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Studies of Proposed Structure in the ρ-Meson Mass*

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Monte Carlo techniques were used to study the dependence of the "observed" ρ-meson mass distribution on the width of the experimental resolution function and on the theoretical shape which the ρ might be expected to assume with infinitely good resolution. By using presently available mass resolutions, we find that structure similar to that of the A_2 meson would already have been observed if the ρ were a dipole resonance, as proposed by Kreps and Moffat. Some other possible mass shapes suggested in part by the $\rho+\rho'$ model which could easily have been missed are considered. Inferences relevant to the A_2 meson are also drawn.

I. INTRODUCTION

THE discovery[1] of mass structure in the A_2 meson, which appears to be characteristic of a dipole or double pole,[1–3] has led to proposals for similar mass structure in other resonances. One such suggestion concerns the ρ meson, for which it has been argued that dipole mass structure would imply the "dipole" shape for the nucleon form factors and offer a simultaneous explanation of polarization observed in pion-nucleon charge exchange.[4] Therefore, it seems of interest to ask whether some of the high-statistics good-resolution experiments on the ρ meson should have seen such structure.

The CERN missing-mass study[5] of ρ mesons produced in π^-p reactions was done at several incident pion momenta between 3.5 and 5.0 GeV/c with mass distributions obtained at several *different* $-t$ (momentum transfer squared) values. The mass resolution was $\pm$14 MeV, and events were plotted in 10-MeV bins. The results at various t values and incident momenta are quite consistent with one another and no obvious dipole structure is visible. The over-all compilation of events from this experiment totals $>$15 000, which serves to define the level of statistics in attempting to answer the question of whether structure should have been observed. Compilations of bubble-chamber experiments with statistics not far different from that of CERN are also available[6] for all charge states. The hint of structure appearing in the neutral ρ in such compilations can be explained by the well-studied ρ^0-ω electromagnetic mixing.

In Sec. II, we investigate with Monte Carlo techniques whether an experiment, with finite resolution, yielding 10 000 events above background would detect specified structure. First, we examine the structure implied by the assumption in Ref. 4 that the S matrix for $I=1$, $J=1$ $\pi\pi$ (elastic) scattering as a function of scattering energy E has a double pole,

$$S=[(E_0-E+i\tfrac{1}{2}\Gamma)/(E_0-E-i\tfrac{1}{2}\Gamma)]^2. \qquad (1)$$

Therefore, the amplitude T has a double pole and the ρ-mass distribution would be (often referred to as the dipole mass shape)

$$|T|^2\propto(E-E_0)^2/[(E-E_0)^2+\tfrac{1}{4}\Gamma^2]^2. \qquad (2)$$

Our analysis leads us to believe that structure such as this would not have been missed in existing experiments. The addition of incoherent and coherent background does not significantly alter this conclusion. This is a reasonably important result since it argues against the idea that the entire exchange-degenerate trajectory upon which the A_2 lies is necessarily a Regge dipole.

Since perhaps it is not obvious, it seems worth emphasizing that by virtue of the nature of a missing-mass type of experiment, the various partial waves of a two-pion system opposite to which the proton recoils are incoherent. Further, these two-pion partial-wave states do not interfere with other multiparticle missing-mass states. This follows since only the recoiling proton is detected and all quantum numbers or degrees of freedom for the missing-mass (m.m.) system are summed or integrated over, including angles in the m.m. rest system. Therefore, this particular type of experiment is exceedingly well suited for studying structure in a particular partial wave—all other states contribute to noninterfering background. Specifically, the problems associated with the undetected ϵ meson, or S-wave $\pi\pi$ resonance, do not obscure the analysis. This is why most of histograms we present are calculated

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[1] G. Chikovani, M. N. Focacci, W. Kienzle, C. Lechanoine, B. Levrat, B. Maglić, M. Martin, P. Schübelin, L. Dubal, M. Fischer, P. Grieder, H. A. Neal, and C. Nef, Phys. Letters **25B**, 44 (1967); H. Benz, G. E. Chikovani, G. Damgaard, M. N. Focacci, W. Kienzle, C. Lechanoine, M. Martin, C. Nef, P. Schübelin, R. Baud, B. Bošnjaković, J. Cotteron, R. Klanner, and A. Weitsch, *ibid.* **28B**, 233 (1968).

[2] J. S. Bell, CERN Report No. 67/721/1-Th. 784 (unpublished).

[3] K. E. Lassila and P. V. Ruuskanen, Phys. Rev. Letters **19**, 762 (1967); J. V. Beaupre, T. P. Coleman, K. E. Lassila, and P. V. Ruuskanen, *ibid.* **21**, 1849 (1968).

[4] R. E. Kreps and J. W. Moffat, Phys. Rev. **175**, 1942 (1968); **175**, 1945 (1968).

[5] H. R. Blieden, D. Freytag, J. Geibel, A. R. F. Hassan, W. Kienzle, F. Lefèbvres, B. Levrat, B. C. Maglić, J. Sequinot, and A. J. Smith, Nuovo Cimento **43**, 71 (1966).

[6] M. Roos, Nucl. Phys. **B2**, 615 (1967); J. Pišút and M. Roos, *ibid.* **B6**, 325 (1968). For recent results of several groups working on the $\pi\pi$ problem, see Proceedings of the Conference on $\pi\pi$ and $K\pi$ Interactions, Argonne National Laboratory, Argonne, Ill., 1969, edited by F. Loeffler and E. Malamud (unpublished).

with parameters characteristic of the old CERN experiment.[5]

If dipole structure should have been visible, perhaps other similar deviations from the pure Breit-Wigner shape would not have been seen. Since the $\rho+\rho'$ model[7] has enjoyed some success in fitting pion-nucleon charge-exchange data in the high-energy Regge region, a two-pole alternative without restriction to coincident poles in S seems reasonable to try. With the ρ' pole restricted to be in the physical ρ region, S can be taken to be of modulus one since all channels except $\pi\pi$ have, experimentally, negligible couplings. When the poles have different real and/or imaginary parts, structure is possible which would not have (as yet) been seen in experiment. The "interference" dip due to a ρ' pole with a small imaginary part would have been missed in all published experiments to date. If the ρ' is not in the ρ region, but higher in mass (though not too far from the ρ region), as some analyses have indicated, the resulting mass curve for the ρ will be, at most, distorted with the high-mass side falling off somewhat more quickly than the low-mass part of the curve.

On the basis of the present analysis, we can make the following comment about the A_2 mass system providing the A_2 m.m. data can be described by two poles in the amplitude for $\pi\rho\to\pi\rho$. (This is probably reasonable for the $\pi\rho$ mode within an $\sim 15\%$ error since it is the dominant A_2 decay mode.) The rather deep dip in the CERN data by itself argues against a pole configuration in which the imaginary parts are drastically different and favors a situation closer to a dipole or especially the case denoted in the literature by A_2^{high} and A_2^{low} since the finite experimental resolution would be less effective in filling the dip.

II. RESULTS FOR MASS DISTRIBUTIONS

A. Dipole

Events are generated according to the dipole mass distribution function of Eq. (1), which uses nonrelativistic notation (this will not affect our conclusions, but will affect our plotted histograms very slightly). For convenience, we rewrite Eq. (2) as

$$F(M)=(M-M_0)^2/[(M-M_0)^2+\tfrac{1}{4}\Gamma^2]^2, \qquad (2')$$

where M_0 is the mass of the "ρ", M is the exact mass of the $\pi\pi$ system, and Γ is the width parameter. We also introduce another mass variable M', defined as the observed mass of the $\pi\pi$ system, obtained from M by Monte Carlo weighting with a Gaussian of width given by the experimental resolution. The resolution[8] ΔM and Γ are then the parameters of this study and are constrained by the requirement that the resulting histogram should be of width comparable to that of the actual experiment. In the calculation (M' is the abscissa in the histograms),

$$M_0=765 \text{ MeV},$$

$$400\leq M\leq 1050 \text{ MeV},$$

and

$$600\leq M'\leq 950 \text{ MeV}.$$

In Fig. 1 we display some representative mass histograms generated for a 10 000-event experiment on the ρ meson for various Γ and resolution values. Shown plotted are results for (a) $\Gamma/2=20$ MeV, $\Delta M=\pm 29.4$ MeV (an average-to-poor bubble-chamber experiment, say); (b) $\Gamma/2=25$ MeV, $\Delta M=\pm 18$ MeV (a reasonable bubble-chamber experiment); and (c) $\Gamma/2=25$ MeV, $\Delta M=\pm 14$ MeV (the CERN m.m. experiment[5] on the ρ meson). Of Figs. 1(a)–1(c), only 1(a) does not show a dip but the result of smearing out the dipole with such a broad resolution function (full width at half-maximum of 59 MeV) produces an 80-MeV-wide flat top which is not characteristic of a single Breit-Wigner resonance. The other two histograms in Fig. 1 show rather deep dips. With the dipole assumption, the existing CERN experiment on the ρ should be like Fig. 1(c) with a statistically significant dip; however, the actual data[5] show no hint of such structure. Thus, existing data,[5,6] we believe, strongly argue against the ρ meson being a dipole. Numerous other calculations with slightly different, and also the same, parameters give results similar to those plotted. Calculations with smaller resolution such as that achieved in the CERN A_2 experiment[1] ($\Delta M=\pm 7.5$ MeV) or that of a high-precision experiment on the ρ^0 performed at Berkeley[9] show dips deeper than that of Fig. 1(c).

The addition of coherent or incoherent background does not alter significantly the above conclusion. Since the background changes from one experiment to another, the simplifying assumption was made that the incoherent background increases linearly from 0.6 to 1 GeV. The ratio of this background at 1 GeV to that at 0.6 GeV was set at 3:1, and the total number of events under this linear curve was taken as 5000 (this is fairly similar to the CERN experiment[5]). In support of our conclusion a sample of the histograms calculated with this background assumption is shown in Fig. 2. Figure 2(a) represents results with $\Gamma/2=25$ MeV, $\Delta M=\pm 18$ MeV [see Fig. 1(b) for the no-background case] and 2(b) represents those with $\Gamma/2=25$ MeV,

[7] R. K. Logan, J. Beaupre, and L. Sertorio, Phys. Rev. Letters **18**, 259 (1967); W. Rarita and B. Schwarzchild, Phys. Rev. **162**, 1378 (1967). For further references and detailed comparison of the $\rho+\rho'$ models with cut models, see C. Michael, Nucl. Phys. **B8**, 431 (1968), who finds that existing data cannot differentiate between these models.

[8] In a strict treatment, particularly for the bubble-chamber experiments, the ΔM should not be fixed but should instead be chosen from a Gaussian distribution. For one case this was done, and there were no statistically significant differences between the distribution generated in this fashion and that generated using a fixed ΔM.

[9] G. Goldhaber, W. R. Butler, D. G. Coyne, B. H. Hall, J. M. MacNaughton, and G. H. Trilling, Phys. Rev. Letters **23**, 1351 (1969).

$\Delta M = \pm 14$ MeV (the value for the CERN experiment of Ref. 5).

For the case of coherent, very slowly varying background introduced through a factor $e^{i\varphi}$ multiplying the dipole form for S in Eq. (1), the conclusions above based on the pronounced dip are unchanged. The main effect of coherent background is to distort the wings of the resonance curve. For $\varphi > 0$, the high-energy side tends to fall to zero faster and the low-energy side slower than when $\varphi = 0$. For the dipole case, the higher-mass peak is then narrower, but if it is made narrow enough to escape detection or fake the ρ^0-ω mixing effect, the distortion of the wings becomes completely incompatible with existing experiments, e.g., Refs. 5 and 6. For a typical experiment, the "nearly" symmetric

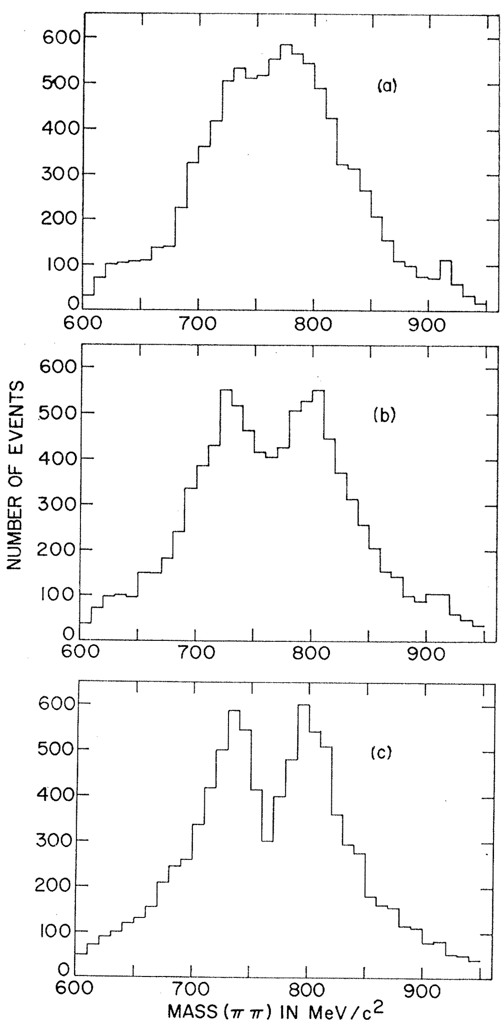

FIG. 1. Monte-Carlo-generated mass distributions for the ρ-mass region under the assumption of Ref. 4 that the ρ meson is a dipole resonance described by Eq. (2′). The histograms plotted are for (a) width parameter in Eq. (2′) $\frac{1}{2}\Gamma = 20$ MeV and experimental resolution $\Delta M = \pm 29.4$ MeV; (b) width $\Gamma/2 = 25$ MeV, $\Delta M = \pm 18$ MeV; and (c) width $\Gamma/2 = 25$ MeV, $\Delta M = \pm 14$ MeV (the resolution in Ref. 5).

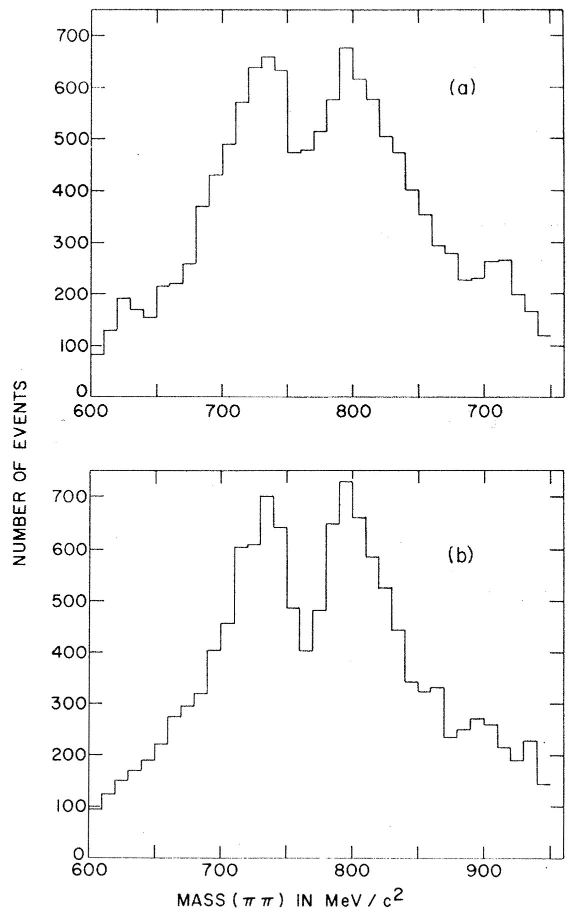

FIG. 2. Histograms calculated for the cases shown in Fig. 1(b) and 1(c) with the addition of a linearly varying incoherent background of 5000 events: (a) width $\Gamma/2 = 25$ MeV and resolution $\Delta M = \pm 18$ MeV, (b) width $\Gamma/2 = 25$ MeV and resolution $\Delta M = \pm 14$ MeV.

shape of the mass distribution within $\sim\Gamma/2$ of the peak limits φ to be less than ~0.3 rad, limiting the background phase shift in the partial wave with the ρ resonance to $\sim -8° < \varphi < \sim 8°$.

B. Noncoincident Poles

We wish to write S of Eq. (1) in more general form as[10]

$$S = e^{i\varphi}\left[\frac{(E-E_1-i\frac{1}{2}\Gamma_1)(E-E_2-i\frac{1}{2}\Gamma_2)}{(E-E_1+i\frac{1}{2}\Gamma_1)(E-E_2+i\frac{1}{2}\Gamma_2)}\right], \qquad (3)$$

with obvious notation, so the $\pi\pi$ $I=1$, $J=1$ partial-wave scattering amplitude has two noncoincident poles (the elasticity, $\eta = 1$). Going through the same Monte Carlo procedures as above (E is replaced by M), we have examined several situations different from the dipole which can be appropriately adjusted to explain

[10] See, e.g, J. S. Bell, CERN Report No. 67/721/1-TH. 784 (unpublished); or K. W. McVoy, *Fundamentals in Nuclear Theory* (International Atomic Energy Agency, Vienna, 1967), p. 419.

electromagnetic form factors and the $\rho+\rho'$ model results in πp charge-exchange scattering. The conclusions at the end of the preceding subsection on the effect of $\varphi\neq 0$ are found to be unchanged so results will be given for $\varphi=0$ only.

(i) $M_1=M_2$, $\Gamma_1\neq\Gamma_2$, $\varphi=0$. In Fig. 3, we present representative results expected in the experiment of Ref. 5 for the case when the poles for the ρ (indicated by the subscript 2) and ρ' (indicated by the subscript 1) have equal real parts but differing distances from the physical cut. Older bubble-chamber experiments, typically with large resolution, would not have seen this type of structure if the Γ_i were considerably different. These calculations lead us to suggest that the CERN experiment should have been able to detect interference between the ρ and a superimposed ρ' of width $\Gamma_1 \gtrsim 10$ MeV. A 10^4-event experiment with resolution $\Delta M = \pm 5$ MeV (which was the resolution achieved in Ref. 9) should be able to detect the interference dip due to $\Gamma_1\approx 4$ MeV. These statements, of course, apply to a description of the ρ by Eq. (3) with $E_1=E_2$.

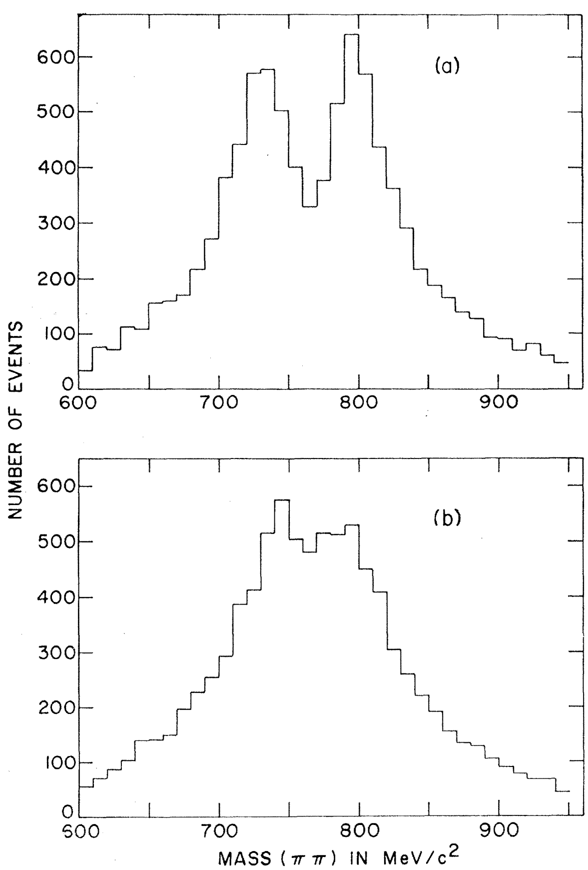

FIG. 3. Monte-Carlo-generated mass distributions for "maximally" overlapping resonances described by the unitary S matrix of Eq. (3) in which $M_1=M_2$ (or $E_1=E_2$). (a) $\Gamma_1/2=20$ MeV, $\Gamma_2/2=30$ MeV, $\varphi=0$. (b) $\Gamma_1/2=4$ MeV, $\Gamma_2/2=55$ MeV, $\varphi=0$. In both cases the resolution in the experiment of Ref. 5, $\Delta M=\pm 14$ MeV, is used.

(ii) $M_1\neq M_2$, $\Gamma_1=\Gamma_2$ *(for both* $\varphi=0$ *and* $\varphi\neq 0$*)*. With M_1 and M_2 in the ρ-mass region, this configuration of poles in Eq. (3) accentuates the width of the interference dip compared with the dipole case so that even a poor-resolution experiment would find a dip. Structure of this type with the real parts M_1 and M_2 different by more than 5–10 MeV appears to be certainly ruled out for the ρ.

(iii) $M_1\neq M_2$, $\Gamma_1\neq\Gamma_2$, $\varphi\cong 0$. In this general case of Eq. (3), although the condition $\varphi\neq 0$ considerably sharpens one of the peaks, small values of φ are preferred by the data since no drastic asymmetry exists in the ρ wings. However, the high-mass side may fall off somewhat more rapidly than the low-mass end,[11] but the limit quoted earlier, $\varphi<0.3$ rad, more than allows for this observed distortion in the DESY[11] experiment.

Numerous possibilities exist under this category. If M_1 is somewhat *above* the ρ-mass region, the only effect will be a faster falloff in the high-energy tail of the ρ-mass spectrum than that given by a simple Breit-Wigner dependence. Thus, the DESY data might indicate the existence of the ρ'.

In the overlapping situation, from the above results we expect that Γ_1 must be small in order for existing experiments to have missed structure. In Fig. 4, we show representative histograms for $M_2=760$, $\Gamma_2/2=50$, $M_1=790$, $\Gamma_1/2=3$ MeV, and $\varphi=0$ with resolution functions corresponding to (a) $\Delta M=\pm 14$ MeV and (b) $\Delta M=\pm 5$ MeV, since this case might well look like the ρ^0-ω mixing expected[12] in the neutral-ρ mass. The structure in Fig. 4(a) is not statistically significant but with smaller resolution, e.g., $\Delta M=\pm 7.5$ MeV, the dip-bump structure following from Eq. (3) near 790 Mev becomes significant. For $\Delta M=\pm 5$ MeV, *with* $\Gamma_1/2=2$ MeV, the same structure visible in Fig. 4(b) is still there but is not significant statistically. In this two-pole description (for ρ and ρ') one expects structure in each component of the isospin multiplet. Therefore, although this last case resembles very closely the result expected from ρ^0-ω interference,[9,12] the presence of a narrow ρ' would show up also in the charged-ρ states, and it would appear important to cross check any ρ^0 structure with experiments on both charged and uncharged ρ mesons. Remarkably, the compilations[6] of $(\pi\pi)^\pm$ mass distributions indicate structure in the ω-mass region even though no effect due to the ω-ρ^0 interference is possible. A single experiment which shows a similar effect is that of Baton and Laurens[13] whose $\pi^-\pi^0$ mass distribution has a dip at $\sim$800 MeV

[11] J. G. Asbury, V. Becker, W. K. Bertram, M. Binkley, E. Coleman, C. L. Jordan, M. Rohde, A. J. S. Smith, and S. C. C. Ting, Phys. Rev. Letters **20**, 227 (1968).

[12] J. Bernstein and G. Feinberg, Nuovo Cimento **25**, 1343 (1962); A. S. Goldhaber, G. C. Fox, and C. Quigg, Phys. Letters **30B**, 249 (1969).

[13] J. P. Baton and G. Laurens, Phys. Rev. **176**, 1574 (1968). Calculations of this type could well have been made for comparison with the work of this group; however, it was not possible to find any information concerning their mass resolution.

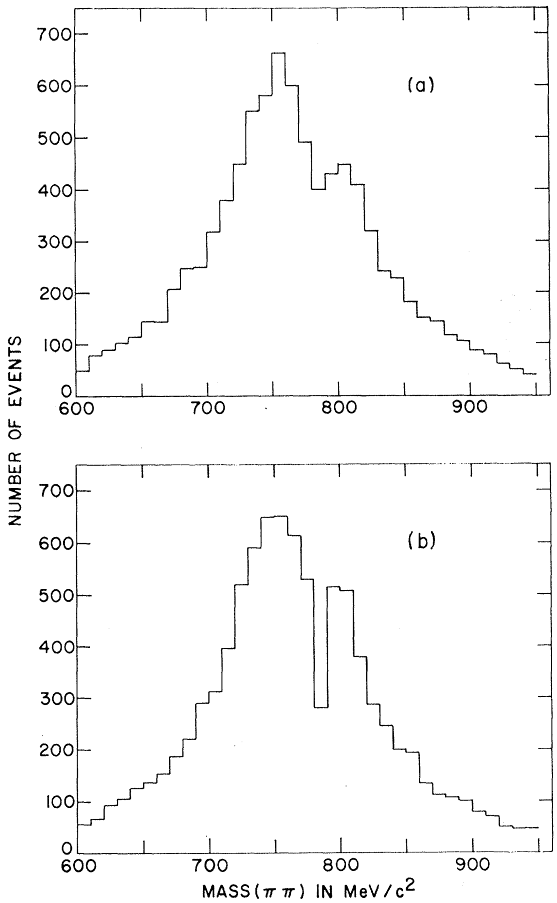

FIG. 4. Monte-Carlo-generated mass distributions for overlapping ρ and ρ' resonances treated via the unitary S matrix of Eq. (3) for $M_2=760$, $\frac{1}{2}\Gamma_2=50$, $M_1=790$, and $\frac{1}{2}\Gamma_1=3$ MeV for (a) $\Delta M=\pm14$ MeV (Ref. 5) and (b) $\Delta M=\pm5$ MeV (Ref. 8).

on the high-mass side of the ρ^-. In the momentum-transfer-squared slice $2<\Delta^2/\mu^2\leq 4$, this dip appears as a 4-standard-deviation effect. Further precise, high-resolution experiments on charged ρ, $(\pi\pi)^\pm$ mass distributions are clearly of interest.

The calculations based on Eq. (3) were compared with those obtained from a K matrix of two pole terms with pole positions and residues determined by the real part and imaginary part, respectively, of the pole locations of S in Eq. (3). Only minor differences having no effect on the conclusions were found.

III. DISCUSSION

The main result drawn from the above work is that the early CERN m.m. experiment on the ρ meson should have been able to find dipole structure if it really were there. This conclusion is completely unaffected by such questions as whether or not an ϵ meson exists, since the reaction $\pi^- p \to (\text{m.m.})^- + p$ produces m.m. states which have isotopic spin one (or two). It is likewise unaffected by the inclusion of off-mass-shell effects, a result anticipated in Sec. I where it was noted that the experiment showed no difference between ρ-mass distributions for various incident momenta and t values. This is not surprising since the ρ-meson mass region is considerably above the 2π threshold and off-mass-shell effects are largest for low masses. In the only case where a scattering process has been studied both on the mass shell ($\pi N \to \pi N$) and off the mass shell (pion exchange in $NN \to NN\pi$ or $N^*N\pi$), the angular distributions for the two cases are in very close agreement.[14]

Though convinced that off-shell corrections would be unimportant, we nevertheless repeated the calculations relevant for the above conclusion with various prescriptions for going off the mass shell included. No essential change in the structure indicated in the figures for the CERN experiment was found. This was checked using each of the three commonly used methods for making off-mass-shell corrections: the Born approximation,[15] the Dürr-Pilkuhn[16] method (DP) which replaces the Born factors by angular momentum barrier-penetration factors, and that of Benecke and Dürr[17] (BD) wherein the penetration factors are essentially replaced by Legendre functions of the second kind (proportional to the partial-wave projections of a particle-exchange amplitude). Appropriate formulas for direct application of these three methods are given by, e.g., Wolf[18] (see also Ref. 13). From these one sees immediately that the on-shell cross sections (those in Figs. 1–4) need only be multiplied by a function (rather complicated for the DP and BD cases) of momentum transfer squared (t), pion mass, and $\pi\pi$ mass. In each case (Born, DP, or BD) this factor varies very slowly and smoothly across the ρ-mass region. This is due in part to the fact that the data[5] were taken over a relatively small t range, with most of the data at $t=-0.12$ and -0.18 (GeV/c)^2 and one run at $t=-0.24$ (GeV/c)^2.

For completeness, we have also examined the $\pi^+\pi^-$ case, which is not as clean for the present purposes as the $\pi^-\pi^0$ case emphasized above. Since the main background "under" the $l=1$ $\pi^+\pi^-$ partial wave is a large S-wave $I=0$ contribution, the study of ρ structure is more difficult. Also, the off-mass-shell correction factors change as the angular momentum l changes, so the $l=0$ and $l=1$ off-shell corrections must be done separately. However, when the BD factors were evaluated for small t, the corrections for $l=0$ and $l=1$ were essentially identical for $650<M_{\pi\pi}<850$ MeV at very small t ($|t|\lesssim 5m_\pi^2$) and were the same within $\sim7\%$ even for

[14] E. Colton *et al.*, Phys. Rev. Letters **17**, 884 (1966). More recent results are given in E. Colton and P. Schlein, in Proceedings of the Conference on $\pi\pi$ and $K\pi$ Interactions Argonne National Laboratory, Argonne, Ill., 1969, edited by F. Loeffler and E. Malamud, p. 1 (unpublished).

[15] See, e.g., H. Pilkuhn, *The Interactions of Hadrons* (North-Holland, Amsterdam, 1967), pp. 279 ff.

[16] H. P. Dürr and H. Pilkuhn, Nuovo Cimento **40**, 899 (1965).

[17] J. Benecke and H. P. Dürr, Nuovo Cimento **56**, 269 (1968).

[18] G. Wolf, Phys. Rev. Letters **19**, 925 (1967); Phys. Rev. **182**, 1538 (1969).

larger $|t|$, e.g., $10m_\pi^2$, the upper momentum-transfer cut in a typical experiment.[19] Such a cut of course serves to enhance the pion-exhange contribution. Therefore, the simplifying assumption was made that the off-shell correction (in the ρ region, at least) was a common factor for $l=0$ and $l=1$ contributions.

To proceed further in the use of $\pi^+\pi^-$ data as a test of the dipole hypothesis, one must known the S-wave phase shifts δ_0^0 and δ_0^2. The latter is reasonably well established as negative, small ($>-20°$), and smoothly varying to $\pi\pi$ masses of 1 GeV. The former, δ_0^0, traditionally has been determined to have any one of the four possible energy dependences: up-down, down-down, up-up, or down-up. The first "up" or "down" refers to the relative size of the two possible solutions for δ_0^0 below $M_{\pi\pi}\cong 0.7$ GeV, the up solution increasing slowly (with magnitude of $\sim 70°$) and the down solution increasing more rapidly to equal the up case at $M_{\pi\pi}\approx 0.7$ GeV. The second up or down describes how the solutions diverge above $M_{\pi\pi}>0.7$ GeV, the down solution increasing very slowly and the up solution faster to cross 90° at ~ 750 MeV.

Arguments now exist in favor of the down solution for the energy dependence of δ_0^0 above 0.7 GeV, i.e., an S wave which varies slowly, but with fairly large magnitude, across the central mass of the ρ.[20] If the preferred up-down or down-down solutions for δ_0^0, along with the dipole model for the ρ, are inserted into the present calculation to describe the $\pi^+\pi^-$ angle-integrated mass distribution, then our conclusion is the same as that above for the $\pi^-\pi^0$ case. The relatively slowly varying S wave produces a broad, smooth background on which the ρ sits and the speculated dipole structure will not be masked seriously. However, if the up solution for δ_0^0 above 0.7 GeV (i.e., down-up or up-up) is used in the calculation, the more rapidly increasing S-wave phase shift results in an $l=0$, $I=0$ (ϵ) $\pi^+\pi^-$ mass peaking in the ρ region. For example, with the up solution of Ref. 19 the ϵ peaks near 750 MeV with width of the same order as that of the ρ, so dipole structure in the ρ can be completely masked or barely indicated by a small bump on the high side of the $\pi^+\pi^-$ mass distribution very much as in Fig. 4(a).

[19] J. H. Scharenguivel *et al.*, Phys. Rev. **186**, 1387 (1969).

[20] G. L. Kane, paper presented at the 1970 Philadelphia Conference on Boson Spectroscopy (unpublished); David Morgan and Graham Shaw, Phys. Rev. D **2**, 520 (1970).

Therefore, if the up solution above $M_{\pi\pi}\approx 0.7$ GeV is correct, practically any structure, regardless of how gross it is, need not be visible in $\pi^+\pi^-$ mass distributions. However, since the more probable solution[20] is the down one, the $\pi^+\pi^-$ data most likely to support our conclusion that dipole structure would have been seen. In any case, further discussion or calculation on the $\pi^+\pi^-$ data is academic for present purposes (and was not put into the original version of this paper for that reason) since the charged-ρ data allow a more direct, clearer analysis.

Comments made in this paper on the A_2 meson as a two-pole system are applicable to the $\pi\rho$ mode and are reliable for $\pi\rho\to\pi\rho$ to the extent that this mode dominates over $K\bar{K}$ and $\pi\eta$, i.e., $|S|\approx 1$. Thus, Eq. (3) might be approximately applicable to, e.g., the CERN A_2 data of Ref. 1 which have a very narrow deep dip at their center. Although these data do not differentiate statistically between some of the two-pole cases discussed here, if the dip (full width at half-minimum = 15 MeV measured with resolution $\Delta M=\pm 7.5$ MeV or $\Gamma_{\rm exp}=15$ MeV) is taken seriously, then the case $M_1\neq M_2$, $\Gamma_1=\Gamma_2$ is somewhat favored over the dipole, which, in turn, is favored over the case $M_1=M_2$, $\Gamma_1\neq\Gamma_2$ for which the considerably narrower "theoretical" dip will be washed out by resolution.

Note added in proof. A recent Durham University report (unpublished) by R. C. Johnson and E. J. Squires essentially repeats the calculation in Ref. 4 with the ρ' trajectory (to use the notation of the present paper) multiplied by a factor (*ad hoc*) which goes to zero at the ρ mass so that the mass distribution is due to the ρ only.

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