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Inelastic Electron-Proton Scattering and a Sum Rule for the Schwinger Term

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It is shown that the form of the one-particle expectation value of the Schwinger term in the equal-time commutator of the electromagnetic current and change densities is determined by relativistic covariance. This allows us to derive a sum rule for the one-proton expectation value of the Schwinger term involving the structure function W_2 measured in inelastic electron-proton scattering. From this sum rule for the Schwinger term one can obtain the sum rule

$$\frac{d}{dq^2}\int_0^\infty d\nu\,\frac{\nu W_2(q^2,\nu)}{q^2}=0\,,$$

where q^2 is the square of the four-momentum transfer, and ν is the energy loss of the electron in the laboratory frame. We compare this equation with the recent experimental data using "scale invariance." We find from this comparison that this sum rule is reasonably well satisfied.

I. INTRODUCTION

THE spin-averaged inelastic electron-proton scattering cross section is characterized by two structure functions $W_1(q^2,\nu)$ and $W_2(q^2,\nu)$. These can be defined in terms of the electromagnetic current $j_\mu(x)$ of the hadrons as follows[1]:

$$\frac{1}{2\pi}\int d^4x\, e^{-iq\cdot x}\sum_S \langle p|[j_\mu(x),j_\nu(0)]|p\rangle$$
$$=W_1(q^2,\nu)\left(\delta_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\right)$$
$$+W_2(q^2,\nu)\left(\frac{p_\mu}{m_p}+\frac{\nu}{q^2}q_\mu\right)\left(\frac{p_\nu}{m_p}+\frac{\nu}{q^2}q_\nu\right).\quad(1.1)$$

Here, $|p\rangle$ is a single-proton state of momentum p_μ, $\sum_S$ denotes the spin average, and q_μ is the four-momentum transfer given by the difference of the initial and final electron four-momenta. Further,

$$q^2=\mathbf{q}^2-q_0^2 \quad\text{and}\quad \nu=-p\cdot q/m_p\,,\qquad(1.2)$$

where m_p is the proton mass. We shall use the laboratory system ($\mathbf{p}=0$), so that $\nu=q_0$ is just the energy loss of the electron. Hereafter, $|p\rangle$ will denote the proton state at rest.

By combining the local current algebra with the commutation relations between the current densities and the Lorentz boost operators, we had derived[2] a set of sum rules for the neutrino-nucleon and electron-nucleon inelastic scattering. In particular, we had obtained the following sum rule[3] for electron-proton scattering:

$$\frac{d}{dq^2}\int_0^\infty d\nu\frac{\nu W_2(q^2,\nu)}{q^2}=0.\qquad(1.3)$$

Our purpose in this paper is twofold: (1) to confront this sum rule with the recent experimental data,[4–6] and (2) to give an alternate derivation of Eq. (1.3) through a route which yields a sum rule for the Schwinger term.[7]

The Schwinger term $S_i(\mathbf{x},\mathbf{y})$ is defined by the equal-time commutator

$$[j_0(x),j_i(y)]=iS_i(\mathbf{x},\mathbf{y})\quad\text{at}\quad x_0=y_0.\qquad(1.4)$$

The expectation value of $S_i(\mathbf{x},\mathbf{y})$ with respect to the spin-averaged single-proton state at rest is of the form

$$\sum_S\langle p|S_i(\mathbf{x},\mathbf{y})|p\rangle=f_p\frac{\partial}{\partial y_i}\delta(\mathbf{x}-\mathbf{y})\,,\qquad(1.5)$$

where f_p is the Schwinger constant for the proton. We show that the higher-derivative terms do not contribute to Eq. (1.5) due to relativistic covariance and the commutation relation

$$[j_0(x),j_0(y)]=0\quad\text{at}\quad x_0=y_0.\qquad(1.6)$$

We then use Eqs. (1.4) and (1.5) to derive the sum rule for f_p,

$$\int_0^\infty d\nu\frac{\nu W_2(q^2,\nu)}{q^2}=\tfrac{1}{2}f_p.\qquad(1.7)$$

The old sum rule in Eq. (1.3) follows by differentiation of Eq. (1.7) with respect to q^2.

In Sec. II, the derivation of the constraint on $S_i(\mathbf{x},\mathbf{y})$ due to relativistic covariance as well as the sum rule for f_p is given. In Sec. III, we compare the sum rule (1.3) with the present experimental data using scale invariance[8] and find that it is reasonably well satisfied.

[1] See, e.g., F. J. Gilman, Phys. Rev. **167**, 1365 (1968).
[2] V. Gupta and G. Rajasekaran, Phys. Rev. **185**, 1940 (1969).
[3] This is Eq. (21) in Ref. (2) since W_2 is same as β.
[4] Rapporteur talk of W. K. H. Panofsky, in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968* edited by S. Prentki and J. Steinberger (CERN, Geneva, 1968), p. 23.
[5] M. Breidenbach *et al.*, Phys. Rev. Letters **23**, 935 (1968).
[6] L. W. Mo, in *Proceedings of the Third International Conference on High-Energy Collisions, Stony Brook, 1969* (Gordon & Breach, New York, 1969).
[7] T. Goto and T. Imamura, Progr. Theoret. Phys. (Kyoto) **14**, 396 (1955); T. Pradhan, Nucl. Phys. **9**, 124 (1958); J. Schwinger, Phys. Rev. Letters **3**, 296 (1959).
[8] J. D. Bjorken, Phys. Rev. **179**, 1547 (1969).

II. COVARIANCE CONSTRAINT AND SUM RULE FOR SCHWINGER TERM

For generality we start with the local vector current algebra of Gell-Mann,[9]

$$[j_0^\alpha(x),j_0^\beta(y)]=if^{\alpha\beta\gamma}j_0^\gamma(x)\delta(\mathbf{x}-\mathbf{y}) \quad \text{at} \quad x_0=y_0\,, \tag{2.1}$$

where α, β, and γ are the $SU(3)$ indices. The Schwinger term is defined by

$$[j_0^\alpha(x),j_i^\beta(y)]=if^{\alpha\beta\gamma}j_i^\gamma(x)\delta(\mathbf{x}-\mathbf{y})+iS_i^{\alpha\beta}(\mathbf{x},\mathbf{y}) \quad \text{at} \quad x_0=y_0\,. \tag{2.2}$$

Commuting both sides of (2.1) with the Lorentz boost operator K_i (i=1, 2, 3) and using

$$i[K_i,j_0^\alpha(x)]=j_i^\alpha(x)-x_i\partial_0 j_0^\alpha(x) \quad \text{at} \quad x_0=0\,, \tag{2.3}$$

we obtain

$$\begin{aligned}&[j_0^\alpha(x),(j_i^\beta(y)-y_i\partial_0 j_0^\beta(y))]\\ &\quad+[(j_i^\alpha(x)-x_i\partial_0 j_0^\alpha(x)),j_i^\beta(y)]\\ &=if^{\alpha\beta\gamma}(j_i^\gamma(x)-x_i\partial_0 j_0^\alpha(x))\delta(\mathbf{x}-\mathbf{y})\\ &\qquad\qquad \text{at} \quad x_0=y_0=0\,.\end{aligned} \tag{2.4}$$

We shall only be concerned with conserved vector currents, that is,

$$\partial_0 j_0^\alpha(x)+\partial_i j_i^\alpha(x)=0, \quad \alpha=1,\,2,\,3 \text{ and } 8\,. \tag{2.5}$$

Now, using (2.2) and (2.5) in (2.4), we find the covariance condition[10] for the Schwinger term,

$$\frac{\partial}{\partial x_i}[x_j S_i^{\beta\alpha}(\mathbf{y},\mathbf{x})]=\frac{\partial}{\partial y_i}[y_j S_i^{\alpha\beta}(\mathbf{x},\mathbf{y})]\,. \tag{2.6}$$

Or, defining the symmetric and antisymmetric combinations

$$S_i^{\pm}(\mathbf{x},\mathbf{y})=\tfrac{1}{2}[S_i^{\alpha\beta}(\mathbf{x},\mathbf{y})\pm S_i^{\beta\alpha}(\mathbf{x},\mathbf{y})]\,, \tag{2.7}$$

we have

$$\frac{\partial}{\partial x_i}[x_j S_i^{\pm}(\mathbf{y},\mathbf{x})]=\pm\frac{\partial}{\partial y_i}[y_j S_i^{\pm}(\mathbf{x},\mathbf{y})]\,. \tag{2.8}$$

The most general local form for $S_i^{\pm}(\mathbf{x},\mathbf{y})$ is

$$\begin{aligned}S_i^{\pm}(\mathbf{x},\mathbf{y})&=f_i^{\pm}(\mathbf{y})\delta(\mathbf{x}-\mathbf{y})\\ &\quad+f_{ij}^{\pm}(\mathbf{y})\frac{\partial}{\partial y_j}\delta(\mathbf{x}-\mathbf{y})+f_{ijk}^{\pm}(\mathbf{y})\frac{\partial^2}{\partial y_j\partial y_k}\delta(\mathbf{x}-\mathbf{y})\\ &\quad+\cdots \text{up to the } n\text{th derivative}.\end{aligned} \tag{2.9}$$

Taking the expectation value with respect to a single particle (which will be taken as the proton) at rest and averaging over spin, we get

$$\begin{aligned}\sum_S \langle p|S_i^{\pm}(\mathbf{x},\mathbf{y})|p\rangle&=f_1^{\pm}\frac{\partial}{\partial y_i}\delta(\mathbf{x}-\mathbf{y})\\ &\quad+f_3^{\pm}\frac{\partial}{\partial y_i}\nabla^2\delta(\mathbf{x}-\mathbf{y})+f_5^{\pm}\frac{\partial}{\partial y_i}\nabla^4\delta(\mathbf{x}-\mathbf{y})+\cdots\,,\end{aligned} \tag{2.10}$$

where $f_1^{\pm}$, $f_3^{\pm}$, ... are constants. We take the expectation value of Eq. (2.8) and use Eq. (2.10). The left-hand side of Eq. (2.8) is

$$(f_1^{\pm}+f_3^{\pm}\nabla^2+f_5^{\pm}\nabla^4+\cdots)\frac{\partial}{\partial x_j}\delta(\mathbf{x}-\mathbf{y}) +x_j(f_1^{\pm}+f_3^{\pm}\nabla^2+f_5^{\pm}\nabla^4+\cdots)\nabla^2\delta(\mathbf{x}-\mathbf{y})\,,$$

whereas the right-hand side becomes, after some manipulation,

$$\pm(f_1^{\pm}+3f_3^{\pm}\nabla^2+5f_5^{\pm}\nabla^4+\cdots)\frac{\partial}{\partial x_j}\delta(\mathbf{x}-\mathbf{y}) \pm x_j(f_1^{\pm}+f_3^{\pm}\nabla^2+f_5^{\pm}\nabla^4+\cdots)\nabla^2\delta(\mathbf{x}-\mathbf{y})\,.$$

Hence, we conclude that $f_1^-=f_3^{\pm}=f_5^{\pm}=\cdots=0$, and

$$\sum_S \langle p|S_i^{\alpha\beta}(\mathbf{x},\mathbf{y})|p\rangle =f_1^{\alpha\beta}\frac{\partial}{\partial y_i}\delta(\mathbf{x}-\mathbf{y}) \quad \text{with} \quad f_1^{\alpha\beta}=f_1^{\beta\alpha}\,. \tag{2.11}$$

Thus, starting from Eq. (2.11) and using Lorentz covariance, we have shown that the spin-averaged single-particle expectation value of $S_i(\mathbf{x},\mathbf{y})$ involves only the first derivative of the δ function.[11] Specializing to the case of the electromagnetic current $j_\mu(x)$, we rewrite Eqs. (2.2) and (2.11) as

$$[j_0(x),j_i(y)]=iS_i(\mathbf{x},\mathbf{y}) \quad \text{at} \quad x_0=y_0 \tag{2.12}$$

and

$$\sum_S \langle p|S_i(\mathbf{x},\mathbf{y})|p\rangle=f_p\frac{\partial}{\partial y_i}\delta(\mathbf{x}-\mathbf{y})\,. \tag{2.13}$$

Using the above result, we now derive the sum rule for f_p. The method[12] followed is essentially the same as

[9] M. Gell-Mann, Physics **1**, 63 (1964).

[10] Equation (2.6) is the symmetric form of the condition III derived earlier; see V. Gupta and G. Rajasekaran, Nucl. Phys. **B10**, 11 (1969).

[11] It can be shown that $S_i^{\alpha\beta}(\mathbf{x},\mathbf{y})$ itself has only the first derivative of the δ function. However, one has to assume, in addition, a particular form for the commutation relation of j_0 with the energy density. This has been shown by D. J. Gross and R. Jackiw, Phys. Rev. **163**, 1688 (1967).

[12] One can obtain the sum rule by the infinite-momentum method by a modification of the procedure used by K. Gottfried, Phys. Rev. Letters **18**, 1174 (1967).

in Ref. 2. We start with the identity

$$\int_0^\infty dx_0 e^{iq_0x_0}\langle f|[\partial_0 A(x_0),B(0)]|i\rangle$$
$$=-\langle f|[A(0),B(0)]|i\rangle$$
$$-iq_0\int_0^\infty dx_0\langle f|[A(x_0),B(0)]|i\rangle e^{iq_0x_0}, \quad (2.14)$$

which holds for all q_0 in the upper half of the complex plane. For $|i\rangle$ and $|f\rangle$ we shall take a single-proton state at rest denoted by $|p\rangle$, and we shall further average over the proton spin. Choose

$$A(x_0)=\int d^3x\, e^{-is\cdot x}j_0(\mathbf{x},x_0)\,,$$
$$B(x_0)=\partial_0 j_0(0,x_0)\,,$$

where j_μ is the electromagnetic current, and take the limit as $q_0\to 0$. The second term on the right-hand side of Eq. (2.14) vanishes in this limit for all $|\mathbf{s}|^2>0$. We therefore get, using the continuity equation,

$$\lim_{q_0\to 0} s_i s_k\int d^4x \exp(-i\hat{q}\cdot x)\,\theta(x_0)\sum_S\langle p|[j_i(x),j_k(0)]|p\rangle$$
$$=is_i\sum_S\int d^3x\, e^{-is\cdot x}\langle p|[j_0(\mathbf{x},0),j_i(0)]|p\rangle, \quad (2.15)$$

where $\hat{q}=(iq_0,\mathbf{s})$. Defining the functions $v_1(q^2,q_0)$ and $v_2(q^2,q_0)$ by

$$\int d^4x\, e^{-iq\cdot x}\theta(x_0)\sum_S\langle p|[j_i(x),j_k(0)]|p\rangle$$
$$=v_1(q^2,q_0)\delta_{ik}+v_2(q^2,q_0)q_iq_k\,, \quad (2.16)$$

and using Eqs. (2.12) and (2.13), we get the low-energy theorem:

$$\lim_{q_0\to 0}(v_1+q^2v_2)=-if_p\,, \quad (2.17)$$

where we have replaced $|\mathbf{s}|^2$ by q^2.

We now assume the unsubtracted dispersion relation[13]:

$$v_1(q^2,q_0)+v_2(q^2,q_0)q^2$$
$$=\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{dq_0'}{q_0'-q_0}\{v_1'(q^2,q_0')+q^2v_2'(q^2,q_0')\}\,, \quad (2.18)$$

where v_1' and v_2' are the absorptive parts defined by

$$\frac{1}{2i}\int d^4x\, e^{-iq\cdot x}\sum_S\langle p|[j_i(x),j_k(0)]|p\rangle$$
$$=v_1'\delta_{ik}+v_2'q_iq_k\,. \quad (2.19)$$

By comparing with Eq. (1.1), we find

$$v_1'+q^2v_2'=-i\pi(q_0^2/q^2)W_2\,. \quad (2.20)$$

Putting $q_0=0$ in Eq. (2.18) and using Eqs. (2.17) and (2.20) and the fact that v_1' and v_2' as defined by Eq. (2.19) are odd functions of q_0, we obtain the sum rule for the Schwinger constant,

$$\int_0^\infty d\nu\frac{\nu W_2(q^2,\nu)}{q^2}=\tfrac{1}{2}f_p\,. \quad (2.21)$$

Note that all our matrix elements should be understood to contain only the connected part. Consequently, f_p is really the expectation value of the operator part of the Schwinger term. The left-hand side of (2.21) is positive and nonzero. This implies that if the unsubtracted dispersion relation in Eq. (2.18) is valid, then the Schwinger term cannot be a c number.

Formally, by differentiating (2.21) with respect to q^2, since f_p is a constant, one obtains

$$\frac{d}{dq^2}\int_0^\infty d\nu\frac{\nu W_2(q^2,\nu)}{q^2}=0\,. \quad (2.22)$$

However, it should be emphasized that Eq. (2.22) can be derived directly from the boosted commutation relation [Eq. (2.4)], as has been shown earlier.[2]

Separating the "elastic" term in Eqs. (2.21) and (2.22), we have[14]

$$R(q^2)+\int_{\nu_t(q^2)}^\infty d\nu\frac{\nu W_2(q^2,\nu)}{q^2}=\tfrac{1}{2}f_p \quad (2.23)$$

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[13] If the sum rule to be derived turns out to be divergent, then this assumption would be suspect. On the other hand, by assuming the unsubtracted dispersion relation for the amplitude $v_1(q^2,q_0)+v_2(q^2,q_0)\mathbf{q}^2$ and following the same procedure, one can get the sum rule

$$\int_0^\infty d\nu\frac{\nu}{q^2}\left[\left(1+\frac{\nu^2}{q^2}\right)W_2-W_1\right]=\tfrac{1}{2}f_p.$$

Or, in terms of the absorption cross section for longitudinal photons (Ref. 1), we have

$$\frac{1}{2\pi^2\alpha}\int_0^\infty d\nu\,\frac{\nu}{q^2}(q^2+\nu^2)^{1/2}|\sigma_{\rm long}(q^2,\nu)|=f_p.$$

However, the convergence of this sum rule seems to be more doubtful.

[14] If one takes the limit $q^2\to 0$, then Eq. (2.23) reduces to

$$\frac{1}{2m_p}+\frac{1}{4\pi^2\alpha}\int_0^\infty\sigma(\nu)d\nu=\tfrac{1}{2}f_p,$$

since the total photoabsorption cross section for photons of energy ν is given by

$$\sigma(\nu)=4\pi^2\alpha\lim_{q^2\to 0}\frac{\nu W_2(q^2,\nu)}{q^2},$$

where $\alpha\approx 1/137$. Recent experimental data (Ref. 6) indicate a decreasing trend for $\sigma(\nu)$ with increasing ν. However, this decrease might turn out to be insufficient to make the above integral convergent. Thus the convergence of (2.23), in the neighborhood of $q^2=0$, seems unlikely. However, for large q^2 the sum rule (2.23) may well turn out to be convergent.

It is interesting, however, to contrast the above result with the sum rule obtained by assuming an unsubtracted dispersion relation for the full Compton amplitude, namely,

$$\frac{1}{2m_p}+\frac{1}{4\pi^2\alpha}\int_0^\infty\sigma(\nu)d\nu=0,$$

which is self-contradictory.

and[15]

$$\frac{dR(q^2)}{dq^2}+\int_{\nu_t(q^2)}^{\infty} d\nu \frac{\partial}{\partial q^2}\left[\frac{\nu W_2(q^2,\nu)}{q^2}\right]=0\,, \quad (2.24)$$

where $R(q^2)$ is a function of the "elastic" form factors of the proton G_E and G_M:

$$R(q^2)=\frac{[G_E(q^2)]^2+(q^2/4m_p{}^2)[G_M(q^2)]^2}{2m_p(1+q^2/4m_p{}^2)} \quad (2.25)$$

and $\nu_t(q^2)$ is the inelastic threshold

$$\nu_t(q^2)=m_\pi+(q^2+m_\pi{}^2)/2m_p\,. \quad (2.26)$$

III. EXPERIMENTAL TEST OF SUM RULE

In principle, $W_2(q^2,\nu)$ can be determined from the inelastic electron-proton scattering experiment, and hence the sum rule can be tested for all values of q^2. However, the published experimental data on this scattering are not complete enough to warrant such a systematic analysis. So, we have attempted a rather rough comparison with experimental data.

For this purpose we exploit the "scale invariance" which was originally suggested by Bjorken,[8]

$$\nu W_2(q^2,\nu)=F_2(\omega),\quad \omega=2m_p\nu/q^2\,, \quad (3.1)$$

where $F_2(\omega)$ is supposed to be a universal function that is conjectured to be valid for large values of ν and q^2. Experimentally,[4–6] this "scale invariance" is found to be valid for $q^2\gtrsim 0.5$ BeV^2 and $\omega\gtrsim 4$. We shall check the sum rule only for q^2 in this region.

First note that scale invariance is supposed to be exact for $q^2\to\infty$. What does the sum rule in Eq. (2.24) imply in this limit? Using Eq. (3.1), Eq. (2.24) becomes

$$\lim_{q^2\to\infty}\left[\frac{dR(q^2)}{dq^2}-\frac{1}{2m_pq^2}\int_1^{\infty} d\omega\frac{d}{d\omega}\{\omega F_2(\omega)\}\right]=0 \quad (3.2)$$

or

$$\frac{dR(q^2)}{dq^2}\xrightarrow[q^2\to\infty]{}\frac{[\omega F_2(\omega)]_{\omega=\infty}}{2m_pq^2}\,, \quad (3.3)$$

where we have put[15] $F_2(1)=0$. If $[\omega F_2(\omega)]_{\omega=\infty}\neq 0$, Eq. (3.3) implies that the "elastic" form factors of the proton increase logarithmically with q^2 for $q^2\to\infty$. Since this seems unlikely, we conclude that

$$\omega F_2(\omega)\xrightarrow[\omega\to\infty]{} 0\,. \quad (3.4)$$

Presently available experimental data[5,6] indicate that $F_2(\omega)$ is, in fact, a decreasing function of ω for $\omega\gtrsim 5$. The validity of Eq. (3.4) can be checked perhaps in the near future.

We next consider q^2 finite but larger than 0.5 BeV^2, for which values scale invariance is consistent with experiment.[5,6] We split the integral in Eq. (2.24) into a low-energy part and a high-energy part:

$$\frac{dR(q^2)}{dq^2}+\int_{\nu_t(q^2)}^{\nu_s} d\nu\frac{\partial}{\partial q^2}\left(\frac{\nu W_2}{q^2}\right)+\int_{\nu_s}^{\infty} d\nu\frac{\partial}{\partial q^2}\left(\frac{\nu W_2}{q^2}\right)=0\,, \quad (3.5)$$

where ν_s is chosen sufficiently high so that in the high-energy integral we can use scale invariance. Thus,

$$\int_{\nu_s}^{\infty} d\nu\frac{\partial}{\partial q^2}\left(\frac{\nu W_2}{q^2}\right)=\frac{1}{2m_pq^2}\omega_sF_2(\omega_s)\,, \quad (3.6)$$

where

$$\omega_s=2m_p\nu_s/q^2\,, \quad (3.7)$$

and we have assumed Eq. (3.4) to be true, which, as we had shown above, is necessary for consistency with the limit $q^2\to\infty$. Hence, for large enough q^2 we have the "finite-energy sum rule"

$$\frac{dR(q^2)}{dq^2}+\int_{\nu_t(q^2)}^{\nu_s} d\nu\frac{\partial}{\partial q^2}\left(\frac{\nu W_2}{q^2}\right)=-\frac{1}{2m_pq^2}\omega_sF_2(\omega_s)\,. \quad (3.8)$$

We have evaluated the sum rule for $q^2=0.9$ BeV^2 and $\omega_s=4$ using the experimental data[4–6] and find

$$\frac{dR(q^2)}{dq^2}+\int_{\nu_t(q^2)}^{\nu_s} d\nu\frac{\partial}{\partial q^2}\left(\frac{\nu W_2}{q^2}\right)\approx -0.87\ \text{BeV}^{-3} \quad (3.9)$$

$$-\frac{1}{2m_pq^2}\omega_sF_2(\omega_s)\approx -0.76\ \text{BeV}^{-3}\,. \quad (3.10)$$

It should be emphasized that these numbers are rough estimates based on the rather meager data available so far. Nevertheless, we regard the agreement with the sum rule encouraging.[16] With more complete data which may be available in the future, one could undertake a direct test of the sum rule without using scale invariance.

Finally, we remark that the sum rule for the Schwinger term (2.21), in the limit of $q^2\to\infty$, using scale invariance, becomes simply

$$\int_1^{\infty} d\omega\, F_2(\omega)=m_pf_p\,. \quad (3.11)$$

The validity of Eq. (3.4) is not enough to guarantee the

[15] In obtaining Eq. (2.24) from Eq. (2.22), we have made use of the vanishing of W_2 at the inelastic threshold.

[16] Earlier, in Ref. 2, on the basis of the then available data (Ref. 4), it was inferred that $(\partial/\partial q^2)[\nu W_2(q^2,\nu)/q^2]$ was negative. Further, since $dR(q^2)/dq^2$ is also negative, it seemed then that one could not satisfy Eq. (2.24), implying that the assumption of unsubtracted dispersion relations was incorrect. However, the more recent data (Refs. 5 and 6) indicate that, for large ω, $F_2(\omega)$ decreases with increasing ω, implying that at or after some large value of ν, $(\partial/\partial q^2)[\nu W_2(q^2,\nu)/q^2]$ changes sign, thus giving a positive contribution which makes it possible to satisfy Eq. (2.24).

convergence of the above integral,[17] though the reverse is true. In any case, the question of the convergence of this integral as well as its numerical evaluation cannot be considered at the present stage of our knowledge of $F_2(\omega)$ for large ω. It would be really interesting if one could determine f_p directly from the experimental data using the sum rule presented here.

[17] For example, the behavior $\omega F_2(\omega) \underset{\omega\to\infty}{\longrightarrow} (\ln\omega)^{-1}$ will lead to a mild divergence of the integral in Eq. (3.11).

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Duality and the Regge-Pole Eikonal Scheme*

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The Regge-pole eikonal scheme is used to generate the partial and total cross sections and Argand diagrams in $\pi^+\pi^-$ and πp scattering at intermediate and low energies. The correspondence between the calculated partial-wave results and the observed s-channel resonances is closer than with the direct use of Regge poles, but the total cross sections at lower energies completely lack the experimentally observed structure.

I. INTRODUCTION

It has been suggested by Drago[1] that if one takes t-channel Regge exchanges (ρ, f, and P) in $\pi^+\pi^-$ scattering and uses the Regge-pole eikonal ansatz, then structure is found in the total cross section that appears to reflect s-channel resonances. This was given as possible further evidence for the hypothesis of Schmid[2] that "the equivalence between t-channel Regge poles and s-channel resonances does not hold on the average, but even locally at each intermediate energy." Schmid examined Argand diagrams (imaginary versus real parts of partial-wave amplitudes) resulting from direct use of Regge exchanges and found circles characteristic of resonances. His conclusion has been criticized for a number of reasons, including the lack of any resonance-like structure in cross sections from Regge amplitudes, and the lack of poles on the second (unphysical) sheet, which are usually associated with resonances. Others[3–5] have examined more Argand diagrams (with the direct use of Regge poles) and have found rather poor correspondence between calculated and experimental results. However, Cohen-Tannoudji *et al.*[6] have proposed an explicit form for partial-wave amplitudes which does have second-sheet poles (this form is similar, on the real axis, to that of the eikonal approach, which probably has branch points on the second sheet). Drago also used the Cohen-Tannoudji form and found results similar to those found with the eikonal approach.

One of the problems in the debate is that the concept of "local duality" has not been well defined. The problem of definition has sometimes been avoided by simply invoking the qualitative nature of results using leading trajectories at intermediate and low energies (in addition to certain other simplifying assumptions). However, at higher energies the cross sections have little or no structure. Further, the signature factor $(1-e^{-i\pi\alpha})$ of Regge amplitudes can easily result in counterclockwise motion on an Argand diagram. Under these circumstances, the use of qualitative results to check a loosely defined concept makes study of the issue difficult.

We decided first to investigate the source of the structure found by Drago and then to extend the Regge-pole eikonal approach to πp scattering where one has nucleon resonances instead of meson resonances. The added degrees of freedom of isospin and parity offer hope that even qualitative results may clarify the situation.

For $\pi^+\pi^-$ scattering we examined both the partial cross sections and the Argand diagrams.[7] We found that the more impressive structure found by Drago in

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