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Modified Scaling Law for Large-Angle πN Scattering*

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The large-angle high-energy πN elastic amplitude is conjectured to obey a modified scaling law, which states that this amplitude is of the form $F(t)H(-t/s)$, where t is the momentum-transfer variable, s the energy variable, and $F(t)$ a vector form factor. The presence of the function H, which depends only on the ratio of large kinematical variables, suggests a connection with Bjorken's scaling law for electroproduction. Such a form is in good agreement with the large-angle data currently available. The conjecture is based on (1) pion current commutators applied to mass-shell amplitudes, and (2) Feynman-graph experiments motivated by recent calculations in the eikonal (small-angle) region.

I. INTRODUCTION

IN the extensive literature on high-energy scattering, one finds relatively few papers on large-angle processes, compared to the large number of papers on diffractive processes. Large-angle scattering is difficult both experimentally (because of the smallness of the cross sections) and theoretically; one standard approach[1] is to use a Glauber multiple-scattering formalism[2] (eikonal approach) and push it to values of the momentum transfer to where the formalism may be of doubtful validity.[3,4]

Nonetheless, some very interesting results have been obtained in this way. A classic example is Chou and Yang's[5] extension of the Wu-Yang conjecture[6] that the pp elastic scattering amplitude is proportional to the square of the electric form factor $F(t)$. Chou and Yang proposed that the Fourier transform of the eikonal $\chi(b)$ was proportional to a product of form factors for the two incident particles involved in elastic scattering, and were able to fit $d\sigma/dt$ for $|t|<1$ GeV^2 and $F(t)$ for $|t|<25$ GeV^2.

Unfortunately, if Chou and Yang are correct, we are still far from the asymptotic region; the measured differential cross section for $|t|\geq 5$ deviates drastically from Chou and Yang's proposed asymptotic limits, even at incident momenta of 30 GeV (see their Fig. 1). It is difficult to assess the significance of calculations of $d\sigma/dt$ out to $|t|\simeq 6$ or more, based on various models of the eikonal $\chi(b)$,[7–10] since it is not clear that the eikonal formalism itself is valid for such large t.[3,4]

In this paper, we explore high-energy large-angle meson-baryon elastic scattering from two complementary points of view. In the first, we reexamine the work of Domokos and Karplus,[11] who studied the consequences of the hypothesis that the equal-time commutator of two meson currents is essentially the time component of the vector current. These authors were not quite correct in their treatment of current commutators with all particles on the mass shell, so that our conclusions differ somewhat from theirs.

The second point of view both confirms and extends the mass-shell current-commutator arguments. In it, we look at Feynman graphs based on a certain effective Lagrangian, the main feature of which is that at high energy, forces between hadrons come from the exchange of massive neutral vector mesons coupled to the baryon current. It is precisely this feature which is important in the many eikonal calculations of high-energy scattering at finite t (small angles).[12–17] However, the graphs which add up to eikonal form for small t do *not* yield the leading terms at asymptotically large t; a different set of graphs must be used, but they share with the eikonal graphs the features that forces are transmitted by vector exchange.

Either of these two points of view yields essentially the same result in the large-angle high-energy scattering

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region. We find the following: In the limit $s\to\infty$, $-t\to\infty$, $-u\to\infty$, with $\zeta\equiv -t/4\nu$ fixed[18] $[\nu=\frac{1}{4}(s-u)]$, the differential cross section for πp elastic scattering obeys

$$\frac{d\sigma_{\rm el}}{dt}\to\frac{G^4}{4\pi s^2}|F(t)|^2|H(\zeta)|^2(1-\zeta^2)\,, \tag{1a}$$

where $G^2/4\pi\simeq 15$ and $H(\zeta)$ is a (complex) function of ζ, nominally of order unity, constrained to vanish at $\zeta=0$ if there are no right-signature fixed poles in πN scattering (see Sec. IV). $F(t)$ is a form factor, given in the second point of view by a set of Feynman graphs already discussed by Levy[17] in connection with the eikonal approximation. The precise nature of $F(t)$ is somewhat model dependent; there are hints that $F(t)$ is the form factor of the baryon current, rather than of the usual vector octet, but in the models we study, the asymptotic behavior of both currents is the same. Kinematically, the models suggest that $F(t)$ is either the magnetic form factor $G_M(t)$ or the Dirac form factor $F_1(t)$; these are the same thing asymptotically. In a comparison of (1a) with pion data (Sec. II), a quite reasonable function $H(\zeta)$ is obtained when the proton magnetic form factor G_M is used for $F(t)$.

Equation (1a) does not, in itself, suggest an immediate generalization to other processes. For $\pi\pi$ elastic scattering, Sec. IV shows that $F(t)$ in (1a) should be replaced by the pion form factor, while there is (as yet) no easy generalization to pp scattering (however, see the discussion in Sec. V).

The appearance of the function $H(\zeta)$ (whose nature depends on detailed dynamics) is reminiscent of the scaling law proposed by Bjorken[19] for electroproduction, involving a highly virtual photon. For mass-shell processes, Bjorken and Paschos[20] have used the parton model for *inelastic* Compton scattering to find that the cross section summed over all final hadron states (an inclusive experiment in Feynman's[21] language) depends on a scale-invariant function of ζ, with no form factors $F(t)$. Our results for *elastic* Compton scattering (an exclusive experiment) look like (1a), with G^4 replaced by e^4. It is tempting to speculate that various powers of $F(t)$ distinguish the scaling-law behavior of inclusive and exclusive cross sections.

The theories discussed in the present paper give little insight into the structure of $H(\zeta)$. In later work, we shall discuss possible dynamic schemes for calculating $H(\zeta)$, which incorporate parton-like models.

II. COMPARISON WITH EXPERIMENT

There are perhaps many readers whose greatest interest in (1a) is as an ansatz to be tested against data,

[18] In the infinite-energy limit, $\zeta=(1-\cos\theta)(3+\cos\theta)^{-1}$, where θ is the c.m. scattering angle.
[19] J. D. Bjorken, Phys. Rev. **179**, 1547 (1969).
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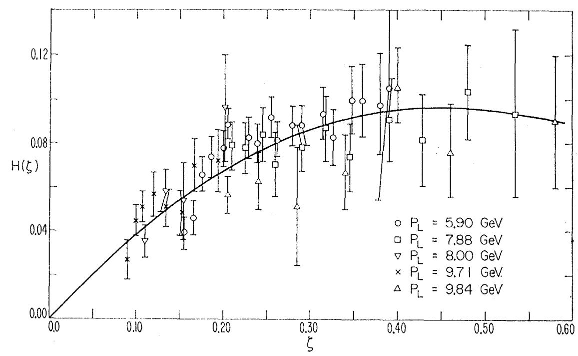


FIG. 1. Experimental test of the modified scaling law. Data points are taken from Refs. 22 and 23, for $|t|$ values from 3 to 12 GeV^2. P_L is the pion laboratory momentum.

quite apart from detailed theoretical motivations. Therefore, we begin with a comparison with experiment. The equation used was very similar to (1a):

$$\frac{d\sigma}{dt}=\frac{G^4}{4\pi}[s-(M+m_\pi)^2]^{-1}[s-(M-m_\pi)^2]^{-1} \times(1-\zeta^2)|G_M(t)|^2|H(\zeta)|^2, \tag{1b}$$

with $G^2/4\pi\simeq 15$, M the proton mass, m_π the pion mass, G_M the magnetic form factor of the proton, and $\zeta=-t/4\nu \simeq -t(2s+t)^{-1}$. The specific form of (1b) is based on the considerations of Sec. IV, which suggest that the A amplitude of pion scattering vanishes asymptotically one power of t faster than the B amplitude, which is proportional to $G_M(t)\nu^{-1}$; therefore, the A amplitude has been set equal to zero in deriving (1b).

For the comparison, data on large-angle $\pi^- p$ elastic scattering cross sections was taken from the work of Orear *et al.*[22] and Owen *et al.*[23] Values of $H(\zeta)$ computed from these works and from Eq. (1b) are shown in Fig. 1; the ranges of s and t covered are $3\leq|t|\leq 13$ GeV^2, $12\leq s\leq 19.4$ GeV^2. The solid line is a least-squares polynomial fit, and has no basic theoretical significance. The line passes through the error bars of all but a few points, and most of these points have $|t|\simeq 3$ GeV^2, where it is very possible that the scaling limit (1b) is not very good; in fact, the cross sections near $|t|=3$ show a dip structure very much like the multiple-scattering breaks found in, e.g., the works of Chou and Yang[8] and Durand and Lipes[7] based on the eikonal approximation. There is, of course, no necessary contradiction between the eikonal (finite-t) theory and our scaling (infinite-t) theory; if our theory is correct, the two regions should overlap in a domain where features common to both the eikonal and the scaling limits appear. Note that $H(\zeta)$

[22] J. Orear, R. Rubinstein, D. B. Scarl, D. H. White, A. D. Krisch, W. R. Frisken, A. L. Read, and H. Ruderman, Phys. Rev. **152**, 1162 (1966).
[23] D. P. Owen, F. C. Peterson, J. Orear, A. L. Read, D. G. Ryan, D. H. White, A. Ashmore, C. J. S. Damerell, W. R. Frisken, and R. Rubinstein, Phys. Rev. **181**, 1794 (1969).

appears to vanish near $\zeta=0$, in accordance with the arguments of Sec. IV.

The scale of $H(\zeta)$ (~ 0.1) is rather small, compared to the Born approximation, which would suggest $H\sim 1$. This is perhaps reminiscent of the small scale of the electroproduction structure function[24] which seems to reflect a rather small mean-square charge per parton (cf. Bjorken and Paschos[20]).

It is difficult to say what the scale of $H(\zeta)$ really should be, at the present crude level of theory, and numerical factors such as $G^4/4\pi$ in (1b) might be differently chosen. For example, if G_M in (1b) is replaced by $G_M\mu^{-1}$ (where $\mu=2.79$ is the proton magnetic moment in Bohr magnetons), $|H(\zeta)|$ as shown in Fig. 1 should be multiplied by μ, which brings the scale of H intriguingly close to that of the electroproduction structure functions.

In the ranges of t and s quoted above, the cross sections vary by almost three orders of magnitude, while the points in Fig. 1 fall on a common line within (roughly) $\pm 20\%$. This is impressive evidence for scaling as described in Eq. (1b).

III. MASS-SHELL CURRENT ALGEBRA

Here we contrast the Bjorken limit for extracting matrix elements of equal-time commutators with the infinite momentum limit of the mass-shell scattering amplitude.

Consider the Fourier transform of the retarded commutator of the currents a, b, and moments q_1, q_2, taken between (covariantly normalized) free-particle states of momenta P_1 and P_2:

$$R(\mathbf{P},Q,\Delta)=i\int d^4x\, e^{iQ\cdot x}\theta(x_0)\times\langle P_2|[J^a(\tfrac{1}{2}x),J^b(-\tfrac{1}{2}x)]|P_1\rangle$$
$$=\int_0^\infty dx_0\, F(x_0,\ldots)e^{iQ_0x_0}, \tag{2}$$

where

$$F(x_0,\ldots)=i\int d^3x\, e^{-i\mathbf{Q}\cdot\mathbf{x}}\langle P_2|[J^a(\tfrac{1}{2}x),J^b(-\tfrac{1}{2}x)]|P_1\rangle \tag{3}$$

and

$$P=\tfrac{1}{2}(p_1+p_2),\quad Q=\tfrac{1}{2}(q_1+q_2),\quad \Delta=q_1-q_2=p_2-p_1. \tag{4}$$

If the equal-time commutator $F(0,\ldots)$ exists, its value can be determined from the Bjorken limit (B)[25]:

$$R(P,Q,\Delta)\xrightarrow[Q_0\to\infty]{}-\frac{F(0,\ldots)}{Q_0}+\text{polynomials}+o\left(\frac{1}{Q_0}\right) \tag{5}$$

provided that $F(x_0,\ldots)$ does not explicitly depend on Q_0, i.e., provided that all other momenta in the problem are held fixed. It is thus necessary, for the B limit to be applied, that q_1^2 and q_2^2 also approach infinity.

Our interest is in the case where the J's are pion currents, and it is necessary for us to keep q_1^2 and q_2^2 fixed, with the value m_π^2. In such a case, since

$$q_1^2=Q^2+Q\cdot\Delta+\tfrac{1}{4}t,\quad q_2^2=Q^2-Q\cdot\Delta+\tfrac{1}{4}t \tag{6}$$

(here $t\equiv\Delta^2$), it follows that either $\mathbf{Q}^2$ or t must become infinite with Q_0^2. In either case, $F(x_0,\ldots)$ does depend on Q_0, and the B limit is not applicable.

To see what can be learned in such circumstances, we use a convenient form of the Jost-Lehmann-Dyson representation, which extends the often-used Deser-Gilbert-Sudarshan (DGS)[26] representation for forward matrix elements, and which was first written in essentially the form below by Nakanishi[27]:

$$T(P,Q,\Delta)=\int\frac{d\lambda^2d\beta d\beta' h(\lambda^2,\beta,\beta';t)}{(Q+\beta P+\frac{1}{2}\beta'\Delta)^2-\lambda^2+i\epsilon}, \tag{7}$$

where T is the time-ordered product corresponding to the retarded commutator R. The weight function h has support in $\lambda^2\geq 0$, $|\beta'|\leq 1$. In the limit $\Delta\to 0$, the DGS representation is obtained.

[We digress to give a simple heuristic derivation of (7). The commutator

$$C(x^2,P\cdot x,\Delta\cdot x,t)=\langle P_2|[J^a(\tfrac{1}{2}x),J^b(-\tfrac{1}{2}x)]|P_1\rangle$$

depends on the listed invariants. As C vanishes for $x^2<0$, it has the representation (like the Källén-Lehmann representation)

$$C=i\int_0^\infty d\lambda^2\Delta(x;\lambda^2)G(P\cdot x,\Delta\cdot x,t;\lambda^2),$$

where $\Delta(x;\lambda^2)$ is the free-field commutator with mass λ. G has a Fourier transform with respect to $P\cdot x$, $\Delta\cdot x$ of arguments β, $\frac{1}{2}\beta'$, respectively. In the cases we are interested in, the time-ordered product can be gotten from C by the replacement $\Delta\to\Delta_F$, the Feynman propagator; Fourier transforming with respect to Q yields (7).]

Now we take up the limit of (7) as $Q_0\to\infty$, with $q_1^2=q_2^2=m_\pi^2$ [here $Q\cdot\Delta=0$, from (6)], defined as the M limit. In terms of the quantities $\nu\equiv P\cdot Q$ and t, we construct a dimensionless quantity $\zeta\equiv -t/4\nu$; in the c.m. frame of P_1 and q_1, we find

$$\nu\xrightarrow[M]{}2Q_0^2/(1+\zeta),\quad Q^2\xrightarrow[M]{}-\tfrac{1}{4}t=\zeta\nu. \tag{8}$$

[24] E. D. Bloom *et al.*, Phys. Rev. Letters **23**, 930 (1969); M. Breidenbach *et al.*, *ibid.* **23**, 935 (1969).

[25] J. D. Bjorken, Phys. Rev. **148**, 1467 (1966).

[26] S. Deser, W. Gilbert, and E. C. G. Sudarshan, Phys. Rev. **115**, 731 (1959); M. Ida, Progr. Theoret. Phys. (Kyoto) **23**, 1151 (1960); N. Nakanishi, *ibid.* **26**, 337 (1961); Suppl. **18**, 70 (1961).

[27] N. Nakanishi, Ref. 26.

Then it is easy to find (we take $P_1^2=P_2^2$, so $P\cdot\Delta=0$)

$$T(P,Q,\Delta)\underset{M}{\longrightarrow}\frac{1}{\nu}\int\frac{d\lambda^2 d\beta d\beta' h(\lambda^2,\beta,\beta';t)}{2\beta+\zeta(1+\beta^2-\beta'^2)}. \qquad (9)$$

But in the Bjorken limit,

$$T(P,Q,\Delta)\underset{B}{\longrightarrow}\frac{1}{Q_0^2}\int d\lambda^2 d\beta d\beta' h(\lambda^2,\beta,\beta';t). \qquad (10)$$

Although (9) and (10) are different, it is plausible that the ratio of the two integrals is a function (of order unity) of ζ only, in the limit $t\to\infty$, if the B limit in (10) leads to a rapidly decreasing function of t. We suppose, then, that the t dependence of $h(\lambda^2,\beta,\beta';t)$ in some sense factors out when t becomes very large, and that there is no delicate cancellation of oscillating terms in (10) which leads to the decrease with t. Our conjecture, then, is that the B limit and the M limit of a scattering amplitude differ by a function of the dimensionless variable ζ, and otherwise show the same asymptotic rate of decrease in t.

IV. APPLICATION TO PION SCATTERING

For pion scattering, (7) is replaced by

$$T(P,Q,\Delta)=\bar{u}(P_2)(A+\gamma\cdot QB)u(P_1), \qquad (11)$$

$$A=P\cdot Q\int\frac{h_A(\lambda^2,\beta,\beta';t)}{(Q+\beta P+\frac{1}{2}\beta'\Delta)^2-\lambda^2+i\epsilon}, \qquad (12a)$$

$$B=\int\frac{h_A(\lambda^2,\beta,\beta';t)}{(Q+\beta P+\frac{1}{2}\beta'\Delta)^2-\lambda^2+i\epsilon}, \qquad (12b)$$

where for convenience we now delete the differentials $d\lambda^2 d\beta d\beta'$. The extra power of $P\cdot Q$ in A allows us to accommodate certain commutation relations to be given below; it also allows for Regge behavior with trajectories rising as high as $J=1$, although we are not concerned with the Regge region presently. Superscripts $(\pm)$ will be used, when necessary, to distinguish the isospin-even and -odd πN amplitudes.

The limit of T is governed by the equal-time commutator of pion currents. If one takes the $Q_0\to\infty$ limit of (11) and (12), the resulting equal-time commutator [see (17) below] is the time component of a $J^{PC}=1^{--}$ current; it would be very surprising if this current had totally dissimilar properties to the usual vector currents which participate in weak and electromagnetic interactions. Many readers may find this simple argument at least as compelling as the special models we study below.

As Domokos and Karplus[11] have pointed out, in standard pseudoscalar πN field theory, the commutator of two pion currents is a piece of the vector current. Consider, for example, the Lagrangian

$$L=L_{\text{free}}+iG\bar{\psi}\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\phi}\psi+\gamma\bar{\psi}\gamma_\mu B^\mu\psi, \qquad (13)$$

which has the usual pion coupling, plus a coupling of the baryon current to a singlet vector meson B (this L will be used in the graphical studies of Sec. V). We easily find

$$\delta(x_0)[J_\pi{}^a(x),J_\pi{}^b(0)]=4iG^2\epsilon_{abc}V_c{}^0(x)\delta_4(x), \qquad (14)$$

where $V_\mu{}^c(x)$ is not exactly the whole vector current, but just the baryonic portion:

$$V_\mu{}^c(x)=\bar{\psi}\gamma_\mu\tfrac{1}{2}\tau^c\psi. \qquad (15)$$

The full vector current $J_\mu{}^c$ has a pion contribution:

$$J_\mu{}^c=V_\mu{}^c+\epsilon_{abc}\phi^a\partial_\mu\phi^b. \qquad (16)$$

With the aid of the Bjorken limit (5) and the commutator (14), Eqs. (11) and (12) yield a sum rule for the isospin-odd spectral functions:

$$\bar{u}(P_2)\tfrac{1}{2}\tau_c\left[P_0\int h_A{}^{(-)}(\lambda^2,\beta,\beta';t)+\gamma_0\int h_B{}^{(-)}(\lambda^2,\beta,\beta';t)\right]u(P_1)=-2G^2\langle P_2|V_0{}^c(0)|P_1\rangle. \qquad (17)$$

To the extent that $V_\mu{}^c$ can be related to an independently measurable physical current, (17) can be used (as in Sec. III) to discuss πN elastic scattering in the M limit of large angles and high energy.

There are two points of view one may take with regard to the difference between $J_\mu{}^c$ and $V_\mu{}^c$. In the first, the fact that $\frac{1}{2}\tau_c$ sits between the baryon fields in (15) is ignored, and one thinks of $V_\mu{}^c$ as somehow related to the baryon current. The graphical studies of Sec. V actually suggest this point of view. In the second, one tries to relate $V_\mu{}^c$ to the isovector current $J_\mu{}^c$ by evaluating the correction terms [see (16)]

$$\langle J_\mu{}^c-V_\mu{}^c\rangle\equiv\epsilon_{abc}\langle P_2|\phi^a(x)\partial_\mu\phi^b(x)|P_1\rangle. \qquad (18)$$

This can actually be expressed in terms of the πN amplitude $T(P,Q,\Delta)$:

$$\langle J_\mu{}^c-V_\mu{}^c\rangle=\frac{\epsilon_{abc}}{(2\pi)^4}\int d^4Q\times\frac{Q_\mu T^{ab}(P,Q,\Delta)}{[(Q+\frac{1}{2}\Delta)^2-m_\pi{}^2][(Q-\frac{1}{2}\Delta)^2-m_\pi{}^2]} \qquad (19)$$

as one sees by looking at Feynman graphs. Unfortunately, when expressions (11) and (12) for T are used in (19), the integral over Q diverges logarithmically if the integrals in (17) do not vanish. The divergence itself can be absorbed into a renormalization constant, leaving the question of finite corrections. Very roughly speaking, the structure of these finite corrections is such that (18)

FIG. 2. Sum of all crossed and uncrossed ladders.

is replaced by something like

$$\langle J_\mu{}^c - V_\mu{}^c\rangle \sim \frac{G^2}{16\pi^2}\ln\left(-\frac{t}{M^2}\right)\langle V_\mu{}^c\rangle. \tag{20}$$

While this equation should not be taken too seriously, it indicates that there might be non-negligible corrections in going from $J_\mu{}^c$ to $V_\mu{}^c$. Nonetheless, within a factor of 2 or 3, $\langle J_\mu{}^c\rangle$ is essentially the same as $\langle V_\mu{}^c\rangle$, except at extremely high t, which is all that we need for our purposes.

Let us simply ignore the complications of such corrections, and identify $\langle J_\mu\rangle$ with $\langle V_\mu\rangle$. Then the sum rule (17) becomes

$$M\int h_A{}^{(-)}(\lambda^2,\beta,\beta';t) = 2G^2[F_2{}^p(t) - F_2{}^n(t)], \tag{21a}$$

$$\int h_B{}^{(-)}(\lambda^2,\beta,\beta';t) = -2G^2[G_M{}^p(t) - G_M{}^n(t)], \tag{21b}$$

where F_2 is the Pauli form factor, defined in terms of the electric and magnetic form factors G_E, G_M by

$$F_2(t) = \frac{G_M(t) - G_E(t)}{1 - t/4M^2}. \tag{22}$$

The corresponding integrals for the $(+)$ amplitudes vanish because $h_{A,B}{}^{(+)}$ is even in β.

In the mass-shell (M) limit of Sec. III, T becomes

$$T(P,Q,\Delta) \underset{M}{\longrightarrow} \bar{u}(p_2)\left[\frac{\int h_A(\lambda^2,\beta,\beta';t)}{2\beta+\zeta(1+\beta^2-\beta'^2)} + \frac{\gamma\cdot Q}{\nu}\frac{\int h_B(\lambda^2,\beta,\beta';t)}{2\beta+\zeta(1+\beta^2-\beta'^2)}\right]u(p_1); \tag{23}$$

integrands in (23) differ [at least for the $(-)$ amplitudes] from those in (17) only by the denominators, and it is not unreasonable to conjecture that the integrals in (23) are equal to the ones in (17), multiplied by functions of ζ which are of order unity.

Let us assume that *both* the $(+)$ and $(-)$ amplitudes behave as follows, in the large-angle high-energy limit:

$$\frac{\int h_A{}^{(\pm)}(\lambda^2,\beta,\beta';t)}{2\beta+\zeta(1+\beta^2-\beta'^2)} \underset{M}{\longrightarrow} G^2 H_A{}^{(\pm)}(\zeta)F_2(t), \tag{24a}$$

$$\frac{\int h_B{}^{(\pm)}(\lambda^2,\beta,\beta';t)}{2\beta+\zeta(1+\beta^2-\beta'^2)} \underset{M}{\longrightarrow} G^2 H_B{}^{(\pm)}(\zeta)G_M(t). \tag{24b}$$

These equations define the functions $H(\zeta)$ (if they exist). Here, to be definite, we choose $F_2(t)$ to be the Dirac form factor of the proton, and $G_M(t)$ to be the magnetic form factor of the proton. There is, of course, no justification for (24) based on current commutators for the $(+)$ amplitudes, but the graphical experiments of the following section give similar results for the $(+)$ and $(-)$ amplitudes. [If (24) is considered to be true only for the $(-)$ amplitudes, it is possible to make predictions only for large-angle charge-exchange cross sections.]

The elastic differential cross sections for $\pi^\pm p$ scattering depend on both invariants A and B. However, granted that $G_M(t)$ and $G_E(t)$ fall at the same rate for large t, Eq. (19) shows that $F_2(t)$ falls one power of t faster than $G_M(t)$ and it is easy to calculate that the A amplitude has an asymptotically vanishing contribution, compared to the B amplitude. The differential cross section becomes, as in Eq. (1),

$$\frac{d\sigma}{dt} \underset{M}{\longrightarrow} \frac{G^4}{4\pi s^2}(1-\zeta)^2|H_B(\zeta)|^2 G_M(t)|^2, \tag{25}$$

where $H_B = H_B{}^{(+)} \pm H_B{}^{(-)}$ for $\pi^\pm$ scattering on protons.

Clearly, $H_B{}^{(-)}(\zeta)$ vanishes at $\zeta=0$, since $h_B{}^{(-)}$ is even in β [see (24b)]. $H_B{}^{(+)}(\zeta)$ may also vanish at $\zeta=0$, according to the following Regge arguments. Suppose that for sufficiently large negative t, all moving singularities (cuts and poles) in the t-channel angular momentum plane retreat to the left of $\zeta=0$. With *t fixed*, any ν^{-1} term in $B^{(+)}$ as $\nu\to\infty$ $(\zeta\to 0)$ must come from a right-signature fixed pole at $J=0$, but this is forbidden by bilinear unitarity.

We conclude that $H_B{}^{(+)}(0)=0$ and $H_B(0)=0$ for both π^+p and π^-p scattering. This conclusion is supported by the experimental data (see Fig. 1).

V. EXPERIMENTS WITH FEYNMAN GRAPHS

Recently, a large number of authors[12–17] have considered high-energy scattering in the eikonal (fixed-t) region, as represented by the sum of an infinite number of Feynman graphs. The basic ingredient of many such calculations (insofar as they apply to strong interactions) is that forces are transmitted by neutral vector mesons coupled to a conserved $SU(3)$ singlet current, presumably the baryon current. The fundamental graphical building block is the sum of all crossed and uncrossed laddres of vector exchange between the baryon lines, as shown in Fig. 2. The spin of the exchanged particle is important; generalized ladder graphs of scalar or pseudoscalar rungs vanish asymptotically compared to the vector ladders. It is for this reason that we have added the vector-meson term in the Lagrangian

FIG. 3. πN scattering in the eikonal (finite-t) approximation.

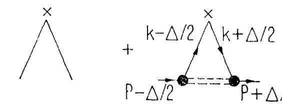
of Eq. (13); otherwise, there would be no pion scattering in the eikonal region at large s. As it is, no *internal* pion lines need be considered in any of the high-energy graphs; such graphs are asymptotically negligible.

More complicated versions of the eikonal scheme are sometimes considered, in which the vector particle is not elementary but is a Regge pole; of course, scalar particles may be used to generate the Regge pole. In this sense, internal pion lines could be important.

In the eikonal region, the important pion scattering graphs are as shown in Fig. 3, where it is understood that only exchanges of an even number of vector mesons are allowed. Both these graphs, and the NN scattering graphs of Fig. 2, are finite even with elementary vector-meson exchange and bare vertices.

Levy[17] has carried these considerations a step further, by looking at graphs for baryonic form factors. For large t (i.e., in leading order of $\log t$), it can be shown that the only important graphs are of the type in Fig. 4, if the baryonic form factors are decreasing.[28] These graphs, unlike those of Figs. 2 and 3, do not converge with bare vertices, of course; they have the usual logarithmic divergence. But self-consistency demands that each of the internal vertices of Fig. 2 have a form factor with the same asymptotic behavior in the meson mass variables as the form factor (Fig. 4) which is constructed from the graphs of Fig. 2. Imposition of this self-consistency requirement makes the graphs in Fig. 4 finite, and allows one to fit a large body of nucleon-nucleon and meson-nucleon scattering data at small t, as well as the asymptotic behavior of form factors at large t, with three adjustable parameters.[17]

FIG. 4. "Eikonal" graphs for vector form factors.



We now turn to the large-angle region of πN scattering, where a quite different set of graphs becomes important, as shown in Fig. 5. These graphs are of the same general type as studied by Fried and Moreno[29] for high-energy electroproduction. It is a straightforward, if lengthy, exercise to show that the eikonal graphs of Fig. 3 are asymptotically small as $s,\ t\to\infty$ (with t/s fixed and finite) compared to those of Fig. 5. (On the other hand, the graphs in Fig. 5 vanish at finite t as $s\to\infty$, and thus are unimportant in the eikonal region.) Figure 5 indicates that the dynamics of πN scattering is determined by the dynamics of off-shell $\bar{N}N$ scattering.

Obviously we can be more general and allow in Fig. 5 any $N\bar{N}$ amplitude, not just the generalized ladders of Fig. 2 with form factors at the vertices. The general conclusions will be unchanged, as long as the graphs of Figs. 4 and 5 are finite. One virtue of the generalized ladder graphs is that their spin dependence is particularly simple, being of the form $\bar{u}\gamma^\mu u\bar{u}\gamma_\mu u$ asymptotically, which simplifies calculations. It is also possible to ignore the loop momentum k in the *numerators* of all integrals because of the high degree of convergence, which further simplifies spin problems.

The off-shell $N\bar{N}$ amplitude satisfies a causal representation of the type given in Sec. III. This, plus the remarks of the preceding paragraph, allows us to write the form-factor graphs of Fig. 4 at large t as

$$F_\mu(t)=\gamma_\mu\left[1+\frac{i}{(2\pi)^4}\int\frac{d^4k\int d\lambda^2 d\beta d\beta' h^{N\bar{N}}(\lambda^2,\beta,\beta';t)}{[(k-\frac{1}{2}\Delta)^2-\mu^2][(k+\frac{1}{2}\Delta)^2-\mu^2][(-k+\beta P+\frac{1}{2}\beta'\Delta)^2-\lambda^2]}\right] \tag{26}$$

and the high-energy limit of the pion scattering amplitude of Fig. 5 as

$$\begin{aligned}T^{ab}(P,Q,\Delta)=-G^2\bar{u}(P_2)\tau_b\tau_a Q u(P_1)&\left[\frac{1}{(P+Q)^2-M^2}+\frac{i}{(2\pi)^4}\right.\\ &\left.\times\int\frac{d^4k\int d\lambda^2 d\beta d\beta' h^{N\bar{N}}(\lambda^2,\beta,\beta';t)}{[(Q+k)^2-u^2][(k-\frac{1}{2}\Delta)^2-\mu^2][(k+\frac{1}{2}\Delta)^2-\mu^2][(-k+\beta P+\frac{1}{2}\beta'\Delta)^2-\lambda^2]}\right]+(b\leftrightarrow a,\ Q\leftrightarrow -Q)\,,\end{aligned} \tag{27}$$

where $h^{N\bar{N}}$ is the spectral function for off-shell $N\bar{N}$ scattering. Note that asymptotically the form factor is pure Dirac type, and the πN amplitude is pure B type (no helicity flip).

The next step is to take the large-t limit of (26), and the large-angle limit of (27). Both integrals can be expressed as weighted integrals over $h^{N\bar{N}}$, after doing the integrals over k and taking limits. In this fashion, one

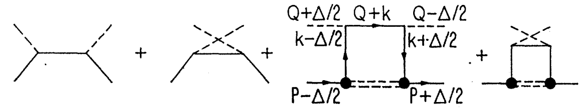


FIG. 5. Dominant graphs for πN scattering in the large-angle ($-t\to\infty$) region.

[28] Graphs of this structure, but with bare vertices, have been studied extensively in quantum electrodynamics: see V. V. Sudakov, Zh. Eksperim. i Teor. Fiz. **30**, 87 (1956) [Soviet Phys. JETP **3**, 65 (1956)]; D. R. Yennie, S. C. Frautschi, and H. Suura, Ann. Phys. (N. Y.) **13**, 379 (1961); M. Cassandro and M. Cini, Nuovo Cimento **34**, 1719 (1964); R. Jackiw, Ann. Phys. (N. Y.) **48**, 292 (1968).

[29] H. M. Fried and H. Moreno, Phys. Rev. Letters **25**, 625 (1970).

finds by saving only leading terms in t that

$$F(t)\to 1+\int d\lambda^2 d\beta d\beta' K(\lambda^2,\beta,\beta';t)h^{N\bar{N}}(\lambda^2,\beta,\beta';t)\,, \quad (28)$$

$$B(\nu,t)\to -\frac{G^2\tau_b\tau_a}{2\nu}\left(\frac{1}{1+\zeta}+\int\frac{Kh^{N\bar{N}}}{\beta+\zeta}\right)+(b\leftrightarrow a,\,\zeta\leftrightarrow -\zeta)\,, \quad (29)$$

where B is the coefficient of $\boldsymbol{Q}$ in the πN amplitude and $\zeta=-t/4\nu$ as before. The kernel K need not be recorded in detail; it is a complicated integral over Feynman parameters. The property of interest to us is that the kernel for the scattering amplitude is $(\beta+\zeta)^{-1}$ times the kernel for the form factor, in leading order of $\log t$.

Were it not for the Born terms [the 1 in (28), the $(1+\zeta)^{-1}$ in (29)], we would have already established a result essentially equivalent to (24). As it is, there must be a cancellation of these Born terms against the integrals in (28) and (29), if the final results are to decrease rapidly in t. Levy[17] has shown that the form factor does decrease, so the cancellation needed for (28) actually occurs. To leading order of $\log t$, the same cancellation occurs in (29) because the dominant asymptotic behavior of the integral comes from the neighborhood of $\beta=1$, for all the graphs of Fig. 5. The lengthy but straightforward proof of this statement uses the techniques already devised for high-energy limits of Feynman graphs in the eikonal calculations,[13,15] and will be omitted. [It is particularly simple for the reader to check this result in the lowest-order box graph of Fig. 5, in which $h^{N\bar{N}}\sim\delta(\beta-1)$; in higher orders, $h^{N\bar{N}}$ has denominators of the type $(\beta-1)t+x$, where x is finite at large t.] As a result, (29) reads

$$B(\nu,t)\to -\frac{G^2\tau_b\tau_a F(t)}{2\nu}\left(\frac{1}{1+\zeta}\right)+(b\leftrightarrow a,\,\zeta\leftrightarrow -\zeta)\,, \quad (30)$$

where the omitted terms are, graph by graph, one order of $\log t$ smaller. [A quick but unjustified way to get to (30) is to change the loop momentum in Fig. 5 by $k=k'+P$, then to set $k'=0$ in the nucleon propagator which joins the two pion vertices.]

While this simple model does illustrate the points made in Sec. IV, there are drawbacks: the scale function $H(\zeta)$ is too simple, and the scale function does not vanish at $\zeta=0$ for $B^{(+)}$. Also, the nonleading terms are only one power of $\log t$ smaller, graph by graph, which means that only at very high t ($\sim$100 GeV^2) would one see the behavior in (30) as dominant.

One feature we hope would survive in a more complete theory is the simple isospin structure, which is totally determined by the two external pion vertices. Because of this simple structure, both $B^{(+)}$ and $B^{(-)}$ have an asymptotic decrease governed by the vector form factor, while the current commutator used in Sec. IV gives direct information only about $B^{(-)}$. Incidentally, it is easy to verify that the Bjorken ($Q_0\to\infty$) limit of the graphs of Fig. 5 is precisely equivalent to the commutator (24), with the matrix elements of the vector current $V_\mu{}^c$ given by $\frac{1}{2}\tau_c\gamma_\mu F(t)$. Note that, in our simple model, $F(t)$ serves as matrix element both for the isovector current and the baryon current.

VI. SUMMARY AND CONCLUSIONS

Let us say that an elastic amplitude shows modified scaling behavior in the large-angle region if it goes like $H(\zeta)F(t)^N$, where $F(t)$ is some vector form factor and N an integer. For πN scattering, both theory and experiment suggest that $N=1$. Figure 1 shows that for $|t|$ in the region 3–12 GeV^2, the elastic πN amplitude follows this modified scaling law rather closely. This experimental fact is perhaps much more compelling than the theoretical motivations given in Secs. III and IV, which compound a number of optimistic assumptions.

It would, of course, be desirable to test this modified scaling behavior in other circumstances. But areas accessible to the theory presented here do not overlap experimentally accessible areas. Thus, there is a great deal of data in NN elastic scattering, but our attempts to interpret the meaning of baryon current commutators, or to sum up suitable graphs, have been fruitless so far. On the other hand, it is easy to extend a theory to large-angle elastic Compton scattering, but the cross sections are tiny.

It would be of considerable interest to see whether NN elastic scattering obeys an empirical modified scaling law (perhaps with $N=2$), but we have not analyzed the data to test this hypothesis.

The very restricted theory presented here might be extended in several directions. First, a dynamical framework for calculating the scale function $H(\zeta)$ in elastic scattering is needed; investigations are now under way with parton models. It would be satisfying to develop a framework which could be extended to the eikonal region and compared with the large body of theoretical ideas used there, so that we could understand what values of t are "finite" and what values are "asymptotically large." It should also be possible to give modified scaling laws for exclusive[21] processes which are slightly inelastic, e.g., $\pi N\to\pi\pi N$, in the kinematic region where all invariants are large. The theory of Sec. III relates this amplitude to the amplitude $\gamma N\to\pi N$, which may itself show modified scaling with $N=1$. Ultimately, it might be possible to add up all the results for exclusive experiments, to discuss inclusive processes like $\pi N\to\pi+$anything, which Feynman[21] argues should show unmodified (i.e., $N=0$) scaling.[30]

Finally, a discussion of scaling laws brings to mind the

[30] N. F. Bali, L. S. Brown, R. D. Peccei, and A. Pignotti, Phys. Rev. Letters **25**, 557 (1970), have discussed experimental evidence for scaling in $pp\to\pi+$anything.

possibility that conformal invariance[31] might be involved. Naive conformal-invariance arguments would suggest that elastic amplitudes depend only on dimensionless ratios asymptotically, which is equivalent to taking $N=0$. This is clearly wrong experimentally. Castell[32] has suggested that particle-mass effects can be partially accounted for by using representations of the conformal group with nonzero mass. This is, of course, questionable because such representations involve mass continua. Castell shows that any elastic differential cross section should behave like

$$\frac{d\sigma}{dt} \to \frac{1}{s^2}(-t)^{-M}H(\cos\theta) \tag{31}$$

in the large-angle region. The integer M depends on the conformal group representations used, but in any case $M \geq 4$. Since form factors fall like t^{-2}, it seems that $M=4$ for πN scattering. The possibility of making such conformal group arguments physically respectable is intriguing, since a great deal of generality would be gained, as contrasted to the very special nature of the theory presented here.

[31] For a review of recent work, see P. Carruthers, Nucl. Phys. (to be published).

[32] L. Castell, Phys. Rev. D 2, 1161 (1970).

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Annihilation of Electron-Positron Pairs into Mesons

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We estimate the cross section for $e^+e^- \to$ mesons using (i) ρ dominance of the electromagnetic current, (ii) known couplings of resonances to states involving the on-shell ρ meson, and (iii) the assumption, for the purpose of obtaining numerical estimates, that the meson couplings vary slowly with virtual ρ mass. We conclude that the cross section at 2-GeV total energy could be comparable to the $e^+e^- \to \mu^+\mu^-$ cross section. We also attempt to estimate a cross section from the statistical model of Bjorken and Brodsky.

I. INTRODUCTION

THE recent commencement of operations at the ADONE facility in Frascati opens an era when multi-GeV energies will be available for hadron production experiments resulting from e^+e^- annihilation. It is of considerable interest, therefore, to speculate upon production mechanisms and consider possible experiments that might be performed at clashing-beam facilities. Some speculations have already been advanced by Cabibbo *et al.*,[1] Ferrara *et al.*,[2] Gatto,[3] and Bjorken and Brodsky.[4]

The discussions in Refs. 1 and 2 are based upon parton model agruments and predict asymptotic total cross sections for hadron production that depend upon the "parton charges," which are unknown. The energy dependence of the cross section is predicted to be s^{-1} (s is the square of the barycentric four-momentum), and it is proposed in Ref. 1 that the high-energy hadron production will be characterized by pairs of "jets" (emanating from the production of "parton pairs"). Three-pion production is strictly forbidden in this model.

The discussion of Ref. 3 deals mainly with particle pair production ($\pi\pi$, $K\bar{K}$, etc.) and is more applicable to

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[1] N. Cabibbo, G. Parisi, and M. Testa, Nuovo Cimento Letters 4, 35 (1970). See also, S. D. Drell, D. J. Levy, and T-M Yan, Phys. Rev. D 1, 1617 (1970).

[2] S. Ferrara, M. Greco, and A. F. Grillo, Nuovo Cimento Letters 4, 1 (1970).

[3] R. Gatto, in *Proceedings of the International Symposium on Electron and Photon Interactions at High Energies* (Springer, Berlin, 1965), p. 106.

[4] J. D. Bjorken and S. J. Brodsky, Phys. Rev. D 1, 1416 (1970).