true for other cases. These comparisons can be made easily by using Tables II and III of this paper with Tables I and II of Ref. 9. However, although TPE improves upon OPE in a comparison with the phenomenological phase parameters, neither OPE+TPE nor OPE+TPE+ΔTPE provide a good fit to these values. Since the vector-meson-exchange contributions are large in the intermediate region, their inclusion is obviously needed.

V. CONCLUSIONS

Since the TPE contribution in the $U(6,6)$ model is highly divergent, this model seems to provide only an effective coupling, and it then significantly affects only the s- and p-wave OPE phase parameters. Thus, the modification of OPE in the $U(6,6)$ model is largely confined to the core.

On the other hand, the TPE contribution in the chiral-dynamical model is obtainable without any serious difficulty, and the additional contribution in this model significantly alters the nucleon-nucleon scattering phase parameters throughout the intermediate region. However, since the agreement between the OPE+TPE phase parameters and the phenomenological values improves in some cases and deteriorates for others by the addition of the chiral-dynamical contribution, a definite conclusion about the chiral-dynamical coupling can be reached only after including the vector-meson-exchange contributions.

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CP Violation: Interference Effects in K^0 Decay*

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The intrinsic magnitude of the CP-violating coupling is not simply related to the decay ratio $(K_L\rightarrow 2\pi)/(K_S\rightarrow 2\pi)$, because of the K_L-K_S overlap. It is this intrinsic coupling which is needed to estimate CP violation in other systems. We examine an explicit model showing the interference effects of the K_L-K_S overlap on the relation of the coupling constants to the above decay ratio. There are interference effects in ϵ which depend on nonmeasurable mass shifts. (Theoretical estimates of these mass shifts are discussed.) However, ϵ' does not depend on these mass shifts and thus can be more easily related to the intrinsic CP-violating coupling.

THE intrinsic magnitude of the CP- or T-violating coupling in the decay of $K_L\rightarrow 2\pi$ is of crucial importance. Theoretical estimates of T-violation effects in other systems such as the neutron electric-dipole moment (EDM)[1,2] depend on knowledge of this coupling. However, because of the overlap of the K_L and K_S,[3]

$$\Delta m = m_S - m_L = -0.47\Gamma_S\,, \tag{1}$$

the observed partial width[3]

$$[\Gamma(K_L\rightarrow 2\pi)/\Gamma(K_S\rightarrow 2\pi)]^{1/2}\simeq 0.002 \tag{2}$$

is not simply related to coupling constants, so that the naive determination of the magnitude of the CP violation from Eq. (2) may be very misleading. Although this fact is well known, it seems to have been repeatedly ignored. Thus we feel that it is useful to present a detailed explicit discussion of the interference effect of the K_L-K_S overlap on the relation of the coupling constants to decay rate (2).

We use the multichannel ND^{-1} formalism[4] to relate the relevant branching ratios to the intrinsic coupling strengths in the case where the CP violation is a weak interaction. (As noted previously,[5] this could equivalently be treated in the Wigner-Weisskopf formalism.) The result[6] for ϵ depends on nonmeasurable mass shifts. Many theoretical estimates[7] of these mass shifts indicate that the K_L-K_S overlap depresses the CP violation, so

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[1] S. L. Glashow, Phys. Rev. Letters **14**, 35 (1965); G. Feinberg and H. S. Mani, Phys. Rev. **137**, 637 (1965); G. Feinberg, *ibid.* **140**, B1402 (1965); G. Barton and E. D. White, *ibid.* **184**, 1660 (1969).

[2] J. K. Baird, P. D. Miller, W. B. Dress, and N. F. Ramsey, Phys. Rev. **179**, 1285 (1969). The authors establish an upper limit $|\mathrm{EDM}| < 5\times10^{-23}$ e cm and indicate that a future experiment should increase their sensitivity to 7×10^{-24} e cm. See also "Search and Discovery," Phys. Today **22**, 56 (November, 1969), for future expectations in improving the measurement of the neutron EDM.

[3] Particle Data Group, Rev. Mod. Phys. **42**, 87 (1970).

[4] P. Coulter and G. Shaw, Phys. Rev. **188**, 2443 (1969).

[5] M. Bander, P. Coulter, and G. Shaw, Phys. Rev. D **2**, 944 (1970).

[6] T. T. Wu and C. N. Yang, Phys. Rev. Letters **13**, 380 (1964). We use the now "standard" definitions of ϵ and ϵ' (see Appendix) which differ by a factor of 2 from the original definitions of Wu and Yang.

[7] K. Nishijima, Phys. Rev. Letters **12**, 39 (1964); T. Truong, *ibid.* **17**, 1102 (1966); K. Kang and D. Land, *ibid.* **18**, 503 (1967).

that the "intrinsic" CP violation may be larger than one would expect from Eq. (2) by a non-negligible factor. However, ϵ' does not depend on these mass shifts, and thus can be more easily related to the intrinsic CP-violating coupling. Thus an accurate measurement of ϵ' is crucial in order to determine the intrinsic CP-violating coupling.

We obtain the result that $\Gamma(K_L\rightarrow 2\pi)$, where the final 2π state has $I=0$, is different from what it would be in the absence of interference by a factor $|F|^2$, where[8]

$$F=(i\Delta m+\tfrac{1}{2}\Gamma_S)^{-1}\left\{\Delta m_L\left[1-\left(\frac{\Gamma_{2S}{}^{(0)}\Gamma_{3S}}{\Gamma_{2L}{}^{(0)}\Gamma_{3L}}\right)^{1/2}\right]\right.$$
$$\left.-(\Delta m_S{}^{(2)}-\tfrac{1}{2}i\Gamma_{2S}{}^{(2)})\left[1-\left(\frac{\Gamma_{2S}{}^{(0)}\Gamma_{2L}{}^{(2)}}{\Gamma_{2L}{}^{(0)}\Gamma_{2S}{}^{(2)}}\right)^{1/2}\right]\right\}. \quad (3)$$

$\Delta m=m_S-m_L$ and $\Delta m_{L(S)}$ is the shift of the K_L (K_S) meson from the mass it would have if it were stable. $\Delta m_S{}^{(2)}$ is the mass shift of K_S due to its decay into the $2\pi(I=2)$ channel. $\Gamma_{2L}{}^{(0)}$ is the partial width for the decay $K_L\rightarrow 2\pi(I=0)$ which would be seen in the absence of interference with K_S. Γ_{3S} is the decay width for $K_S\rightarrow$ three-body decay channels of K_L. We expect the square-root terms to be of order unity if the CP violation for $K_S\rightarrow 3\pi$ is comparable to that for $K_L\rightarrow 2\pi$.[9] The widths $\Gamma_{2S}{}^{(0)}$, $\Gamma_{2S}{}^{(2)}$, and Γ_{3L} are not changed much by the interference because of the smallness of the CP violation. We only expect appreciable changes in CP-violation widths, e.g., $\Gamma(K_L\rightarrow 2\pi)$. The sign of the square-root terms is written as though all the couplings were positive. The actual sign can be determined by letting $\sqrt{\Gamma_{ij}}$ carry the sign of the coupling between channels i and j.

The major uncertainties in Eq. (3) are the mass shifts Δm_L and $\Delta m_S{}^{(2)}$. There have been many theoretical attempts to compute Δm. The usual assumption[7] is that the major part of Δm is a result of the coupling of K_S to the $2\pi(I=0)$ channel. If this is a valid approximation, one would expect $|F|\ll 1$, so that the effect of the interference of K_L and K_S would be to *suppress* the decay $K_L\rightarrow 2\pi$. This would imply that theoretical estimates of the neutron EDM may be *larger* than previously expected if the weak interaction violates T invariance.[1]

On the other hand, theoretical calculations of mass shifts in the K^0 system are in a generally unsatisfactory state. For example, an attempt by Rockmore[10] to compute Δm_L using Sakurai's weak-interaction Hamiltonian resulted in values of Δm_L which are negative and $|\Delta m_L|\approx 3|\Delta m|$. In this case $|\Delta m_L/\Delta m|>1$ and we might have $|F|>1$. We make no attempt here to compute Δm_L; however, our model does indicate that it would not be surprising if $|\Delta m_L|$ were large.

The last term in Eq. (3) involving the $|\Delta I|=\frac{3}{2}$ transitions is the term which is computed in the models of Truong[11] and Barshay[12] and more recently by Kamal and Kenny.[13] All of these authors conclude that the $K^0\rightarrow 2\pi(I=2)$ amplitude is not enough to give a value of Reϵ compatible with experiment.

To summarize, the crucial factors for determining the intrinsic magnitude of the CP-violating couplings in ϵ are the mass shifts Δm_L and $\Delta m_S{}^{(2)}$. These quantities cannot be measured and theoretical calculations of Δm_L and $\Delta m_S{}^{(2)}$ are unreliable. However, we will see that ϵ' does not depend on the mass shifts. In deriving Eq. (3), we assume that the CP violation occurs entirely in the weak interactions. If the CP violation occurs in electromagnetic or semistrong interactions, the form of Eq. (3) can be more complicated, but an evaluation of F will still require a knowledge of mass shifts.

In our model we assume that the K_L and K_S mesons appear as bound states in two high-mass channels with opposite eigenvalues of CP. The K_S and K_L are then coupled to the $2\pi(I=0,2)$ and 3π channels, respectively, via the weak interactions. We lump all of the three-body decay channels of K_L into a single channel which we treat as a quasi-two-body channel. The potential term describing the three-body decays can be as complicated as needed to simulate their effect in the vicinity of the K^0 mass. We then introduce T-violating forces to couple the two CP eigenstates. In order to describe this system we need a 5-channel model.

We write the Born (or generalized potential) matrix as[8]

$$B=\left\{\begin{array}{cc} \begin{matrix} B_2{}^{(0)} & 0 & 0\\ 0 & B_2{}^{(2)} & 0\\ 0 & 0 & B_3 \end{matrix} & \begin{pmatrix} g_{2S}{}^{(0)} & ig_{2L}{}^{(0)}\\ g_{2S}{}^{(2)} & ig_{2L}{}^{(2)}\\ ig_{3S} & g_{3L} \end{pmatrix}\dfrac{1}{E+E_0} \\ \dfrac{1}{E+E_0}\begin{pmatrix} g_{2S}{}^{(0)} & g_{2S}{}^{(2)} & -ig_{3S}\\ -ig_{2L}{}^{(0)} & -ig_{2L}{}^{(2)} & g_{3L} \end{pmatrix} & \begin{matrix} B_S & 0\\ 0 & B_L \end{matrix} \end{array}\right\}, \quad (4)$$

where E is the total energy in the center-of-mass system. Since we are interested in computing the amplitude in a narrow energy range, the detailed energy dependence of the Born terms is unimportant, and we expect to be able

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[8] We use the subscripts 2, 3, S, and L to label the 2π, three-body K_S, and K_L channels respectively. Superscripts 0 and 2 are used to label the 2π $I=0$ and $I=2$ channels, respectively. Γ's with one subscript are used to label the total widths.

[9] For estimates of relevant phase-space factors see J. D. Jackson, in *Brandeis Summer Institute 1962 Lectures in Theoretical Physics, Elementary Particle Physics and Field Theory I*, edited by K. W. Ford (Benjamin, New York, 1963), p. 263.

[10] R. Rockmore, Phys. Rev. **185**, 1847 (1969).

[11] T. Truong, Phys. Rev. Letters **13**, 358 (1964).

[12] S. Barshay, Phys. Rev. **149**, 1229 (1966).

[13] A. Kamal and B. Kenny, Phys. Rev. **186**, 1473 (1969).

to approximate any one of them by a simple pole with an appropriately determined residue. The position of the pole is determined essentially by the range of interaction of the forces connecting the initial and final state and by the thresholds of the initial and final states. If we assume that the uncoupled channel in which K_L and K_S appear as bound states are high-mass channels with equal mass (by *CPT* invariance) and that these channels are coupled to the 2π and 3π channels via the short-range weak interactions, then we expect that the pole positions for all the nondiagonal terms can be taken equal. We do not expect the pole position for the strong diagonal interactions to be the same as for the nondiagonal interactions and we leave these Born terms in a general form. In the absence of all nondiagonal forces, the K_L and K_S mesons (by *TCP* invariance) will be produced with the same mass and we expect B_S and B_L to be identical. We also expect that the inequality $\Gamma(K_L\rightarrow 3\pi)\ll\Gamma(K_S\rightarrow 2\pi)$ is primarily due to smaller phase space in the 3π channel and hence g_{3L} and $g_{2S}^{(0)}$ should be comparable in magnitude.[9]

T-violating forces can also be introduced through a direct coupling of the 2π and 3π channels. In order to to obtain the experimental value for $\Gamma(K_L\rightarrow 2\pi)$ with this type coupling, we would have to choose the coupling approximately 10^{-3} times as strong as one of the strong diagonal Born terms. We thus identify a direct coupling between the 2π and 3π channels with *T* violations in electromagnetic[14] or semistrong interactions.[15] We also considered a model of this type. The simplest things to do is to make a pole approximation for the coupling between the 2π and 3π channels, If we take the pole position to be the same as for the weak coupling connecting the 2π and 3π states to K_S and K_L, respectively, then we obtain the result presented in Eq. (3). However, we cannot justify choosing the same pole position to describe both the short-range weak interaction and the longer-range semistrong or electromagnetic interaction. Choosing the pole position at different energies greatly complicates the equations, and we were not able to find a simple expression like Eq. (3) entirely in terms of mass shifts and coupling constants to describe the effect of the interference. The reason for the difficulty is that the Born term coupling the 2π and 3π states has no relation to mass shifts and widths that would exist in the absence of overlap. Simple estimates for the factor F show that if the pole position for the 2π-3π coupling is close to the physical region and the pole position for the weak coupling is much farther away, then $|F|$ can be of the order of one even if $|\Delta m_L/\Delta m|$ is very small. Superweak couplings[16] can also be introduced by considering a direct coupling between K_L and K_S ($g_{LS}\neq 0$). This type of *T* violation is irrelevant for our investigation since it will probably never be seen except in K^0 decay.

We can obtain a complete description of this model by solving the multichannel ND^{-1} equations.[17] We can decouple the diagonal matrix elements of N and D from the nondiagonal matrix elements by making a subtraction in D at $E=E_0$, the pole position of the off-diagonal matrix elements of B. We assume that B_S and B_L produce zeros in D_S and D_L near the K^0 mass according to

$$D_S=D_L=d(m_K-E)\,, \tag{5}$$

where m_K would be the mass of K^0 if it were stable.

The solution is straightforward but tedious, and here we will only state the relevant results of the calculation (see the Appendix). The masses of K_L and K_S will be shifted from m_K according to

$$\begin{aligned}\Delta m_S&=m_S-m_K\\&=-\tfrac{1}{2}\alpha_2^{(0)}\Gamma_{2S}^{(0)}-\tfrac{1}{2}\alpha_2^{(2)}\Gamma_{2S}^{(2)}-\tfrac{1}{2}\alpha_3\Gamma_{3S}\,,\end{aligned} \tag{6}$$

$$\begin{aligned}\Delta m_L&=m_L-m_K\\&=-\tfrac{1}{2}\alpha_3\Gamma_{3L}-\tfrac{1}{2}\alpha_2^{(0)}\Gamma_{2L}^{(0)}-\tfrac{1}{2}\alpha_2^{(2)}\Gamma_{2L}^{(2)}\,.\end{aligned} \tag{7}$$

To a good approximation we can write

$$\Delta m_S=-\tfrac{1}{2}\alpha_2^{(0)}\Gamma_{2S}^{(0)}-\tfrac{1}{2}\alpha_2^{(2)}\Gamma_{2S}^{(2)}\,, \tag{6$'$}$$

$$\Delta m_L=-\tfrac{1}{2}\alpha_3\Gamma_{3L}\,. \tag{7$'$}$$

We define

$$\Delta m_S^{(0)}=-\tfrac{1}{2}\alpha_2^{(0)}\Gamma_{2S}^{(0)}\,,\quad \Delta m_S^{(2)}=-\tfrac{1}{2}\alpha_2^{(2)}\Gamma_{2S}^{(2)}\,. \tag{8}$$

The α's can be computed if the B's are known. For example, for $\alpha_2^{(0)}$ we find

$$\alpha_2^{(0)}=\frac{C_2^{(0)}\cot\delta_0-1}{C_2^{(0)}+\cot\delta_0}\,, \tag{9}$$

where

$$C_2^{(0)}=-(\mathrm{Re}D_{2S}^{(0)})/(\rho_2N_{2S}^{(0)}) \tag{10}$$

and δ_0 is the background $\pi\pi$ $I=0$ phase shift which can be found from $B_2^{(0)}$. ρ_2 is a kinematical factor for the 2π $l=0$ partial wave. $\Gamma_{2L}^{(0)}$ is the partial width for the decay $K_L\rightarrow 2\pi(I=0)$ which would exist in the absence of overlap with K_S. $\Gamma_{2L}^{(0)}$ is also related to the background phase shift in the $2\pi(I=0)$ channel according to

$$\Gamma_{2L}^{(0)}=\gamma_{2L}^{(0)}(C_2^{(0)}\sin\delta_0+\cos\delta_0)^2\,, \tag{11}$$

where $\gamma_{2L}^{(0)}$ would be the partial width in the absence of overlap and background,

$$\gamma_{2L}^{(0)}=\rho_2(g_{2L}^{(0)})^2\varphi_L/[d(E+E_0)]\,, \tag{12}$$

where $\varphi_L=-D_{3L}/g_{3L}$ is independent of g_{3L}. Entirely analogous equations are found for the other widths. We

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[14] J. Bernstein, G. Feinberg, and T. D. Lee, Phys. Rev. **139**, B1650 (1965).

[15] T. D. Lee and L. Wolfenstein, Phys. Rev. **138**, B1490 (1965). The 2π-3π coupling can also be nonzero if the weak interactions contain a $\Delta S=0$, *CP*-violating, *P*-violating, and *G*-parity-violating piece. In this event the 2π-3π coupling will be very small and we neglect this possibility.

[16] L. Wolfenstein, Phys. Rev. Letters **13**, 562 (1964).

[17] For a discussion of the multichannel ND^{-1} equations see J. Fulco, G. Shaw, and D. Wong, Phys. Rev. **137**, B1242 (1965).

note that Truong's[7] expression for the mass shift,

$$\Delta m_S = -\tfrac{1}{2}\Gamma_S \cot\delta_0 \,, \tag{13}$$

is the special case of Eq. (9) where $C_2^{(0)} \gg |\cot\delta|$. We have an additional parameter in our expression for the mass shift, which has the advantage of not predicting an infinite value of Δm as $\delta_0 \to 0$.

We can compute the rate for $K_L \to 2\pi(I=0)$ relative to $K_L \to 3\pi$ by computing the ratio $(T_{L2}^{(0)}/T_{L3})$. We find

$$|T_{L2}^{(0)}/T_{L3}| = \left| \frac{(\Gamma_{2L}^{(0)})^{1/2}(m_K-E) - \frac{1}{2}\Gamma_{3S}^{1/2}[(\Gamma_{2L}^{(0)}\Gamma_{3S})^{1/2} + (\Gamma_{2S}^{(0)}\Gamma_{3L})^{1/2}](\alpha_3+i) + \frac{1}{2}(\Gamma_{2S}^{(2)})^{1/2}[(\Gamma_{2S}^{(0)}\Gamma_{2L}^{(2)})^{1/2} - (\Gamma_{2S}^{(2)}\Gamma_{2L}^{(0)})^{1/2}](\alpha_2^{(2)}+i)}{\Gamma_{3L}^{1/2}(m_K-E) + \frac{1}{2}(\Gamma_{2S}^{(0)})^{1/2}[(\Gamma_{2L}^{(0)}\Gamma_{3S})^{1/2} - (\Gamma_{2S}^{(0)}\Gamma_{3L})^{1/2}](\alpha_2^{(0)}+i) + \frac{1}{2}(\Gamma_{2S}^{(2)})^{1/2}[(\Gamma_{2L}^{(2)}\Gamma_{3S})^{1/2} - (\Gamma_{2S}^{(2)}\Gamma_{3L})^{1/2}](\alpha_2^{(2)}+i)} \right| . \tag{14}$$

The signs are written here as though all the couplings were positive. To obtain the actual sign we must give the $\Gamma^{1/2}$ the sign of the appropriate coupling; e.g., $\mathrm{sgn}(\Gamma_{2S}^{(0)})^{1/2} = \mathrm{sgn}(g_{2S}^{(0)})$. If we use the inequalities $\Gamma_{2S}^{(0)} \gg \Gamma_{3L} \gg \Gamma_{2L}^{(2)} \gg \Gamma_{3S}$ (assuming that $g_{2L}^{(2)}$ is of the same order of magnitude as g_{3S}) and evaluate Eq. (14) at $E = m_L$, we obtain

$$|T_{L2}^0/T_{L3}| = (\Gamma_{2L}^{(0)}/\Gamma_{3L})^{1/2}|F| \,, \tag{15}$$

where F is given by Eq. (3) as follows:

$$F = (i\Delta m + \tfrac{1}{2}\Gamma_S)^{-1}\left\{ \Delta m_L \left[1 - \left(\frac{\Gamma_{2S}^{(0)}\Gamma_{3S}}{\Gamma_{2L}^{(0)}\Gamma_{3S}} \right)^{1/2} \right] - (\Delta m_S^{(2)} - \tfrac{1}{2} i \Gamma_{2S}^{(2)}) \left[1 - \left(\frac{\Gamma_{2S}^{(0)}\Gamma_{2L}^{(2)}}{\Gamma_{2L}^{(0)}\Gamma_{2S}^{(2)}} \right)^{1/2} \right] \right\} . \tag{3}$$

We have neglected terms of order $(\Gamma_{2L}^{(0)}/\Gamma_{2S}^{(0)})$ in obtaining Eq. (3). We interpret $|F|^2$ as the change in the partial width $\Gamma(K_L \to 2\pi(I=0))$ from $\Gamma_{2L}^{(0)}$ which would be observed in the absence of the overlap of K_L and K_S.[18]

A commonly used way to compute mass shifts is to write an unsubtracted dispersion relation for the mass shift in terms of the width of the interaction.[7] This procedure leads to the qualitative estimate $|\Delta m_L| \ll |\Delta m_S|$. In our model, Δm_L is determined by the product $\alpha_3\Gamma_{3L}$. Since the effective two-body threshold for the three-body decay channel of K_L is expected to be near the K^0 mass, we expect the effective background phase shift δ_3 will be small. Then $\alpha_3 \approx C_3$ (if $|\cot\delta_3| \gg |C_3|$). If Γ_{3L} is small for kinematical reasons, then C_3 is large for the same reasons and the product $C_3\Gamma_{3L}$ can be large (on the same order of magnitude as $\alpha_2^{(0)}\Gamma_{2S}^{(0)}$). For $\delta_3 \neq 0$ the mass shift can still be sizable, with sign depending on δ_3 as well as C_3.[19]

We compute the expressions for ϵ and ϵ' and find

$$\epsilon = \left(\frac{\Gamma_{2L}^{(0)}}{\Gamma_{2S}^{(0)}} \right)^{1/2} F \,, \tag{16}$$

$$\epsilon' = \frac{ie^{i(\delta_2-\delta_0)}}{\sqrt{2}} \left[\left(\frac{\Gamma_{2L}^{(2)}}{\Gamma_{2S}^{(0)}} \right)^{1/2} - \frac{(\Gamma_{2S}^{(2)}\Gamma_{2L}^{(0)})^{1/2}}{\Gamma_{2S}^{(0)}} \right] . \tag{17}$$

As noted earlier, ϵ' is independent of the (nonmeasurable) mass shifts. Thus an accurate measurement of ϵ' is very important. However, unless the two terms in Eq. (17) add, or one dominates, it is still difficult to estimate the intrinsic CP-violation strengths.

APPENDIX

Many of the results quoted in the main body of this paper can be derived in a two-channel model. The generalization to more channels is straightforward.

We will review briefly the multichannel ND^{-1} equations.[17] We assume that the unphysical cut terms B are known. Then the scattering amplitude may be written as (E = total center-of-mass energy)

$$A = ND^{-1} \,, \tag{A1}$$

where

$$N_{ij}(E) = B_{ij}(E) + \sum_k \frac{1}{\pi} \int_{E_k}^{\infty} \left[B_{ik}(E') - \frac{E-E_S}{E'-E_S} B_{ik}(E) \right] \times \frac{\rho_k(E')N_{kj}(E')}{E'-E} dE' \tag{A2}$$

and

$$D_{ij}(E) = \delta_{ij} - \frac{E-E_S}{\pi} \int_{E_i}^{\infty} \frac{\rho_i(E')N_{ij}(E')}{E'-E_S} \frac{dE'}{E'-E-i\epsilon} . \tag{A3}$$

We have normalized $D_{ij}(E_S) = \delta_{ij}$ by making a subtraction in the D-function at $E = E_S$. E_i is the threshold for the ith channel and ρ_i is a kinematical factor.

We assume [see discussion after Eq. (4)] a Born matrix of the form

$$B = \begin{pmatrix} B_{11}(E) & g_{12}/(E+E_0) \\ g_{12}/(E+E_0) & B_{22}(E) \end{pmatrix} , \tag{A4}$$

where the second channel is a high-mass channel which would have a bound state at $E = M_B$ in the absence of

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[18] To interpret F, we keep the derivation of Eq. (14) general and do not require that the stable masses of K_L and K_S be identical. Then the m_K in Eq. (14) is the stable mass of K_S, m_{K_S}. We can eliminate interference effects by formally separating m_{K_S} and m_{K_L} by many widths Γ_S. In doing this, we obtain the result $|T_{L2}^{(0)}/T_{L3}| = (\Gamma_{2L}^0/\Gamma_{3L})^{1/2}$. The factor F is therefore the change in this ratio due to the interference.

[19] The C's can only be computed by using a model. For a simple calculation assuming a pole approximation without diagonal forces see Ref. 4.

coupling to channel 1. We can simplify the equation by making the subtraction in D at $E=-E_0$. Then the N-functions can be written as

$$N_{ii}(E)=B_{ii}(E)+\frac{1}{\pi}\int_{E_i}^{\infty} B_{ii}(E')-\frac{E+E_0}{E'+E_0}B_{ii}(E) \times\frac{\rho_i(E')N_{ii}(E')}{E'-E}dE' \quad \text{(A5)}$$

and for $i\neq j$

$$N_{ij}(E)=g_{ij}F_i(E)\,, \quad \text{(A6)}$$

where

$$F_i(E)=\frac{1}{E+E_0}+\frac{1}{\pi}\int_{E_i}^{\infty} B_{ii}(E')-\frac{E+E_0}{E'+E_0}B_{ii}(E) \times\frac{\rho_i(E')F_i(E')}{E'-E}dE'\,. \quad \text{(A7)}$$

The D-functions are found from Eq. (A3). For the off-diagonal D-functions, we may write

$$D_{ij}(E)=-g_{ij}\varphi_i(E)\,, \quad \text{(A8)}$$

where

$$\varphi_i(E)=\frac{E+E_0}{\pi}\int_{E_i}^{\infty}\frac{\rho_i(E')F_i(E')}{E'+E_0}\,\frac{dE'}{E'-E-i\epsilon}\,. \quad \text{(A9)}$$

In the energy range of interest ($E\approx m_B\ll E_2$, where φ is real), we separate φ_1 into its real and imaginary parts:

$$\varphi_1(E)=C_1(E)+i\rho_1F_1(E)\,. \quad \text{(A10)}$$

We assume that for $E\approx m_B$ it is a good approximation to write

$$D_{22}(E)=d(m_B-E)\,. \quad \text{(A11)}$$

After making the approximation in Eq. (A11), we find that the S-matrix element for the first channel may be written as

$$S_{11}=e^{2i\delta}\frac{m_R-E+i\frac{1}{2}\Gamma}{m_R-E-i\frac{1}{2}\Gamma}\,, \quad \text{(A12)}$$

where

$$\rho_1N_{11}/D_{11}=e^{i\delta}\sin\delta\,, \quad \text{(A13)}$$

$$\Gamma=\Gamma_1[(c\sin\delta+\cos\delta)F_1(E+E_0)]^2\,, \quad \text{(A14)}$$

$$c=C_1/(\rho_1F_1)\,, \quad \text{(A15)}$$

$$\Gamma_1=2\rho_1g_{12}{}^2\varphi_2/[d(E+E_0)]\,, \quad \text{(A16)}$$

$$m_R=m_B-\tfrac{1}{2}\alpha\Gamma\,, \quad \text{(A17)}$$

$$\alpha=(c\cot\delta-1)/(c+\cot\delta)\,, \quad \text{(A18)}$$

where Γ_1 is the width the resonance would have if the background phase shift δ were zero. Equations (6) and (7) are generalizations of Eq. (A17) for the case of three open channels and two closed channels.

If we generalize to a 5-channel problem with a Born matrix given by Eq. (4), the relevant N and D matrix elements are still determined from equations similar to Eqs. (A6), (A7), (A3), (A8), and (A9). The N and D matrices will have the form

$$N=\begin{bmatrix} N_2{}^{(0)} & 0 & 0 & g_{2S}{}^{(0)}F_2{}^{(0)} & ig_{2L}{}^{(0)}F_2{}^{(0)} \\ 0 & N_2{}^{(2)} & 0 & g_{2S}{}^{(2)}F_2{}^{(2)} & ig_{2L}{}^{(2)}F_2{}^{(2)} \\ 0 & 0 & N_3 & ig_{3S}F_3 & g_{3L}F_3 \\ g_{2S}{}^{(0)}F_S & g_{2S}{}^{(0)}F_S & -ig_{3S}F_S & N_S & 0 \\ -ig_{2L}{}^{(0)}F_L & -ig_{2L}{}^{(2)}F_L & g_{3L}F_L & 0 & N_L \end{bmatrix}, \quad \text{(A19)}$$

$$D=\begin{bmatrix} D_2{}^{(0)} & 0 & 0 & -g_{2S}{}^{(0)}\varphi_2{}^{(0)} & -ig_{2L}{}^{(0)}\varphi_2{}^{(0)} \\ 0 & D_2{}^{(2)} & 0 & -g_{2S}{}^{(2)}\varphi_2{}^{(2)} & -ig_{2L}{}^{(2)}\varphi_2{}^{(2)} \\ 0 & 0 & D_3 & -ig_{3S}\varphi_3 & -g_{3L}\varphi_3 \\ -g_{2S}{}^{(0)}\varphi_S & -g_{2S}{}^{(2)}\varphi_S & +ig_{3S}\varphi_S & D_S & 0 \\ +ig_{2L}{}^{(0)}\varphi_L & +ig_{2L}{}^{(2)}\varphi_L & -g_{3L}\varphi_L & 0 & D_L \end{bmatrix}, \quad \text{(A20)}$$

where $\varphi_2{}^{(0)}$, $\varphi_2{}^{(2)}$, and φ_3 are complex in the K^0 region while φ_S and φ_L ($\varphi_S=\varphi_L$ in this case) are real. The diagonal N-functions are found from Eq. (A5) while the F's are given by Eq. (A7). The D-functions are found from Eqs. (A3), (A8), and (A9). The expressions for the partial widths for decay into a given channel are given by Eqs. (A14) and (A16) while the mass shifts are a sum of partial mass shifts of the form determined in Eqs. (A17) and (A18).

In order to find the ratio $|T_{L2}{}^{(0)}/T_{L3}|$ in Eq. (14) we must invert the D matrix, perform the appropriate matrix multiplications, and rewrite the amplitudes by using the definitions of the widths. The result is Eq. (14).

Once we have found the amplitudes $T_{L2}{}^{(0)}$, $T_{L2}{}^{(2)}$, $T_{S2}{}^{(0)}$, and $T_S{}^{(2)}$, we can compute the amplitudes $A(K_L\to\pi^{+,0}\pi^{-,0})$ and $A(K_S\to\pi^{+,0}\pi^{-,0})$ by taking the appropriate isospin combinations. We can then calculate ϵ and ϵ' [as given in Eqs. (16) and (17)] by using the relations

$$\eta_{+-}=A(K_L\to\pi^+\pi^-)/A(K_S\to\pi^+\pi^-)=\epsilon+\epsilon'\,, \quad \text{(A21)}$$

$$\eta_{00}=A(K_L\to\pi^0\pi^0)/A(K_S\to\pi^0\pi^0)=\epsilon-2\epsilon'\,. \quad \text{(A22)}$$