essentially the same results (within 5% of the values of $c$ and $g$). By fitting the data[8] at 9.8, 12.8, and 14.8 GeV/$c$ simultaneously, using only values of $|t|$ less than 0.6 (GeV/$c$)$^2$, we obtained[10]

$$c=2.52\pm0.04\ \mathrm{mb}^{1/2}/(\mathrm{GeV}/c)\,,$$

$$g=3.03\pm0.06\ (\mathrm{GeV}/c)^{-2}\,,$$

with a $\chi^2$ of 21 for 30 data.

We then calculated $F_{K^+}(t)$ from[2,3]

$$F_p(t)F_{K^+}(t)=(\mathrm{const})[a_{K^+p}(t)+\tfrac{1}{2}a_{K^+p}\otimes a_{K^+p}|_t+\cdots]\,,$$

using 200 terms in the sum. The numerical values[10] for the various form factors are listed in Table II.

From Table II it is easily seen that the kaon form factors fall off slower than either the proton or pion form factors. Thus, the kaon is smaller than either the proton or the pion. The rms radius of the kaon is found to be 0.39±0.03 F.[10] This is not in good agreement with the vector-dominance model,[5] which gives

$$F_K(t)=\frac{1}{2}\frac{M_\rho^2}{M_\rho^2-t}+\frac{1}{6}\frac{M_\omega^2}{M_\omega^2-t}+\frac{1}{3}\frac{M_\phi^2}{M_\phi^2-t}\,,$$

and

$$\langle r^2\rangle^{1/2}=[6F_K'(0)]^{1/2}=0.58\ \mathrm{F}\,.$$

TABLE II. List of the proton form factor $F_p$, the pion form factor $F_\pi$, and the kaon form factor $F_K$. Values of $F_p$ and $F_\pi$ are from Ref. 3.

| $\|t\|$ (GeV/$c$)$^2$ | $F_p$ | $F_\pi$ | $F_K$ |
|---|---|---|---|
| 0.0 | 1.000 | 1.000 | 1.000 |
| 0.1 | 0.810 | 0.846 | 0.948±0.006 |
| 0.2 | 0.665 | 0.728 | 0.892±0.011 |
| 0.3 | 0.553 | 0.636 | 0.833±0.014 |
| 0.4 | 0.466 | 0.564 | 0.773±0.018 |
| 0.5 | 0.399 | 0.505 | 0.712±0.020 |
| 0.6 | 0.347 | 0.455 | 0.651±0.021 |

[10] Only statistical errors are included.

# Current Algebra and the Weak Radiative Decays of Hyperons*

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The recently reported measurement of the asymmetry parameter in the weak radiative decay $\Sigma^+\to p\gamma$ suggests that the parity-violating amplitude in the decay may be large. Here we show that no known theories can account for this result.

THE first experimental determination of the asymmetry[1] parameter for $\Sigma^+\to p\gamma$ suggests that the decay distribution of the proton may exhibit a large asymmetry ($\alpha=-1.03_{-0.42}^{+0.52}$). Even though the measurement is based on very few events, the result is rather surprising. The authors of Ref. 1 point out that of more than six theoretical studies of weak radiative decay, only one, by Ahmed,[2] predicts a large asymmetry for $\Sigma^+\to p\gamma$. Here we wish to show that Ahmed's result arises from an inconsistency in his analysis; and that when the inconsistency is removed, his theory yields a small asymmetry in accord with all the other theories. Thus there are at present no theories in good agreement with the experimental result of Ref. 1.

With the usual assumptions of octet dominance and $CP$ invariance for the weak Hamiltonian, Hara[3] has shown that the parity-violating (p.v.) amplitudes for $\Sigma^+\to p\gamma$ and $\Xi^-\to\Sigma^-\gamma$ are zero for a current×current weak interaction. Further, if $R$ conjugation is also imposed, then all the p.v. amplitudes are zero in the symmetry limit.[4] In the presence of symmetry breaking, the p.v. amplitudes can be evaluated by using the baryon pole model[4,5] with phenomenologically determined p.v. weak-vertex parameters. The procedure leads to very small p.v. amplitudes and asymmetry parameters which are two orders of magnitude smaller than the reported experimental value.[1]

Although Ahmed[2] uses the current×current model of weak interactions and current algebra, his p.v. amplitudes for $\Sigma^+\to p\gamma$ and $\Xi^-\to\Sigma^-\gamma$ do not vanish in the symmetry limit. The inconsistency arises from his extrapolation procedure for the amplitudes, and this is explicitly pointed out in the following.

By applying the reduction technique and the hypothesis of partial conservation of axial-vector current (PCAC), Ahmed relates the amplitudes for the process $\alpha\to\beta+\pi^0+\gamma$ to the amplitudes for $\alpha\to\beta+\gamma$ in the soft-pion limit.[6,7] Thus

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[1] L. K. Gershwin, M. Alston-Garnjost, R. O. Bangerter, A. Barbaro-Galtierei, T. S. Mast, F. T. Solmitz, and R. D. Tripp, Phys. Rev. **188**, 2077 (1969).

[2] M. A. Ahmed, Nuovo Cimento **58A**, 728 (1968).

[3] Y. Hara, Phys. Rev. Letters **12**, 378 (1964).

[4] R. H. Graham and S. Pakvasa, Phys. Rev. **140**, B1144 (1965).

[5] G. Calcucci and G. Furlan, Nuovo Cimento **21**, 679 (1961); J. C. Pati, Phys. Rev. **130**, 2097 (1963); L. R. Ram Mohan, *ibid.* **179**, 1561 (1969).

[6] We are using the Dirac-Pauli metric: $p^2=(\mathbf{p},iE)^2=-m^2$, and Hermitian Dirac matrices $\gamma_\mu{}^\dagger=\gamma_\mu$.

[7] H. Sugawara, Phys. Rev. Letters **15**, 870 (1965); M. Suzuki, *ibid.* **15**, 986 (1965); Y. Hara, Y. Nambu, and J. Schechter, *ibid.* **16**, 380 (1966); L. S. Brown and C. M. Sommerfield, *ibid.* **16**, 751 (1966); S. Badier and C. Bouchiat, Phys. Letters **20**, 529 (1966).

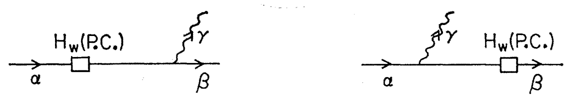


FIG. 1. Parity-conserving pole diagram for $\alpha \to \beta + \gamma$.

$$\lim_{q_\mu \to 0} \{-i(2q_0)^{1/2} \langle \beta(p_2); \gamma(k); \pi^0(q) | H_w(0) | \alpha(p_1) \rangle\}$$
$$= \lim_{q_\mu \to 0} m(\alpha \to \beta\pi\gamma)$$
$$= -f_\pi^{-1} \langle \beta(p_2); \gamma(k) | [F_5^{(3)}, H_w(0)] | \alpha(p_1) \rangle$$
$$+ \lim_{q_\mu \to 0} \left\{ i f_\pi^{-1} q_\mu \times \int d^4x\, e^{-iq\cdot x} \langle \beta; \gamma | T(A_\mu(x) H_w(0)) | \alpha \rangle \right\}, \quad (1)$$

where $f_\pi$ is the pion decay constant.

The equal-time commutator in the first term on the right-hand side of Eq. (1) can be evaluated using current algebra and the current×current theory, and we have

$$\langle \beta(p_2); \gamma(k) | H_w(0) | \alpha(p_1) \rangle = (-i\sqrt{2} f_\pi) \lim_{q_\mu \to 0} \{ m(\alpha \to \beta\pi^0\gamma) + f_\pi^{-1} q_\mu R_\mu \}, \quad (2)$$

where $i f_\pi^{-1} q_\mu R_\mu$ is the second term of Eq. (1).

The procedure is to evaluate the time-ordered product in $R_\mu$ by using single-particle intermediate states and to estimate $m(\alpha \to \beta\pi^0\gamma)$ by using perturbation theory[8] (Low's procedure) to relate it to the nonradiative process $\alpha \to \beta\pi$. The nonradiative process $\alpha \to \beta\pi^0$ is in turn evaluated by using PCAC, the soft-pion technique, and current algebra.[7]

For the parity-conserving (p.c.) amplitude for $\alpha \to \beta\gamma$, we use the matrix elements of $H_w^{\text{p.v.}}$ on the right-hand side of Eq. (1) because of the extra pion on the right side. Following the Sugawara-Suzuki calculation,[7] it is assumed that the matrix elements of $H_w^{\text{p.v.}}$ between two baryon states is zero. Then the $q_\mu R_\mu$ term of Eq. (2) does not contribute. The Low theorem,[8] the soft-pion technique, and the Sugawara-Suzuki calculation to relate $m(\alpha \to \beta\pi^0)_{\text{p.v.}}$ to $m(\alpha \to \beta)_{\text{p.c.}}$ reduces the calculation of the p.c. amplitude for the pure photonic decay mode to the evaluation of the Born diagrams of Fig. 1.

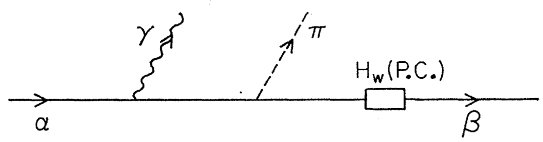


FIG. 2. Feynman diagram used for evaluating the parity-conserving amplitude for $\alpha \to \beta + \pi^0 + \gamma$.

[8] F. E. Low, Phys. Rev. **110**, 974 (1958); S. L. Adler and Y. Dothan, *ibid.* **151**, 1267 (1966).

This is precisely the result of Pestieau,[9] who obtains it by applying the Low theorem directly to the amplitude for $\alpha \to \beta\gamma$. This part of the calculation does not differ in principle from the pole model calculations[4,5] and the usual estimates for the p.c. weak-vertex parameters would ensure the correct order of magnitude for the decay rate.

For the p.v. amplitude for $\alpha \to \beta\gamma$, Ahmed evaluates the right-hand side of Eq. (2) by using the p.c. two-body weak vertices. The $q_\mu R_\mu$ term can be evaluated easily. The term $m(\alpha \to \beta\pi^0\gamma)$ now reduces to six Feynman diagrams obtained by permuting the weak, electromagnetic, and pion vertices in Fig. 2.

If we use derivative coupling of the pion to the baryons it is easy to show that the single-particle pole diagrams of $m(\alpha \to \beta\pi^0\gamma)$ cancel the single-particle poles in the time-ordered product $f_\pi^{-1} q_\mu R_\mu$ after use is made of the Goldberger-Treiman relation to express the axial-vector couplings in $q_\mu R_\mu$ as pion couplings. This cancellation is valid provided the masses of the internal and external baryons are the same. This is just a comparison of Feynman poles and poles in dispersion relations. It should be noted that the derivative coupling is needed for strong interactions in order to identify the axial-vector matrix elements in the Born amplitude as being the strong interaction matrix elements in pole diagrams.

Ahmed gets a different result because he uses a nonderivative coupling for the strong $\bar{B}B\pi$ vertex and compares $m(\alpha \to \beta\pi\gamma)$ with $f_\pi^{-1} q_\mu R_\mu$ when the amplitudes are off the mass shell of the pion and the internal baryons. It is known that the derivative and the nonderivative couplings are inequivalent when the particles are off the mass shell. Thus a comparison of the same amplitude extrapolated in two different ways leads Ahmed to a large p.v. amplitude for $\alpha \to \beta\gamma$. Within the framework of current algebra and the current×current model, the divergence of the axial-vector current should be used as the interpolating field for the pions; this in turn requires that the derivative coupling be used consistently throughout the calculation.

In the light of the above arguments it is felt that if future experiments do confirm the large asymmetry in the decay distribution of $\Sigma^+ \to p^+\gamma$, then a theoretical explanation of this cannot come from the present understanding of the symmetry-breaking effects on weak interactions. An experimental confirmation of the present value of the asymmetry parameter is therefore of great interest.

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[9] J. Pestieau, Phys. Rev. **160**, 1555 (1967).