tion that the mass of the beam particle is negligible may not be true when kaons produce low-mass states.

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Veneziano Formula, K_A Meson Decays, and Weinberg Limits of K_{l4} Form Factors

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s- and d-wave ratios of $g_{K_AK\rho}$ and $g_{K_AK^*\pi}$ couplings have been determined using Veneziano representations for $K_A\pi \to K\pi$ amplitudes. The results are in agreement with those obtained by the hard-meson techniques of current algebra. Assuming that the axial-vector current is dominated by K_A- and K-meson poles only, we obtain the K_{l4} form factors from $K\pi \to K_A\pi$ and $K\pi \to K\pi$ scattering amplitudes when the K_A and one of the K mesons are taken off shell. Current-algebra constraints due to Weinberg, when both pions are soft, are then used to fix the arbitrary parameters of the theory. We find that consistency of our results with current-algebra predictions requires the presence of nonleading Veneziano amplitudes.

I. INTRODUCTION

PRESCRIPTIONS for writing down an amplitude with crossing symmetry and Regge behavior without unitarity for four-point functions have been given by Veneziano.[1] Lovelace[2] has shown how this amplitude could even be used for off-shell amplitudes. Further, a host[3–8] of chiral-symmetry and current-algebra predictions have been obtained by using the Veneziano representations and consistency conditions derived from partial conservation of axial-vector current (PCAC).[9] This raises the hope that the Veneziano representation, if coupled to unitarity, may lead to a good understanding of elementary-particle interactions.

In this paper we first consider the Veneziano representations for $K\pi \to K_A\pi$ scattering amplitudes and obtain the s-to-d-wave ratios of the coupling constants $g_{K_AK\rho}$ and $g_{K_AK^*\pi}$. Our results are in agreement with the calculations of Lai,[10] who used the hard-meson techniques of the current-algebra approach due to Weinberg and Schnitzer.[11] Next, assuming that the Veneziano form for the $K\pi \to K_A\pi$ scattering amplitude can be used even when K_A is taken off the mass shell, we discuss the K_{l4} form factors. For a complete knowledge of these form factors, we also consider $K\pi \to K\pi$ scattering; this particular amplitude appears because the K meson can interact with two pions, continue as a K-meson, and finally couple to lepton pairs.[12,13]

In writing down the Veneziano representations for K_{l4} decay, we use the same forms of amplitudes as used in $K\pi \to K_A\pi$ and $K\pi \to K\pi$ scattering amplitudes, except for the arbitrary parameters. We then attempt to determine these constants from the constraints due to current algebra when both pions are made soft, as calculated by Weinberg. We find that of the three form factors F_i ($i=1, 2, 3$), the form factor F_3, which can be shown to arise only through virtual $K\pi \to K\pi$ scattering under the assumption of kaon PCAC,[3–7] is consistent with the current-algebra prediction, and the arbitrary parameter associated with it is uniquely given, whereas the form factors F_1 and F_2, if determined from $K\pi \to K_A\pi$ scattering amplitudes, are inconsistent with the

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current-algebra predictions. To make our Veneziano amplitudes consistent with current-algebra results, we need to introduce an additional (nonleading) Veneziano amplitude. The determination of the parameters associated with F_1 and F_2 is discussed in Sec. V.

II. DECAY WIDTHS OF K_A MESON

The invariant amplitude $T_\lambda(s,t)$ for

$$K^+(k)+\pi^-(-k^+)\to K_{A\lambda}(l)+\pi^-(k^-) \quad (1)$$

can be written as $T_\lambda(s,t)=(k+k^-)_\lambda C(s,t)+(k-k^-)_\lambda \times D(s,t)$, when $s=(k-k^+)^2$, $t=(k^++k^-)^2$, and $u=(k-k^-)^2$; k, k^+, etc., are the four-momenta of the particles as shown in Eq. (1). The helicity decomposition of our amplitudes can be written down, from which we easily infer the Regge high-energy behavior. Consistent with this, we assume[14,15] the following Veneziano representations for C and D:

$$C(s,t,u)=a[\alpha_\rho(t)-\alpha_{K^*}(s)]\frac{\Gamma(1-\alpha_{K^*}(s))\Gamma(1-\alpha_\rho(t))}{\Gamma(2-\alpha_{K^*}(s)-\alpha_\rho(t))} \quad (2)$$

and

$$D(s,t,u)=b[1-\alpha_{K^*}(s)-\alpha_\rho(t)]\times\frac{\Gamma(1-\alpha_{K^*}(s))\Gamma(1-\alpha_\rho(t))}{\Gamma(2-\alpha_{K^*}(s)-\alpha_\rho(t))}, \quad (3)$$

where a and b are two arbitrary parameters. We can now use Eqs. (2) and (3) to obtain the decay parameters of the K_A meson. Considering ρ and its daughter ϵ in the t channel and K^* and its daughter κ in the s channel, we can easily obtain the real residues from (2) and (3). We then compare with the corresponding residues from Feynman amplitudes for the same pole diagrams. We note here the contributions to the amplitudes due to ρ, ϵ, κ, and K^* poles.

These are, for the C amplitude,

$$\frac{\alpha'}{\alpha_\rho(t)-1}\{\tfrac{1}{2}\gamma_{\rho\pi\pi}[H_S-H_D(m_{K_A}{}^2+m_K{}^2-t-2s)]-\gamma_{K_A\epsilon K}\gamma_{\epsilon\pi\pi}\} \quad (4)$$

and

$$\frac{\alpha'}{\alpha_{K^*}(s)-1}\left\{\tfrac{1}{2}\gamma_{K^*K\pi}\left[H_S'\left(1-\frac{m_K{}^2}{m_{K^*}{}^2}\right)-H_D'\left(m_{K_A}{}^2+2m_K{}^2-2t-s-\frac{m_K{}^2}{m_{K^*}{}^2}(m_{K_A}{}^2+s)\right)\right]+\gamma_{K_A\kappa\pi}\gamma_{K\kappa\pi}\right\}, \quad (5)$$

and for the D amplitude,

$$-\frac{\alpha'}{\alpha_\rho(t)-1}\left\{\tfrac{1}{2}\gamma_{\rho\pi\pi}[3H_S+H_D(m_{K_A}{}^2+m_K{}^2-t-2s)]-\gamma_{K_A\epsilon K}\gamma_{\epsilon\pi\pi}\right\}, \quad (6)$$

$$\frac{\alpha'}{\alpha_{K^*}(s)-1}\left\{\tfrac{1}{2}\gamma_{K^*K\pi}\left[H_S'\left(3+\frac{m_K{}^2}{m_{K^*}{}^2}\right)+H_D'\left(m_{K_A}{}^2+2m_K{}^2-2t-s-\frac{m_K{}^2}{m_{K^*}{}^2}(m_{K_A}{}^2+s)\right)\right]-\gamma_{K_A\kappa\pi}\gamma_{K\kappa\pi}\right\}, \quad (7)$$

where α', the universal slope of the linear trajectory, is $1/(2m_\rho{}^2)$; H_S and H_D are the s- and d-wave K_A-meson couplings to ρ and K; and H_S' and H_D' are similar couplings to K^* and π. The couplings are given by

$$H_S(\epsilon^A\cdot\epsilon^\rho)-2H_D(\epsilon^A\cdot p_\rho)(\epsilon^\rho\cdot p_A) \quad (p_A+p_K=p_\rho),$$

$$\gamma_{K_A\epsilon K}(p_\epsilon+p_K)\cdot\epsilon^A \quad (p_A+p_K=p_\epsilon),$$

and similar expressions. We neglect terms of the order of $m_\pi{}^2/m_K{}^2$.

Now if we use Adler's PCAC consistency condition when the kaon or the pion is taken off-shell, we obtain $m_{K_A}{}^2+m_K{}^2=2m_{K^*}{}^2$. Further, using $\alpha_{K^*}(m_K{}^2)=\frac{1}{2}$, we find from (4) and (6) and from (5) and (7), respectively, that

$$H_S/H_D=-m_\rho{}^2 \quad (8a)$$

and

$$H_S'/H_D'=-m_\rho{}^2. \quad (8b)$$

The values of the coupling of κ and ϵ are given by[16]

$$\gamma_{K_A\epsilon K}\gamma_{\epsilon\pi\pi}=0, \quad (9a)$$

$$\gamma_{K_A\kappa\pi}\gamma_{K\kappa\pi}=-am_\rho{}^2m_K{}^2/m_{K^*}{}^2. \quad (9b)$$

From our analysis it turns out that

$$a=b=-\gamma_{\rho\pi\pi}H_D=\gamma_{K^*K\pi}H_D'. \quad (10)$$

III. VENEZIANO AMPLITUDES FOR K_{l4} FORM FACTORS

Consider now the decay

$$K(k)\to\pi_a(k^+)+\pi_b(k^-)+l\nu(l); \quad (11)$$

as usual, the axial-vector current A_λ is written as

$$A_\lambda=P_\lambda F_1(s,t,u)+Q_\lambda F_2(s,t,u)+(k-P)_\lambda F_3(s,t,u), \quad (12)$$

[14] These forms coincide with those for the $\pi^++\pi^-\to\pi^-+A_1{}^+$ scattering Veneziano amplitudes. In that case, C becomes antisymmetric and D symmetric under $s\leftrightarrow t$, as it should be. The predicted high-energy behavior of C and D from Eqs. (2) and (3) are consistent with those of A and B amplitudes given in Ref. 8. See also Refs. 5 and 6.

[15] There is a set of amplitudes due to C. J. Goebel, M. L. Blackmon, and K. C. Wali, Phys. Rev. **182**, 1487 (1969), which differs from our D amplitudes. This is discussed in Sec. V.

[16] Our assumption of retaining only the leading terms leads to a definite product of ϵ and κ couplings from each analysis which is not the case in Ref. 8.

when $P=(k^+ + k^-)$, $Q=(k^+ - k^-)$. We introduce the variable $s_l=(K-P)^2=$ the invariant lepton pair's mass squared; $s+t+u=s_l+2m_\pi^2+m_K^2$. We assume that the axial-vector current is dominated by K_A- and K-meson poles. F_1 and F_2 receive contributions only from the K_A pole. Assuming that the structure of the Veneziano forms remains unaltered[2] when one particle is taken off shell, we continue the K_A mass to s_l and use the same forms of amplitudes for F_1 and F_2 as given in Eqs. (2) and (3) except for the arbitrary constant a. To represent the off-shell behavior[7] of F_1 and F_2 in the s_l variable, we write $\tilde{a}=\tilde{a}(s_l)$. The physical region for s_l is $0<s_l<(m_K-2m_\pi)^2$. In isospace,

$$F_i(stu)=\delta_{ab}F_i^+ + \tfrac{1}{2}[\tau_a,\tau_b]F_i^-, \tag{13}$$

and because of $s\leftrightarrow u$ symmetry we have

$$F_i^\pm(stu)=\pm(-1)^{i+1}F_i^\pm(uts). \tag{14}$$

In writing down the Veneziano forms, we assume further that an exotic resonance in $I=\frac{3}{2}$ does not exist. Thus from our knowledge of the $K\pi\to K_A\pi$ amplitude, we assume[17] the following forms for the F_i's:

For process I, $K^+\to\pi^+ + \pi^- + K_A^+$,

$$\begin{aligned}F_1^{\rm I}&=\tilde{a}V_1(\alpha_{K^*}(s),\alpha_\rho(t)),\\ F_2^{\rm I}&=\tilde{a}V_2(\alpha_{K^*}(s),\alpha_\rho(t));\end{aligned} \tag{15}$$

and for process II, $K^+\to\pi^0+\pi^0+K_A^+$,

$$\begin{aligned}F_1^{\rm II}&=\tfrac{1}{2}\tilde{a}[V_1(\alpha_{K^*}(s),\alpha_\rho(t))+V_1(\alpha_{K^*}(u),\alpha_\rho(t))],\\ F_2^{\rm II}&=\tfrac{1}{2}\tilde{a}[V_2(\alpha_{K^*}(s),\alpha_\rho(t))-V_2(\alpha_{K^*}(u),\alpha_\rho(t))].\end{aligned} \tag{16}$$

Further, for process III, $K_2^0\to\pi^-+\pi^0+K_A^+$, the F_i's can be easily written down using the $\Delta T=\frac{1}{2}$ rule. The functions V_1 and V_2 are given by

$$\begin{aligned}V_1(\alpha_{K^*}(s),\alpha_\rho(t))&=\tfrac{1}{2}[4\alpha_{K^*}(s)-2\alpha_\rho(t)-1]B(\alpha_{K^*}(s),\alpha_\rho(t)),\\ V_2(\alpha_{K^*}(s),\alpha_\rho(t))&=\tfrac{1}{2}[2\alpha_\rho(t)-1]B(\alpha_{K^*}(s),\alpha_\rho(t)),\\ \text{and}\qquad & \\ B(\alpha_{K^*}(s),\alpha_\rho(t))&=\frac{\Gamma(1-\alpha_{K^*}(s))\Gamma(1-\alpha_\rho(t))}{\Gamma(2-\alpha_{K^*}(s)-\alpha_\rho(t))}.\end{aligned} \tag{17}$$

IV. WEINBERG LIMITS

We now discuss the behavior of the amplitude when both the pions are made soft, when $t\to 0$ and $s=m_K^2-2k\cdot k^+$ and $u=m_K^2-2k\cdot k^-$, keeping the first-order terms in k^+ and k^-. In the limit $k^+\to 0$, $k^-\to 0$, we then find from (15)–(17) that

$$F_i^{\rm I}=0=F_i^{\rm II},\quad i=1,2, \tag{18}$$

where I and II denote the various physical decay amplitudes defined earlier.

Combining (18) with the current-algebra constraint of Weinberg,[12] we deduce that A as defined in Ref. 12 must vanish. Thus, our choice (17) for V_1, etc., with a single leading Veneziano amplitude does not satisfy the current-algebra result. We propose that additional Veneziano terms must be taken into consideration. Thus we replace (15) by

$$\begin{aligned}F_1^{\rm I}&=\tilde{a}V_1(\alpha_{K^*}(s),\alpha_\rho(t))+\lambda_1 B(\alpha_{K^*}(s),\alpha_\rho(t)),\\ F_2^{\rm I}&=\tilde{a}V_2(\alpha_{K^*}(s),\alpha_\rho(t))+\lambda_2 B(\alpha_{K^*}(s),\alpha_\rho(t)),\end{aligned} \tag{19}$$

and corresponding symmetric terms in the other amplitudes II and III. λ_1 and λ_2 are arbitrary parameters. Imposing the PCAC consistency condition when $k^+\to 0$, we find $\lambda_1=\lambda_2$. This corresponds, however, to adding a nonleading Veneziano amplitude to the invariant $D(s,t)$ function in our $K\pi\to K_A\pi$ scattering amplitudes.

With $\lambda_1=\lambda_2=\lambda=A/\pi$, we then immediately recover the Weinberg limits, i.e.,

$$\begin{aligned}&F_1^{\rm I}=F_2^{\rm I}=A,\\ &F_1^{\rm II}=A,\quad F_2^{\rm II}=0,\\ &F_1^{\rm III}=0,\quad F_2^{\rm III}=-A.\end{aligned} \tag{20}$$

To complete our discussion of current-algebra constraints, we now discuss the behavior of F_3 and note that, in principle, it receives contributions from both K- and K_A-meson poles. The asymptotic behavior of the K_A-pole contribution to F_3 cannot be determined in a simple fashion since F_3 does not occur in the physical $K\pi\to K_A\pi$ amplitude. Following Arnowitt *et al.*,[6] we invoke field-current identities, kaon PCAC, and the absence of satellite terms in the $K\pi\to K\pi$ amplitude to argue that the K_A-pole contribution to F_3 vanishes identically. Thus the K-meson pole alone contributes to F_3, which therefore has the following structure for processes I and II:

$$\begin{aligned}F_3^{\rm I}&=\tilde{c}C(\alpha_{K^*}(s),\alpha_\rho(t)),\\ F_3^{\rm II}&=\tfrac{1}{2}\tilde{c}[C(\alpha_{K^*}(s),\alpha_\rho(t))+C(\alpha_{K^*}(u),\alpha_\rho(t))],\end{aligned} \tag{21}$$

where

$$C(x,y)=\frac{\Gamma(1-x)\Gamma(1-y)}{\Gamma(1-x-y)}.$$

Thus near the K-meson pole the full structure of the amplitude F_3 is

$$\frac{\tilde{c}}{s_l-m_K^2}C(\alpha_{K^*}(s),\alpha_\rho(t)). \tag{22}$$

For consistency, we note that kaon PCAC and the absence of satellite terms would lead to only nonleading

[17] We could have started with different forms for our Veneziano amplitudes consistent with high-energy behavior as discussed by Riazuddin and Fayyazuddin (Ref. 8). This would entail a number of parameters which could be determined using the same principles as used in this paper. Our present choice in conjunction with current-algebra constraints suggests the presence of nonleading Veneziano amplitudes also. Our final conclusions are similar to those obtained in Ref. 8.

contributions to F_1 and F_2. This agrees with our modification of Eq. (19) to get agreement with the Weinberg results.

For process II, we find that $F_3^{\rm II}$ reduces to

$$-\tfrac{1}{2}\pi\alpha'\tilde{c}\frac{k\cdot(k^+ + k^-)}{k\cdot(k^+ + k^-)}. \qquad (23)$$

While taking the limit, we keep the first-order terms in k^+ and k^- both in the $(s_l - m_K^2)$ factor and in the Γ functions appearing in (21). Comparing this with the current-algebra result $F_3^{\rm II} \to B$, we find

$$B = -\tfrac{1}{2}\pi\alpha'\tilde{c}, \qquad (24)$$

which determines the arbitrary parameter $\tilde{c}$. In a similar way, using (24) and taking the limits, we find from (21) that

$$F_3^{\rm III} \to B\frac{k\cdot(k^+ - k^-)}{k\cdot(k^+ + k^-)}, \qquad (25)$$

and $F_3^{\rm I}$ can be calculated using the $\Delta T = \frac{1}{2}$ rule. This checks the validity of our assumptions of the form of F_3 given in Eq. (21), etc.

The presence of the extra nonleading terms modifies Eqs. (8) and (9) and we get

$$\begin{aligned} H_S/H_D &= H_S'/H_D' = -m_\rho^2(1+2\lambda/a), \\ \gamma_{K_A\epsilon K}\gamma_{\epsilon\pi\pi} &= m_\rho^2\lambda, \qquad (26) \\ \gamma_{K_A\kappa\pi}\gamma_{K\kappa\pi} &= -m_\rho^2(m_K^2 a - m_\rho^2\lambda)/m_{K^*}^2. \end{aligned}$$

V. CONCLUSIONS

We finally write down the complete structure of the axial-vector current for process I as

$$\begin{aligned} A_\lambda = {} & \frac{g_{K_A}}{s_l - m_{K_A}^2}\{P_\lambda[\tfrac{1}{2}\tilde{a}(4\alpha_{K^*}(s) - 2\alpha_\rho(t) - 1) + \lambda] \\ & + Q_\lambda[\tfrac{1}{2}\tilde{a}(2\alpha_\rho(t) - 1) + \lambda]\}B(\alpha_{K^*}(s), \alpha_\rho(t)) \\ & + \frac{\tilde{c}}{s_l - m_K^2}(P-k)_\lambda[1 - \alpha_{K^*}(s) - \alpha_\rho(t)] \\ & \times C(\alpha_{K^*}(s), \alpha_\rho(t)), \qquad (27) \end{aligned}$$

where the factors $g_{K_A}/(m_{K_A}^2 - s_l)$ and $1/(m_K^2 - s_l)$ are introduced to demonstrate explicitly the dominance[18] of the axial-vector current by K_A- and K-meson poles. Since $\Delta s_l/m_{K_A}^2 \simeq 0.03$, we can replace $1/(m_{K_A}^2 - s_l) \simeq 1/m_{K_A}^2$. The factor $1/(m_K^2 - s_l)$ is due to a K-meson pole term which is the singularity near the physical region of K decay.

Further, the constants a and λ should in principle be functions of s_l so as to represent the off-shell behavior of the amplitudes F_i in the s_l variable. If one assumes that $\tilde{a}(s_l)$, etc., do not depend strongly on s_l, then one can continue the $\tilde{a}(s_l)$ near $s_l = m_{K_A}^2$ and determine the $\tilde{a}$ directly from the $K\pi \to K_A\pi$ on-shell scattering amplitudes. In this case $\tilde{a}$ will of course coincide with a of Eq. (10) and its value will be given by various coupling constants as discussed in Sec. II. It is tempting to assume further that the variation of λ with s_l is such that near $s_l = m_{K_A}^2$, $\lambda(s_l)$ vanishes and the off-shell $K\pi \to K_A\pi$ amplitudes occurring in K_{l4} decay reduce to the on-shell Veneziano forms for $K\pi \to K_A\pi$ scattering,[19] given by Eqs. (2) and (3). It is then needless to mention that $\lambda_1 = \lambda_2 = A/\pi$ has been only determined near $s_l = m_K^2$, which is close to the physical region of K decay.

There is an alternative method of writing down the amplitudes, starting from a somewhat different interpretation of leading terms due to Goebel, Blackmon, and Wali.[15] They construct amplitudes with terms of the form $\Gamma(n-\alpha_{K^*})\Gamma(m-\alpha_\rho)/\Gamma(n+m-p-\alpha_{K^*}-\alpha_\rho)$, where n, m, and p are integers. This corresponds to giving a finite value to our λ; in particular, $\lambda = -a/2$ seems indicated by their analysis. This would lead to $H_S = H_S' = 0$, and if found to be true experimentally, may indicate that their interpretation of what is a leading term is more plausible than ours.

In our discussions we have completely ignored the question of fixed poles in electromagnetic and weak amplitudes.

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[18] This assumption is equivalent to the field-current identity. This hypothesis has also been adopted in Ref. 8.

[19] One obtains such a situation in the $\pi\pi \to \pi A_1$ amplitude. See H. Schnitzer, Ref. 5.