dropped.[5] Likewise, the conformal-invariant matter-field Lagrangians prescribed by Isham, Salam, and Strathdee can be expressed as manifest scalar densities under general coordinate transformations by making use of (2) and (4) and by employing covariant-derivative notation for tensor and spinor[6] fields. In fact, one obtains conformal-invariant matter-field Lagrangians simply by adapting the algorithm for securing physical Lagrangians in general relativity:

(a) Write the Lagrangian in a form valid for curvilinear coordinates in Minkowski space-time with an unspecified metric tensor.

(b) Assume that the Lagrangian so stated remains appropriate for curved Riemannian space-time geometry.

(c) Set the metric tensor and affine connection equal to the conformally Minkowskian forms (2) and (4).

Thus, the Isham-Salam-Strathdee prescription for conformal-invariant matter-field Lagrangians is simply a concomitant of general coordinate covariance if space-time has conformally Minkowskian geometry on the level of hadron physics. Observable experimental features of this space-time geometry are not indicated by the theory of measurement in classical general relativity because χ must be a quantum field according to the dilaton theory.[1,7]

[5] Note that by setting the disposable constant $\langle\chi^2\rangle$ equal to the universal constant $3/4\pi c^3 G$, the dilaton Lagrangian (7) becomes Einstein's Lagrangian for general relativity (with a finite cosmological constant if $\kappa \neq 0$). One may speculate on whether the dilaton theory relates to the quantum theory of general relativity.

[6] W. L. Bade and H. Jehle, Rev. Mod. Phys. **25**, 714 (1953), and references therein, especially V. Bargmann, Sitzber. Preuss. Akad. Wiss. Physik Math. Kl. 346 (1932).

[7] An attempt to ascertain the observable physical consequences of a conformally Minkowskian space-time on the level of hadron physics was made for the special case of de Sitter space-time by P. Roman *et al.*, Nuovo Cimento **42**, 193 (1966); **45**, 268 (1966). For a discussion of the problem of conciliating the non-invariance of causality conditions under conformal transformations with a field-theoretic treatment of hadron physics, see D. Boulware, L. S. Brown, and R. D. Peccei, Phys. Rev. D **2**, 293 (1970). Finally, for a discussion of the epistemological aspect involved in establishing whether space-time is non-Minkowskian on the subatomic level, see G. Rosen, Nuovo Cimento **16**, 966 (1960).

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Comment on the Spin Precession of the Schiff Satellite in the Brans-Dicke Theory*

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It is noted that the two versions of the Brans-Dicke (BD) theory yield different results for the de Sitter precession $\mathbf{\Omega}_{DS}$ of an orbiting gyroscope, while the Lense-Thirring precession $\mathbf{\Omega}_{LT}$ is unchanged, thus making the results unit dependent. This apparent inconsistency is resolved by showing that there exists a third term $\mathbf{\Omega}_\phi$ which arises from an anomalous scalar force term in the equations of motion. This latter term naturally combines with $\mathbf{\Omega}_{DS}$ to make their sum independent of units, and agrees with previously published results.

Recently O'Connell[1] has written down an expression for the precession of the spin axis of a gyroscope in the Brans-Dicke (BD) theory. Other calculations,[2,3] using independent methods, have verified this expression. In arriving at his results, O'Connell has taken advantage of the two versions[4] of the BD theory by calculating the de Sitter term $\mathbf{\Omega}_{DS}$ in the unbarred units and the Lense-Thirring term $\mathbf{\Omega}_{LT}$ in the barred units. By reexpressing either term in the units of the other and combining, one would expect to obtain the same final result.[4,5] However, as shown below, $\mathbf{\Omega}_{DS}$ changes under a units transformation, while $\mathbf{\Omega}_{LT}$ does not. Since O'Connell chooses to express his results in unbarred units ($\alpha=1$, below), they are unaffected by the above observation. However, the question does arise as to how the correct result is to be obtained for the barred units.

In order to resolve this situation the spin procession is analyzed in terms of a general formulation of the BD theory whose units are specified by a parameter α. It is found that $\mathbf{\Omega}_{DS}$ depends on α, while $\mathbf{\Omega}_{LT}$ does not. In addition an anomalous scalar force in the equations

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[1] R. F. O'Connell, Phys. Rev. Letters **20**, 69 (1968).

[2] D. R. Brill, Z. Naturforsch. **22a**, 1336 (1967).

[3] D. C. Wilkins, Ann. Phys. (N.Y.) (to be published).

[4] R. H. Dicke, Phys. Rev. **125**, 2163 (1962).

[5] Actually, this is strictly true only for the dimensionless product $\Omega\Delta t$; Ω, which has inverse-time units, will be scaled by $\lambda^{-(1-\alpha)/2}$ [Eq. (2)]. However, λ differs from unity by first-order quantities, and therefore its effect on scaling Ω will be of second order. In any case, the change considered in the text is obviously not an over-all scale change.

of motion gives rise to a term $\mathbf{\Omega}_\phi$ (also dependent upon α) whose sum with $\mathbf{\Omega}_{\rm DS}$ is independent of α and agrees with the results of O'Connell for all choices of units.

Schiff[6] has found the contributions to the angular velocity of precession in the Einstein theory to be

$$\begin{aligned}\mathbf{\Omega}_{\rm T}&=\tfrac{1}{2}(\mathbf{f}\times\mathbf{v})\,,\\ \mathbf{\Omega}_{\rm DS}&=(3m/2r^3)(\mathbf{r}\times\mathbf{v})\,,\\ \mathbf{\Omega}_{\rm LT}&=(G_0I/c^2r^5)[3\mathbf{r}(\boldsymbol{\omega}\cdot\mathbf{r})-r^2\boldsymbol{\omega}]\,,\end{aligned}\tag{1}$$

where M, I, and ω are, respectively, the mass, moment of inertia, and angular velocity of the earth; $\mathbf{f}$ is any constraint caused by nonmetric forces; $\mathbf{\Omega}_{\rm T}$ is the Thomas term. In Einstein's theory, nonmetric is synonymous with nongravitational, but in the BD theory the scalar field introduces, in general, *a nonmetric gravitational* constraining force which will contribute to this term [see Eq. (6)].

In order to make our calculation completely general, we shall first write down the BD field equations in terms of a parameter α, which specifies the particular units, and then go on to show that our results are independent of α. The general units transformation is specified by the relations[7]

$$\begin{aligned}g_{ij}&\rightarrow\bar{g}_{ij}=\lambda^{1-\alpha}g_{ij}\,,\\ m&\rightarrow\bar{m}=\lambda^{-(1-\alpha)/2}m\,,\end{aligned}\tag{2}$$

and the BD action principle[8] becomes

$$0=\delta\int d^4x(-\bar{g})^{1/2}\phi_0\lambda^\alpha\{\bar{R}-\tfrac{1}{2}[2\omega+3(1-\alpha^2)]\lambda^{-2}\lambda^{,\bar{k}}\lambda_{,k}+(16\pi/c^4)\phi_0^{-1}\lambda^{-\alpha}\bar{L}_{\bar{m}}\}\,,\tag{3}$$

where all barred operations and functionals are taken relative to the barred metric. Equation (3) leads to the following field equations[9]:

$$\begin{aligned}\bar{G}_{ij}&=(8\pi\phi_0^{-1}/c^4)\xi^{-1}{}_{\bar{m}}\bar{T}_{ij}+\zeta(\alpha)\xi^{-2}(\xi_{,i}\xi_{,j}-\tfrac{1}{2}\bar{g}_{ij}\xi^{,\bar{k}}\xi_{,k})+\xi^{-1}(\xi_{,i;\bar{j}}-\bar{g}_{ij}\bar{\Box}\xi)\,,\\ \xi^{,\bar{j}}{}_{;\bar{j}}&=(8\pi\phi_0^{-1}/c^4)[\alpha{}_{\bar{m}}\bar{T}/(2\omega+3)]\,,\end{aligned}\tag{4}$$

where

$$\begin{aligned}&\xi\equiv\lambda^\alpha\,,\\ &\zeta(\alpha)\equiv[2\omega+3(1-\alpha^2)]/2\alpha^2\,,\\ &{}_{\bar{m}}\bar{T}_{ij}\equiv-2(-\bar{g})^{1/2}\frac{\partial}{\partial\bar{g}^{ij}}[(-\bar{g})^{1/2}\bar{L}_{\bar{m}}]\,.\end{aligned}\tag{5}$$

Equation (3) also gives the equations of motion in the case of a free particle as

$$D\bar{u}^i/D\bar{\tau}=-(\ln\bar{m})_{,k}(\bar{u}^k\bar{u}^i+\bar{g}^{ki})\,,\tag{6}$$

where $d\bar{\tau}^2=-d\bar{s}^2=-\bar{g}_{ij}dx^idx^j$, and $\bar{\eta}_{ij}=\mathrm{diag}(-+++)$.

It is not difficult to show that in the linear approximation Eq. (4) yields, for the rotating earth, the following results:

$$\begin{aligned}\bar{h}_{00}&=(2m/r)[(2\omega+3+\alpha)/(2\omega+4)]\,,\\ \bar{h}_{\beta\beta}&=(2m/r)[(2\omega+3-\alpha)/(2\omega+4)]\,,\\ \bar{h}_{0\beta}&=-(2G_0I/c^2)[(2\omega+3)/(2\omega+4)](\mathbf{\Omega}\times\mathbf{r})_\beta\,,\\ \xi&\equiv\lambda^\alpha=1+(2m/r)[\alpha/(2\omega+4)]\,,\end{aligned}\tag{7}$$

in standard notation.

Schiff[6] has shown that for a line element of the form

$$ds^2=-c^2dt^2[1-2\alpha(m/r)+2\beta(m/r)^2+\cdots]+(dr^2+r^2d\Omega^2)[1+2\gamma(m/r)+\cdots]\,,$$

the de Sitter term is modified by a factor $\frac{1}{3}(\alpha+2\gamma)$. Thus, if we use Eq. (7) to determine α and γ, we have

$$\mathbf{\Omega}_{\rm DS}=[(6\omega+9-\alpha)/(6\omega+12)](3m/2r^3)(\mathbf{r}\times\mathbf{v})\,.\tag{8}$$

Since the equations of motion for a free particle are not, in general, geodesic in the BD formalisms [see Eq. (6)], there will be an additional contribution $\mathbf{\Omega}_\phi$ associated with the "external force" (Thomas) term $\mathbf{\Omega}_{\rm T}$ of Eqs. (1), viz.,

$$\mathbf{\Omega}_\phi=\tfrac{1}{2}\mathbf{f}_\phi\times\mathbf{v}=-\tfrac{1}{2}\nabla(\ln\bar{m})\times\mathbf{v}\,,\tag{9}$$

where $\mathbf{f}_\phi$ is the three-force given by the right-hand side of Eq. (6), and the second equality is obtained by evaluating this expression in the rest frame of the particle. Using the expression for $\bar{m}$ given by Eq. (2) and the last of Eqs. (7) in Eq. (9), we find

$$\mathbf{\Omega}_\phi=\tfrac{1}{2}[(\alpha-1)/(2\omega+4)](m/r^3)(\mathbf{r}\times\mathbf{v})\,.\tag{10}$$

Equation (10) is a gravitational-type term since it vanishes as $m\rightarrow0$. It should therefore be added to $\mathbf{\Omega}_{\rm DS}$ and not to $\Omega_{\rm T}$, so that we have the total contribution

$$\begin{aligned}``\mathbf{\Omega}_{\rm DS}"&=\mathbf{\Omega}_{\rm DS}+\mathbf{\Omega}_\phi\\ &=(3m/2r^3)[(3\omega+4)/(3\omega+6)](\mathbf{r}\times\mathbf{v})\,,\end{aligned}\tag{11}$$

in agreement with O'Connell's results for $\mathbf{\Omega}_{\rm DS}$.[1]

Finally, we would like to point out that according to Eqs. (7), $\bar{h}_{0\beta}$, and therefore $\mathbf{\Omega}_{\rm LT}$, is independent of the parameter α, that is, of the particular BD formalism.

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[6] L. I. Schiff, Proc. Natl. Acad. Sci. U. S. **46**, 871 (1960).

[7] R. E. Morganstern, Phys. Rev. D **1**, 2969 (1970). Note that $\alpha=1$ corresponds to the original unbarred units of the BD theory, while $\alpha=0$ corresponds to the barred units of Ref. 2.

[8] C. Brans and R. H. Dicke, Phys. Rev. **124**, 925 (1961).

[9] These equations hold for $\alpha=0$ provided that the limit is properly taken, viz.,

$$\lim_{\alpha\to0}\xi=\lim_{\alpha\to0}e^{\alpha\ln\lambda}=1\,,\quad \lim_{\alpha\to0}\alpha^{-1}\xi_{,i}=\lim_{\alpha\to0}(\ln\lambda)_{,i}e^{\alpha\ln\lambda}=(\ln\lambda)_{,i}\,.$$

In this limit, Eqs. (4) reduce to the barred equations of Ref. 4.