

General Local Interactions and Tests of $V-A$ Theory in Neutrino Scattering Processes*

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(Received 28 September 1970)

Neutrino scattering processes are studied in the framework of general local current-current interaction. The scattering amplitudes for such processes are expressed in a simple factorized form in terms of helicity form factors for the current vertex functions. These expressions explicitly display the full kinematic content of the local current-current interaction and are used as the basis for a systematic study of possible ways to test the $V-A$ structure of the weak current in high-energy neutrino scattering processes. A direct test in an "inclusive experiment" consists of measuring the outgoing lepton polarization. Alternative tests from angular and spin correlations are possible only in "exclusive experiments." Several examples of this latter type of experiment are given: (i) pure lepton scattering processes, (ii) neutrino scattering off spin-zero (nuclei) targets, (iii) quasi-elastic scattering off polarized-nucleon targets, (iv) quasi-elastic hyperon production (decay asymmetry), and (v) single-pion production in the N^* region.

I. INTRODUCTION

ALL present experimental evidence in weak decay processes is consistent with the $V-A$ theory of weak interactions.¹ The weak vector (V) and axial-vector (A) currents played an essential role in the remarkable theoretical developments beginning with the conserved-vector-current hypothesis and culminating in the successes of "current algebra."^{1,2}

The weak decay processes all involve rather limited ranges in the energy and momentum-transfer variables. The neutrino scattering processes, which are just becoming experimentally feasible (both in existing laboratories and with the projected new generation of accelerators) promise to extend the range of these variables to entirely new territories. This will open up vast domains of the weak interaction hitherto unavailable for our scrutiny.³ The first obvious questions are whether at these high energies and large momentum transfers, these processes are still describable by an effective current-current local interaction Hamiltonian and, if so, whether this interaction is still a V and A combination. These questions must, in principle, be answered by experiments in the affirmative before further comparison

with more detailed theoretical predictions based on this basic structure together with other dynamical assumptions can be made truly meaningful.

Consequences of the local $V-A$ current-current interaction in neutrino scattering processes have been studied before.^{4,5} Pais and Treiman, in particular, have exhibited the "full content" of the locality assumption for such processes by giving the energy and angular distributions for the outgoing lepton.⁵ Assuming the most general form of local (nonderivative) interaction, we study in this paper in considerable detail the resulting energy and angle spectrums as well as angular and spin correlations in an *arbitrary* neutrino scattering process with the specific aim of seeking particular cases in which the presence of any local scalar (S), pseudo-scalar (P), and tensor (T) admixture to the $V-A$ interaction can be experimentally detected.

The systematic investigation of *all* possible processes of this type for our particular purpose is facilitated by a helicity-like formalism^{6,7} which (a) suggests the most natural variables to use in analyzing such processes, (b) defines helicity form factors for arbitrary current vertices⁷ which are natural generalizations of the familiar G_E and G_M form factors for nucleon electromagnetic current vertex, and (c) allows us to write down a general expression for the scattering amplitude which compactly displays all the kinematical contents of current-current interaction in a factorized form (in terms of the helicity form factors) and leads to expres-

* Supported in part by the U. S. Atomic Energy Commission, under Contract No. AT(11-1)-264.

† Research sponsored by the National Science Foundation under Grant No. GP-16147.

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¹ For a review and references, see, for instance, T. D. Lee and C. S. Wu, *Annual Review of Nuclear Sciences* (Annual Reviews, Palo Alto), Vol. 15 (1965) and Vol. 16 (1966); or R. Marshak, Riazuddin, and C. P. Ryan, *Weak Interactions in Particle Physics* (Wiley-Interscience, New York, 1969).

² For a review and references, see S. L. Adler and R. F. Dashen, *Current Algebra* (Benjamin, New York, 1968).

³ For a recent review, see A. Pais, in Proceedings of the Conference on Expectations for Particle Reactions at the New Accelerators, Madison, Wis., 1970 (unpublished).

⁴ T. D. Lee and C. N. Yang, *Phys. Rev.* **126**, 2239 (1963).

⁵ A. Pais and S. Treiman, Contribution to the *Anniversary Volume Dedicated to N. N. Bogoliubov* ("Nauka," Moscow, 1969).

⁶ I. Muzinich, J. M. Wang, and L. L. Wang, *Phys. Rev. D* **2**, 1985 (1970).

⁷ The essence of the method used in the present paper is the same as that of Ref. 6 although the role played by the form factors is largely by-passed in the latter.

sions for angular and spin correlation functions in the form of simple matrix products.

In Sec. II we spell out our basic assumptions. In Sec. III we introduce the helicity expansion for the current vertices functions. In Sec. IV we give the general expressions for the transition amplitude and intensity distribution of an arbitrary neutrino scattering process with general local interactions. It is pointed out, as is undoubtedly known to others, that if the lepton mass can be neglected, the most unambiguous way to distinguish the $V-A$ interaction from the other types (S , T , or P) is to look at the polarization of the outgoing lepton. For neutrino-initiated reactions, this should be purely left-handed if $V-A$ is the only interaction involved and purely right-handed if any combination of S , T , and P interaction is responsible. For antineutrino-initiated reactions, the result is the converse. Since the measurement of lepton (mainly muon) polarization at very high energies is an extremely difficult task, we examine possibilities for testing the $V-A$ theory through angular and (target) spin correlation experiments. The method used is explained at the end of Sec. IV and specific examples are given in Sec. V. These neutrino processes include (i) pure leptonic scatterings, (ii) quasi-elastic scattering off spin-zero (nuclear) targets, (iii) quasi-elastic scattering off polarized-nucleon targets, (iv) decay angular correlations in hyperon production, and (v) angular correlations in single-pion production in the N^* region. These tests involve the measurement of the outgoing lepton angular distribution (one variable only) in either the differential cross section [(i) and (ii)] or in certain asymmetry functions [(iii)-(v)]. For maximum statistics, data obtained at different incident energies can be combined and integrated over to obtain the needed spectrum.

For high-energy scattering processes, the lepton mass can, for all practical purposes, be neglected. In Appendix A we briefly indicate the lepton-mass correction effects. In Appendix B we describe some properties of the helicity form factors and give the explicit relations to the conventional invariant form factors for the case of spin- $\frac{1}{2}$ particles. In Appendix C we give some detailed formulas on the single-pion-production process discussed in the text.

II. BASIC ASSUMPTIONS

We are interested in the neutrino scattering processes (Fig. 1)

$$\left\{ \begin{array}{c} \nu \\ \bar{\nu} \end{array} \right\} + A \rightarrow \left\{ \begin{array}{c} l \\ \bar{l} \end{array} \right\} + B, \quad (1)$$

where l stands for either the electron or muon and A and B can be leptons (pure leptonic processes) or hadronic systems (semileptonic processes). In the latter case, A is usually a single-particle state while B may be a single-particle or multiparticle hadronic system with

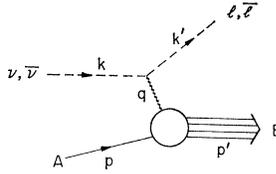


FIG. 1. A general neutrino scattering process.

or without additional lepton pairs. For definiteness, we shall consider explicitly the neutrino-initiated processes [first line in (1)] and refer to the A , B systems as hadrons; all considerations obviously remain valid for the antineutrino-initiated reactions and for the case where A , B are leptons with little change in the resulting formulas. We shall remark on the necessary changes at the appropriate places. We denote by k , k' , p , p' the 4-momenta and λ , λ' , σ , σ' the polarization indices of the states ν , l , A , and B , respectively. In general, charge labels will be omitted for conservation of indices.

We assume a general local current-current interaction. The transition amplitude for the process (1) (first line) is of the form

$$f = (G/\sqrt{2}) \langle k'\lambda' | j^\dagger(0) | k\lambda \rangle \langle p'\sigma' | J(0) | p\sigma \rangle, \quad (2)$$

where $j(x)$ and $J(x)$ are the weak currents associated with the ν - l (leptonic) and A - B ("hadronic") vertices, respectively. These currents can be an arbitrary combination of the five types of possible currents— S , V , T , A , and P .

Since the incoming neutrinos usually come from pion (or K -meson) decay in flight and are known to be purely left-handed, we can assume they are described by the two-component theory with

$$\langle k'\lambda' | j^\dagger(0) | k\lambda \rangle = \bar{u}_{\lambda'}(k') \Gamma(1+\gamma_5) u_\lambda(k), \quad (3)$$

where Γ stands for some combination of 1, γ^μ , and $\sigma^{\mu\nu}$ and $\lambda \equiv -\frac{1}{2}$ because of the factor $(1+\gamma_5)$. Since Eq. (3) automatically implies parity nonconservation in these processes, the distinction between the scalar and pseudo-scalar currents as well as the vector and axial-vector currents at the hadronic vertex [Eq. (2)] becomes unnecessary. From now on, J stands for some combination of S (scalar as well as pseudoscalar), V (vector as well as axial-vector), and T (tensor) currents, and Eq. (2) becomes

$$f_{\lambda'\sigma',\sigma} = (G/\sqrt{2}) \left[\langle k'\lambda' | s^\dagger | k, -\frac{1}{2} \rangle \langle p'\sigma' | S | p\sigma \rangle + \langle k'\lambda' | v_\mu^\dagger | k, -\frac{1}{2} \rangle \langle p'\sigma' | V^\mu | p\sigma \rangle + \langle k'\lambda' | t_{\mu\nu}^\dagger | k, -\frac{1}{2} \rangle \langle p'\sigma' | T^{\mu\nu} | p\sigma \rangle \right]. \quad (4)$$

The first factor of each term is given by an expression of the form (3).

We shall neglect the lepton mass for most of our considerations since we are mainly interested in high-energy regions where the existing theory has not been tested before. The lepton mass can be easily incorporated, however, in the ensuing considerations. Appendix A indicates how this can be done should it become neces-

sary. We shall not consider radiative corrections which modify strict locality, nor shall we consider second-order weak interaction effects for which we know of no reliable estimates.

To begin with, we study the kinematical structure of each of the current vertex functions appearing in Eq. (4)

III. CURRENT VERTEX FUNCTIONS

The current vertex function⁸ $\langle p'\lambda' | J(0) | p\lambda \rangle$ is related to the decay current matrix element $\langle p'\lambda', \bar{p}\bar{\lambda} | J(0) | 0 \rangle$ by crossing. It is a familiar fact that in such decay processes, the kinematics of the angular and spin correlations are much simplified if the transition amplitudes are expressed in terms of the *center-of-mass* (c.m.) variables of the (p', \bar{p}) pair and if $\lambda', \bar{\lambda}$ are chosen to be the helicity indices.⁹ This suggests that for the matrix element $\langle p'\lambda' | J(0) | p\lambda \rangle$, it is most natural to choose our variables in the *brick-wall* (BW) frame in which the spacelike 4-momentum transfer $q = p - p'$ is of the standard form $(0, 0, 0, \sqrt{q^2})$. Referring back to Eq. (2) or (4), because of the relation $q = p - p' = k' - k$ we see that both current vertices can be simplified at the same time by this choice. Furthermore, since the transition amplitude f is a Lorentz invariant (j and J are always contracted), there is no loss of generality in choosing a particular frame to evaluate the vertex functions.

A. Definition of States

In analogy to the well-known Jacob-Wick⁹ c.m. system helicity states, we proceed to define¹⁰ the particle states $|p\lambda\rangle$ and $|p'\lambda'\rangle$ in the BW frame as follows: the *standard* BW-frame state for the "incoming" (or first) particle is defined to be

$$|p_s\lambda\rangle = B_3(u) |0\lambda\rangle, \quad (5)$$

while that for the "outgoing" (or second) particle is

$$|p'_s\lambda'\rangle = B_3(-u') |0, -\lambda'\rangle. \quad (6)$$

Here $B_3(u)$ denotes a boost along the positive 3-direction characterized by the hyperbolic angle u , and $|0, \lambda\rangle$ are the usual rest-frame angular momentum states. The standard BW-frame vectors (p_s, p'_s) are of the form

$$\begin{aligned} p_s &= M(\cosh u, 0, 0, \sinh u), \\ p'_s &= W(\cosh u', 0, 0, -\sinh u'), \end{aligned} \quad (7)$$

with $M^2 = -p^2$, $W^2 = -p'^2$ and

$$\begin{aligned} M \sinh u &= (q^2 - M^2 + W^2)/2\sqrt{q^2}, \\ W \sinh u' &= (q^2 - W^2 + M^2)/2\sqrt{q^2}, \end{aligned} \quad (8)$$

⁸ For earlier studies of the current vertex functions see, for example, L. Durand, III, P. C. DeCelles, and R. B. Marr, Phys. Rev. **126**, 1882 (1962); M. Scadron, *ibid.* **165**, 1640 (1968); M. Bander, *ibid.* **173**, 1568 (1968); T. L. Trueman, *ibid.* **182**, 1569 (1969).

⁹ M. Jacob and G. C. Wick, Ann. Phys. (N. Y.) **7**, 404 (1959).

¹⁰ The choice of definition of states used here is one of the possibilities enumerated in J. Strathdee, J. F. Boyce, R. Delbourgo, and A. Salam, Trieste Report No. IC/67/9 (unpublished).

so that $q = p - p'$ is of the standard form $(0, 0, 0, \sqrt{q^2})$. A *general* configuration of the vectors (p, p') in the BW frame can be obtained from the standard vectors, Eq. (7), by a *common* $SO(2, 1)$ transformation which leaves the vector q invariant (i.e., Lorentz transformations involving the 0, 1, 2 axes only). Denoting this transformation by $O(\psi, \phi)$, we have

$$\begin{aligned} |p\lambda\rangle &= O(\psi, \phi) |p_s\lambda\rangle, \\ |p'\lambda'\rangle &= O(\psi, \phi) |p'_s\lambda'\rangle, \end{aligned} \quad (9)$$

where $O(\psi, \phi)$ is chosen to be a boost along the 1-axis by the (hyperbolic) angle ψ followed by a rotation around the 3-axis by the angle ϕ , i.e.,

$$O(\psi, \phi) = R_3(\phi) B_1(\psi). \quad (10)$$

The general BW-frame momenta (p, p') are therefore parametrized as¹⁰

$$\begin{aligned} p &= M(\cosh u \cosh \psi, \cosh u \sinh \psi \cos \phi, \\ &\quad \cosh u \sinh \psi \sin \phi, \sinh u), \\ p' &= W(\cosh u' \cosh \psi, \cosh u' \sinh \psi \cos \phi, \\ &\quad \cosh u' \sinh \psi \sin \phi, -\sinh u'). \end{aligned} \quad (11)$$

With these definitions, let us turn to the various vertex functions that enter Eqs. (2) and (4).

B. Scalar Current

Using definition (9), the scalar vertex function in the brick-wall frame can be written

$$\begin{aligned} \langle p'\lambda' | S | p\lambda \rangle &= \langle p'_s\lambda' | O^{-1}(\psi, \phi) SO(\psi, \phi) | p_s\lambda \rangle \\ &= \langle p'_s\lambda' | S | p_s\lambda \rangle = S_{\lambda'\lambda}(q^2), \end{aligned} \quad (12)$$

where we used the fact that $O^{-1}SO = S$ (S is a scalar) and that p_s and p'_s depend only on the invariant variable q^2 , Eq. (7). The second line in Eq. (12) defines the "scalar form factor" or "reduced matrix element" $S_{\lambda'\lambda}(q^2)$. This equation shows that the vertex function $\langle p'\lambda' | S | p\lambda \rangle$ is independent of the variables ψ and ϕ . It is shown in Appendix B that

$$S_{\lambda'\lambda}(q^2) = \delta_{0, \lambda + \lambda'} S_\lambda(q^2), \quad (13)$$

which expresses angular momentum conservation.

C. Vector Currents

Let the unit vectors in the BW frame be $\{e_{(0)}^\mu, e_{(1)}^\mu, e_{(2)}^\mu, e_{(3)}^\mu\}$. We define

$$e_{(\pm 1)}^\mu = (\mp e_{(1)}^\mu - i e_{(2)}^\mu) / \sqrt{2} \quad (14)$$

and use the set $\{e_{(\alpha)}^\mu; \alpha = +1, 0, -1, 3\}$ as our basis vectors. They satisfy the orthonormality and completeness conditions:

$$\begin{aligned} e_{(\alpha)}^{\mu*} e_{(\beta)}^\mu &= \delta_\alpha^\beta, \\ e_{(\alpha)}^{\mu} e_{(\alpha)}^{\nu*} &= g^{\mu\nu}, \end{aligned} \quad (15)$$

where $e^{(0)} = -e_{(0)}$ and $e^{(i)} = e_{(i)}$, $i = \pm 1, 3$. The vector

current can be written as

$$V^\mu = e_{(\alpha)}^\mu V^{(\alpha)} = e_{(3)}^\mu V^{(3)} + e_{(m)}^\mu V^{(m)}, \quad (16)$$

where $m = +1, 0, -1$ and $V^{(\alpha)} = e^{(\alpha)\mu} V_\mu$. In analogy with the scalar case, we can now write down the general expression for the vector vertex function in the BW frame:

$$\langle p'\lambda' | V^\mu(0) | p\lambda \rangle = \langle p_s'\lambda' | O^{-1}(\psi, \phi) V^\mu(0) O(\psi, \phi) | p_s\lambda \rangle \\ = e_{(\alpha)}^\mu D(\psi, \phi)^\alpha_\beta V_{\lambda'\lambda}^{(\beta)}(q^2), \quad (17)$$

where

$$V_{\lambda'\lambda}^{(\alpha)}(q^2) = \langle p_s'\lambda' | V^{(\alpha)}(0) | p_s\lambda \rangle, \quad (18)$$

$$D(\psi, \phi)^\alpha_\beta = 1, \quad D(\psi, \phi)^\alpha_m = D(\psi, \phi)^m_\alpha = 0,$$

$$D(\psi, \phi)^m_n = e^{-im\phi} \bar{d}(\psi)^m_n, \quad (19)$$

and

$$\bar{d}(\psi)^m_n = \begin{bmatrix} \frac{1}{2}(1 + \cosh\psi) & -\sinh\psi/\sqrt{2} & \frac{1}{2}(1 - \cosh\psi) \\ -\sinh\psi/\sqrt{2} & \cosh\psi & \sinh\psi/\sqrt{2} \\ \frac{1}{2}(1 - \cosh\psi) & \sinh\psi/\sqrt{2} & \frac{1}{2}(1 + \cosh\psi) \end{bmatrix}.$$

Equation (17) is an expansion of the vertex functions in terms of the form factors $V_{\lambda'\lambda}^{(\beta)}$, which depend only on the invariant variable q^2 (together with possible "internal variables" if the final state is a complex system). The (ψ, ϕ) dependence of the vector vertex function is explicitly exhibited in the D -functions. Angular-momentum-conservation conditions again impose the constraints:

$$V_{\lambda'\lambda}^{(3)}(q^2) = \delta_{0, \lambda+\lambda'} V_{\lambda'\lambda}^{(3)}(q^2), \\ V_{\lambda'\lambda}^{(m)}(q^2) = \delta_{m, \lambda+\lambda'} V_{\lambda'\lambda}^{(m)}(q^2), \quad m = +1, 0, -1. \quad (20)$$

The last equation indicates that the index m has the physical interpretation of being the "helicity" of the current (with 4-momentum q).

D. Tensor Current

We define basis tensors¹¹ $\{e_{(m)}^{\mu\nu}, \tilde{e}_{(m)}^{\mu\nu}; m = +1, 0, -1\}$:

$$e_{(m)}^{\mu\nu} = i[e_{(3)}^\mu e_{(m)}^\nu - e_{(m)}^\mu e_{(3)}^\nu]/\sqrt{2}, \quad (21) \\ \tilde{e}_{(m)}^{\mu\nu} = -\frac{1}{2}i\epsilon^{\mu\nu\lambda\sigma} e_{(m)\lambda\sigma}.$$

They satisfy the orthonormality and completeness conditions:

$$e_{(m)}^* \cdot e^{(n)} = -\tilde{e}_{(m)}^* \cdot \tilde{e}^{(n)} = \delta_m^n, \quad (22) \\ e_{(m)}^* \cdot \tilde{e}^{(n)} = \tilde{e}_{(m)}^* \cdot e^{(n)} = 0,$$

$$e_{(m)\mu\nu} e_{(m)\lambda\sigma}^* - \tilde{e}_{(m)\mu\nu} \tilde{e}_{(m)\lambda\sigma}^* = \frac{1}{2}(g_{\mu\lambda}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\lambda}). \quad (23)$$

It is easy to see that under the $SO(2,1)$ transformations (10), the two sets $\{e_{(m)}^{\mu\nu}\}$ and $\{\tilde{e}_{(m)}^{\mu\nu}\}$ transform separately as vectors in the $(0,1,2)$ subspace. Thus, for example,

$$O(\psi, \phi)_\lambda^\mu O(\psi, \phi)_\sigma^\nu e_{(m)\lambda\sigma} = e_{(m)\mu\nu} D(\psi, \phi)^n_m, \quad (24)$$

where $D(\psi, \phi)$ is given by Eq. (19).

¹¹ The factors of i adopted in the following definitions are to render the resulting form factors real whenever time-reversal invariance can be applied.

The antisymmetric tensor current operator may be written as

$$T^{\mu\nu} = e_{(m)}^{\mu\nu} T^{(m)} + \tilde{e}_{(m)}^{\mu\nu} \tilde{T}^{(m)}, \quad (25)$$

where

$$T_{(m)} = e_{(m)}^{\mu\nu} T_{\mu\nu}, \quad (26) \\ \tilde{T}_{(m)} = -\tilde{e}_{(m)}^{\mu\nu} T_{\mu\nu}.$$

Using (24) and (25) we obtain

$$\langle p'\lambda' | T^{\mu\nu}(0) | p\lambda \rangle = e_{(m)}^{\mu\nu} D(\psi, \phi)^m_n T_{\lambda'\lambda}^{(n)}(q^2) \\ + \tilde{e}_{(m)}^{\mu\nu} D(\psi, \phi)^m_n \tilde{T}_{\lambda'\lambda}^{(n)}(q^2), \quad (27)$$

where the tensor form factors are defined by

$$T_{\lambda'\lambda}^{(m)}(q^2) = \langle p_s'\lambda' | T^{(m)}(0) | p_s\lambda \rangle, \quad (28) \\ \tilde{T}_{\lambda'\lambda}^{(m)}(q^2) = \langle p_s'\lambda' | \tilde{T}^{(m)}(0) | p_s\lambda \rangle.$$

Again we have the constraints

$$T_{\lambda'\lambda}^{(m)}(q^2) = \delta_{m, \lambda+\lambda'} T_{\lambda'\lambda}^{(m)}(q^2), \quad (29) \\ \tilde{T}_{\lambda'\lambda}^{(m)}(q^2) = \delta_{m, \lambda+\lambda'} \tilde{T}_{\lambda'\lambda}^{(m)}(q^2).$$

E. Remarks

(a) The expansions (12), (17), and (27) separate out the dependence of the vertex function on the (ψ, ϕ) variables in the form of D functions (reflecting the Lorentz-transformation properties of the specific current involved) from the dependences on q^2 and other internal variables which are determined by dynamics.

(b) What we have called $S_{\lambda'\lambda}$, $V_{\lambda'\lambda}^{(3)}$, $V_{\lambda'\lambda}^{(m)}$, $T_{\lambda'\lambda}^{(m)}$, and $\tilde{T}_{\lambda'\lambda}^{(m)}$ can in a certain sense be regarded as the "helicity amplitudes" for the process⁶ (Fig. 2)

$$J + A \rightarrow B, \quad (30)$$

on account of the physical meaning given to the index (m) . For convenience, let us call them *helicity form factors*. These form factors are the natural generalizations of the familiar G_E and G_M form factors to general currents and arbitrary states A and B . They diagonalize the (unpolarized) intensity distribution function for the general process (1) (as will be shown in Sec. IV) and, when q^2 is small and a nonrelativistic reduction procedure makes sense, are simply related to the familiar *multipole moments* with the associated physical interpretations.¹² Some further properties of these form factors are briefly enumerated in Appendix B.

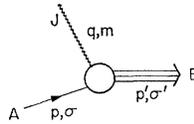


FIG. 2. The effective current-hadron scattering process.

¹² For a discussion of G_E and G_M , see F. J. Ernst, R. G. Sachs, and K. C. Wali, Phys. Rev. **119**, 1105 (1960). Generalization of G_E and G_M to the arbitrary-spin case and the relation to multipole moments are studied by Durand *et al.*, Ref. 8. However, this paper used the Jacob-Wick definition of the states and chose a BW frame in which $p_3 = -p_3'$ (instead of $p_3 = p_3' = \sqrt{q^2}$ and $p_0 = p_0'$). As a result, many of the appealing features of the helicity form factors are lost.

We should emphasize, however, the polarization indices λ, λ' are helicities in the BW frame and should not be identified with the Jacob-Wick helicities⁹ defined for two-particle states in the c.m. frame for scattering processes.

IV. TRANSITION AMPLITUDE AND INTENSITY DISTRIBUTIONS

We can now use the results of Sec. III to evaluate the transition amplitude, Eq. (4), for the general local interaction. In the brick-wall frame we can choose the coordinate axes such that the momenta (p, p') associated with the A - B vertex are of the *standard* form (p_s, p'_s), Eq. (7). Then, using (12), (17), and (27) for the current vertices and the orthonormality conditions for the basis vectors, we obtain

$$\begin{aligned} f_{\lambda'\sigma',\sigma}(\psi, \phi, q^2, \dots) &= (G/\sqrt{2}) j_{\lambda,(\beta)}^*(q^2) D^*(\psi, \phi) \alpha_{\beta} J_{(\alpha)\sigma'\sigma}(q^2, \dots) \\ &= (G/\sqrt{2}) \{ D^*(\psi, \phi) m_n [v_{\lambda,(\beta)}^{(n)*} V_{(m)\sigma'\sigma} + t_{\lambda,(\beta)}^{(n)*} T_{(m)\sigma'\sigma} \\ &\quad + \tilde{t}_{\lambda,(\beta)}^{(n)*} \tilde{T}_{(m)\sigma'\sigma}] + s_{\lambda'} S_{\sigma'\sigma} + v_{\lambda,(\beta)}^{(3)*} V_{(3)\sigma'\sigma} \}. \end{aligned} \quad (31)$$

Note that Eq. (31) (first line) appears in a "factorized" form consisting of two vertex functions each depending only on q^2 (and its own internal variables, if any) and they are connected by a D -function depending on the variables (ψ, ϕ) which specify the relative coordinates of the two vertices (cf. Fig. 1).

To make the choice of variables clearer, we recall that our BW frame is chosen such that

$$\begin{aligned} q &= p - p' = k' - k = (0, 0, 0, \sqrt{q^2}), \\ p &= (p^0, 0, 0, p), \\ k &= k(\cosh\psi, \sinh\psi \cos\phi, \sinh\psi \sin\phi, -1), \end{aligned} \quad (32)$$

where

$$\begin{aligned} p &= (q^2 - M^2 + W^2)/2\sqrt{q^2}, \quad p^0 = (p^2 + M^2)^{1/2}, \\ k &= (q^2 + m_l^2)/2\sqrt{q^2}, \end{aligned}$$

M being the mass of the target particle A , m_l that of the lepton l , and W the effective mass of the system B (see Fig. 3). These variables are related to the *laboratory frame* incoming neutrino energy ϵ , the outgoing lepton energy ϵ' and scattering angle θ_L , and the magnitude of the 3-momentum transfer $|\mathbf{q}_L|$ by

$$\begin{aligned} \cosh\psi &= (\epsilon + \epsilon')/|\mathbf{q}_L|, \\ \sinh\psi &= [(\sqrt{q^2})/|\mathbf{q}_L|] \cot\frac{1}{2}\theta_L, \end{aligned} \quad (33)$$

in the approximation of $m_l=0$.

The leptonic form factors are, of course, explicitly known from (3). Straightforward calculations yield

$$\begin{aligned} v_{-\frac{1}{2}}^{(-1)} &= t_{\frac{1}{2}}^{(0)} = \tilde{t}_{\frac{1}{2}}^{(0)} = -\sqrt{2} s_{\frac{1}{2}} \\ &= -2(2q^2)^{1/2} (1 + m_l^2/q^2)^{1/2} \end{aligned} \quad (34a)$$

and

$$\begin{aligned} v_{\frac{1}{2}}^{(0)} &= v_{\frac{1}{2}}^{(3)} = -\frac{1}{2} t_{-\frac{1}{2}}^{(-1)} = -\frac{1}{2} \tilde{t}_{-\frac{1}{2}}^{(-1)} \\ &= 2m_l(1 + m_l^2/q^2)^{1/2}, \end{aligned} \quad (34b)$$

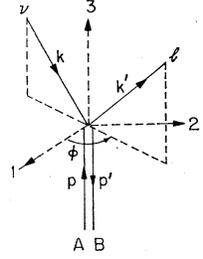


FIG. 3. The brick-wall-frame kinematics.

and all other form factors vanish due to the conditions (13), (20), and (29). We note also that the form factors given in Eq. (34b) are proportional to the lepton mass. In the limit $m_l=0$, which is a good approximation for high-energy processes, the only surviving (leptonic) form factors are those in Eq. (34a), and they are all proportional to $\sqrt{q^2}$. The transition amplitude (31) can, therefore, be written

$$f_{\frac{1}{2}\sigma',\sigma} = -2G(\sqrt{q^2}) [S_{\sigma'\sigma} + e^{im\phi} \bar{d}(\psi) m_0 T_{(m)\sigma'\sigma}], \quad (35a)$$

$$f_{-\frac{1}{2}\sigma',\sigma} = -2G(\sqrt{q^2}) e^{im\phi} \bar{d}(\psi) m_{-1} V_{(m)\sigma'\sigma}, \quad (35b)$$

where, for simplicity, we have written $T_{(m)}$ for the combination $T_{(m)} - \tilde{T}_{(m)}$ and absorbed a factor $(-1/\sqrt{2})$ into the definition of $S_{\sigma'\sigma}$.

Equation (35) exhibits very compactly the full kinematic content of the general local current-current interaction for all neutrino scattering processes. The most obvious feature of this equation is the separation of the S - T and V interactions according to the helicity of the outgoing lepton. This is expected [as can be seen by a close examination of (3)] and holds true for all possible targets A and final states B . This points to the most unambiguous way of testing the structure of the local current-current interaction at high energies and large momentum transfers: to the extent that the lepton mass can be neglected, a purely left-handed outgoing lepton indicates V interaction, a purely right-handed one indicates S - T interaction, and the coexistence of both helicities indicates a mixture of the two. Although this sounds very simple, the practical difficulties of measuring the polarization of the very high-energy outgoing lepton (muon in almost all planned experiments) are quite formidable though perhaps not entirely insurmountable.

We are thus led to explore the more detailed structure of Eq. (35) and to seek to distinguish the two types of interaction through angular or (target) spin correlation experiments. To this end, we write down the transition probability for the general reaction (1) when the lepton helicity is not observed:

$$I = \rho_{\sigma\tau}^A \rho_{\sigma'\tau'}^B (f_{-\frac{1}{2}\sigma',\sigma} f_{-\frac{1}{2}\tau',\tau}^* + f_{\frac{1}{2}\sigma',\sigma} f_{\frac{1}{2}\tau',\tau}^*). \quad (36)$$

Here ρ^A and ρ^B are the density matrices for the states A and B , respectively. Substituting (35) into (36) and using the explicit expressions for $\bar{d}(\psi)$, we obtain⁶ the

following general distribution in the variables (ψ, ϕ) :

$$I = 2G^2q^2 \{ \sinh^2\psi I_1 + (1 - \cosh\psi)^2 I_2 + (1 + \cosh\psi)^2 I_3 \\ + \sinh\psi(1 - \cosh\psi)(\cos\phi I_4 + \sin\phi I_5) \\ + \sinh\psi(1 + \cosh\psi)(\cos\phi I_6 + \sin\phi I_7) \\ + \sinh^2\psi(\cos 2\phi I_8 + \sin 2\phi I_9) \}, \quad (37)$$

where the coefficients I_i are given by

$$I_1 = V^{(0)}V^{(0)*} + T^{(+)}T^{(+)*} + T^{(-)}T^{(-)*} + T^{(0)}T^{(0)*} - SS^*, \\ I_2 = \frac{1}{2}[V^{(+)}V^{(+)*} + (T^{(0)} + S)(T^{(0)} + S)^*], \\ I_3 = \frac{1}{2}[V^{(-)}V^{(-)*} + (T^{(0)} - S)(T^{(0)} - S)^*], \\ I_4 = -\sqrt{2} \operatorname{Re}[V^{(+)}V^{(0)*} + (T^{(+)} - T^{(-)})(T^{(0)} + S)^*], \\ I_5 = \sqrt{2} \operatorname{Im}[V^{(+)}V^{(0)*} + (T^{(+)} + T^{(-)})(T^{(0)} + S)^*], \quad (38) \\ I_6 = -\sqrt{2} \operatorname{Re}[V^{(0)}V^{(-)*} + (T^{(+)} - T^{(-)})(-T^{(0)} + S)^*], \\ I_7 = \sqrt{2} \operatorname{Im}[V^{(0)}V^{(-)*} + (T^{(+)} + T^{(-)})(-T^{(0)} + S)^*], \\ I_8 = -\frac{1}{2} \operatorname{Re}[V^{(+)}V^{(-)*} + 2T^{(+)}T^{(-)*}], \\ I_9 = \frac{1}{2} \operatorname{Im}[V^{(+)}V^{(-)*} + 2T^{(+)}T^{(-)*}].$$

For simplicity, in Eq. (38) we have (a) used (\pm) in place of (± 1) for the superscript (m) and (b) omitted a common factor consisting of the density matrices $\rho^A \rho^B$. In other words, each term in Eq. (38) stands for

$$J^{(\alpha)}J^{(\beta)*} \equiv \rho_{\sigma\tau}^A \rho_{\sigma'\tau'}^B J_{\sigma'\sigma}^{(\alpha)} J_{\tau'\tau}^{(\beta)*}. \quad (39)$$

We note a few features of Eqs. (37)–(39). (a) The form of the nine-term distribution function is the same for V and T - S interactions in general.^{3,5} (b) When the state B consists only of a single particle or includes all possible final states, the form factors can be taken to be real provided time-reversal invariance holds. In that case, the $\sin\phi$ terms (I_5 , I_7 , and I_9) vanish and we have a six-term distribution function. In all of these processes, time-reversal invariance can be tested by measuring the asymmetry in ϕ with all other variables integrated over.¹³ (c) The first three terms in (37) (I_1 , I_2 , and I_3), which are the only surviving terms in the unpolarized quasi-two-body cross sections, are diagonal in the helicity form factors. The other terms involve simple interference terms. The same distribution functions when written in terms of the lab or c.m. frame variables and the conventional invariant form factors are invariably so complicated as to be almost untractable except in the simplest cases.

As a result of the difference in the outgoing lepton helicity for the S - T and V - A interactions, the lepton current has helicity 0 in the first case and -1 in the second. This is reflected in the appearance of the function $\bar{d}(\psi)^{m_0}$ in Eq. (35a) and $\bar{d}(\psi)^{m_{-1}}$ in (35b). In an experiment where the hadronic current vertexes are nonvanishing for all three values of the current helicity m ($m = 1, 0, -1$), the differential distribution function

¹³ To observe the asymmetry, one needs a preferred direction with respect to which the asymmetry is defined. This can only be supplied by the polarization vector of either A (polarized target) or B (final polarization measured).

is always of the general form (37), which does not distinguish the two types of interactions. This is the case in all “inclusive experiments,”¹⁴ where part of the final states are left undetected and in “exclusive experiments”¹⁴ involving particles of nonzero spin and with no information available on the polarization of the initial and final hadrons. On the other hand, if the polarization indices (σ, σ') for the hadrons are suitably limited (either because the particles are spinless or that one or more of them are polarized) so that the hadronic current helicity m is restricted ($m = \sigma + \sigma'$), then the distribution function for the two cases are distinguishable [$\bar{d}(\psi)^{m_0}$ versus $\bar{d}(\psi)^{m_{-1}}$]. This means that different terms in the general distribution function Eq. (37) vanish, depending on which type of current are responsible for the decay process under these special circumstances. In the next section we explicitly work out several examples of this latter type as possible places to look for tests of the V - A interaction. These examples also serve as illustrations from which it should be apparent how the general formalism developed earlier can be applied to any other special cases of interest.

We remark at this point some necessary sign changes in the formulas presented so far if they are to be applied to antineutrino scattering. The first change comes in Eqs. (34) where all the signs of the polarization indices (λ' and m) should be reversed. In addition, all form factors appearing in Eq. (34a) change sign. These changes imply that the right-hand sides of Eqs. (35) change sign and $\bar{d}(\psi)^{m_{-1}}$ is replaced by $\bar{d}(\psi)^{m_1}$. These, in turn, imply that in Eq. (37) we should replace $\sinh\psi$ by $(-\sinh\psi)$ and interchange $(1 - \cosh\psi)$ and $(1 + \cosh\psi)$.

V. EXAMPLES

A. Lepton-Lepton “Elastic” Scattering

We first consider the reaction

$$\nu_l + l' \rightarrow l + \nu_{l'}, \quad (40)$$

which is among the experiments being contemplated at NAL.¹⁵ The intensity distribution for this reaction can be easily obtained from the general formulas of the previous section by setting $V^{(-)} = v^{(-)}$, $S = \frac{1}{2}s$, and all other form factors zero (in particular, note $T^{(m)} = \bar{l}^{(m)} - \bar{l}'^{(m)} = 0$). We get

$$I(q^2, \psi) = 8G^2q^4(1 + \cosh\psi)^2 \quad (41)$$

if V - A interaction still holds at high energies. The S - P interaction, if present, would contribute

$$I(q^2, \psi) = 8G'^2q^4, \quad (42)$$

¹⁴ R. Feynman, Phys. Rev. Letters **23**, 1415 (1969); in *Proceedings of the Third International Conference on High-Energy Collisions, Stony Brook, 1969*, edited by C. N. Yang *et al.* (Gordon and Breach, New York, 1969).

¹⁵ National Accelerator Laboratory, Summer Study, 1968 and 1969 (unpublished).

where we wrote G' in place of G to allow for a different coupling constant. The tensor interaction does not contribute to this process.¹⁶

We also mention another lepton-lepton interaction which will be investigated experimentally for its own right as well as in connection with the search for the intermediate boson.¹⁵ This is neutrino lepton-pair creation in the Coulomb field of nuclei, e.g.,

$$\nu_l + Z \rightarrow l + \bar{l}' + \nu_{l'} + Z. \quad (43)$$

Here, because of the photon interaction, the strict locality of the four-fermion interaction is modified. However, it has been shown¹⁷ that the μ^- and μ^+ energy spectra are markedly different for $V-A$ and $S-P$ interactions. (Again the tensor current, even if it is present, does not contribute to this process.¹⁶)

B. Quasi-Elastic Scattering off Spinless Targets

The cases where both A and B in reaction (1) consist of spinless particles (nuclei) offer the only possibilities for testing the $V-A$ interaction in semileptonic processes from the lepton intensity distribution alone.

Consider the quasi-elastic process where both A and B are single particles of spin 0. There is only one form factor for each type of current and we get, from Eqs. (37) and (38),

$$I = 2G^2 q^2 [(|V|^2 + |T|^2 - |S|^2) \sinh^2 \psi + \frac{1}{2} |T+S|^2 (1 - \cosh \psi)^2 + \frac{1}{2} |T-S|^2 (1 + \cosh \psi)^2]. \quad (44)$$

We should therefore observe a pure $\sinh^2 \psi$ distribution if the vector ($V-A$ in usual language) action is the only one present. Deviation from such a distribution indicates the presence of tensor or scalar admixtures. This case is completely analogous to that of the K_{e3} decay where the hyperbolic angle ψ should be replaced by the decay angle θ in the c.m. system of the leptons.¹⁸

The case where the final state B consists of two spinless particles is also, in principle, capable of distinguishing the $V-A$ interaction from the other possibilities. There are only three independent vector-axial-vector form factors, $V_{0m}^{(m)}$, entering the nine-term intensity distribution (37). We shall not enter into the details here because of the lack of practical applications.

¹⁶ The vanishing of the tensor contribution to four-point lepton processes (of which this is one example) can be proved in another way. By using the Fierz transformation, one obtains

$$\begin{aligned} & \bar{\psi}_l \sigma_{\mu\nu} (1 + \gamma_5) \psi_{l'} \bar{\psi}_{l'} (1 - \gamma_5) \sigma^{\mu\nu} \psi_l \\ &= -\frac{1}{2} \bar{\psi}_{l'} (1 - \gamma_5) \sigma_{\mu\nu} (1 + \gamma_5) \psi_l \bar{\psi}_l \sigma^{\mu\nu} \psi_{l'} \\ & \quad + \frac{1}{2} \bar{\psi}_{l'} (1 - \gamma_5) (1 + \gamma_5) \psi_{l'} \bar{\psi}_l + \frac{1}{2} \bar{\psi}_{l'} (1 - \gamma_5) \gamma_5 (1 + \gamma_5) \psi_l \bar{\psi}_l \gamma_5 \psi_{l'} \\ &= 0. \end{aligned}$$

We thank Dr. K. Fujikawa (see Ref. 17) for pointing out this proof to us.

¹⁷ K. Fujikawa, thesis, Princeton University, 1970 (unpublished).

¹⁸ The latest experimental upper limits for $|S/V|$ and $|T/V|$ are 0.23 and 0.58, respectively, in K_{e3} decay. See D. Botterill *et al.*, Phys. Rev. **174**, 1661 (1968).

It is perhaps worth noting, however, that this type of process is again very closely related to the K_{e4} decay¹⁹ in its kinematics.

C. Quasi-Elastic Scattering off Polarized Target

We now consider the process

$$\nu_l + N \rightarrow l + N', \quad (45)$$

where the initial nucleon is polarized. The final particle N' can be any spin- $\frac{1}{2}$ baryon. Our standard BW frame is related to the laboratory frame by a boost along the recoil N' direction which can be chosen conveniently to be the *negative* 3-axis. Let us further choose the polarization vector of the target N to lie in the 1-3 plane and form an angle θ with the *positive* 3-axis in the laboratory system. The scattering amplitudes depend on four variables which we choose to be q^2 , ψ , ϕ , and θ .

The general formulas (37)–(39) can be applied to this case with the following substitutions for the initial and final density matrices:

$$\rho_{\sigma\tau}^A = \frac{1}{2} (1 + \not{p} \boldsymbol{\sigma} \cdot \mathbf{n})_{\sigma\tau} = \sum_{\kappa} d_{\sigma\kappa}^{\frac{1}{2}}(\theta) \not{p}_{\kappa} d_{\kappa\tau}^{\frac{1}{2}}(-\theta), \quad (46)$$

$$\rho_{\sigma'\tau'}^B = \delta_{\sigma'\tau'},$$

where \not{p} is the polarization of the target, $\not{p}_{\pm\frac{1}{2}} = \frac{1}{2}(1 \pm \not{p})$, and $d^{1/2}(\theta)$ is the usual spin- $\frac{1}{2}$ rotational matrix. Substituting into (39), one obtains

$$\begin{aligned} J^{(0)} J^{(0)*} &= \frac{1}{2} (|J_{-+}|^2 + |J_{+-}|^2) \\ & \quad + \not{p} \cos \theta \times \frac{1}{2} (|J_{-+}|^2 - |J_{+-}|^2), \\ J^{(\pm)} J^{(\pm)*} &= \frac{1}{2} (1 \pm \not{p} \cos \theta) |J_{\pm, \pm}|^2, \\ J^{(+)} J^{(0)*} &= \frac{1}{2} \not{p} \sin \theta J_{++} J_{+-}^*, \\ J^{(0)} J^{(-)*} &= \frac{1}{2} \not{p} \sin \theta J_{-+} J_{--}^*, \\ J^{(+)} J^{(-)*} &= 0, \end{aligned} \quad (47)$$

where we have omitted the superscript (m) on the right-hand side since $m = \sigma' + \sigma$. The results can be applied to each term in Eq. (38), yielding

$$\begin{aligned} I_i &= a_i + \not{p} \cos \theta b_i \quad \text{for } i = 1, 2, 3 \\ &= \not{p} \sin \theta a_i \quad \text{for } i = 4, 5, 6, 7 \\ &= 0 \quad \text{for } i = 8, 9. \end{aligned} \quad (48)$$

Equation (48), when substituted into Eq. (37), gives rise to a ten-term combined correlation function in the variables ψ , ϕ , and θ . The contribution of the vector

¹⁹ A. Pais and S. Treiman, Phys. Rev. **168**, 1858 (1968); **178**, 2365 (1969).

form factors to the coefficient functions are

$$\begin{aligned}
a_1 &= \frac{1}{2}(|V_{-+}|^2 + |V_{+-}|^2), \\
b_1 &= \frac{1}{2}(|V_{-+}|^2 - |V_{+-}|^2), \\
a_2 &= b_2 = \frac{1}{4}|J_{++}|^2, \\
a_3 &= -b_3 = \frac{1}{4}|J_{--}|^2, \\
a_4 &= -(1/\sqrt{2}) \operatorname{Re} V_{++} V_{+-}^*, \\
a_5 &= (1/\sqrt{2}) \operatorname{Im} V_{++} V_{+-}^*, \\
a_6 &= -(1/\sqrt{2}) \operatorname{Re} V_{-+} V_{--}^*, \\
a_7 &= (1/\sqrt{2}) \operatorname{Im} V_{-+} V_{--}^*.
\end{aligned} \tag{49}$$

The four vector form factors appearing above are essentially the usual G_E , G_M , and their axial-vector counterparts. The exact relations between the two sets are given in Appendix B. It is clear from (49) that if this process is mediated only by the usual vector current then there exist four relations among the ten coefficient functions a_i , b_i . The two more useful ones among these relations are already explicitly displayed in the third and fourth equations in (49). From Eqs. (37)–(39), (47), and (48), it is straightforward to verify that the presence of any scalar or tensor currents will spoil these relations. This suggests the possibility of testing the structure of the basic interaction by measuring the correlation functions and checking these relations among the coefficients. In practice, it is very unlikely that the complete correlation distributions (37), (38), and (48) can be obtained experimentally. It is therefore desirable to see how much of the phase-space volume can be integrated over without destroying the relevant information to be extracted.

First of all, the ϕ dependence is not of particular interest for our purpose; thus it can be integrated over. The resulting differential cross section (for fixed incident energy) is

$$\begin{aligned}
\frac{d\sigma}{dq^2} &= \frac{G^2 q^2}{32\pi\epsilon^2 M^2} \{ \sinh^2\psi a_1(q^2) + (1 - \cosh\psi)^2 a_2(q^2) \\
&\quad + (1 + \cosh\psi)^2 a_3(q^2) + p \cos\theta [\sinh^2\psi b_1(q^2) \\
&\quad + (1 - \cosh\psi)^2 b_2(q^2) + (1 + \cosh\psi)^2 b_3(q^2)] \}, \tag{50}
\end{aligned}$$

where ϵ , the lab neutrino energy, is given in terms of the BW-frame variables by

$$\epsilon = (1/4M) \{ (q^2 - M^2 + M'^2) + [q^2 + (M' + M)^2]^{1/2} \times [q^2 + (M' - M)^2]^{1/2} \cosh\psi \}.$$

The first three terms (a_i) give the spectrum function for *unpolarized target*,

$$A_+(q^2, \psi) = \left. \frac{d\sigma}{dq^2} \right|_{\theta=0} + \left. \frac{d\sigma}{dq^2} \right|_{\theta=\pi}. \tag{51}$$

The last three terms (b_i) can be isolated by forming the *asymmetry function*

$$A_-(q^2, \psi) = \left. \frac{d\sigma}{dq^2} \right|_{\theta=0} - \left. \frac{d\sigma}{dq^2} \right|_{\theta=\pi}. \tag{52}$$

The q^2 and ψ dependences can be separated only by measuring A_{\pm} at many different incident energies. (At fixed energy, the two variables are related.) With these available, one can divide A_{\pm} by the common factor ($G^2 q^2 / 32\pi\epsilon^2 M^2$) and integrate over the experimentally available range of the variable q^2 at fixed ψ . The resulting ψ spectra for the two cases are

$$\sinh^2\psi \begin{Bmatrix} \langle a_1 \rangle \\ \langle b_1 \rangle \end{Bmatrix} + (1 - \cosh\psi)^2 \begin{Bmatrix} \langle a_2 \rangle \\ \langle b_2 \rangle \end{Bmatrix} + (1 + \cosh\psi)^2 \begin{Bmatrix} \langle a_3 \rangle \\ \langle b_3 \rangle \end{Bmatrix}, \tag{53}$$

where

$$\langle a_i \rangle = \int dq^2 a_i(q^2) \quad \text{and} \quad \langle b_i \rangle = \int dq^2 b_i(q^2).$$

From (49) we infer that if the weak current remains a vector at high energies, we should expect

$$\langle a_2 \rangle = \langle b_2 \rangle, \quad \langle a_3 \rangle = -\langle b_3 \rangle. \tag{54}$$

We also point out that if second-class currents are absent, then $V_{+-} = V_{-+}$ (cf. Appendix B) and consequently [cf. Eq. (49)],

$$b_1 = \langle b_1 \rangle = 0. \tag{55}$$

It is worth noting that although we integrate over q^2 in order to gain maximum statistics, *the tests (54) and (55) are free from any assumption on the q^2 dependence of the form factors.*

For antineutrino scattering, the coefficients of the $(1 \pm \cosh\psi)$ terms are interchanged. Consequently, the relevant relations are

$$\langle a_2 \rangle = -\langle b_2 \rangle, \quad \langle a_3 \rangle = \langle b_3 \rangle, \quad \text{and} \quad \langle b_1 \rangle = 0. \tag{56}$$

D. Quasi-Elastic Hyperon Production

Here we consider processes of the type

$$\nu + N \rightarrow l + Y \quad \searrow \quad N' + \pi, \tag{57}$$

where N is a target nucleon (unpolarized) and Y a hyperon. We choose the hyperon decay plane to be the 1-3 plane and denote by θ the decay angle of N' in the hyperon c.m. frame. The over-all process (57) is specified by the four independent variables q^2 , ψ , ϕ , and θ .

The density matrixes for the initial and final states are

$$\begin{aligned}
\rho_{\sigma\tau}{}^A &= \frac{1}{2} \delta_{\sigma\tau}, \\
\rho_{\sigma'\tau'}{}^B &= \sum_{\kappa} d_{-\sigma'\kappa}{}^{\frac{1}{2}}(\theta) |a_{\kappa}|^2 d_{\kappa, -\tau'}{}^{\frac{1}{2}}(-\theta) \\
&= |a|^2 \times \frac{1}{2} (1 - \alpha \sigma \cdot \mathbf{n})_{\sigma'\tau'}, \tag{58}
\end{aligned}$$

where $a_{\pm\frac{1}{2}}$ are the Y -decay amplitudes for \pm helicity outgoing N' [they are simply $(1/\sqrt{2})$ times the sum and difference of the conventional a_s, a_p], respectively; $|a|^2 = |a_+|^2 + |a_-|^2 = |a_s|^2 + |a_p|^2$; α is the asymmetry parameter ($|a_+|^2 - |a_-|^2$) $|a|^{-2} = 2(\operatorname{Re} a_s a_p^*) |a|^{-2}$; and \mathbf{n} is the polarization vector with components $(\sin\theta, 0, \cos\theta)$.

The similarity between this process and the polarized-target case is obvious [compare Eq. (58) with (46)]. All considerations of Sec. V C can be carried over with very little change. In particular, the combined correlation distribution in the variables ψ , ϕ , and θ is given by Eqs. (37) and (48). The coefficients $\{a_i, b_i\}$ are expressed in terms of the vector form factors by formulas similar to Eqs. (49) with the following changes: (a) an over-all factor of $\frac{1}{2}|a|^2 = \frac{1}{2}(|a_s|^2 + |a_p|^2)$ is inserted, (b) the form factors V_{+-} and V_{-+} are interchanged, and (c) $b_1, b_2, b_3, a_4, a_5, a_6$, and a_7 acquire an additional minus sign. The phase space for the present process involves one more integration in the θ variable. The two spectrum functions $A_{\pm}(q^2, \psi)$ are obtained in this case by integrating over the entire θ range for A_+ and taking the difference of a twofold division of events with $0 < \theta < \frac{1}{2}\pi$ and $\frac{1}{2}\pi < \theta < \pi$ for A_- (asymmetry of N' with respect to N in the Y c.m. frame). Again the q^2 variable may be integrated over in the manner described before. The relations which serve as tests of the $V-A$ structure of the weak current are now (56) for the neutrino-initiated reactions and (54) for the anti-neutrino-initiated reactions.

It may be interesting to note that similar tests of the $V-A$ interaction can be carried out in polarized-hyperon β decay. Essentially all the above analyses go through if the variables (ψ, ϕ) are replaced by the decay angles of the lepton in the (l, ν) c.m. frame. Data on these processes are being accumulated at such a rate that it appears such an analysis may be feasible in the very near future.²⁰

E. Single-Pion Production in N^* Region

Finally, we consider the single-pion production process

$$\nu + N \rightarrow l + N' + \pi, \quad (59)$$

with the final $(N'\pi)$ invariant mass in the low-energy region where N^* dominates. As a first approximation, we assume that the final $(N'\pi)$ system is in a pure $J^P = \frac{3}{2}^+$ state (N^*) in its c.m. frame. Then the initial and final density matrices can be written

$$\begin{aligned} \rho_{\sigma\tau}^A &= \frac{1}{2} \delta_{\sigma\tau} \\ \rho_{\sigma'\tau'}^B &= |a|^2 \sum_{\kappa=\pm\frac{1}{2}} d_{-\tau'\kappa}^{3/2}(\theta) d_{-\sigma'\kappa}^{3/2}(\theta), \end{aligned} \quad (60)$$

where $|a|$ stands for the magnitude of the N^* decay matrix element $\langle N^* | N'\pi \rangle$. Substituting (60) into (39), one can obtain the expected distribution in the variables ψ , ϕ , and θ from (37) and (38).

As before, we integrate over the ϕ variable obtaining a six-term distribution expressed by

$$\begin{aligned} I_i &= a_i + b_i \cos^2\theta \quad \text{for } i=1, 2, 3 \\ &= 0 \quad \text{for } i=4, 5, 6, 7, 8, 9. \end{aligned} \quad (61)$$

²⁰ For example, J. Lindquist *et al.* (Chicago-Washington-Ohio State-Argonne collaboration), in Proceedings of the Fifteenth International Conference on High-Energy Physics, Kiev, 1970 (unpublished).

The vector-current contributions to a_i, b_i can be easily worked out; they are

$$\begin{aligned} 3a_1 &= b_1 = \frac{3}{4} [|V_+^{(0)}|^2 + |V_-^{(0)}|^2], \\ a_2 &= \frac{1}{8} [|V_+^{(+)}|^2 + |V_-^{(+)}|^2], \\ b_2 &= \frac{3}{8} [|V_+^{(+)}|^2 - |V_-^{(+)}|^2], \\ a_3 &= \frac{1}{8} [|V_-^{(-)}|^2 + |V_+^{(-)}|^2], \\ b_3 &= \frac{3}{8} [|V_-^{(-)}|^2 - |V_+^{(-)}|^2]. \end{aligned} \quad (62)$$

Here the form factors are labeled by the current helicity (m) and initial N polarization, σ , with $\sigma' = m - \sigma$ omitted. The first equation in (62) can be used as a test of the $V-A$ interaction in the same manner as described in the previous two sections.

In order to improve on the (rather drastic) approximation made above, we may include into our consideration correction terms brought about by other partial waves. In the N^* region, the $J^P = \frac{1}{2}^{\pm}$ states are expected to have some influence. An analysis may be carried out retaining the three partial waves $\frac{1}{2}^{\pm}$ and $\frac{3}{2}^+$. The procedure used is the same as before, the algebra becomes slightly more involved. We give the detailed results in Appendix C. The combined ψ, ϕ , and θ correlation distribution consists of 19 independent terms. Since the number of form factors proliferates with the inclusion of more states, we found it convenient to assume time-reversal invariance. This enables us to invoke the Watson's theorem to fix the phases of the form factors in terms of the measured (πN) phase shifts, thus reducing the number of unknowns. This way, the 19 coefficients in the correlation function can be expressed in terms of three known phase shifts and 14 (modulus of) vector form factors if $V-A$ interaction alone contributes. Thus, five relations among the 19 coefficients exist and again serve as the basis for a test of the $V-A$ interaction at high energies. For the details of this calculation we again refer the reader to Appendix C. Here we only note that the procedure used is, in a sense, the reverse of that used by Pais and Treiman in K_{e4} and Ξ_{14} decays.¹⁹ There, the $V-A$ interaction is assumed and the phase shifts treated as unknowns to be solved from the coefficients of the correlation function.

F. Conclusions

We have given a formula for the general intensity distribution function for an arbitrary neutrino scattering process in the local current-current picture in terms of helicity form factors. We have discussed various ways to test whether the basic $V-A$ interaction structure of the weak-interaction Hamiltonian deduced from low-energy decay experiments still remains valid for high-energy neutrino scattering processes. All considerations are independent of dynamical assumptions.²¹ It is quite evident that none of these proposed tests are easy to carry out in the laboratory. The analyses of

²¹ There is one exception. That is the omission of other partial waves in the $(N'\pi)$ system in Sec. V E. This assumption can be independently checked experimentally.

this paper showed, however, that there does not seem to be any easier alternative for settling this very important question. The "inclusive experiments" (without detection of the outgoing lepton polarization) which are the most practical reactions to measure in the laboratory can serve to test some aspects of locality^{4,5} but *not* the $V-A$ structure of the weak interaction. The "exclusive experiments" examined in detail in this paper as well as the measurement of the outgoing lepton polarization mentioned previously are undoubtedly much harder experiments. We hope, however, with the rapid advancement in experimental techniques some of these tests will be carried out.²²

Aside from the proposed tests, we hope the analyses in this paper also succeed in demonstrating the usefulness of the BW-frame variables⁶ (ψ, ϕ) and the helicity form factors, in describing all types of weak scattering processes. The variables $\sinh\psi$ and $\cosh\psi$, which appear widely in our formulas, are rather simply related to the more familiar laboratory variables through Eq. (33). The helicity form factors, which are just generalizations of the familiar G_E and G_M , can be used in other types of weak and electromagnetic processes to great advantage.²³

ACKNOWLEDGMENTS

We would like to thank Professor S. B. Treiman for an enlightening discussion. One of us (W. K. T.) thanks Dr. I. M. Muzinich and Dr. L. L. Wang for discussions on the use of the BW frame and Dr. P. Frampton for discussions and reading of the manuscript; the other author (T. P. C.) thanks Professor A. Pais for a useful conversation. We also thank Professor Carl Kaysen for hospitality at The Institute for Advanced Study.

APPENDIX A

We briefly indicate the lepton-mass corrections to the various formulas in the text. This is rather easy to do in our formalism. Thus, substituting Eqs. (34a) and (34b) into Eq. (31), the general scattering amplitudes are [compare with Eq. (35)]

$$\begin{aligned} f_{\frac{1}{2}\sigma',\sigma} &= -2G(q^2+m_l^2)^{1/2} [S_{\sigma'\sigma} + e^{im\phi} \bar{d}(\psi)^{m_0} T_{(m)\sigma'\sigma}] \\ &\quad + \sqrt{2} G m_l (1+m_l^2/q^2)^{1/2} \\ &\quad \times [V_{\sigma'\sigma}^{(3)} + e^{im\phi} \bar{d}(\psi)^{m_0} V_{(m)\sigma'\sigma}], \quad (\text{A1}) \\ f_{-\frac{1}{2}\sigma',\sigma} &= -2G[(q^2+m_l^2)^{1/2} V_{(m)\sigma'\sigma} \\ &\quad - \sqrt{2} m_l (1+m_l^2/q^2)^{1/2}] e^{im\phi} \bar{d}(\psi)^{m-1}. \end{aligned}$$

The intensity distribution can then be obtained from (A1) and (36). The distribution function is again of the form Eq. (37) but with I_i containing more terms than those given in Eq. (38) due to contributions from the second terms in Eqs. (A1). They can be easily worked

out when necessary, we do not give the complete expressions here.

As a consequence of the lepton-mass terms, the outgoing lepton polarization is not 100% for pure $V-A$ or $S-T$ interactions. The correction is roughly of the order m_l^2/q^2 [see Eq. (A1)]. Similarly, the distribution functions (41), (44), (49), and (62) and the constraints (54), (56), and (62) all are subject to additional corrections of this order. For electron-neutrino interactions, m_l^2/q^2 is for all practical purposes zero. For muon-neutrino interactions, $m_l^2/q^2=0.01$ for $q^2=1$ (GeV/c)².

APPENDIX B

We briefly enumerate a few relevant properties of the helicity form factors defined in the text. We recall the definition of such form factors:

$$\begin{aligned} J_{\lambda'\lambda}^{(m)} &= \langle p_s' \lambda' | J^{(m)}(0) | p_s \lambda \rangle \\ &= \langle 0 - \lambda' | B_3(-u') J^{(m)}(0) B_3(u) | 0 \lambda \rangle. \quad (\text{B1}) \end{aligned}$$

Here, as in the text, $J^{(m)}$ denotes the m -component of some general current operator. The m -index is defined such that

$$[J^{(m)}(0), J_3] = m J^{(m)}(0), \quad (\text{B2})$$

where J_3 denotes the angular momentum operator along the 3-axis. (An exception to this statement is $V^{(3)}$ which corresponds to $m=0$ but, for obvious reasons, we cannot, and did not, label it $V^{(0)}$.) Sandwich Eq. (B2) between the states $\langle p_s' \lambda' |$ and $| p_s \lambda \rangle$. We get $(\lambda + \lambda') J_{\lambda'\lambda}^{(m)} = m J_{\lambda'\lambda}^{(m)}$, which implies

$$J_{\lambda'\lambda}^{(m)} = \delta_{m, \lambda + \lambda'} J_{\lambda'\lambda}^{(m)}. \quad (\text{B3})$$

The hadronic states $| p_s \lambda \rangle$ and $| p_s' \lambda' \rangle$ have definite transformation properties under the space- and time-inversion operators. It is therefore meaningful to separate the S , V , and T currents in the text into the usual S , P , V , A , and T currents which also have definite transformation properties under these discrete transformations. One can then derive consequences due to covariance under these transformations. This is easy to do with the explicit definition (B1); we shall not go into it explicitly here but rather confine ourselves to a few remarks. Constraints on the form factors due to these symmetries arise only if all the momenta (both the initial and final states may be composite systems with internal momenta) lie in a plane [so that the operators $U_P R_2(\pi)$ and $U_T R_2(\pi)$ will leave the momenta unchanged]. Otherwise one only relates form factors at different momentum values. As is well known, parity relates $J_{\lambda'\lambda}^{(m)}$ to $J_{-\lambda',-\lambda}^{(-m)}$ and time-reversal invariance yields information on the phase of the form factors and, in the special case where the initial and final states are identical, symmetry relations in the initial and final variables.

As shown in the text, the helicity form factors diagonalize the unpolarized cross-section formula and yield simple expressions for spin- and angular-correlation functions. The relation between these form factors and the conventional invariant form factors can be

²² A quick look at the list of proposed experiments at the National Accelerator Laboratory should offer much encouragement to one's optimism on this point.

²³ See, for instance, an application to the β decay of polarized baryons: P. H. Frampton and Wu-ki Tung, Phys. Rev. D (to be published).

worked out easily for any particular case from Eq. (B1) according to the specific way the latter are defined. They can be also related to the nonrelativistic multipole moments familiar in atomic and nuclear physics. To do this, simply observe that the operator $B_3(-u')J^{(m)}(0) \times B_3(u)$ in Eq. (B1) can be decomposed into a sum of terms irreducible under the rotation group $O(3)$ (with parity). When sandwiched between the rest-frame states in (B1), one obtains reduced matrix elements which are generalizations of the multipole moments.⁸

Finally we give the explicit relations between the vector and axial-vector helicity form factors and the conventional invariant form factors for spin- $\frac{1}{2}$ particles. From the definition

$$V_{\lambda,\lambda'} = \bar{u}_{\lambda'}(p_s') [\gamma^\mu f_1 + i\sigma^{\mu\nu} q_\nu f_2 + q^\mu f_3 + \gamma^\mu \gamma_5 g_1 + i\sigma^{\mu\nu} q_\nu \gamma_5 g_2 + q^\mu \gamma_5 g_3] u_\lambda(p_s), \quad (B4)$$

$$q^\mu = p_s^\mu - p_s'^\mu,$$

straightforward calculations yield

$$V_{-\lambda,\lambda}^{(3)} = (\Delta M f_1 + q^2 f_3)(1 + 4\bar{M}^2/q^2)^{1/2} - 2\lambda(2\bar{M} g_1 + q^2 g_3)(1 + \Delta M^2/q^2)^{1/2} \quad (\lambda = \pm \frac{1}{2}),$$

$$V_{-\lambda,\lambda}^{(0)} = (2\bar{M} f_1 - q^2 f_2)(1 + \Delta M^2/q^2)^{1/2} - 2\lambda(\Delta M g_1 - q^2 g_2)(1 + 4\bar{M}^2/q^2)^{1/2}, \quad (B5)$$

$$V_{\frac{1}{2},\frac{1}{2}}^{(+1)} = (2q^2)^{1/2} [-(f_1 + 2\bar{M} f_2)(1 + \Delta M^2/q^2)^{1/2} + (g_1 + \Delta M g_2)(1 + 4\bar{M}^2/q^2)^{1/2}],$$

$$V_{-\frac{1}{2},-\frac{1}{2}}^{(-1)} = (2q^2)^{1/2} [-(f_1 + 2\bar{M} f_2)(1 + \Delta M^2/q^2)^{1/2} - (g_1 + \Delta M g_2)(1 + 4\bar{M}^2/q^2)^{1/2}],$$

where $\Delta M = M' - M$ and $\bar{M} = \frac{1}{2}(M + M')$.

The vector parts of the helicity form factors $V^{(0)}$ and $V^{(\pm 1)}$ can be readily recognized as multiples of the conventional G_E and G_M form factors, respectively.

APPENDIX C

We give some detailed results on single-pion production in the N^* region. The process under consideration is

$$\nu + N \rightarrow \mu + N' + \pi. \quad (C1)$$

In our convention, the total momentum of the $(N'\pi)$ system is p' . Let us denote the relative momentum of this two-particle state by r and the N' polarization by κ . This process is described by five independent variables (ψ, ϕ, q^2, θ , and W), where W is the invariant mass of the $N'\pi$ system. In the region where W is near or below the two-pion production threshold, we assume the matrix elements of the current operators between the states $(N'\pi)$ and N are dominated by the $(N'\pi)$ system in the $J^P = \frac{1}{2}^\pm, \frac{3}{2}^+$ states. The helicity form factors which enter into Eqs. (31), (35), and (38) can be written

$$\langle p_s'; r\kappa | J^{(m)} | p_s \sigma \rangle = \langle 0; r,\kappa | B_3(-u') J^{(m)} B(u) | 0 \sigma \rangle$$

$$= \sum_{\sigma'} d_{-\sigma',\kappa}^{\frac{1}{2}}(\theta) [J_{\sigma'\sigma}^{(m)(\frac{1}{2}^+)} + 2\kappa J_{\sigma'\sigma}^{(m)(\frac{1}{2}^-)}]$$

$$+ \sum_{\sigma'} d_{-\sigma',\kappa}^{\frac{3}{2}}(\theta) J_{\sigma'\sigma}^{(m)(\frac{3}{2}^+)}, \quad (C2)$$

where the second superscript indicates the J^P state of the $(N'\pi)$ system.

In the region of W variable of interest to us, elastic unitarity in the $(N'\pi)$ channel holds. If one assumes time-reversal invariance, as is consistent with present experimental evidences in high-energy scattering, then the phases of the form factors in (C2) are identical to the elastic scattering phase shifts in the appropriate $(N'\pi)$ states. For simplicity in notation, let us denote the phase shifts in the $\frac{1}{2}^+, \frac{1}{2}^-,$ and $\frac{3}{2}^+$ states by α, β, γ , and the form factors $J_{\sigma'\sigma}^{(m)(J^P)}$ in these states by $\alpha_{\sigma}^{(m)}, \beta_{\sigma}^{(m)}, \gamma_{\sigma}^{(m)}$, respectively. Then, substituting (C2) into Eqs. (37) and (38) yields the combined correlation distribution in ψ, ϕ, θ , with the result

$$I_i = a_i + b_i \cos^2 \theta + c_i \cos \theta \quad \text{for } i=1, 2, 3$$

$$= (a_i + c_i \cos \theta) \sin \theta \quad \text{for } i=4, 5, 6, 7 \quad (C3)$$

$$= a_i \sin^2 \theta \quad \text{for } i=8, 9.$$

We give the contribution of the vector-current form factors to the coefficients $\{a_i, b_i, c_i\}$:

$$a_1 = (\alpha_+^0)^2 + (\alpha_-^0)^2 + (\beta_-^0)^2 + \frac{1}{4} [(\gamma_+^0)^2 + (\gamma_-^0)^2] - (\beta_+^0 \gamma_+^0 - \beta_-^0 \gamma_-^0) \cos(\beta - \gamma),$$

$$b_1 = 3 \{ \frac{1}{4} [(\gamma_+^0)^2 + (\gamma_-^0)^2] + (\beta_+^0 \gamma_+^0 - \beta_-^0 \gamma_-^0) \cos(\beta - \gamma) \},$$

$$c_1 = 2(\alpha_+^0 \beta_+^0 - \alpha_-^0 \beta_-^0) \cos(\alpha - \beta),$$

$$a_2 = \frac{1}{2} [(\alpha_+^+)^2 + (\beta_+^+)^2] + \frac{1}{8} [(\gamma_+^+)^2 + (\gamma_-^+)^2],$$

$$b_2 = \frac{3}{2} \{ \frac{1}{4} [(\gamma_+^+)^2 - (\gamma_-^+)^2] - \beta_+^+ \gamma_+^+ \cos(\beta - \gamma) \},$$

$$c_2 = -\alpha_+^+ \beta_+^+ \cos(\alpha - \beta) + \gamma_+^+ \alpha_+^+ \cos(\gamma - \alpha),$$

$$a_3 = \frac{1}{2} [(\alpha_-^-)^2 + (\beta_-^-)^2] + \frac{1}{8} [(\gamma_+^-)^2 + (\gamma_-^-)^2] - \frac{1}{2} \beta_-^- \gamma_-^- \cos(\beta - \gamma),$$

$$b_3 = \frac{3}{2} \{ \frac{1}{4} [(\gamma_-^-)^2 - (\gamma_+^-)^2] + \beta_-^- \gamma_-^- \cos(\beta - \gamma) \},$$

$$c_3 = \alpha_-^- \beta_-^- \cos(\alpha - \beta) + \gamma_-^- \alpha_-^- \cos(\gamma - \alpha),$$

$$a_4 = -\sqrt{2} [(\alpha_+^+ \beta_+^0 + \beta_+^+ \alpha_+^0) \cos(\alpha - \beta) + \frac{1}{2} (\gamma_+^+ \alpha_+^0 + \alpha_+^+ \gamma_+^0 + \sqrt{3} \gamma_+^+ \alpha_-^0) \cos(\gamma - \alpha)], \quad (C4)$$

$$c_4 = -(\sqrt{\frac{3}{2}}) \{ \gamma_-^+ \gamma_-^0 + [\sqrt{3} (\beta_+^+ \gamma_+^0 + \gamma_+^+ \beta_+^0) - \gamma_-^+ \beta_-^0] \times \cos(\beta - \gamma) \},$$

$$a_5 = \sqrt{2} [(\alpha_+^+ \beta_+^0 - \beta_+^+ \alpha_+^0) \sin(\alpha - \beta) + \frac{1}{2} (\gamma_+^+ \alpha_+^0 + \alpha_+^+ \gamma_+^0 + \sqrt{3} \gamma_+^+ \alpha_-^0) \sin(\gamma - \alpha)],$$

$$c_5 = (\sqrt{\frac{3}{2}}) [\sqrt{3} (\beta_+^+ \gamma_+^0 - \gamma_+^+ \beta_+^0) + \gamma_-^+ \beta_-^0] \sin(\beta - \gamma),$$

$$a_6 = -\sqrt{2} [(\alpha_-^0 \beta_-^- + \beta_-^0 \alpha_-^-) \cos(\alpha - \beta) + \frac{1}{2} (\gamma_-^0 \alpha_-^- - \alpha_-^0 \gamma_-^- - \sqrt{3} \alpha_+^0 \gamma_+^-) \cos(\gamma - \alpha)],$$

$$c_6 = -(\sqrt{\frac{3}{2}}) \{ -\gamma_+^0 \gamma_+^- + [\sqrt{3} (\beta_-^0 \gamma_-^- + \gamma_-^0 \beta_-^-) - \beta_+^0 \gamma_+^-] \cos(\beta - \gamma) \},$$

$$a_7 = \sqrt{2} [(\alpha_-^0 \beta_-^- - \beta_-^0 \alpha_-^-) \sin(\alpha - \beta) + \frac{1}{2} (\gamma_-^0 \alpha_-^- + \alpha_-^0 \gamma_-^- - \sqrt{3} \alpha_+^0 \gamma_+^-) \sin(\gamma - \alpha)],$$

$$c_7 = (\sqrt{\frac{3}{2}}) [\sqrt{3} (\beta_-^0 \gamma_-^- - \gamma_-^0 \beta_-^-) - \beta_+^0 \gamma_+^-] \sin(\beta - \gamma),$$

$$a_8 = \frac{1}{4} \sqrt{3} [\frac{1}{2} (\gamma_-^+ \gamma_-^- + \gamma_+^+ \gamma_+^-) + (\beta_+^+ \gamma_+^- - \gamma_-^+ \beta_-^-) \times \cos(\beta - \gamma)],$$

$$a_9 = \frac{1}{4} \sqrt{3} (-\beta_+^+ \gamma_+^- - \gamma_-^+ \beta_-^-) \sin(\beta - \gamma).$$

The 19 coefficients are given in terms of three known

phase shifts and 14 unknown form factors. Thus, in principle there exist five relations among the correlation coefficients which can provide as tests of the $V-A$ interaction. It is not hard to see from Eqs. (C4) however, that these relations are rather complicated in general. In the text we considered the zeroth-order approximation of setting $\alpha_{\sigma}^{(m)} = \beta_{\sigma}^{(m)} = 0$ and obtained one simple constraint relation $a_1 = 3b_1$. One can improve

on these results by attempting to solve the equations in (C4) in the approximation that $(\alpha_{\sigma}^{(m)}/\gamma_{\sigma}^{(m)})$ and $(\beta_{\sigma}^{(m)}/\gamma_{\sigma}^{(m)})$ are small but nonzero. We shall not do this here explicitly. It can be carried out in a straightforward manner when the result is called for. Let us simply state that the aforementioned relation $a_1 = 3b_1$ still holds to the first-order approximation in the small parameters (α/γ) and (β/γ) .

Broken Scale Invariance and Kinematic Moments of Electroproduction Cross Sections*

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(Received 10 September 1970)

Expressions for high-energy limits of electroproduction cross sections weighted by $-q^2\omega$, $\omega\nu^{-2}$, and q^4 at constant s , θ , and ν , respectively, are derived from sum rules in the broken-scale-invariance scheme of Ciccariello, Gatto, Sartori, and Tonin.

THE structure functions $W_{1,2}(q^2, \nu)$ which determine the cross section for electron-nucleon scattering were predicted by Bjorken¹ to have the following scale-invariant limits for highly inelastic scattering ($-q^2$, $\nu \equiv p \cdot q \rightarrow \infty$):

$$\begin{aligned} MW_1(q^2, \nu) &\rightarrow F_1(\omega) < \infty, \\ (\nu/M)W_2(q^2, \nu) &\rightarrow F_2(\omega) < \infty, \end{aligned} \quad (1)$$

where p and M are the four-momentum and mass, respectively, of the nucleon, q is the four-momentum transferred to the nucleon, and $\omega \equiv -q^2/\nu$ is fixed. Both $F_{1,2}$ are non-negative. In terms of these, the highly inelastic cross section assuming one-photon exchange is

$$\begin{aligned} \frac{d^2\sigma}{d|q^2|d\nu} &= \frac{\pi\alpha^2}{ME^2q^4} \left\{ \frac{2\nu\omega}{M} F_1(\omega) \right. \\ &\quad \left. + \left[4E \left(E - \frac{\nu}{M} \right) + q^2 \right] \frac{M}{\nu} F_2(\omega) \right\}. \end{aligned} \quad (2)$$

Ciccariello, Gatto, Sartori, and Tonin (CGST) derived the following sum rules for $F_{1,2}$ from a set of current commutators which they had constructed assuming broken scale invariance²:

$$\begin{aligned} \int_0^2 d\omega \omega F_1(\omega) &= \frac{2}{3}C_1, \\ \int_0^2 d\omega F_2(\omega) &= -\frac{2}{3}C_2, \end{aligned} \quad (3)$$

* Supported in part by the U. S. Atomic Energy Commission.
¹ J. D. Bjorken, Phys. Rev. **179**, 1547 (1969). My conventions are identical to his, excepting the definition of ν . A typographical error of the factor M is corrected.

where $C_{1,2}$ are unknown constants. Recently Mack published³ an expression relating $[\omega]_{q^2} \equiv \langle \omega \rangle_{q^2} d\sigma/d|q^2|$ for large E and large constant $-q^2$ to C_2 . The purposes of this paper are to derive similar kinematic moments of the differential cross section (KM's) for $[-q^2\omega]_s$, where $s = (p+q)^2 = M^2 + 2\nu + q^2$ is fixed; for $[\omega\nu^{-2}]_{\theta}$ where θ , the laboratory angle between the electron's incident and final directions, is fixed; for $[q^4]_{\nu}$; and for $[q^4]_{\text{total}}$; and to discuss their implications. The KM's would be kinematic averages if they were divided by the integrated cross section. For instance, $[q^4]_{\nu} = \langle q^4 \rangle_{\nu} d\sigma/d\nu$; however, $d\sigma/d\nu$ and $d\sigma/ds$ must be defined delicately because of the q^{-4} factor in (2).

First the derivation of Mack's average will be reviewed. If (2) is valid for sufficiently large, fixed $-q^2$,

$$\begin{aligned} \langle \omega \rangle_{q^2} \frac{d\sigma}{d|q^2|} &\equiv \int d\nu \omega \frac{d^2\sigma}{d|q^2|d\nu} = \frac{\pi\alpha^2}{ME^2q^4} \int_{\omega_0}^2 d\omega \\ &\quad \times \left\{ \frac{2q^4}{M\omega} F_1 + \left[4E \left(E + \frac{q^2}{M\omega} \right) - q^4 \right] M F_2 \right\}, \end{aligned} \quad (4)$$

where $\omega_0 = -4Eq^2/(Mq^2 + 4ME^2)$. If ωF_1 and F_2 are less singular than ω^{-1} as $\omega \rightarrow 0$, the term in the integrand proportional to E^2 dominates in the limit $E \rightarrow \infty$. Therefore,

$$\begin{aligned} \lim_{E \rightarrow \infty} \left\langle \frac{-q^2}{\nu} \right\rangle_{q^2} \frac{d\sigma}{d|q^2|} &= \frac{4\pi\alpha^2}{q^4} \int_0^2 d\omega F_2 \\ &= \frac{4\pi\alpha^2}{q^4} \left(-\frac{2}{3}C_2 \right). \end{aligned} \quad (5)$$

² S. Ciccariello, R. Gatto, G. Sartori, and M. Tonin, Phys. Letters **30B**, 546 (1969).

³ G. Mack, Phys. Rev. Letters **25**, 400 (1970).