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Patterns in Direct-Channel Helicity Amplitudes Caused by Crossed-Channel Exchanges*

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We discuss a set of patterns for Reggeon exchanges in elastic scattering. Each pattern corresponds to a particular set of t-channel quantum numbers, and allows only certain s-channel helicity amplitudes $f_{cd;ab}{}^s$ to be present to order s^α. These dominant amplitudes are those such that $|c\pm a|=n$, and $|d\pm b|=m$, where n and m are integers, and the pattern is labeled by n,m and the choice of $+$ or $-$ in each case. Any Regge residue may be constructed as a linear combination of the basic "$-$" patterns; any Regge residue which vanishes sufficiently rapidly at $t=0$ may be constructed as a linear combination of the basic "$+$" patterns. Correct behavior for both s- and t-channel amplitudes at $t=0$ and t-channel thresholds is ensured by the formalism.

I. INTRODUCTION

It has been customary to interpret high-energy results in terms of crossed-channel exchanges. Thus the patterns of t-channel helicity amplitudes created by exchange of particular quantum numbers, and their implications for various experimental quantities, are well known.[1] Recent developments in theory have made it useful to study these patterns in terms of direct-channel amplitudes. In particular, incorporation of spin into "duality"-type models requires knowledge of those couplings for the direct-channel towers which will yield particular t-channel quantum numbers.

One might think that patterns which seem simple in terms of t-channel amplitudes may be hopelessly complicated in terms of s-channel amplitudes. The work of Gilman, Pumplin, Schwimmer, and Stodolsky (GPSS)[2] has shown us that this is not necessarily so. They show that in elastic scattering residues for Pomeranchukon exchange (a natural-parity Reggeon) can be chosen in such a way that only those direct-channel helicity amplitudes with zero helicity flip survive to order s^α. In other words, we can choose Regge residues in such a way that the Reggeon contribution obeys a simple pattern in the s channel.

In this paper we demonstrate that the results of GPSS can be extended to a number of different direct-channel patterns, and also to exchanges with quantum numbers different from the Pomeranchukon. For any exchange, in elastic scattering, we find a set of possible s-channel patterns. Any Regge residue can be expressed simply in terms of the members of this set. The residues thus obtained explicitly incorporate proper behavior at $t=0$ and at t-channel thresholds.

For s-channel helicity amplitudes $f_{cd,ab}{}^s$ we can define helicity differences $x=c-a$, $x'=d-b$, and helicity sums $y=c+a$, $y'=d+b$. A pattern of the type discussed here ensures that only s-channel helicity amplitudes with $|x|=n$ (or $|y|=n$) for some integer n be present to order s^α. A different pattern (with different integer n') may apply to the other vertex (x' or y'). (In this language, the result of GPSS suppresses amplitudes with $x\neq0$, $x'\neq0$.) Patterns of this type are naturally studied in terms of the amplitudes derived in the preceeding paper[3] (referred to hereafter as I), and we use the techniques and results of that work.

Studies of $O(4)$ have shown that, at $t=0$, exchange of an $O(4)$ representation with quantum number M will allow only those s-channel helicity amplitudes with $x=x'=M$ to survive.[4] Thus $O(4)$ also relates particular s-channel patterns to particular t-channel patterns. Our results are complementary to these in several ways: (i) We deal only with t-channel states of single quantum numbers [whereas all $O(4)$ representations with $M>0$ contain parity mixtures], and (ii) we deal with all small t, not just $t=0$. Compatibility between the two theories is assured by the fact that our results for $n>0$ vanish at $t=0$ and thus do not have to fall into the $O(4)$ classification scheme.

The general organization of the paper is as follows. Section II describes some details of the two special cases pp and $\rho\rho$ scattering, which aided us in formulating the general case. The analyticity properties of helicity amplitudes near $t=0$ and t-channel threshold are summarized in Sec. III. In Sec. IV we propose a basis set of Regge vertices, and demonstrate that these have the correct analyticity properties and provide patterns of energy suppression in the direct-channel amplitudes. We return to our special cases (pp and $\rho\rho$ scattering) in Sec. V, and discuss them in terms of the pattern formalism. Behavior of our solution for lower orders of s is briefly discussed in Sec. VI; however, this cannot be treated adequately without including contributions from the daughter trajectories. Comparison with low-order perturbation theory is given in the Appendix.

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[1] K. Gottfried and J. D. Jackson, Nuovo Cimento **33**, 309 (1964).

[2] F. G. Gilman, J. Pumplin, A. Schwimmer, and L. Stodolsky, Phys. Letters **31B**, 387 (1970).

[3] Lorella M. Jones and D. G. Ravenhall, preceding paper, Phys. Rev. D **3**, 690 (1971).

[4] This rule by now is common knowledge. It appears to have been first stated by R. F. Sawyer in Phys. Rev. Letters **18**, 1212 (1967).

II. SPECIAL CASES

The existence of the patterns discussed in the following sections was first noticed by the authors in detailed studies of elastic pp and $\rho\rho$ (or deuteron-deuteron) scattering. In this section we point out some features of these examples which help one to develop intuition for the general case.

A. Proton-Proton Scattering

There are five independent helicity amplitudes remaining after imposition of symmetry relations[5] (parity, time reversal, and Pauli principle in the s channel; or parity, charge conjugation, and time reversal in the t channel). These are

$$f_1\equiv f_{++;++},\quad f_2\equiv f_{++;--},\quad f_3\equiv f_{+-,+-},\\ f_4\equiv f_{+-,-+},\quad f_5\equiv f_{++;+-}, \tag{2.1}$$

where $\pm$ indicates helicities $\pm\frac{1}{2}$.

Exchange of a Reggeon of spin S in the t channel, with parity $P=\eta_P(-1)^S$ and charge conjugation $C=\eta_C(-1)^S$, produces the constraints on the t-channel amplitudes listed in Table I. After crossing, the resulting s-channel amplitudes to the leading power in s are those listed in Table II. The exchange $\eta_P=1$, $\eta_C=-1$ does not occur for pp scattering. Both $\eta_P=-1$ exchanges involve only one independent t-channel amplitude. This means that their s-channel amplitudes are uniquely determined; notice that $\eta_P=-1$, $\eta_C=+1$ exchanges populate only direct-channel helicity-flip amplitudes, whereas $\eta_P=-1$, $\eta_C=-1$ exchanges populate only helicity-nonflip amplitudes (to this order in s).

For an exchange with $\eta_P=\eta_C=1$ (such as the Pomeranchukon), the presence of three independent t-channel amplitudes means that in general all five of the s-channel amplitudes are populated, with the relationships given. As GPSS have pointed out, it is possible to choose a particular relationship between the t-channel residues so that only $f_1{}^s$ and $f_3{}^s$ are nonzero to order s^α. Although this relationship can be found by working with invariant amplitudes (as GPSS did), it is simplest just to study the Trueman-Wick[6] crossing matrix at infinite momentum,

$$f_{cd;\,ab}{}^s=\sum d_{A'a}{}^{\frac{1}{2}}(\pi-\chi^0)d_{b'b}{}^{\frac{1}{2}}(\chi^0)d_{c'c}{}^{\frac{1}{2}}(\chi^0)\\ \times d_{D'd}{}^{\frac{1}{2}}(\pi-\chi^0)f_{c'A';\,D'b'}{}^t, \tag{2.2}$$

where

$$\cos\chi^0=\left(\frac{t}{t-4M^2}\right)^{1/2}.$$

It is then clear that there exists a choice of f^t's to make $f_{++;--}{}^s$, $f_{+-;-+}{}^s$, and $f_{++;+-}{}^s$ all equal to zero, and that it will be such that the ratios $f_1{}^t/f_3{}^t$, and $f_1{}^t/f_5{}^t$ can be expressed as ratios of trigonometric functions of χ^0. It is also clear that this is not the only possible choice of direct-channel helicity pattern. One can also find a relation between t-channel residues which results in only $f_2{}^s$ and $f_4{}^s$, the double-flip amplitudes, being nonzero to order s^α.

The crossing matrix for pp scattering is so simple that the combined amplitudes defined in I are not especially advantageous for this study. However, because we will return in Sec. V to compare the results obtained above by "brute force" with our general formula, we take this opportunity to list the combined amplitudes for the process. The symmetries T (s channel) or C (t channel), when imposed on the combined amplitudes of I, require that $f_{JJ'MM'}{}^{s(-,-)}$ and $f_{JJ'MM'}{}^{(+,+)}$ be zero unless $2J_a+2J_b-J-J'$ is an even integer. Thus we have only the amplitudes with $J=J'=1$ and with $J=J'=0$. The additional constraint of the Pauli principle (s channel) or time-reversal invariance (t channel) imposes the condition that these amplitudes be symmetric in M and M'. It is then simple to count that $f_{11MM'}{}^{(\sigma\sigma)}$ has four independent components, and $f_{00}{}^{(\sigma\sigma)}$ has one, making five in all.

B. ρ-ρ Scattering

Direct application of the symmetries appropriate to elastic scattering of identical particles reduces the number of independent helicity amplitudes from $3^4=81$ to 17. The detailed examination of this case, in the same

TABLE I. Relations between t-channel p-p amplitudes arising from exchange of a Reggeon of spin S, with parity $P=\eta_P(-1)^S$, charge conjugation $C=\eta_C(-1)^S$. The symbol $\simeq$ indicates equality for the highest power of $\cos\theta_t$.

Amplitude	$\eta_P=1$, $\eta_C=1$	$\eta_P=-1$, $\eta_C=-1$	$\eta_P=1$, $\eta_C=-1$	$\eta_P=-1$, $\eta_C=1$
$f_1{}^t=f_{++;++}{}^t$	$f_1{}^t$	0	0	$f_1{}^t$
$f_2{}^t=f_{++,--}{}^t$	$f_1{}^t$	0	0	$-f_1{}^t$
$f_3{}^t=f_{+-,+-}{}^t$	$f_3{}^t$	$f_3{}^t$	0	0
$f_4{}^t=f_{+-,-+}{}^t$	$\simeq-f_3{}^t$	$\simeq f_3{}^t$	0	0
$f_5{}^t=f_{++,+-}{}^t$	$f_5{}^t$	0	0	0
Number of independent amplitudes	3	1	0	1

TABLE II. Relations between s-channel p-p amplitudes obtained by crossing the t-channel relations of Table I, to leading powers in s only. The column $\eta_P=1$, $\eta_C=-1$ is not included, since no such exchanges are possible.

Amplitude	$\eta_P=1$, $\eta_C=1$	$\eta_P=-1$, $\eta_C=-1$	$\eta_P=-1$, $\eta_C=1$
$f_1{}^s=f_{++,++}{}^s$	$f_1{}^s$	$f_1{}^s$	0
$f_2{}^s=f_{++,--}{}^s$	$f_2{}^s$	0	$f_2{}^s$
$f_3{}^s=f_{+-,+-}{}^s$	$\simeq f_1{}^s$	$\simeq -f_1{}^s$	0
$f_4{}^s=f_{+-,-+}{}^s$	$\simeq -f_2{}^s$	0	$\simeq f_2{}^s$
$f_5{}^s=f_{++,+-}{}^s$	$f_5{}^s$	0	0

[5] M. L. Goldberger, M. T. Grisaru, S. W. MacDowell, and D. Y. Wong, Phys. Rev. 120, 2250 (1960).

[6] T. L. Trueman and G. C. Wick, Ann. Phys. (N. Y.) 26, 322 (1964).

TABLE III. Numbers of helicity amplitudes, for ρ-ρ scattering, for various Reggeon exchanges, to order s^{α}. (The numbers listed in the third row add to more than 17 because not all contributions are independent.)

Amplitude	$\eta_P=\eta_C=1$	$\eta_P=\eta_C=-1$	$\eta_P=1$, $\eta_C=-1$	$\eta_P=-1$, $\eta_C=1$
Independent t-channel	10	3	1	3
Nonzero s-channel				
{nonflip	4	2	0	0
{flip	13	6	4	8

manner as for pp scattering, has been carried through, as a guide and check on the general method discussed in later sections. The details are clearly too lengthy to be included here, and we summarize the main features.

First, this case is more interesting that that of pp scattering because all exchanges are allowed, and the unnatural-parity ($\eta_P=-1$) exchanges (which were limited to one amplitude each in pp scattering) now have several possible independent residues. The number of independent t-channel amplitudes involved in each case is listed in Table III. Second, all cases except $\eta_P=\eta_C=+1$ are still simple enough to reveal helicity patterns similar to those found in pp scattering.

For example, the "pion" exchange $\eta_P=-1$, $\eta_C=+1$[7] automatically populates only those s-channel helicity amplitudes with some helicity flip at each vertex [Eq. (4.3) of I]. By choice of the relative magnitudes of the three t-channel residues, it is possible to suppress some s-channel amplitudes to orders s^{α} just as we did with the $\eta_P=\eta_C=+1$ case in pp scattering. Now, however, the choice would appear to be a suppression of either the helicity-flip-one vertices or of the helicity-flip-two vertices. Since the three independent s-channel amplitudes involved are $f_{11;00}{}^s$, $f_{11;0-1}{}^s$, and $f_{11;-1-1}{}^s$, the choice might also be expressed in terms of suppression of helicity *sum* zero or one. This feature has a natural place in our general solution.

The case of Pomeranchukon exchange (or more generally any $\eta_P=\eta_C=+1$ exchange), however, involves ten independent t-channel amplitudes, and is too intractable for a direct solution. The combined amplitudes greatly facilitate the consideration of this case.

We observe that counting of independent combined amplitudes for this case is formally identical to the corresponding enumeration for nonrelativistic scattering, using states of total spin. The numbers of nonzero independent combined amplitudes $f_{JJ'MM'}{}^{s(-,-)}$ and $f_{JJ'MM'}{}^{t(+,+)}$ allowed by the symmetries are, respectively: for $J=J'=2$, $M\geq M'$ (9); for $J=2$, $J'=0$, $M\geq 0$ (3); for $J=J'=1$, $M\geq M'$ (4); and for $J=J'=0$ (1). The numbers in parentheses add up to 17, as expected.

The two examples will be further discussed in Sec. V, when we exhibit the forms taken by our general solution. We now go on to examine kinematic constraints which any model must obey.

[7] As we are ignoring isospin, g parity does not matter and the impossibility of a $\rho\rho\pi$ vertex is irrelevant. By "pion" we mean any $\eta_P=-1$, $\eta_C=+1$ exchange.

III. BEHAVIOR OF AMPLITUDES NEAR $t=0$ AND t-CHANNEL THRESHOLDS

Any model to be used in the small-t region, such as the Regge model, must incorporate proper behavior near $t=0$ (for elastic scattering, this is the forward point). If the t-channel thresholds happen to be near to the s-channel physical region, proper behavior at them must be included also. In any case, it is useful to have a model which automatically both provides the correct singularity in each helicity amplitude and satisfies the constraint equations at these singular points. In the paragraphs below, we list the desired behavior for elastic scattering.

A. Singularities at $t=0$

For the t-channel helicity amplitudes, the behavior found by Wang[8] must apply:

$$\begin{aligned} f_{cA;Db}{}^t &\sim \sqrt{t} \quad \text{if } |D-b-c+A| \text{ is odd} \\ &\sim 1 \quad \text{if } |D-b-c+A| \text{ is even}. \end{aligned} \tag{3.1}$$

These results can be multiplied by powers of t without changing the analyticity properties of the amplitude.

In a Regge model, this behavior is assigned to the Regge residue function, which is assumed to factor in the form $\beta_{cA}\beta_{Db}$. The singularity structure given in Eq. (3.1) may be ensured by either of two possible mechanisms:

$$\text{(i)} \quad \begin{aligned} \beta_{Db} &\sim (\sqrt{t})f(t) \quad \text{for } D-b \text{ odd} \\ &\sim g(t) \quad \text{for } D-b \text{ even} \end{aligned}$$

or

$$\text{(ii)} \quad \begin{aligned} \beta_{Db} &\sim f(t) \quad \text{for } D-b \text{ odd} \\ &\sim (\sqrt{t})g(t) \quad \text{for } D-b \text{ even}. \end{aligned}$$

Study of perturbation-theory models for nucleon-nucleon and ρ-ρ scattering (see the Appendix) indicates that if the exchanged Reggeon has normal parity $P=(-1)^S$, the residues follow pattern (i), whereas if the exchange has abnormal parity $P=-(-1)^S$, the residues follow pattern (ii).

The s-channel helicity amplitudes $f_{cd;ab}{}^s$ must vanish in the forward direction in a manner dictated by angular momentum conservation:

$$f_{cd;ab}{}^s \sim (\sin \tfrac{1}{2}\theta_s)^{|a-b-c+d|} \sim (\sqrt{t})^{|a-b-c+d|}. \tag{3.2}$$

Because each s-channel helicity amplitude is a linear combination of t-channel helicity amplitudes, this behavior provides a constraint equation which the t-channel amplitudes must satisfy in addition to the behavior of (3.1). Conversely, if the s-channel amplitudes are found to have appropriate behavior, the

[8] L. L. Wang, Phys. Rev. **142**, 1187 (1966).

constraint equation is automatically satisfied. [Note that the behavior given in Eq. (3.2) can be multiplied by any power of t without violating any principles of analyticity.]

B. Singularities at t-Channel Threshold

Behavior of the t-channel helicity amplitudes near threshold is given by standard angular momentum arguments.[8] For spin S exchanged in the t channel, the contribution to the t-channel helicity amplitude $f_{cd;ab}{}^t$ looks like

$$f_{cA;Db}{}^t = T_{cA;Db}{}^S(t) d_{D-b;c-A}{}^S(\theta_t) .$$

For D,b a particle-antiparticle pair, the behavior of the partial-wave amplitude $T_{cA;Db}{}^S(t)$ near the threshold $t=4m_D{}^2$ is $T_{cA;Db}{}^S \sim q^L$ [L is the lowest orbital angular momentum allowed the particle-antiparticle pair, and q is their relative momentum $q=\frac{1}{2}(t-4m_D{}^2)^{1/2}$]. The t-channel scattering angle behaves like $\cos\theta_t \sim 1/q$, so the over-all behavior of the helicity amplitude is $f_{cA;Db}{}^t \sim q^{L-S}$.

For exchanges with $CP=+1$, the lowest angular momentum is $L=S-2J_D$, and we expect $f_{cA;Db}{}^t \sim q^{-2J_D}$. When $CP=-1$, $L=S-2J_D+1$ so that

$$f_{cA;Db}{}^t \sim q^{-2J_D+1} .$$

Our combined amplitudes $f_{JJ'KK'}{}^{t(\sigma_1,\sigma_2)}$ with $\sigma_1=-$ represent the particle-antiparticle pair combined into spin J. For this combination, an exchange with $C=(-1)^S$ has a minimum orbital momentum $L=S-J$, whereas exchanges with $C=-(-1)^S$ have $L=S-J+1$. The expected threshold behavior for these combinations is then q^{-J} and q^{-J+1}, respectively.

There is no reason for s-channel helicity amplitudes to have kinematic singularities near t-channel threshold points. The absence of such singularities gives us another set of constraint equations for the t-channel helicity amplitudes to obey.

IV. BASIC SETS OF REGGE RESIDUES

The detailed algebra of Pomeranchukon exchange in pp scattering showed us that the ratios of t-channel residues needed to produce given s-channel helicity patterns are simple functions of $\cot(\frac{1}{2}\chi^0)$. Here χ^0 is the crossing angle for the appropriate mass, necessarily taken in the limit $s\to\infty$, since suppression involves only leading powers of s. Thus, the structure of the combined amplitudes introduced in I, and their simple crossing relation,

$$f_{JJ'MM'}{}^{s(\sigma_1,\sigma_2)} = \sum_{KK'} d_{KM}{}^J(\chi_c) \times d_{K'M'}{}^{J'}(\chi_b) f_{JJ'KK'}{}^{t(-\sigma_1,-\sigma_2)} , \quad (4.1)$$

suggest the following form for the f^t appropriate to helicity-flip suppression:

$$f_{JJ'KK'}{}^{t(+,+)} = s^\alpha g_J g_{J'} d_{0K}{}^J(-\chi_c{}^0) d_{0K'}{}^{J'}(-\chi_b{}^0) , \quad (4.2)$$

giving

$$f_{JJ'MM'}{}^{s(-,-)} = s^\alpha g_J g_{J'} d_{0M}{}^J(\chi_c-\chi_c{}^0) \times d_{0M'}{}^{J'}(\chi_b-\chi_b{}^0) . \quad (4.3)$$

Because $\chi-\chi^0\to 0$ as $s\to\infty$, and $d_{0M}{}^J(\beta)\sim(\sin\beta)^{|M|}$, this form clearly achieves its s-channel objective.

It can be interpreted physically as follows: as $s\to\infty$, the pattern is defined by the nonzero set of amplitudes $f_{JJ'00}{}^s(s\to\infty, t)$. Crossing to the t channel at $s\to\infty$, this produces the pattern (4.2). So far as leading powers of s are concerned, this may be taken to be the form of f^t at finite s also. Crossing back to the s channel then produces the result shown.

This argument suggests that there is a family of such helicity patterns. We now present them, and demonstrate that they possess the requisite symmetries.

A. Generalized t-Channel Amplitudes

We assume that all s^α contributions to the t-channel amplitudes take the form

$$f_{JJ'KK'}{}^{t(\sigma_1,\sigma_2)} \sim s^\alpha \Sigma g_J{}^n g_{J'}{}^{n'} \beta_n{}^{KJ}(\sigma_1) \beta_{n'}{}^{K'J'}(\sigma_2) . \quad (4.4)$$

The couplings $g_J{}^n$ and $g_{J'}{}^{n'}$ are functions of t regular at $t=0$ and t-channel thresholds; Σ is the Regge signature factor, $\Sigma=(1\pm e^{-i\pi\alpha})/\sin\pi\alpha$. The Regge vertex functions, generalized from (4.2), are defined by

$$\beta_n{}^{K,J}(\sigma=-) = (\surd t)^{2J_a+n}[d_{nK}{}^J(-\chi_c{}^0) + \eta_P(-1)^{2J_a-J+n} d_{-nK}{}^J(-\chi_c{}^0)] , \quad (4.5)$$

$$\beta_n{}^{K,J}(\sigma=+) = (\surd t)^n[d_{nK}{}^J(-\chi_c{}^0) + \eta_P(-1)^{2J_a-J+n} d_{-nK}{}^J(-\chi_c{}^0)] . \quad (4.6)$$

As in the special case $n=0$, $\chi_c{}^0$ is the limit of the crossing angle χ_c as $s\to\infty$, so that $\cos\chi_c{}^0=[t/(t-4m_a{}^2)]^{1/2}$.

B. Behavior under Replacement of K by $-K$

In Sec. IV of paper I, a number of symmetries of the amplitudes $f_{JJ'KK'}{}^{t(\sigma_1,\sigma_2)}$ in order s^α were derived. Use of (4.4) allows us to translate these into symmetries of the residue functions β:

$$\beta_n{}^{K,J}(-) = \eta_C(-1)^K \beta_n{}^{-K,J}(-) , \quad \eta_C = \eta_P(-1)^{2J_a-J} , \quad (4.7a)$$

$$\beta_n{}^{K,J}(+) = \eta(-1)^K \beta_n{}^{-K,J}(+) , \quad \eta = \eta_P(-1)^{2J_a-J} . \quad (4.7b)$$

The rotation functions $d_{\lambda\mu}{}^J$ have the property

$$d_{\lambda\mu}{}^J = (-1)^{\lambda-\mu} d_{-\lambda-\mu}{}^J .$$

Substitution into (4.5) and (4.6) then shows that the vertices suggested have the symmetry required by (4.7).

C. Energy Dependence in s Channel

Use of the crossing relations (4.1) derived in I, with the t-channel amplitudes of Eq. (4.4), gives s-channel amplitudes of the form

$$f_{JJ'MM'}{}^{s(-\sigma_1,-\sigma_2)}\sim s^\alpha \Sigma g_J{}^n g_{J'}{}^{n'}(\sqrt{t})^{n+n'}(\sqrt{t})^{\varphi_1+\varphi_2} \times[d_{nM}{}^J(\chi_c-\chi_c{}^0)+\eta_P(-1)^{2J_a-J+n}d_{-nM}{}^J(\chi_c-\chi_c{}^0)] \times[d_{n'M'}{}^{J'}(\chi_b-\chi_b{}^0)+\eta_P(-1)^{2J_b-J'+n'} \times d_{-n'M'}{}^J(\chi_b-\chi_b{}^0)]. \quad (4.8)$$

Here we have used the shorthand function φ_i as follows:

$$\varphi_i=\begin{cases}0 & \text{if } \sigma_i=+\\ 2J_i & \text{if } \sigma_i=-.\end{cases} \quad (4.9)$$

In the limit as s approaches infinity, the difference $\chi_c-\chi_c{}^0$ approaches zero in such a way that

$$\sin(\chi_c-\chi_c{}^0)\sim i(\sqrt{t})m_a/s. \quad (4.10)$$

Each factor

$$[d_{nM}{}^J(\chi_c-\chi_c{}^0)+\eta_P(-1)^{2J_a-J+n}d_{-nM}{}^J(\chi_c-\chi_c{}^0)]$$

vanishes like $[\sin(\chi_c-\chi_c{}^0)]^{||M|-n|}$. Thus we see that for a given n and n' the only amplitudes $f_{JJ'MM'}{}^{s(\sigma_1,\sigma_2)}$ which survive to power s^α are those with $|M|=n$ and $|M'|=n'$.

When $\sigma_1=\sigma_2=+$, these dominating amplitudes are ones with s-channel helicity difference (at the t-channel vertices) of n and n', respectively. If either σ is $-$, the sum of the helicities must be n, etc. Hence each integer n specifies a pattern for its vertex. Because the only particles we are presently able to scatter elastically are baryons, pseudoscalars, and photons, the only integers of practical importance are 0, 1, and 2.

D. Analytic Behavior near $t=0$

We now check that the powers of $\sqrt{t}$ inserted in our postulated Regge vertices (4.5) and (4.6) ensure the analytic behavior near $t=0$ established in Sec. III. First, note that because of the relationship between $d_{nK}{}^J$ and $d_{-nK}{}^J$, the combination

$$[d_{nK}{}^J(-\chi_c{}^0)+\eta_P(-1)^{2J_a-J+n}d_{-nK}{}^J(-\chi_c{}^0)]$$

contains either all even or all odd powers of $\sqrt{t}$. (This is good because we do not want the vertex functions to have mixed singularity structure.)

Second, we observe that at $t=0$, $\chi^0=\frac{1}{2}\pi$, and for this special value the d functions have an additional symmetry, $d_{nK}{}^J(-\frac{1}{2}\pi)=(-1)^{J+K}d_{-nK}{}^J(-\frac{1}{2}\pi)$. Hence the combination

$$V_{nK}{}^J\equiv d_{nK}{}^J(-\chi_c{}^0)+\eta_P(-1)^{2J_a-J+n}d_{-nK}{}^J(-\chi_c{}^0)$$

has the following properties near $t=0$:

$$V_{nK}{}^J=0 \quad \text{if}\begin{cases}K=0,\ \eta_P(-1)^{2J_a-J}=-1\\ n=0,\ \eta_P(-1)^{2J_a-J}=-1;\end{cases} \quad (4.11\text{a})$$

$$V_{nK}{}^J\sim 1 \quad \text{if}\begin{cases}K\neq 0,\ n\neq 0, & \eta_P(-1)^{2J_a+n-K}=+1,\\ K=0,\ J-n \text{ even}, & \eta_P(-1)^{2J_a-J}=+1,\\ n=0,\ J-K \text{ even}, & \eta_P(-1)^{2J_a-J}=+1;\end{cases} \quad (4.11\text{b})$$

$$V_{nK}{}^J\sim\sqrt{t} \quad \text{if}\begin{cases}K\neq 0,\ n\neq 0, & \eta_P(-1)^{2J_a+n-K}=-1,\\ K=0,\ J-n \text{ odd}, & \eta_P(-1)^{2J_a-J}=+1,\\ n=0,\ J-K \text{ odd}, & \eta_P(-1)^{2J_a-J}=+1.\end{cases} \quad (4.11\text{c})$$

We conclude that, provided the vertex exists, $V_{nK}{}^J\sim 1$ if $\eta_P(-1)^{2J_a+n-K}=+1$ and $V_{nK}{}^J\sim\sqrt{t}$ if $\eta_P(-1)^{2J_a+n-K}=-1$. Thus the behavior of the vertex functions near $t=0$ can be summarized as follows:

$$\beta_n{}^{K,J}(-)=(\sqrt{t})^{2J_a+n}V_{nK}{}^J\sim(\sqrt{t})f(t) \quad \text{if } \eta_P(-1)^{-K}=-1$$
$$=(\sqrt{t})^{2J_a+n}V_{nK}{}^J\sim g(t) \quad \text{if } \eta_P(-1)^{-K}=+1;$$
$$\beta_n{}^{K,J}(+)=(\sqrt{t})^{n}V_{nK}{}^J\sim(\sqrt{t})f'(t) \quad \text{if } \eta_P(-1)^{2J_a-K}=-1$$
$$=(\sqrt{t})^{n}V_{nK}{}^J\sim g'(t) \quad \text{if } \eta_P(-1)^{2J_a-K}=+1.$$

(Here f, f', g, and g' are arbitrary functions of t.)

We recall that the residue function must have a square-root singularity when either (i) the helicity difference at the vertex is odd and $\eta_P=+1$, or (ii) the helicity difference at the vertex is even and $\eta_P=-1$. Suppose the character σ at the t-channel vertex is $-$. Then K is the difference in helicities. Our criterion then requires the residue $(\sqrt{t})^{2J_a+n}V_{nK}{}^J$ to behave like $\sqrt{t}$ if $\eta_P(-1)^K=-1$ and to be regular if $\eta_P(-1)^K=+1$. We have seen that this is satisfied.

If, on the other hand, the character at the vertex is $+$, the integer K is the sum of helicities, not the difference. However, the helicity difference is odd or even according to whether $K+2J_a$ is odd or even. We then require the residue $(\sqrt{t})^nV_{nK}{}^J$ to have square-root behavior if $\eta_P(-1)^{2J_a+K}=-1$, and regular behavior if $\eta_P(-1)^{2J_a+K}=+1$. Again, we have seen that our residue has this behavior.

To determine whether the constraint equations have been satisfied, we examine the behavior of the s-channel amplitudes given by Eq. (4.8). Use of (4.10) indicates that the highest remaining power of s behaves like

$$f_{JJ'MM'}{}^{s(-\sigma_1,-\sigma_2)}\sim(\sqrt{t})^{n+n'}(\sqrt{t})^{\varphi_1+\varphi_2} \times(\sqrt{t})^{|n-|M||}(\sqrt{t})^{|n'-|M'||}s^{\alpha-x}, \quad (4.12)$$

where $x=||M|-n|+||M'|-n'|$. The exponent of $\sqrt{t}$ is then $|M|+|M'|+\varphi_1+\varphi_2+2\times$(positive integer or zero), independent of J and J'. The helicity amplitudes [which may be reexpressed in terms of our combined amplitudes by Eq. (2.9c) of I] will therefore also have this dependence on $\sqrt{t}$.

For $\sigma_1=\sigma_2=+$, the behavior near $t=0$ is

$$f_{cd;ab}{}^s\sim(\sqrt{t})^{|a-c|+|d-b|}g^+(t)\,.$$

This is compatible with the analyticity requirement

$$f_{cd;ab}{}^s\sim(\sqrt{t})^{|a-b-c+d|}\,.$$

For the case $\sigma_1=\sigma_2=-$, involving helicity sums, the behavior is

$$f_{cd;ab}{}^s\sim(\sqrt{t})^{|a+c|+|d+b|+2J_a+2J_b}g^-(t)\,.$$

Again this is compatible with analyticity. Hence we can be assured that all $t=0$ constraint equations have been satisfied.

E. Behavior at t-Channel Thresholds

Near the t-channel threshold $t=4m_a^2$, $\cos\chi_c^0\to\infty$. In this limit,

$$d_{-nK}{}^J(-\chi_c{}^0)\approx(-1)^n d_{nK}{}^J(-\chi_c{}^0)\,,$$

and $d_{nK}{}^J\propto q^{-J}$. The phase factor involved in the definitions (4.5) and (4.6) of the vertices simplifies because of the symmetry conditions (4.7). Thus the leading singularity given to t-channel amplitudes $f_{JJ'KK'}{}^t$ is

$$q^{-J}(1+\eta_C)\quad\text{for}\quad\sigma=-\ \text{vertices}\qquad(4.13\text{a})$$

and

$$q^{-J}(1+\eta)\quad\text{for}\quad\sigma=+\ \text{vertices}.\qquad(4.13\text{b})$$

Formula (4.13a) clearly agrees with the threshold behavior derived from angular momentum considerations for $\sigma=-$ vertices in Sec. III B. To examine the $\sigma=+$ combinations, we reconstruct helicity amplitudes according to Eq. (2.9c) of I. The most singular contribution comes from the maximum J value allowed by the Clebsch-Gordan coefficient, i.e., $J=2J_a$. Thus we find that formula (4.13b) predicts a singularity of q^{-2J_a} for exchanges with $CP=+$, and behavior of q^{-2J_a+1} for exchanges with $CP=-$. This also agrees with the considerations in Sec. III.

Because the angle $\chi_c-\chi_c{}^0$ is regular at $t=4m_a^2$, the s-channel amplitudes have no singularities at this point. Hence the t-channel Regge residues automatically satisfy the threshold constraint equations. This in itself makes the model interesting.

F. Patterns as Basis Set for General Regge Residue

We have seen in Secs. IV A–IV E that the residues given in (4.5) and (4.6) have many desirable kinematic properties, and produce a set of patterns in the energy dependence of the s-channel helicity amplitudes. The question naturally arises whether these vertices form a basis for all Regge residues; i.e., is it possible to express all Regge residues in the form of a linear combination of either the $\sigma=+$ or $\sigma=-$ vertices?

To prove an expansion of this type, we must not only show that there are as many independent patterns as independent amplitudes, and demonstrate how to make the expansion, but we must also be sure that the patterns given have the "minimal" possible behavior at $t=0$. This is to ensure that all expansion coefficients are regular at $t=0$, as postulated in (4.4). On the basis of comparisons with perturbation theory for low-spin cases (see the application to NN scattering in Sec. V) we know that the expression given in Eq. (4.5) (for t-channel $\sigma=-$ vertices) is nonminimal. As far as we can tell, the solution given in Eq. (4.6) for t-channel $\sigma=+$ vertices is minimal in the sense that no perturbation-theory exchange produces Regge residues (and s-channel amplitudes to order s^α) which vanish less rapidly at $t=0$. Hence we will limit the discussion of basis states for Regge residues to the $\sigma=+$ vertices. However, if a minimal modification of (4.5) is found, the same arguments will apply to it.

For a given J the set of residues $\{\beta_{n_1}{}^{K,J}\}$ is clearly linearly independent from the set $\{\beta_{n_2}{}^{K,J}\}$, provided $n_1\neq n_2$. We can therefore count independent patterns by counting n's: For each J there are $J+1$ independent patterns. Similarly for each J, J', and K' there are $J+1$ independent amplitudes $f_{JJ'KK'}{}^{t(\sigma_1,\sigma_2)}$. Thus there are enough patterns to span all the vertices. It would appear that the easiest way to express a general Regge contribution in terms of the patterns would be to identify its s-channel helicity amplitudes with those of a general expansion.

Our patterns for the t-channel $\sigma=+$ case are quite similar to those which Klein[9] has obtained, starting from a different motivation and using a more elegant formalism. He has treated the expansion problem in much more detail, and has included the effects of daughter trajectories and unequal masses. We feel that recent developments in the direct channel warrant a new study of these formulas, and that the simplifications introduced by restricting the study to elastic scattering make the work accessible to a broader range of readers. To our knowledge, no previous study has been made of s-channel patterns caused by the $\sigma=-$ type vertices.

V. EXAMPLES: PATTERNS IN NUCLEON-NUCLEON AND ρ-ρ SCATTERING

We now make detailed comparison with the results of nucleon-nucleon and ρ-ρ scattering that were summarized in Sec. II.

A. Nucleon-Nucleon Scattering

1. Pion-Trajectory Exchange ($\eta_P=-$, $\eta_C=+$)

For the t-channel $\sigma=+$ vertex,

$$\eta_C=+=(-1)^{2J_a-J}=-(-1)^J.$$

Hence the s^α contribution will appear only in $J=1$ amplitudes. The residue functions then have the form

$$\beta_n{}^{K,1}(+)\sim(\sqrt{t})^n[d_{nK}{}^1(-\chi_c{}^0)-(-1)^n d_{-nK}{}^1(-\chi_c{}^0)].$$

[9] S. Klein, Phys. Rev. D 1, 609 (1970).

The $n=0$ contribution will vanish. Thus we have only one pattern, $n=1$. Note that the vertex will contribute only to the $K=\pm1$ amplitudes and hence to $f_{++;++}{}^t$ and $f_{++;--}{}^t$. The behavior

$$\beta_1{}^{1,1}(+)\sim(\sqrt{t})[d_{11}{}^1(-\chi_c{}^0)+d_{-11}{}^1(-\chi_c{}^0)]\sim\sqrt{t}$$

is just what perturbation theory gives for one-pion exchange.

Now consider the other character, $\sigma=-$. Vertices of this type require $\eta=(-1)^{2J_a-J}$ for the contribution to be of order s^α. Since $\eta=-$, we must have $J=0$ and hence only $n=0$ is allowed. The vertex will then behave like

$$\beta_0{}^{K,0}(-)\sim(\sqrt{t})^1(d^0+d^0)\sim\sqrt{t}.$$

Only $K=0$ is allowed but K is now the helicity difference; thus again the exchange only populates amplitudes $f_{++;++}{}^t$ and $f_{++;--}{}^t$. The enumeration of independent f^t must be independent of σ, of course, although convenience usually suggests a particular one.

2. A_1-Trajectory Exchange ($\eta_P=-$, $\eta_C=-$)

Again we begin with the $\sigma=+$vertex. From

$$\eta_C=(-1)^{2J_a-J}$$

we see that $J=0$. Only $n=0$ will then be allowed, and only $K=0$ states will be populated. Thus only $f_{+-;+-}{}^t$ and $f_{+-;-+}{}^t$ will be present to order s^α and these do not vanish at $t=0$ (i.e., each vertex behave like 1 at $t=0$ and the t-channel threshold).

For the other possibility, $\sigma=-$, the identity

$$\eta=(-1)^{2J_a-J}=+1$$

requires $J=1$. Again only $n=1$ is allowed. For this case the individual vertices given by Eq. (4.5) behave like t, and the t-channel amplitudes vanish like t^2. This is clearly nonminimal behavior.

3. Natural-Parity, Natural-C-Parity Exchange

Many important trajectories fall into this category. The Pomeranchukon and the ρ meson are sample ones with different isospins.

Consider first the $\sigma=+$ case. Here we find $J=1$ for the s^α contributions. Both $n=0$ and $n=1$ patterns may exist. The vertex for the $n=0$ case behaves like $t^0[2d_{0K}{}^1(-\pi/2)]$, so that for nucleon-antinucleon helicities $+\frac{1}{2}$, $+\frac{1}{2}$, with $K=1$, it is $\sim t^0$. For helicities $+\frac{1}{2}$, $-\frac{1}{2}$, with $K=0$, the zero of the d function suppresses the vertex in the desired manner so that it acts like $\sqrt{t}$. This is the pattern found by GPSS in pion-nucleon and nucleon-nucleon scattering. The other case, $n=1$, enhances direct-channel amplitudes with helicity flip. It gives $(\sqrt{t})d_{10}{}^1\sim\sqrt{t}$ for the $+\frac{1}{2}$, $-\frac{1}{2}$ vertex and $(\sqrt{t})[d_{11}{}^1(-\chi_c{}^0)-d_{-11}{}^1(-\chi_c{}^0)]\sim t$ for the $+\frac{1}{2}$, $+\frac{1}{2}$ vertex.

For the $\sigma=-$ case, we again have $J=1$ and both $n=0$ and $n=1$ allowed. Now the vertices for the $n=0$ case behave like $\sqrt{t}$ and t for nucleon-antinucleon helicities $+\frac{1}{2}$, $-\frac{1}{2}$ and $+\frac{1}{2}$, $+\frac{1}{2}$, respectively. If the pattern is given by $n=1$, the corresponding behaviors for $+\frac{1}{2}$, $-\frac{1}{2}$ and $+\frac{1}{2}$, $+\frac{1}{2}$ are $t\sqrt{t}$ and t, respectively. Again, the $\sigma=-$ patterns are nonminimal near $t=0$.

Just as the Pomeranchukon has long been reputed to populate mainly the direct-channel helicity-nonflip amplitudes, the ρ meson is generally believed to populate mainly the direct-channel flip amplitudes. Thus it is possible that the ρ meson may be approximately classified as a $\sigma=+$, $n=1$ or $\sigma=-$, $n=0$ pattern. While deviations from this pattern are quite important in data fitting, assignment of an approximate pattern may facilitate construction of dual models for the exchange in question.

4. Natural-Parity, Unnatural-C-Parity Exchange

For both $\sigma=+$ and $\sigma=-$ vertices, only $J=1$ occurs. The selection rules on the vertices (4.7) then forbid either kind of vertex.

B. ρ-ρ Scattering

We review a few of the properties mentioned in Sec II.

1. Natural Parity, Unnatural C-Parity

Because of the exact symmetry associated with $\sigma=-$ vertices, $\eta=-1=(-1)^{2J_i-J}$, we see that only $f_{11MM'}{}^{t(-,-)}$ will be populated. The relations good to highest power in cos θ_t show that for this exchange, $f_{1100}{}^{t(-,-)}=0$ and $f_{111-1}{}^{t(-,-)}=f_{1111}{}^{t(-,-)}$, so that there is only one independent t-channel amplitude.

2. "Pion" Exchange ($\eta_P=-1$, $\eta_C=+1$)

Again because $\eta=-1$, the $(-\,-)$ amplitudes offer the quickest counting, and we have only $f_{11MM'}{}^{t(-,-)}$ as the nonzero $(-\,-)$ amplitudes. There are now three independent components to $f_{11MM'}{}^{t(-,-)}$. These may be classified into $n=0$ and $n=1$ patterns. Although this pattern offers a simple classification of pion exchange, the $t=0$ behavior of the vertices is distinctly nonminimal.

To obtain $t=0$ behavior compatible with perturbation theory, one must examine the $(+\,+)$ amplitudes. Since $\eta_C=+=(-1)^{2J_i-J}$, only $f_{22MM'}{}^{(+,+)}$, $f_{20M0}{}^{(++)}$, and $f_{0000}{}^{(++)}$ can be populated to order s^α. But from the formula (4.6) we find that there are no $J=0$ vertices, and no $n=0$ vertices for the $J=2$ case. The only nonzero contributions are for $n=1$, $K=1$, 2 and $n=2$, $K=1$, 2. A vertex with $t=0$ behavior agreeing with elementary pseudoscalar exchange is given by the $n=1$ pattern.

3. *Natural-Parity, Natural-Charge Conjugation*

For the case $\eta_P=+1$, $\eta_C=+1$, $\eta=+1$, if we use the $\sigma_1=\sigma_2=+$ t-channel amplitudes, we have the exact result that $f_{JJ'MM'}{}^{t(++)}$ depends only on the magnitude of M and M'. To the highest power in $\cos\theta_t$, we must have both J and J' even. We are still left with $J=J'=2$, $M\geq M'\geq 0$ (6); $J=2$, $J'=0, M\geq 0$ (3); and $J=J'=0(1)$, giving ten amplitudes in all. The enumeration of the allowed patterns of helicity suppression given in Sec. IV follows the same form as this. Hence among the 17 s-channel amplitudes we have produced just ten distinct helicity patterns.

VI. LOWER-ORDER TERMS

From Eq. (4.12) we see that the contribution from the leading power of the Reggeon is reduced to

$$f_{JJ'MM'}{}^{s(\sigma_1,\sigma_2)}\sim s^{\alpha-||M|-n|-||M'|-n'|}.$$

For some helicity states this gives a reduction of many powers of s. However, there is no reason to believe, in general, that the entire contribution to these amplitudes will be suppressed by more than one or two powers of s.

The reason for this is that the coefficients of $s^{\alpha-1}$ and $s^{\alpha-2}$ in the combined amplitudes $f_{JJ'KK'}{}^{t(\sigma_1,\sigma_2)}$ are ordinarily not simple d functions. Once the Regge residue is given by Eq. (4.5) or (4.6), the contributions of the parent Regge trajectory are determined to all orders in s by the behavior

$$f_{c'A';D'b'}{}^t\sim T_{c'A';D'b'}{}^{\alpha}d_{D'-b';c'-A'}{}^{\alpha}(\theta_t)$$
$$=R_{c'A';D'b'}s^{\alpha}+\bar{R}_{c'A';D'b'}s^{\alpha-1}+\tilde{R}_{c'A';D'b'}s^{\alpha-2}\cdots.$$

We can now calculate the combined amplitudes by using Eq. (2.8d) of I. Although

$$\Sigma(J_aJ_a-\bar{A}'c'|JK)(J_bJ_b-\bar{D}'b'|J'K')\times C_{\bar{A}'A'}{}^{J_a}(\sigma_1,t)C_{\bar{D}'D'}{}^{J_b}(\sigma_2,t)R_{c'A';D'b'}=g_J{}^ng_{J'}{}^{n'}\beta_n{}^{KJ}(\sigma_1)\beta_{n'}{}^{K'J'}(\sigma_2)$$

by construction, the other sums such as

$$\Sigma(J_aJ_a-\bar{A}'c'|JK)(J_bJ_b-\bar{D}'b'|J'K')\times C_{\bar{A}'A'}{}^{J_a}(\sigma_1,t)C_{\bar{D}'D'}{}^{J_b}(\sigma_2,t)\tilde{R}_{c'A';D'b'}$$

will in general have no such simple form. Hence they do not collaborate with the crossing matrix to produce a suppressed energy dependence. As a consequence, if all the contributions of the parent trajectory are considered, the suppressed helicity amplitudes will in general be suppressed by only one or two powers of s.

Proper treatment of lower powers requires inclusion of contributions from daughter trajectories. These are determined at $t=0$ by 0(4) considerations and may be continued away from that point. Readers interested in this aspect of the problem should consult the work of Klein[9] or that of Bitar and Tindle.[10]

[10] K. Bitar and G. Tindle, Phys. Rev. **175**, 1835 (1968).

VII. SUMMARY AND CONCLUSIONS

In elastic scattering, it is possible to choose Regge residues in such a way that the contributions to direct-channel helicity amplitudes have strength s^{α} only when the helicity indices at a t-channel vertex add, or subtract, to give a particular integer. In some reactions where the contributions for a particular exchange have only one vertex function (as pion exchange in NN scattering) this takes place automatically. General Regge contributions may be linear combinations of these patterns.

The fact that a pattern has special properties in both the s and t channels may make patterns particularly useful in dual models. Further work along these lines is in progress.

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APPENDIX: PROPERTIES OF VERTEX FUNCTIONS NEAR $t=0$

Analyticity considerations [such as listed in Eq. (3.1)] tell the $t=0$ behavior of entire amplitudes, but they do not in themselves determine the properties of factorized vertex functions. Such properties can be found (i) by using factorization and the analytic structure of several related reactions, or (ii) by examining perturbation-theory models. In this appendix we show that both (i) and (ii) lead us to adopt the behavior listed in Sec. II a.

Factorization

The natural-parity exchanges in the reaction $AB\to AB$ can be isolated by studying $\pi B\to\pi B$. The behavior found by Wang[8] for this amplitude is

$$f_{00;\lambda_1\lambda_2}{}^t\sim 1 \quad \text{if } \lambda_1-\lambda_2 \text{ is even}$$
$$\sim\sqrt{t} \quad \text{if } \lambda_1-\lambda_2 \text{ is odd}.$$

TABLE IV. Perturbation-theory results for the behavior of vertices.

Exchange	Coupling	Helicity indices	Behavior
Nucleon-nucleon vertices			
pion	$\bar{N}\gamma_5N$	$\frac{1}{2}$ $\frac{1}{2}$	$\sqrt{t}$
A_1	$\bar{N}\gamma_\mu\gamma_5N$	$\frac{1}{2}-\frac{1}{2}$	1
scalar	$\bar{N}N$	$\frac{1}{2}$ $\frac{1}{2}$	1
ρ-ρ vertices			
scalar	$\rho_{1\mu}\rho_2{}^{\mu}$	00	1
		11	1
pseudoscalar	$\epsilon_{\mu\nu\rho\sigma}\rho_1{}^{\mu}\rho_2{}^{\nu}P_1{}^{\rho}P_2{}^{\sigma}$	11	$\sqrt{t}$
axial-vector	$\epsilon_{\mu\nu\rho\sigma}\rho_1{}^{\mu}\rho_2{}^{\nu}(P_1+P_2)^{\rho}$	11	$\sqrt{t}$
		10	t

Thus, barring some $\sqrt{t}$ dependence at the $\pi\pi$ vertex, we feel that natural-parity couplings to a particle-antiparticle pair will have square-root dependence if the helicity difference in the t-channel center-of-mass system is odd, and regular behavior otherwise.

Likewise, the unnatural-parity exchanges can be isolated by studying $\pi B \to \sigma B$ (here σ is a scalar particle with the same mass as the pion). The behavior found by Wang[8] for these amplitudes is

$$f_{00;\lambda_1\lambda_2}{}^t \sim 1 \quad \text{if } \lambda_1-\lambda_2 \text{ is odd}$$
$$\sim\sqrt{t} \quad \text{if } \lambda_1-\lambda_2 \text{ is even.}$$

This leads us to expect that unnatural-parity couplings to a particle-antiparticle pair will have square-root dependence if the helicity difference is even, and regular behavior otherwise.

Perturbation Theory

To check the above conclusions, we compute a number of vertices in perturbation theory, for the two examples discussed in the text—$p\bar{p}$ scattering, and ρ-ρ scattering—and list them in Table IV. All examples are computed in the t-channel center-of-mass system.

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Asymptotic Helicity Conservation and Pion-Nucleon Dynamics

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Motivated by the hypothesis of the asymptotic conservation of helicity, we exploit the hypothesis that the P and P' Regge trajectories decouple from the A^+ amplitude for pion-nucleon scattering. From the vantage point of recent theoretical developments, viz., the concept of duality and the Freund-Harari conjecture, a set of dispersion relations is selected, by means of which we have shown, in good agreement with what is known, the correlation between low-energy resonances in the direct channel and non-Pomeranchukon t-channel exchanges. Furthermore, we have shown that there exists a dynamical limit in which the t-channel (ρ and σ) exchanges decouple, thus providing us with new insight into the relationship between our present approach and the Chew-Low model, the reciprocal-bootstrap, and the strong-coupling theories. Finally, some remarks of a speculative nature are given.

I. INTRODUCTION

THE use of dispersion relations ushered in a new phase in the study of hadron dynamics[1]; however, dispersion relations as originally used could not provide a detailed picture of strong interactions. Subsequently the Mandelstam representation was conceived and the idea of the analytic S matrix was developed.[1] Thus various attempts were made to explain the main feature of low-energy hadron-hadron scattering through the contributions of the nearby singularities unitarized via procedures such as the N/D method. For example, in the case of pion-nucleon scattering, the N and N^* poles in the s and u channels and the ρ and σ poles in the t channel were considered. However, the nature of the approximations made in such approaches was not at all clear, and moreover there existed the problem of possible double counting, as was more recently emphasized through the concept of duality.[2] The current-algebra approach,[3] on the other hand, while mainly providing interesting low-energy theorems, suffers from ambiguities which arise from the nature of the Schwinger and the so-called σ terms, and the problems of pion-mass extrapolation. Similarly, chiral dynamics,[4] though

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[1] See, for example, G. F. Chew, *S-Matrix Theory of Strong Interactions* (Benjamin, New York, 1961).

[2] See, for example, H. Harari, Lectures given at the Brookhaven Summer School in Elementary Particle Physics, 1969 (unpublished), and references quoted therein.

[3] See, for example, S. L. Adler and R. F. Dashen, *Current Algebra* (Benjamin, New York, 1968), and references quoted therein.

[4] See, for example, W. A. Bardeen and B. W. Lee, in *Nuclear and Particle Physics*, edited by B. Margolis and C. S. Lam (Benjamin, New York, 1968), and references quoted therein.