the text [Eq. (3.24)] seems to be one of the simplest. However, it may be worthwhile to consider others as well. One such is given by

$$\mathfrak{L}_{fg}=\lambda(-\det g)^{-1/2}+\lambda'(-\det f)^{-1/2}+\mu(-\det f)^{-\alpha}\times(-\det g)^{-\beta}[-\det\tfrac{1}{2}(f+g)]^{-\gamma}, \quad \text{(A1)}$$

where μ, α, β, and γ are parameters which must be fixed in terms of the "cosmological" constants λ and λ'. The following paragraphs are concerned with developing the criteria whereby the parameters μ, α, β, and γ are fixed.

First, notice that general covariance by itself imposes only the restriction

$$\alpha+\beta+\gamma=\tfrac{1}{2}. \quad \text{(A2)}$$

Further conditions are obtained by expanding the Lagrangian

$$\mathfrak{L}_f+\mathfrak{L}_g+\mathfrak{L}_{fg}$$

in powers of the quantized fields $F^{\mu\nu}$ and $h^{\mu\nu}$ which were defined by (3.23). In this expansion we require the terms linear in $F^{\mu\nu}$ and $h^{\mu\nu}$ to vanish (absence of tadpoles) and that the quadratic terms define a sensible propagator (absence of ghosts).

The determinants in (A1) may be typically expanded[26] according to the formula

$$\begin{aligned}(\det f)^{-\alpha}&=e^{-\alpha\,\mathrm{Tr}\,\ln(1+\kappa F)}\\&=e^{-\alpha\,\mathrm{Tr}(\kappa F-\frac{1}{2}\kappa^2F^2+\cdots)}\\&=1-\alpha\kappa\,\mathrm{Tr}F+\tfrac{1}{2}\alpha\kappa^2[\mathrm{Tr}F^2+\alpha(\mathrm{Tr}F)^2]+\cdots.\end{aligned} \quad \text{(A3)}$$

We want to show that $\mathfrak{L}_{fg}$ provides a mass term for one of the two particles. On expanding this up to quadratic terms, the constant and linear pieces are

$$\begin{aligned}\mathfrak{L}_{fg}=(\lambda+\lambda'+\mu)-\{\tfrac{1}{2}\lambda+\mu(\beta+\tfrac{1}{2}\gamma)\}\,\mathrm{Tr}(\kappa_g h)\\-\{\tfrac{1}{2}\lambda'+\mu(\alpha+\tfrac{1}{2}\gamma)\}\,\mathrm{Tr}(\kappa_f F)+(\text{terms quadratic in } h,F)\\+(\text{higher-order terms}).\end{aligned} \quad \text{(A4)}$$

The linear pieces should be eliminated leaving only the quadratic ones, thus imposing the constraints

$$\tfrac{1}{2}\lambda+\mu(\beta+\tfrac{1}{2}\gamma)=0, \quad \text{(A5)}$$

$$\tfrac{1}{2}\lambda'+\mu(\alpha+\tfrac{1}{2}\gamma)=0, \quad \text{(A6)}$$

which when added together imply, on using Eq. (A2),

$$\lambda+\lambda'+\mu=0. \quad \text{(A7)}$$

[26] To simplify the computations it is convenient to use a Euclidean-space metric. This has the effect of replacing the $\eta^{\mu\nu}$ in Eqs. (3.40) and (3.41) by the Kronecker delta $\delta^{\mu\nu}$. The minus signs must also be removed from in front of the determinants. For example, $(-\det f)^{-\alpha}$ becomes $(\det f)^{-\alpha}$. This change involves no loss of generality and the correct Minkowski signature can easily be inserted at the end.

Notice that the constant term in Eq. (A4) is eliminated simultaneously with the linear terms.

The computation of the second-order quadratic terms is straightforward but tedious. The result on substituting the above constraints is

$$\mathcal{L}_{fg}=\frac{\lambda\lambda'}{8(\lambda+\lambda')}[\mathrm{Tr}(\kappa_f F-\kappa_g h)]^2+\frac{(\lambda+\lambda')\gamma}{8}[\mathrm{Tr}(\kappa_f F-\kappa_g h)^2]+(\text{higher-order terms}). \quad \text{(A8)}$$

A similar expansion must be performed for the quadratic kinetic terms of the f and g fields. These appear in the form

$$\mathcal{L}=\tfrac{1}{4}(h_{\mu\nu,\alpha}h_{\mu\nu,\alpha}-h_{\mu\mu,\alpha}h_{\nu\nu,\alpha}+2h_{\mu\mu,\alpha}h_{\alpha\nu,\nu}-2h_{\mu\nu,\alpha}h_{\nu\alpha,\mu}), \quad \text{(A9)}$$

with a similar expression for the F field.

From (A8) it is clear that the bilinear terms in $\mathcal{L}$ are diagonalized by the fields $\tilde{F}$ and $\tilde{h}$ defined by

$$\begin{aligned}(\kappa_f^2+\kappa_g^2)^2\tilde{F}^{\mu\nu}&=\kappa_f F^{\mu\nu}-\kappa_g h^{\mu\nu},\\(\kappa_f^2+\kappa_g^2)^{1/2}\tilde{h}^{\mu\nu}&=\kappa_g F^{\mu\nu}+\kappa_f h^{\mu\nu},\end{aligned} \quad \text{(A10)}$$

in terms of which the pure spin-2 part of the mass term appears as

$$\mathcal{L}_{fg}{}^{(2)}=-\tfrac{1}{4}M^2\sum_{i,j=1}^{3}\tilde{F}_{ij}\tilde{F}_{ij}, \quad \text{(A11)}$$

with

$$M^2=-\tfrac{1}{2}(\lambda+\lambda')\gamma(\kappa_f^2+\kappa_g^2). \quad \text{(A12)}$$

Obviously none of the quantities above is necessarily a generally covariant tensor. The associated diagonalized[27] *tensor* fields are

$$\tilde{f}^{\mu\nu}=(f^{\mu\nu}-g^{\mu\nu}), \quad \text{(A13)}$$

$$\tilde{g}^{\mu\nu}=\left(1+\frac{\kappa_g^2}{\kappa_f^2}\right)^{-1}\left(g^{\mu\nu}+\frac{\kappa_g^2}{\kappa_f^2}f^{\mu\nu}\right), \quad \text{(A14)}$$

which are related to $\tilde{F}$ and $\tilde{h}$ by

$$\tilde{f}^{\mu\nu}=(\kappa_f^2+\kappa_g^2)^{1/2}\tilde{F}^{\mu\nu}, \quad \text{(A15)}$$

$$\tilde{g}^{\mu\nu}=\eta_{\mu\nu}+\frac{\kappa_f\kappa_g}{(\kappa_f^2+\kappa_g^2)^{1/2}}\tilde{h}_{\mu\nu}. \quad \text{(A16)}$$

The parameters α, β, and μ can be eliminated by conditions (A5)–(A7). The parameter γ can be eliminated by requiring that no spin-0 ghost should appear. To see this, a tedious calculation is necessary. One must set up the spin-0 part of the propagator matrix—in the center-of-mass frame—for the fields $\tilde{F}^{\mu\nu}$ defined by (A10). According to (A8) and (A9), this propagator is defined by the bilinear form

$$\tfrac{1}{8}(\tilde{F}^{44}\tilde{F}^{ii})\begin{pmatrix}M_{11}^2 & M_{21}^2\\ M_{12}^2 & \tfrac{4}{3}p^2+M_{22}^2\end{pmatrix}\begin{pmatrix}\tilde{F}^{44}\\ \tilde{F}^{ii}\end{pmatrix}, \quad \text{(A17)}$$

where

$$\begin{aligned}M_{11}^2&=(\kappa_f^2+\kappa_g^2)\left[\frac{\lambda\lambda'}{\lambda+\lambda'}+(\lambda+\lambda')\gamma\right],\\M_{12}^2&=M_{21}^2=(\kappa_f^2+\kappa_g^2)\frac{\lambda\lambda'}{\lambda+\lambda'},\\M_{22}^2&=(\kappa_f^2+\kappa_g^2)\left[\frac{\lambda\lambda'}{\lambda+\lambda'}+\tfrac{1}{3}(\lambda+\lambda')\gamma\right].\end{aligned} \quad \text{(A18)}$$

In order that no ghosts should appear—in fact no spin-0 states of any kind—the determinant of (A17) must be independent of p^2; i.e., $M_{11}^2=0$. Thus the parameter γ must satisfy the condition

$$\lambda\lambda'/(\lambda+\lambda')+(\lambda+\lambda')\gamma=0 \quad \text{(A19)}$$

and the mass of the spin-2 meson (A12) is given by

$$M^2=\tfrac{1}{2}\lambda\lambda'/(\lambda+\lambda')(\kappa_f^2+\kappa_g^2). \quad \text{(A20)}$$

Apparently this mixing model requires that both cosmological constants shall be nonvanishing.

In summary, we find the rather surprising result that the parameters α, β, γ, and μ which specify the mixing term are completely fixed in terms of the two cosmological constants,

$$\begin{aligned}\alpha&=\tfrac{1}{2}(2\lambda+\lambda')\lambda'/(\lambda+\lambda')^2,\\\beta&=\tfrac{1}{2}\lambda(\lambda+2\lambda')/(\lambda+\lambda')^2,\\\gamma&=-\lambda\lambda'/(\lambda+\lambda')^2,\\\mu&=-(\lambda+\lambda').\end{aligned} \quad \text{(A21)}$$

A constraint on the relative values of the two cosmological constants is provided by (A20), which gives the heavy graviton mass.

An interpolating field $\mathfrak{F}_\mu{}^\nu$ for the heavy graviton in this model can be defined by similar arguments to those used in Sec. III. The analog of Eq. (3.28) is

$$\mathfrak{F}_\mu{}^\nu=(\kappa_f/M^2)\{-\lambda(-g)^{-1/2}\delta_\mu{}^\nu+[\mathcal{L}_{fg}-\lambda'(-f)^{-1/2}-\lambda(-g)^{-1/2}]\times[\beta\delta_\mu{}^\nu+\gamma(f+g)_{\mu\alpha}{}^{-1}g^{\alpha\nu}]\}.$$

[27] The insertions of Eqs. (A10) show clearly that the true weak and strong gravitational coupling constants are, respectively,

$$\tilde{\kappa}_g=\kappa_f\kappa_g/(\kappa_f^2+\kappa_g^2)^{\frac{1}{2}} \quad\text{and}\quad \tilde{\kappa}_f=\kappa_f^2/(\kappa_f^2+\kappa_g^2)^{\frac{1}{2}}.$$

Experimentally, of course, $\tilde{\kappa}_g\ll\tilde{\kappa}_f$ and inversion of the above equations shows that $\kappa_g\ll\kappa_f$. Essentially then, $(\kappa_f^2+\kappa_g^2)^{\frac{1}{2}}$ may be set equal to κ_f and Eq. (A14) becomes simply $\tilde{g}_{\mu\nu}=g_{\mu\nu}$. These coupling-constant renormalizations have an exact analog in the ρ-γ case where, as shown by Eqs. (2.11) and (2.13),

$$\tilde{e}=eg/(e^2+g^2)^{\frac{1}{2}} \quad\text{and}\quad \tilde{g}=g^2/(e^2+g^2)^{\frac{1}{2}}.$$