and

$$6g^{7}{}_{\bar{8}\bar{5}\bar{5}}=\frac{\pm(\sqrt{\Delta})\mu_8}{F_K F_\kappa}\left[\frac{-\cos\varphi}{\sqrt{(2C_A)}}+\frac{\sin\varphi}{2\sqrt{C_B}}\right],$$
$$6g^{7}{}_{\bar{9}\bar{5}\bar{5}}=\frac{\pm(\sqrt{\Delta})\mu_9}{F_K F_\kappa}\left[\frac{\sin\varphi}{\sqrt{(2C_A)}}+\frac{\cos\varphi}{2\sqrt{C_B}}\right]. \qquad \text{(C30)}$$

Equation (C29) was first obtained by Pande.[16] One may also easily verify that Eqs. (C26a) and Eq. (C21) are identical to the results of Ref. (23) [Eqs. (19) and (20)] with the notational changes $S_K/S_\pi=\sqrt{(Z_K/Z_\pi)}$ and $S_\kappa/S_\pi=-\sqrt{(Z_\kappa/Z_\pi)}$. Again, unlike Ref. (23), no *a priori* assumption of $(3,3^*)+(3^*,3)$-symmetry breaking has been assumed here.

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Electromagnetic Corrections to Nucleon Transitions of the Vector and Axial-Vector Currents*

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We examine the matrix elements of the divergences of the vector and axial-vector current between nucleon states. These matrix elements are related to the nucleon mass difference and the corrections to the Goldberger-Treiman relation, respectively. For the nucleon mass difference we indicate that for the sign of this quantity to be understood in terms of the electromagnetic interaction requires (i) comparable longitudinal and transverse virtual photon-nucleon cross sections, or (ii) $\sigma_t{}^p(q^2,\nu)-\sigma_t{}^n(q^2,\nu)<0$ over a large region of the $(|q^2|,\nu)$ plane, where q^2 is the spacelike virtual-photon mass and ν is the photon energy (this requirement is contraindicated by experimental data at $q^2=0$), or (iii) fixed J-plane poles at $J=0, I=1$ in the virtual Compton amplitude. We also estimate the electromagnetic correction to the Goldberger-Treiman relation, and it is shown to be very small.

I. INTRODUCTION

In this paper we discuss the transition matrix elements $\langle p|V_\mu{}^{(+)}(0)|n\rangle$ and $\langle p|A_\mu{}^{(+)}(0)|n\rangle$ of the vector and axial-vector currents between nucleon states in the presence of the electromagnetic interaction. Our interest in these matrix elements stems from the observation that the matrix elements of the divergence of these currents are related to the nucleon mass difference and the corrections to the Goldberger-Treiman formula. Neither of these quantities is well understood on a theoretical basis.

For the nucleon mass difference we obtain the usual Cottingham formula,[1] assuming that the mass difference is electromagnetic and the interaction is treated to lowest order in $\alpha=1/137$. Assuming that the total cross sections for longitudinally polarized photons or nucleons is suppressed relative to that for transverse polarization, we discuss the extreme difficulty of obtaining the correct sign for $\delta M=M_p-M_n$. Here it is pointed out that if the recently reported[2] qualitative character of the total photon-nucleon cross section $[\sigma(\gamma p)-\sigma(\gamma n)>0$ for physical photons of energy 4–18 GeV$]$ can be extrapolated for virtual photons, then the deep-inelastic region, which is an important region for the nucleon mass shift, will contribute with the wrong sign to δM. We conclude that to have the possibility of understanding the sign of δM in terms of electromagnetism, we must have (i) comparable longitudinal and transverse cross sections, or (ii) $\sigma_t{}^p(q^2,\nu)-\sigma_t{}^n(q^2,\nu)<0$ for a large region of the $(|q^2|,\nu)$ plane, or (iii) fixed poles at $J=0$ $I=1$ in the virtual Compton amplitude. The first two of these possibilities can be examined in the forthcoming experiments at SLAC.

We have also examined the radiative corrections to the Goldberger-Treiman formula for π^+ decay. They are estimated to be very small, $\sim\alpha/4\pi$ relative to the observed correction ~ 0.1. In accord with our expectation, the origin of this correction is to be sought in hadron dynamics and not in electromagnetism.

II. VECTOR CURRENT

First we consider the matrix elements of the vector current between proton and neutron states, which has the general form

$$\langle p(p')|V_\mu{}^{(+)}(0)|n(p)\rangle=u(p')\tau^+[\gamma_\mu F_1(t)+i\sigma_{\mu\nu}(p'-p)_\nu F_2(t)+(p'-p)_\mu F_3(t)]u(p). \qquad (2.1)$$

The divergence is specified by

$$\langle p(p')|-i\partial_\mu V_\mu{}^{(+)}(0)|n(p)\rangle=u(p')\tau^+[\delta M F_1(t)+tF_3(t)]u(p),$$

where $t=(p'-p)^2$ and $\delta M=M_p-M_n$. If the current is

* Research sponsored in part by the Air Force Office of Scientific Research, Office of Aerospace Research, U. S. Air Force, under AFOSR Grant No. 69-1629.

[1] W. N. Cottingham, Ann. Phys. (N. Y.) **25**, 424 (1963).

[2] D. O. Caldwell *et al.*, Phys. Rev. Letters **25**, 609 (1970); **25**, 613 (1970).

conserved, $\partial_\mu V_\mu^{(+)}(x)=0$, then $D^V(t)=\delta M F_1(t)+tF_3(t)=0$. If we set $t=0$, we must have either $\delta M=0$, the usual realization of isospin symmetry, or $F_3(t)$ has a pole at $t=0$ with residue $-\delta M F_1(0)$ corresponding to a zero-mass Goldstone boson with $J^P=0^+$. In the absence of any such state, we will assume that the symmetry is realized in the usual way with $\delta M=0$.

To study the breaking of isospin symmetry, we have in general[3]

$$\partial_\mu V_\mu^{(+)}(0)=i[H(0),{}^VQ^{(+)}], \tag{2.2}$$

with $H(x)$ the Hamiltonian density and

$${}^VQ^{(+)}=\int d^3x\, V_0^{(+)}.$$

The Hamiltonian can be written as $H(x)=H_0(x)+H'(x)$, where $H_0(x)$ commutes with ${}^VQ^{(+)}$. We will assume that the symmetry-breaking piece H' is due to electromagnetism so $H'=eJ_\mu A_\mu$ with J_μ the electromagnetic current and $\Box A_\mu=eJ_\mu$. To lowest order in e^2 the effective Hamiltonian is

$$H'(0)=\frac{e^2}{2(2\pi)^4}\int d^4q\,\Delta_{\mu\nu}(q)T_{\mu\nu}(q), \tag{2.3}$$

where $\Delta_{\mu\nu}(q)=-g_{\mu\nu}/q^2$ is the photon propagator and

$$T_{\mu\nu}(q)=\int d^4x\, e^{-iq\cdot x}T(J_\mu(x)J_\nu(0)). \tag{2.4}$$

It follows from (2.2) and the assumption that we will treat isospin breaking to lowest order in e^2 that

$$\begin{aligned}\langle p|-i\partial_\mu V_\mu^{(+)}(0)|n\rangle&=\langle p|[H(0),{}^VQ^{(+)}]|n\rangle\\&=\langle p|H(0)|p\rangle-\langle n|H(0)|n\rangle.\end{aligned}$$

Utilizing the projection operator

$${}^VP=\sum_{\text{spins}}\frac{2M^2}{4M^2-t}u(p')\bar{u}(p),$$

we obtain for $D^V(t)=\delta MF_1(t)+tF_3(t)$

$$D^V(t)=-\frac{e^2}{2(2\pi)^4}{}^VP\int\frac{d^4q}{q^2}g^{\mu\nu}[\langle p(p')|T_{\mu\nu}(q)|p(p)\rangle-(p\to n)]. \tag{2.5}$$

At $p'=p$ and $t=0$, we have $D^V(0)=\delta M$, since $F_1(0)=1+O(e^2)$. From (2.5), if we perform a Wick rotation, $q_0\to iq_0$, and the angular integration, there results the Cottingham formula, which we write in general as

$$\delta M^{(I)}=\tfrac{1}{4}\pi\int_0^{-\infty}\frac{dq^2}{q^2}\int_{-1}^{+1}dy(1-y^2)^{1/2}q^2T_\mu{}^{\mu(I)}(q^2,i\nu),$$
$$\nu=p\cdot q,\quad y=\nu/\sqrt{(-q^2)}. \tag{2.6}$$

Here

$$T_{\mu\nu}{}^{(I)}(q^2,\nu)=T_1{}^{(I)}(q^2,\nu)\left(-g_{\mu\nu}+\frac{q_\mu q_\nu}{q^2}\right)+\frac{T_2{}^{(I)}(q^2,\nu)}{M^2}\left(p_\mu-\frac{p\cdot q}{q^2}q_\mu\right)\left(p_\nu-\frac{p\cdot q}{q^2}q_\nu\right). \tag{2.7}$$

$T_{1,2}{}^{(I)}(q^2,\nu)$ are the usual amplitudes for forward virtual Compton scattering, and $I=0, 1, 2$ labels the t-channel isospin so $\delta M^{(0)}$ corresponds to the self-mass and $\delta M^{(1)}$ to the mass difference. The advantage of going to the forward direction $p'=p$ is that the absorptive parts of the forward Compton amplitudes $\mathrm{Im}T_{1,2}{}^{(I)}(q^2,\nu)=\pi W_{1,2}{}^{(I)}(q^2,\nu)$ are just the inelastic structure functions for electron-nucleon scattering which can be measured experimentally.

It is worth remarking at this point that besides the Cottingham approach, one can also analyze the contributions to δM by postulating an unsubtracted dispersion relation for $D^V(t)$ which is given by (2.5). Then we have for the mass shift

$$\delta M=\frac{1}{\pi}\int_{(2\mu)^2}^{\infty}\frac{dt}{t}\,\mathrm{Im}D^V(t). \tag{2.8}$$

This approach would be particularly interesting if there were a large 0^+ continuum or discrete state which enhances the absorptive part $\mathrm{Im}F_3(t)$ and hence δM. This corresponds to just the usual tadpole model[4] of electromagnetic mass differences. While this method emphasizes the t-channel states, it has the disadvantage of not directly analyzing δM in terms of experimentally accessible quantities.

Now we will discuss the implications of the recent experiments at SLAC[5] for the mass shift. From the Cottingham formula (2.6) one can compute the Born-term (nucleon-pole) contribution with the result $\delta M_{\text{Born}}{}^{(1)}=+0.8$ MeV. The contribution from specific nucleon resonances can also be estimated and is found to be very small. Hence, if we are to understand $\delta M^{(1)}$, the large $-q^2$ region must become important. From the Cottingham formula (2.6) we see that the Regge region $|y|\to\infty$ for the Compton amplitude is not particularly important since $|y|<1$ in (2.6). However, Regge behavior of the amplitudes is important, as emphasized by Harari,[6] for analyzing the amplitudes $T_{1,2}{}^{(I)}(q^2,\nu)$ in terms of their connection with the absorptive parts $W_{1,2}{}^{(I)}(q^2,\nu)$, for Regge analysis indicates the need for subtractions in the dispersion relations. From such an analysis one concludes that $T_1{}^{(I)}(q^2,\nu)\to\beta_1{}^{(I)}(q^2)\times\nu^{\alpha^{(I)}}(0)$, $T_2{}^{(I)}(q^2,\nu)\to\beta_2{}^{(I)}(q^2)\nu^{\alpha^{(I)}(0)-2}$ as $\nu\to\infty$, $-q^2$ fixed. Since $\alpha^{(I)}(0)>0$ for $I=0$, 1, $T_1{}^{(I)}(q^2,\nu)$ requires

[3] H. Pagels, University of N. Carolina report, 1966 (unpublished); D. J. Gross and R. Jackiw, Phys. Rev. **163**, 1688 (1967).

[4] S. Coleman and S. L. Glashow, Phys. Rev. **134**, B671 (1964).

[5] R. E. Taylor, in Proceedings of the Fourth International Symposium on Electron and Photon Interactions at High Energy, Daresbury Nuclear Physics Laboratory, 1969 (unpublished).

[6] H. Harari, Phys. Rev. Letters **17**, 1303 (1966).

a subtraction while $T_2^{(I)}(q^2,\nu)$ does not, and this introduces an unknown subtraction term.

The region which is of potential importance in the determination of $\delta M^{(I)}$ corresponds to deep-inelastic electron-nucleon scattering, $-q^2 \to \infty$, $\omega = -q^2/\nu$ fixed. In configuration space this corresponds to region near the light cone. Of importance in this connection is the experimental observation that the structure functions have a simple behavior in this limit, at least for the proton, $W_1^{(p)}(q^2,\omega) = F_1^{(p)}(\omega)$, $q^2 W_2^{(p)}(q^2,\omega) = -\omega F_2^{(p)}(\omega)$ as $-q^2 \to \infty$. It has also been reported[5] that the observed longitudinal combination $\omega F_L^{(p)}(\omega) = F_2^{(p)}(\omega) - \omega F_1^{(p)}(\omega)$ is consistent with $F_L^{(p)}(\omega) = 0$.

The absence of a longitudinal amplitude suggests that we consider the ratio of the longitudinal to transverse cross sections

$$R^{(I)}(q^2,\nu) = \frac{\sigma_l^{(I)}(q^2,\nu)}{\sigma_t^{(I)}(q^2,\nu)}. \tag{2.9}$$

The relation between the cross sections and $T_{1,2}^{(I)}(q^2,\nu)$ is $\pi W_{1,2}^{(I)} = \operatorname{Im} T_{1,2}^{(I)}$, with

$$W_T^{(I)} = W_1^{(I)}, \quad W_L^{(I)} = W_2^{(I)}\left(1 - \frac{\nu^2}{M^2 q^2}\right) - W_1^{(I)},$$

$$4\pi^2\alpha W_T^{(I)} = (\nu - \tfrac{1}{2}|q^2|)\sigma_t^{(I)},$$

$$4\pi^2\alpha W_L^{(I)} = (\nu - \tfrac{1}{2}|q^2|)\sigma_l^{(I)}.$$

In the scaling region which is observed to set in at rather low $-q^2 \simeq (1\text{ GeV})^2$, experiments indicate that $R^{(p)}(q^2,\nu) \simeq 0$.

We will assume that, for $|q^2| > |q_0^2| \cong (1\text{ GeV})^2$, $R^{(I)}(q^2,\nu) = 0$ or $W_L^{(I)}(q^2,\nu) = 0$. As already remarked, this is consistent with the data on the proton and can be tested for the neutron as well. This assumption implies that for $|q^2| > |q_0^2|$, $W_2^{(I)}(1 - \nu^2/M^2 q^2) - W_1^{(I)} = 0$ and hence

$$T_2^{(I)}(q^2,\nu)(1 - \nu^2/M^2 q^2) - T_1^{(I)}(q^2,\nu) = P^{(I)}(q^2,\nu), \tag{2.10}$$

where $P(q^2,\nu)$ is a polynomial in ν^2. If there are fixed J-plane poles in $T_{1,2}^{(I)}(q^2,\nu)$ at $J=0$ with residues $R_{1,2}^{(I)}(q^2)$, then, taking the $\nu \to \infty$ limit of (1.10), we can identify the polynomial in terms of these residues:

$$\begin{aligned} P^{(I)}(q^2,\nu) &= P^{(I)}(q^2) \\ &= -[R_1^{(I)}(q^2) + R_2^{(I)}(q^2)/M^2 q^2]. \end{aligned} \tag{2.11}$$

Since the combination (2.10) is a longitudinal amplitude, we have a kinematical constraint $R^{(I)}(0) = 0$.[7] From (2.7), (2.10), and (2.11), we obtain

$$T_\mu{}^{\mu(I)}(q^2,\nu) = 3P^{(I)}(q^2) - 2T_2^{(I)}(q^2,\nu)\left(1 - \frac{\nu^2}{M^2 q^2}\right). \tag{2.12}$$

Writing the unsubtracted dispersion relation for $T_2^{(I)}(q^2,\nu)$ in terms of $\omega = -q^2/\nu$, we have

$$T_2^{(I)}(q^2,\omega) = -\omega^2 \int_0^4 \frac{d\omega'^2 W_2^{(I)}(q^2,\omega')}{\omega'^2(\omega'^2 - \omega^2)}. \tag{2.13}$$

Substituting (2.13), and (2.12) into (2.6), we can explicitly perform the y integration, and substituting $x = -q^2/\omega^2$ and normalizing with $M^2 = 1$, we find[8]

$$\delta M^{(I)} = \tfrac{3}{8}\pi^2 \int_0^{-\infty} dq^2 P^{(I)}(q^2) + \pi^2 \int_0^2 d\omega\, \omega \int_0^\infty dx\, x G(x) W_2^{(I)}(-x\omega^2, \omega), \tag{2.14}$$

where $G(x) = (1+x)(1+1/x)^{1/2} - \tfrac{3}{2} - x > 0$, $x > 0$. The first term represents the contribution of the residues of fixed poles which we ignore for the moment.

We now consider the second term. The contribution to $\delta M^{(I)}$ can be written as

$$\delta M^{(I)} = \delta M_L^{(I)} + \delta M_H^{(I)},$$

with

$$\delta M_L^{(I)} = \tfrac{1}{4}\pi \int_0^{-q_0^2} \frac{dq^2}{q^2} T^{(I)}(q^2),$$

$$\delta M_H^{(I)} = \tfrac{1}{4}\pi \int_{-q_0^2}^{-\infty} \frac{dq^2}{q^2} T^{(I)}(q^2),$$

$$T^{(I)}(q^2) = 2\int_0^1 dy (1-y^2)^{1/2} q^2 T_\mu{}^{\mu(I)}(q^2, i\nu),$$

where $-q_0^2 \simeq (1\text{ GeV})^2$ corresponds to the onset of scaling behavior. The low-$|q^2|$ region contributing to $\delta M_L^{(I)}$ is well approximated by the nucleon pole and a few inelastic states, and one finds roughly $\delta M_L^{(1)} \sim \delta M_L^{(0)} \sim 1$ MeV. If we assume that $W_2^{(I)}(\omega,q^2)$ scales for $|q^2| > |q_0^2|$, then $xW_2^{(I)}(-x\omega, \omega) = F_2^{(I)}(\omega)/\omega$ as $x \to \infty$. If the scaling function $F_2^{(I)}(\omega)$ is nontrivial and $W_L^{(I)} = 0$, then it is known that $\delta M^{(I)}$ is divergent.[9] Using

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[7] It has been conjectured that the residues of fixed poles are polynomials in q^2 [T. P. Cheng and Wu-Ki Tung, Phys. Rev. Letters **24**, 851 (1970)]. This would imply that $P(q^2)$ is a polynomial in q^2 and, since $P(0) = 0$, we must have $P(q^2) = Aq^2 + Bq^4 \cdots$. Hence either $P(q^2) = 0$ or the contribution of the fixed pole to δM is at least quartically divergent.

[8] Strictly speaking, the lower limit of the x integration should be $x_0 = -q_0^2/\omega^2$ since we assume the absence of the longitudinal amplitude only for $|q^2| > |q_0^2|$.

[9] H. Pagels, Phys. Rev. **185**, 1990 (1969); R. Jackiw, R. Van Royen, and G. B. West, Phys. Rev. D **2**, 2473 (1970). The general condition that there be no logarithmic divergence is given by $2q^2 T_1^{(I)}(q^2,\infty) + \int_0^2 d\omega [F_2^{(I)}(\omega) + \omega F_1^{(I)}(\omega)] = 0$, $q^2 \to -\infty$. If $T_L^{(I)}(q^2,\nu)$ obeys an unsubtracted dispersion relation, then we can obtain the subtraction term from the dispersion relation $T_1^{(I)}(q^2,\infty) = \int (d\nu^2/\nu^2)[W_2^{(I)}(q^2,\nu) - W_L^{(I)}(q^2,\nu)]$. Assuming for the moment that as $-q^2 \to \infty$, $W_L^{(I)}(q^2,\nu) \to F_L^{(I)}(\omega) + H_L^{(I)}(\omega)/q^2$, ω fixed, we have $q^2 T_1^{(I)}(q^2,\infty) = 2\int_0^2 (d\omega/\omega)[-q^2 F_L^{(I)}(\omega) - \omega F_2^{(I)}(\omega) + H_L^{(I)}(\omega)]$. For no quadratic divergence we must have $F_L^{(I)}(\omega) = F_2^{(I)}/\omega - F_1^{(I)} = 0$, so the condition for no logarithmic divergence reads $\int_0^2 d\omega [F_2^{(I)}(\omega) + 2H_L^{(I)}(\omega)/\omega] = 0$. Since in our analysis we make the stronger assumption that $W_L^{(I)}(q^2,\nu) = 0$ for $|q^2| > (1\text{ GeV})^2$ so that $H_L^{(I)}(\omega) = 0$, we must have $F_2^{(I)}(\omega) = 0$ for no divergence. These assumptions on the longitudinal amplitude can be tested experimentally. If the ratio $R^{(I)}(q^2,\nu)$ given by (2.8) is vanishingly small near $|q^2| \sim |q_0^2| \sim (1\text{ GeV})^2$, then we expect $F_L^{(I)} = 0$, $H_L^{(I)} = 0$.

$G(x)=3/8x+O(1/x^2)$ as $x\to\infty$, we identify this divergent piece from (2.14) as

$$\delta M^{(I)}{}_{\rm div}=\tfrac{3}{8}\pi^2 \ln\left(\frac{\Lambda^2}{|q_0^2|}\right)\int_0^2 d\omega\, F_2{}^{(I)}(\omega)\,. \quad (2.15)$$

From the data on the proton, $\int_0^2 d\omega\, F_2{}^p(\omega)=(2\alpha/\pi)\times(0.18)$, so that

$$\delta M^p{}_{\rm div}=\frac{3\alpha}{4\pi}\ln\left(\frac{\Lambda^2}{|q_0^2|}\right)(0.18)\,, \quad (2.16)$$

which for $\Lambda\sim 100$ GeV, $|q_0|\sim 1$ GeV gives $\delta M^p{}_{\rm div}\sim 1$ MeV.

For the case of interest ($I=1$ corresponding to the proton-neutron mass difference), it is clear that in the absence of fixed poles or large longitudinal cross sections the explanation of the mass shift, if it is due to electromagnetism, resides in second term of (2.14). Since $G(x)>0$ and $x>0$, it is evident that, to obtain the correct sign of the mass shift, $W_2{}^{(1)}(q^2,\nu)$ must be negative for a substantial region of the $-q^2,\nu$ plane. We conclude that $\sigma_t{}^p(q^2,\nu)-\sigma_t{}^n(q^2,\nu)<0$ for some large region of $-q^2$ and ν. This proposition can be tested in forthcoming experiments at SLAC.

For the case of physical photons, $q^2=0$, it has been reported[2] that $\sigma_t{}^p(0,\nu)-\sigma_t{}^n(0,\nu)>0$ for photon energies in the transresonance region 4 GeV$<\nu<$18 GeV. If this qualitative feature is maintained when extrapolated to the region $|q^2|>|q_0{}^2|$, it would seem unlikely that the continuum is the explanation of the mass shift.

We might also remark that the scattering experiments with physical photons are consistent with Regge behavior[2] and that one finds (reduced residue of $A_2{}^0$)/(reduced residue of Pomeranchukon)$\sim 1/20$. If this is any indication of the relative strength of the $I=0$ to $I=1$ amplitudes in the high-energy region relevant to the mass shift, and if $\delta M_{\rm div}{}^p\sim 1$ MeV is an indication of contribution from $|q^2|>|q_0{}^2|$, we would expect this region to contribute in magnitude about 0.05 MeV to the mass difference, which is far too small. Such an argument should be taken with a grain of salt in view of the extrapolation involved.

In conclusion, if $\delta M=M_p-M_n$ is to be understood in terms of the Cottingham formula, we must have (i) comparable longitudinal and transverse cross sections so that $R^{(1)}(q^2,\nu)=\sigma_l{}^{(1)}(q^2,\nu)/\sigma_t{}^{(1)}(q^2,\nu)\simeq 1$ (in which case the considerations given here do not apply), (ii) $\sigma_t{}^p(q^2,\nu)-\sigma_t{}^n(q^2,\nu)<0$ for some large region of the $-q^2,\nu$ plane, $|q^2|>|q_0{}^2|$ (already contraindicated for physical photons), or (iii) fixed poles at $J=0$, $I=1$.

III. AXIAL-VECTOR CURRENT

The matrix element of the axial-vector current between nucleon states is specified by

$$\langle p(p')|A_\mu{}^{(+)}(0)|n(p)\rangle = u(p')i\gamma_5\tau^+[\gamma_\mu g_A(t)-q_\mu h_A(t)]u(p) \quad (3.1)$$

and the divergence by

$$\langle p(p')|i\partial_\mu A_\mu{}^{(+)}(0)|n(p)\rangle = u(p')i\gamma_5\tau^+ D^A(t)u(p)\,,$$
$$D^A(t)=(M_p+M_n)g_A(t)+th_A(t)\,. \quad (3.2)$$

If the current is conserved, $D^A(t)=0$; and upon setting $t=0$, we must have either $M_p+M_n=0$ or $h_A(t)$ has a pole at $t=0$ with residue$=(M_p+M_n)g_A(0)$. The pole in $h_A(t)$ corresponds to the presence of a zero-mass, $J^P=0^-$ Goldstone boson. We will assume that the conservation of the axial-vector current is realized by a Goldstone boson which is to be identified with the pion. In the absence of strict conservation of the axial-vector current, the pion acquires its observed mass of 140 MeV.

The divergence of the axial-vector current can be expressed by

$$\partial_\mu A_\mu{}^{(+)}(x)=i[H'(x),{}^AQ^{(+)}]\,, \quad (3.3)$$

where $H'(x)$ is the term in the Hamiltonian density that breaks ${}^AQ^{(+)}$ symmetry. For this piece we assume $H'(x)=H^{\rm had}(x)+H^{\rm em}(x)$, where $H^{\rm had}(x)$ breaks chiral $SU(2)$ but preserves isospin symmetry and $H^{\rm em}(x)$ is the electromagnetic interaction which is effectively given by (2.3). Assuming that the electromagnetic current is $J_\mu(x)=J_\mu{}^3(x)+J_\mu{}^8(x)/\sqrt{3}$, where the space integral of $J_0{}^\alpha(x)$ is a generator of $SU(3)$, and assuming the $SU(3)\times SU(3)$ current algebra, we have $[J_\mu(x),{}^AQ^{(+)}]={}^AJ_\mu{}^{(+)}(x)$ at equal times. Hence

$$[H^{\rm em}(0),{}^AQ^{(+)}]=-\frac{e^2}{2(2\pi)^4}\int\frac{d^4q}{q^2}g_{\mu\nu}\,{}^AT_{\mu\nu}{}^{(+)}(q)\,, \quad (3.4)$$

$${}^AT_{\mu\nu}{}^{(+)}(q)=\int d^4x\, e^{-iq\cdot x}[T({}^AJ_\mu{}^{(+)}(x)J_\nu(0)) + T(J_\mu(x)\,{}^AJ_\nu{}^{(+)}(0))].$$

To examine the matrix elements of the divergence of the axial-vector current between nucleon states, we will utilize the projection operator

$${}^AP=\sum_{\rm spins}\frac{-2M^2}{t}\bar{u}(p)i\gamma_5 u(p')$$

and from (3.4) with (3.2) and (3.3) we obtain

$$D^A(t)=D^A{}_{\rm had}(t)+D^A{}_{\rm em}(t)\,, \quad (3.5)$$

where

$$D^A{}_{\rm em}(t)=\frac{e^2\,{}^AP}{2(2\pi)^4}\int\frac{d^4q}{q^2}g^{\mu\nu}[{}^A\Gamma_{\mu\nu}{}^{(+)}(q,k) + {}^A\Gamma_{\nu\mu}{}^{(+)}(-q,-k)] \quad (3.6)$$

and

$${}^A\Gamma_{\mu\nu}{}^{(+)}(q,k)=\int d^4x\, e^{-iq\cdot x}\langle p(p')| \times T({}^AJ_\mu{}^{(+)}(x)J_\nu(0))|u(p)\rangle\,. \quad (3.7)$$

Here $k=p+q-p'$. The term $D^A{}_{\rm had}(t)$ represents the contribution of $[H^{\rm had}(x),{}^AQ^{(+)}]$ to the divergence and, in the absence of a detailed knowledge of the dynamics of chiral symmetry breaking, this term is difficult to estimate.

As a consequence of the assumption that the symmetry limit is attained by a Goldstone pion, we must extract the pion-pole term from the right-hand side of (3.5) to study the effects of symmetry breaking. This point has been emphasized by Dashen and Weinstein.[10] Defining $\langle 0|A_\mu{}^{(+)}(0)|\pi^+(p)\rangle = f_\pi p_\mu$, then this pole term is extracted as

$$D^A(t)-\frac{\sqrt{2}g_{pn\pi^+}f_\pi\mu^2}{\mu^2-t}=[D^A{}_{\rm had}(t)+D^A{}_{\rm em}(t)]_{\rm nonpole}. \quad (3.8)$$

Here μ is the mass of π^+ and $g_{pn\pi^+}$ is the physical coupling of the π^+ to nucleons. Using $D^A(0)=(M_p+M_n)g_A$, we have for the quantity $\Delta\equiv 1-(M_n+M_p)g_A/(\sqrt{2}g_{pn\pi}f_\pi)$, to lowest order in the symmetry breaking,

$$-\Delta=\frac{1}{2Mg_A}[D^A{}_{\rm had}(0)+D^A{}_{\rm em}(0)]_{\rm nonpole}. \quad (3.9)$$

The observed value of the corrections to the Goldberger-Treiman formula is $\Delta=+0.08\pm0.02$.

First we will examine the contribution of $D^A{}_{\rm em}(0)$ given by (3.6) in the approximation of retaining just the nucleon and pion poles. For simplicity, we may assume that the axial-vector current in the expression ${}^A\Gamma_{\mu\nu}{}^{(+)}\times(q,k)$ is conserved since this approximation will introduce errors of order $\Delta\sim+0.1$ relative to other pieces. The Ward identities for ${}^A\Gamma_{\mu\nu}{}^{(+)}(q,k)$ are

$$\begin{aligned} q^\mu\,{}^A\Gamma_{\mu\nu}{}^{(+)}(q,k)&=i\langle p(p')|{}^AJ_\nu{}^{(+)}(0)|n(p)\rangle,\\ k^\nu\,{}^A\Gamma_{\mu\nu}{}^{(+)}(q,k)&=i\langle p(p')|{}^AJ_\mu{}^{(+)}(0)|n(p)\rangle. \end{aligned} \quad (3.10)$$

As a first step, we include the nucleon and pion pole terms in ${}^A\Gamma_{\mu\nu}{}^{(+)}(q,k)$ without form factors and anomalous magnetic moment couplings. Then we have

$$\begin{aligned} {}^A\Gamma_{\mu\nu}{}^{(+)}(q,k)=ig_Au(p')\bigg[&\gamma_\nu\frac{(\mathbf{p}+\mathbf{q}+M)}{(p+q)^2-M^2}i\gamma_5\left(\gamma_\mu+\frac{2Mq_\mu}{q^2}\right)\\ &+\frac{2Mi\gamma_5}{t}\left(\frac{(2q-k)_\nu q_\mu}{q^2}-g_{\mu\nu}\right)\bigg]u(p), \end{aligned} \quad (3.11)$$

which satisfies the Ward identities (3.10). The first term is the nucleon pole and the second the pion pole at $t=0$, which, as already remarked, must be extracted from the final expression since its contribution can be lumped into the observed residue, $\mu^2 f_\pi$. Substituting (3.11) into (3.6) and removing the π-pole term, we find from (3.9) $\Delta=\Delta_{\rm had}+\Delta_{\rm em}$, with $\Delta_{\rm em}=-D^A{}_{\rm em}(0)|_{\rm nonpole}/2Mg_A$,

$$\Delta_{\rm had}=-\frac{D^A{}_{\rm had}(0)|_{\rm nonpole}}{2Mg_A},$$

where

$$\Delta_{\rm em}\underset{t\to0}{\longrightarrow}\frac{ie^2}{2(2\pi)^4}\int\frac{d^4q}{q^2}\left[\frac{1}{s-M^2}+\frac{1}{u-M^2}+\frac{k^2-q^2}{t}\left(\frac{1}{s-M^2}-\frac{1}{u-M^2}\right)\right].$$

Here $s=(p+q)^2$, $u=(p'-q)^2$, $t=(q-k)^2$, and $p'+k=p+q$. This integral is logarithmically divergent, with the result

$$\Delta_{\rm em}=-\frac{\alpha}{4\pi}\ln\frac{\Lambda^2}{M^2}+(\text{finite terms}). \quad (3.12)$$

Had we introduced form factors with poles at the 1^- and 1^+ vector-meson masses, this calculation would be rendered finite. Even for $\Lambda\sim100$ GeV, $\Delta_{\rm em}$ is but a faction of a percent. This crude calculation suggests that a more refined treatment would not radically alter the conclusion that $\Delta_{\rm em}$ is very small, and hence the explanation for the observed Δ must lie in the domain of hadron dynamics.[11]

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