Comments and Addenda

The Comments and Addenda section is for short communications which are not of such urgency as to justify publication in Physical Review Letters *and are not appropriate for regular articles. It includes only the following types of communications: (1) comments on papers previously published in* The Physical Review *or* Physical Review Letters; *(2) addenda to papers previously published in* The Physical Review *or* Physical Review Letters, *in which the additional information can be presented without the need for writing a complete article. Manuscripts intended for this section may be accompanied by a brief abstract for information retrieval purposes. Accepted manuscripts will follow the same publication schedule as articles in this journal, and galleys will be sent to authors.*

Dilaton Field Theory and Conformally Minkowskian Space-Time*

GERALD ROSEN
Drexel University, Philadelphia, Pennsylvania 19104
(Received 8 September 1970)

It is observed that conformal invariance of Lagrangians describing matter fields is a concomitant of general coordinate covariance, if space-time is a conformally Minkowskian Riemannian geometry on the level of hadron physics. The dilation field of Isham, Salam, and Strathdee is the mathematical object which fixes the local length-time scale in the prescribed system of coordinates.

RECENTLY, Isham, Salam, and Strathdee[1] have shown that the introduction of a real scalar "dilaton" field enables one to construct conformal-invariant Lagrangians for massive matter fields. To convert an ordinary Lagrangian into its conformal-invariant counterpart, one multiplies all matter fields that appear in the Lagrangian by appropriate powers of χ, adds suitable terms proportional to $\chi^{-1}\chi_{,\mu}$ to the gradients of the matter fields, and finally affixes the dilaton Lagrangian[2]

$$L_\chi = \tfrac{1}{2}\bar{g}^{\rho\sigma}\chi_{,\rho}\chi_{,\sigma} + \kappa\chi^4 \tag{1}$$

to the modified matter-field Lagrangian. Such a prescription for constructing conformal-invariant Lagrangians is patently *ad hoc* within the conventional Minkowski space-time framework for field theory. Our purpose here is to point out that Isham-Salam-Strathdee Lagrangians with conformal invariance appear naturally, as a consequence of general coordinate covariance, if space-time is not Minkowskian but rather conformally Minkowskian in the sense of Riemannian geometry.[3,4]

If space-time is conformally Minkowskian, coordinate systems exist for which the metric tensor takes the form

$$g_{\mu\nu} = \langle\chi^2\rangle^{-1}\chi^2\bar{g}_{\mu\nu}, \tag{2}$$

with $\langle\chi^2\rangle$ denoting a disposable positive constant, $\chi=\chi(x)$ denoting a positive function of the coordinates, and $\bar{g}_{\mu\nu}\equiv\text{diag}[-1,1,1,1]$ denoting the Minkowski metric tensor. The form (2) is preserved under the 15-parameter group of conformal coordinate transformations with χ (identified as the dilaton field), transforming as

$$\chi' = |\det(\partial x^{\mu'}/\partial x^\nu)|^{-1/4}\chi \tag{3}$$

for $x^{\mu'}=x^{\mu'}(x)$ a conformal coordinate transformation. From the conformally Minkowskian metric tensor (2), one obtains the affine connection

$$\Gamma_{\mu\nu}{}^{\sigma} = \chi^{-1}(\delta_\mu{}^\sigma\chi_{,\nu} + \delta_\nu{}^\sigma\chi_{,\mu} - \bar{g}_{\mu\nu}\bar{g}^{\sigma\rho}\chi_{,\rho}), \tag{4}$$

the Ricci tensor

$$R_{\mu\nu} = 2\chi^{-1}\chi_{,\mu\nu} - 4\chi^{-2}\chi_{,\mu}\chi_{,\nu} + \chi^{-1}\bar{g}^{\rho\sigma}(\chi_{,\rho\sigma} + \chi^{-1}\chi_{,\rho}\chi_{,\sigma})\bar{g}_{\mu\nu}, \tag{5}$$

and the curvature scalar

$$R = 6\langle\chi^2\rangle\chi^{-3}\bar{g}^{\rho\sigma}\chi_{,\rho\sigma}. \tag{6}$$

In view of (2) and (6), the dilaton Lagrangian (1) is a scalar density under general coordinate transformations,

$$L_\chi = -\tfrac{1}{12}\langle\chi^2\rangle R\sqrt{(-g)} + \langle\chi^2\rangle^2\kappa\sqrt{(-g)}, \tag{7}$$

where an additive pure-divergence term has been

* Work supported by a National Science Foundation grant.
[1] C. J. Isham, A. Salam, and J. Strathdee, Phys. Rev. D 2, 685 (1970), and references therein.
[2] Greek indices run 0, 1, 2, 3 with the summation convention employed, $\bar{g}^{\rho\sigma}\equiv\text{diag}[-1,1,1,1]$, and $\chi_{,\rho}\equiv\partial\chi/\partial x^\rho$.
[3] For example, L. P. Eisenhart, *Riemannian Geometry* (Princeton U. P., Princeton, N. J., 1949), pp. 89–92.
[4] The space-time geometry is conformally Minkowskian if and only if the Weyl-Schouten condition $R_{\rho\mu\nu\sigma}=\frac{1}{2}(g_{\rho\sigma}R_{\mu\nu}-g_{\rho\nu}R_{\mu\sigma}+g_{\mu\nu}R_{\rho\sigma}-g_{\mu\sigma}R_{\rho\nu})+\frac{1}{6}(g_{\rho\nu}g_{\mu\sigma}-g_{\rho\sigma}g_{\mu\nu})R$ is satisfied, and in turn the Weyl-Schouten condition guarantees existence of coordinate systems for which the metric tensor takes the form (2). Moreover, it follows from the Weyl-Schouten condition that space-time is simply Minkowskian if and only if the Ricci tensor (5) equals zero identically; only then can one find a conformal coordinate transformation for which (3) produces $\chi'\equiv1$ at all space-time points. It is interesting to note that all of the homogeneous and isotropic cosmological models of the universe can be cast in conformally Minkowskian form [see G. E. Tauber, J. Math. Phys. **8**, 118 (1967); F. Gürsey, Ann. Phys. (N. Y.) **24**, 211 (1963); S. Deser, *ibid.* **59**, 248 (1970)].

dropped.[5] Likewise, the conformal-invariant matter-field Lagrangians prescribed by Isham, Salam, and Strathdee can be expressed as manifest scalar densities under general coordinate transformations by making use of (2) and (4) and by employing covariant-derivative notation for tensor and spinor[6] fields. In fact, one obtains conformal-invariant matter-field Lagrangians simply by adapting the algorithm for securing physical Lagrangians in general relativity:

(a) Write the Lagrangian in a form valid for curvilinear coordinates in Minkowski space-time with an unspecified metric tensor.

(b) Assume that the Lagrangian so stated remains appropriate for curved Riemannian space-time geometry.

(c) Set the metric tensor and affine connection equal to the conformally Minkowskian forms (2) and (4).

Thus, the Isham-Salam-Strathdee prescription for conformal-invariant matter-field Lagrangians is simply a concomitant of general coordinate covariance if space-time has conformally Minkowskian geometry on the level of hadron physics. Observable experimental features of this space-time geometry are not indicated by the theory of measurement in classical general relativity because χ must be a quantum field according to the dilaton theory.[1,7]

[5] Note that by setting the disposable constant $\langle\chi^2\rangle$ equal to the universal constant $3/4\pi c^3G$, the dilaton Lagrangian (7) becomes Einstein's Lagrangian for general relativity (with a finite cosmological constant if $\kappa\neq 0$). One may speculate on whether the dilaton theory relates to the quantum theory of general relativity.

[6] W. L. Bade and H. Jehle, Rev. Mod. Phys. **25**, 714 (1953), and references therein, especially V. Bargmann, Sitzber. Preuss. Akad. Wiss. Physik Math. Kl. 346 (1932).

[7] An attempt to ascertain the observable physical consequences of a conformally Minkowskian space-time on the level of hadron physics was made for the special case of de Sitter space-time by P. Roman *et al.*, Nuovo Cimento **42**, 193 (1966); **45**, 268 (1966). For a discussion of the problem of conciliating the non-invariance of causality conditions under conformal transformations with a field-theoretic treatment of hadron physics, see D. Boulware, L. S. Brown, and R. D. Peccei, Phys. Rev. D **2**, 293 (1970). Finally, for a discussion of the epistemological aspect involved in establishing whether space-time is non-Minkowskian on the subatomic level, see G. Rosen, Nuovo Cimento **16**, 966 (1960).

PHYSICAL REVIEW D VOLUME 3, NUMBER 2 15 JANUARY 1971

Comment on the Spin Precession of the Schiff Satellite in the Brans-Dicke Theory*

R. E. MORGANSTERN
Lunar Science Institute, 3303 Nasa Road 1, Houston, Texas 77058
(Received 24 July 1970)

It is noted that the two versions of the Brans-Dicke (BD) theory yield different results for the de Sitter precession $\mathbf{\Omega}_{DS}$ of an orbiting gyroscope, while the Lense-Thirring precession $\mathbf{\Omega}_{LT}$ is unchanged, thus making the results unit dependent. This apparent inconsistency is resolved by showing that there exists a third term $\mathbf{\Omega}_\phi$ which arises from an anomalous scalar force term in the equations of motion. This latter term naturally combines with $\mathbf{\Omega}_{DS}$ to make their sum independent of units, and agrees with previously published results.

RECENTLY O'Connell[1] has written down an expression for the precession of the spin axis of a gyroscope in the Brans-Dicke (BD) theory. Other calculations,[2,3] using independent methods, have verified this expression. In arriving at his results, O'Connell has taken advantage of the two versions[4] of the BD theory by calculating the de Sitter term $\mathbf{\Omega}_{DS}$ in the unbarred units and the Lense-Thirring term $\mathbf{\Omega}_{LT}$ in the barred units. By reexpressing either term in the units of the other and combining, one would expect to obtain the same final result.[4,5] However, as shown below, $\mathbf{\Omega}_{DS}$ changes under a units transformation, while $\mathbf{\Omega}_{LT}$ does not. Since O'Connell chooses to express his results in unbarred units ($\alpha=1$, below), they are unaffected by the above observation. However, the question does arise as to how the correct result is to be obtained for the barred units.

In order to resolve this situation the spin procession is analyzed in terms of a general formulation of the BD theory whose units are specified by a parameter α. It is found that $\mathbf{\Omega}_{DS}$ depends on α, while $\mathbf{\Omega}_{LT}$ does not. In addition an anomalous scalar force in the equations

* Prepared at the Lunar Science Institute under the joint support of the Universities Space Research Association and the National Aeronautics and Space Administration Manned Spacecraft Center under Contract No. NSR 09-051-001. The Lunar Science Institute Contribution number was No. 5.

[1] R. F. O'Connell, Phys. Rev. Letters **20**, 69 (1968).

[2] D. R. Brill, Z. Naturforsch. **22a**, 1336 (1967).

[3] D. C. Wilkins, Ann. Phys. (N.Y.) (to be published).

[4] R. H. Dicke, Phys. Rev. **125**, 2163 (1962).

[5] Actually, this is strictly true only for the dimensionless product $\Omega\Delta t$; Ω, which has inverse-time units, will be scaled by $\lambda^{-(1-\alpha)/2}$ [Eq. (2)]. However, λ differs from unity by first-order quantities, and therefore its effect on scaling Ω will be of second order. In any case, the change considered in the text is obviously not an over-all scale change.