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N-Point Functions in Chiral $SU(3)\times SU(3)$ Current Algebra*

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Hard-meson techniques are presented for calculating processes involving octets (nonets) of mesons under the assumptions of chiral $SU(3)\times SU(3)$ $[U(3)\times U(3)]$ current-algebra commutation relations, partial conservation of axial-vector currents, conservation and partial conservation of vector currents, and single-meson saturation of intermediate sums. Using these conditions and the usual smoothness hypothesis, the general procedure for constructing the arbitrary N-point function is given. If, in addition, one assumes that the "σ commutators" (i.e., the commutators of the time components of the currents with the scalar fields) are single-particle dominated, it is inconsistent to assume only octets of particles. A consistent formalism involving nonets of particles can be constructed, however. The spin-zero mesons must then belong to the $(3,3^*)+(3^*,3)$ representation. Hence the $(3,3^*)+(3^*,3)$ symmetry-breaking condition is deduced from the current-algebra conditions when combined with pole dominance.

I. INTRODUCTION

RECENTLY, a considerable amount of work has been done in the development of hard-pion techniques using $SU(2)\times SU(2)$ current algebra.[1,2] This work has been successfully applied to a number of processes, including an excellent fit to the $\pi\pi$ $I=0$ and $I=2$ S-wave phase shifts from threshold up to 1 GeV.[3] The hard-pion method was also recently extended to the algebra involving strangeness-changing currents in order to analyze the K_{l3} decay.[4,5] In the present paper we wish to extend the hard-meson method further, so that one can compute an arbitrary N-point function using the chiral $SU(3)\times SU(3)$ current algebra. In this analysis no *a priori* assumptions are made about the type of symmetry breakdown occurring, chiral or ordinary; nor do we impose any specific symmetry on the mass spectrum or interaction structure aside from the conditions demanded by current algebra (and the symmetries strictly preserved by the strong interaction, such as conservation of isotopic spin, G parity, and ordinary parity).

The physics involved in the study of the $SU(2)\times SU(2)$ algebra rested on a few basic assumptions. These were (1) that there exist currents which satisfy the equal-time commutation relations of Gell-Mann, i.e., the chiral algebra of $SU(2)\times SU(2)$, (2) that these currents also satisfy both PCAC (partial conservation of the axial-vector currents) and CVC (conservation of the vector currents), (3) that the vacuum expectation values of the T-products of an arbitrary number of currents would be saturated by a few low-lying single-particle states, and (4) that the resulting particle vertex functions are smooth and thus may be approximated by a low-order polynomial in the momenta of the single particles involved.

The extensions we will consider here are such that the present program may be developed for an arbitrary underlying Lie algebra. However, we shall focus our attention on $SU(3)\times SU(3)$ [or $U(3)\times U(3)$]. In addition, we shall impose the partial conservation of the strangeness-changing vector current PCVC [to account for $SU(3)$ breakdown], as well as PCAC and CVC for the conserved vector currents. We shall refer to these collectively as PCC. In generalizing assumption (3) above, we will introduce four nonets of particles. These are (a) the vector mesons (ρ, K^*, ω, and φ), (b) the axial-vector mesons (A_1, K_A, D, and E), (c) the scalar mesons (δ, κ, σ, and η_V), and (d) the pseudoscalar mesons (π, K, η, and η').[6] Along with them we will consider a ninth vector as well as an axial-vector current. The conservation of the ninth vector current is taken to represent the conservation of baryon number.

In Sec. II we formalize the above assumptions and display the effective Lagrangian to be used in calculating three-point functions. We then impose the current-algebra conditions to obtain equations relating to the

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6 The existence and J^P values of all of these mesons have not been established. However, modification of the above choices does not effect the general formalism presented in this paper.

coupling constants. In Sec. III we solve these equations. In Secs. IV–VI we develop this program further, which then allows one to calculate an arbitrary N-point function. Here, we find that if one requires single-meson dominance of the "σ commutators" then it is necessary to consider nonets of particles rather than octets. It is for this reason that we have allowed for nonets of currents in Secs. II and III.

II. THREE-POINT FUNCTIONS

We start by considering the vacuum expectation value of a T bracket of three currents:

$$F^{\alpha\beta\gamma}{}_{abc}(x,y,z)=\langle 0|T(V^{\alpha}{}_a(x)V^{\beta}{}_b(y)V^{\gamma}{}_c(z))|0\rangle\,, \quad (2.1)$$

where $V^{\alpha}{}_a$, etc., are vector or axial-vector currents. The indices a, b, and c run from 1 to l if our algebra involves l generators. Since we are dealing with chiral algebras, it is convenient to use barred indices to denote axial-vector currents. Thus, if we consider $U(3)\times U(3)$, a will run from 1 to 9 for the vector currents and $\bar{1}$ to $\bar{9}$ for the axial-vector currents. The above T-product may be expanded in its six time orderings of which one corresponds to $x^0>y^0>z^0$. For this case, upon using closure, we obtain

$$F^{\alpha\beta\gamma}{}_{abc}(x,y,z)=\sum_{n,m}\langle 0|V^{\alpha}{}_a(x)|n\rangle\langle n|V^{\beta}{}_b(y)|m\rangle \times\langle m|V^{\gamma}{}_c(z)|0\rangle\,. \quad (2.2)$$

We now assume that the sum over intermediate states is saturated by single particles, which may be scalar, pseudoscalar, vector, or pseudovector. This assumption is a generalization of the vector-dominance hypothesis for the isotopic vector current.

Let a state describing a spin-zero meson be denoted by

$$|n\rangle=|sqa\rangle\,,$$

where s denotes spin zero, q its momentum, and a its internal symmetry classification. Thus for nonets of scalar and pseudoscalar mesons $a=i=1\cdots 9$ denotes the scalar particles while $a=\bar{\imath}=\bar{1}\cdots\bar{9}$ the pseudoscalar meson (e.g., $a=\bar{3}$ refers to a π^0 meson). Similarly, we will denote a spin-one meson by

$$|m=|vqa\lambda\rangle\,,$$

where v refers to spin one, q and a as above, while λ denotes its helicity. The various one-particle to vacuum matrix elements of the currents encountered in Eq. (2.2) serve to define the coupling strengths of these currents to the particles. Thus F_{ab} and g_{ab} are defined by the equations

$$\langle 0|V^{\mu}{}_a(0)|sqb\rangle=iq^{\mu}F_{ab}N_{sb}(q)\,, \quad (2.3a)$$

$$\langle 0|V^{\mu}{}_a(0)|vqb\lambda\rangle=g_{ab}\epsilon^{\mu}(\lambda)N_{vb}(q)\,, \quad (2.3b)$$

where N_{sb}, etc., are the Bose normalization factors,[7] and $\epsilon^{\mu}(\lambda)$ are the polarization vectors of helicity λ. We note that $g_{i\bar{j}}=g_{\bar{j}i}=F_{i\bar{j}}=F_{\bar{j}i}=0$ from parity considerations. Furthermore, we have

$$F_{ij}=F_i\delta_{ij}\,,\quad i,\,j=1,\,\ldots,\,9$$

where conservation of all the vector currents except the strangeness changing implies

$$F_i=0\,,\quad i=1,\,2,\,3,\,8,\,9$$

while

$$F_i=F_\kappa\,,\quad i=4,\,5,\,6,\,7\,. \quad (2.4a)$$

In addition, one has

$$F_{\bar{\imath}\bar{j}}=F_{\bar{\imath}}\delta_{\bar{\imath}\bar{j}}\,,\quad \bar{\imath},\,\bar{j}=1,\,\ldots,\,7$$

with

$$F_{\bar{\imath}}=F_\pi\,,\quad \bar{\imath}=\bar{1},\,\bar{2},\,\bar{3}$$

$$F_{\bar{\imath}}=F_K\,,\quad \bar{\imath}=\bar{4},\,\bar{5},\,\bar{6},\,\bar{7}$$

and

$$F_{\bar{8}\bar{j}}=F_{\bar{9}\bar{j}}=F_{\bar{j}\bar{8}}=F_{\bar{j}\bar{9}}=0\,,\quad \bar{j}=\bar{1},\,\ldots,\,7\,. \quad (2.4b)$$

With our choice of particle labeling for the η and η' mesons, we may write

$$F_{\bar{8}\bar{8}}=F_{\bar{8}\eta}\,,\quad F_{\bar{8}\bar{9}}=F_{\bar{8}\eta'}\,,\quad F_{\bar{9}\bar{8}}=F_{\bar{9}\eta}\,,\quad F_{\bar{9}\bar{9}}=F_{\bar{9}\eta'} \quad (2.4c)$$

since $|s,q,\bar{8}\rangle$ and $|s,q,\bar{9}\rangle$ are the η and η' states, respectively.

The g_{ab} are taken to satisfy

$$g_{ij}=g_i\delta_{ij}\,,\quad i,\,j=1,\,\ldots,\,7$$

with

$$g_i=g_\rho\,,\quad i=1,\,2,\,3$$

$$g_i=g_{K^*}\,,\quad i=4,\,5,\,6,\,7$$

and

$$g_{8j}=g_{9j}=g_{j8}=g_{j9}=0\,,\quad j=1,\,\ldots,\,7\,. \quad (2.5a)$$

Again, with our choice of particle labeling for the ω and φ mesons[8] we set

$$g_{88}=g_{8\omega}\,,\quad g_{89}=g_{8\varphi}\,,\quad g_{98}=g_{9\omega}\,,\quad g_{99}=g_{9\varphi}\,. \quad (2.5b)$$

Similar equations may be written for the $g_{\bar{\imath}\bar{j}}$ with the substitutions

$$g_{\bar{\imath}}=g_A\,,\quad \bar{\imath}=1,\,2,\,3$$

$$g_{\bar{\imath}}=g_{K_A}\,,\quad \bar{\imath}=4,\,5,\,6,\,7 \quad (2.5c)$$

$$g_{\bar{8}\bar{8}}=g_{\bar{8}D}\,,\quad g_{\bar{8}\bar{9}}=g_{\bar{8}E}\,,\quad g_{\bar{9}\bar{8}}=g_{\bar{9}D}\,,\quad g_{\bar{9}\bar{9}}=g_{\bar{9}E}\,.$$

It was shown in Ref. 1 that the assumption of single-particle saturation allows one to calculate three-point functions by the device of employing an effective Lagrangian. This Lagrangian is to be cubic in interacting fields and is to be used only to first order in the coupling constants. Furthermore, the currents are to be constructed using the Heisenberg field operators via the field-current identity. The most general effective Lagrangian involving spin-zero and spin-one fields

[7] We normalize states so that $N_{sb}(q)=[2\omega_{sb}(q)(2\pi)^3]^{-1/2}$, where $\omega_{sb}(q)=(q^2+\mu_b{}^2)^{1/2}$; $N_{vb}(q)=[2\omega_{vb}(b)(2\pi)^3]^{-1/2}$, where $\omega_{vb}(q)=(q^2+m_b{}^2)^{1/2}$; μ_b and m_b are the masses of spin-zero and spin-one particles, respectively. The polarization vectors obey $\epsilon^{\mu*}(\lambda)\epsilon_\mu(\lambda')=\delta_{\lambda\lambda'}$ and $q_\mu\epsilon^\mu(\lambda)=0$.

[8] This choice was also made in R. Arnowitt, M. H. Friedman, and P. Nath, Phys. Letters 27B, 657 (1968).

satisfying the smoothness condition[9] may be written as

$$\mathcal{L}=\mathcal{L}_{(2)}+\mathcal{L}_{(3)}\,, \tag{2.6}$$

where

$$\mathcal{L}_{(2)}=-s^{\mu}{}_{a}\partial_{\mu}s_{a}+\tfrac{1}{2}(s^{\mu}{}_{a}s_{\mu a}-\mu_{a}{}^{2}s_{a}{}^{2})-\tfrac{1}{2}G^{\mu\nu}{}_{a}(\partial_{\mu}v_{\nu a}-\partial_{\nu}v_{\mu a})+\tfrac{1}{4}G^{\mu\nu}{}_{a}G_{\mu\nu a}-\tfrac{1}{2}m_{a}{}^{2}v^{\mu}{}_{a}v_{\mu a} \tag{2.7a}$$

and

$$\mathcal{L}_{(3)}=g^{1}{}_{abc}v_{\mu a}s^{\mu}{}_{b}s_{c}+g^{2}{}_{abc}s_{\mu a}s_{\nu b}G^{\mu\nu}{}_{c}+g^{3}{}_{abc}v_{\mu b}s_{a}v^{\mu}{}_{c}+g^{4}{}_{abc}s_{a}G^{\mu\nu}{}_{b}G_{\mu\nu c}+g^{5}{}_{abc}v_{\mu b}s_{\nu a}G^{\mu\nu}{}_{c}+g^{6}{}_{abc}v_{\mu a}v_{\nu b}G^{\mu\nu}{}_{c}+g^{7}{}_{abc}s_{a}s_{b}s_{c}+g^{8}{}_{abc}s_{a}s^{\mu}{}_{b}s_{\mu c}+\epsilon_{\mu\nu\alpha\beta}[h^{1}{}_{abc}v^{\mu}{}_{a}v^{\nu}{}_{b}+h^{2}{}_{abc}s^{\mu}{}_{a}s^{\nu}{}_{b}+h^{3}{}_{abc}v^{\mu}{}_{a}s^{\nu}{}_{b}+h^{4}{}_{abc}s_{a}G^{\mu\nu}{}_{b}]G^{\alpha\beta}{}_{c}\,, \tag{2.7b}$$

where we choose $\epsilon^{0123}=+1$ (our metric has signature $+2$). Here the $(s_{\mu a},s_{a})$ and $(G_{\mu\nu a},v_{\mu a})$ are to be varied independently to yield first-order coupled differential equations. One has that (s_{0a},s_{a}) and (G_{0ia},v_{ia}) are the canonically conjugate pairs of variables for the spin-zero and spin-one fields, respectively. The coupling constants $g^{i}{}_{abc}$ and $h^{i}{}_{abc}$ are arbitrary at this point of the analysis. *A priori* we require only that these coupling constants take on values that maintain the invariance under isotopic spin, G parity, and parity.

The currents are given in terms of the Heisenberg fields by

$$V^{\mu}{}_{a}(x)=g_{ab}v^{\mu}{}_{a}+F_{ab}\partial^{\mu}s_{b}\,, \tag{2.8}$$

where g_{ab} and F_{ab} are the coupling strengths defined in Eqs. (2.3). In order to carry out the algebraic restrictions to be imposed on these currents, it will be convenient to make use of the equations of motion obtained from the Lagrangian equations (2.7). For the spin-zero fields, we have

$$-\partial_{\mu}s^{\mu}{}_{a}+\mu_{a}{}^{2}s_{a}=\delta\mathcal{L}_{(3)}/\delta s_{a}\,, \tag{2.9a}$$

$$s^{\mu}{}_{a}=\partial^{\mu}s_{a}-\delta\mathcal{L}_{(3)}/\delta s_{\mu a}\,, \tag{2.9b}$$

while for the spin-one fields

$$\partial_{\nu}G^{\mu\nu}{}_{a}+m_{a}{}^{2}v^{\mu}{}_{a}=\delta\mathcal{L}_{(3)}/\delta v_{\mu a} \tag{2.10a}$$

$$G^{\mu\nu}{}_{a}=(\partial^{\mu}v^{\nu}{}_{a}-\partial^{\nu}v^{\mu}{}_{a})-2\delta\mathcal{L}_{(3)}/\delta G_{\mu\nu a}. \tag{2.10b}$$

The currents given by Eqs. (2.8) are to be subjected to the requirements of the current algebra. These are (1) the equal-time canonical commutation relations

$$\delta(x^{0}-y^{0})[V^{0}{}_{a}(x),V_{b}{}^{\mu}(y)]=iC_{abc}\delta^{4}(x-y)V^{\mu}{}_{c}(y)+c\text{-No. ST}\,, \tag{2.11}$$

where c-No. ST means "c-number Schwinger terms," and C_{abc} are the structure constants of the algebra being consider $[SU(3)\times SU(3)$ or $U(3)\times U(3)]$. We also have (2) the divergence conditions

$$\partial_{\mu}V^{\mu}{}_{a}(x)=F_{ab}\mu_{b}{}^{2}s_{b}(x)\,, \tag{2.12}$$

where the index b is to be summed.

[9] We here use first-order formalism, and the smoothness condition is the same as that of Refs. 1 and 2.

As has been shown in Refs. 1 and 2, Eqs. (2.11) and (2.12) need only be satisfied to first order in the coupling constants if we are only interested in three-point functions. In order to implement Eq. (2.11), it is more convenient to express the currents in terms of the Heisenberg canonical variables by using the field equations. One is then to evaluate the commutators to first order in the coupling constants by means of the canonical commutation relations. The spatial components of the currents are immediately expressable in terms of canonical variables and are given by

$$V^{i}{}_{a}(x)=g_{ab}v^{i}{}_{b}(x)+F_{ab}\partial^{i}s_{b}(x)\,. \tag{2.13}$$

We note that they only contain terms linear in the fields and are of zeroth order in the coupling constants. For the time components, we find, upon using the equations of motion (2.9) and (2.10),

$$V^{0}{}_{a}(x)=V^{0}{}_{a(1)}(x)+V^{0}{}_{a(2)}(x)\,, \tag{2.14}$$

where $V^{0}{}_{a(1)}(x)$ is linear in canonical variables while $V^{0}{}_{a(2)}(x)$ is quadratic. The former is of zeroth order in the coupling constants, while the latter is of first order. They are explicitly given by

$$V^{0}{}_{a(1)}(x)=(g_{ab}/m_{b}{}^{2})\partial_{i}G^{i0}{}_{b}-F_{ab}S_{0b}\,, \tag{2.15a}$$

$$V^{0}{}_{a(2)}(x)=-Z_{1abc}s_{0b}s_{c}+Z_{2abc}v_{ib}G^{i0}{}_{c}+Z_{4abc}\partial_{i}s_{b}G^{i0}{}_{c}+Z_{3abc}s_{b}\partial_{i}G^{i0}{}_{c}+2\epsilon_{ijk}(Y_{1abc}v^{i}{}_{b}\partial^{j}v^{k}{}_{c}+Y_{2abc}\partial^{i}s_{b}\partial^{j}v^{k}{}_{c})\,, \tag{2.15b}$$

where

$$Z_{1abc}=(g_{ad}/m_{d}{}^{2})g^{1}{}_{dbc}+2F_{ad}g^{8}{}_{cdb}\,, \tag{2.16a}$$

$$Z_{2abc}=-2(g_{ad}/m_{d}{}^{2})g^{6}{}_{dbc}+F_{ad}g^{5}{}_{dbc}\,, \tag{2.16b}$$

$$Z_{3abc}=\left(2\frac{g_{ad}}{m_{d}{}^{2}}g^{3}{}_{bdc}+F_{ad}g^{1}{}_{cdb}\right)\frac{1}{m_{c}{}^{2}}\,, \tag{2.16c}$$

$$Z_{4abc}=-(g_{ad}/m_{d}{}^{2})g^{5}{}_{bdc}-2F_{ad}g^{2}{}_{dbc}\,, \tag{2.16d}$$

$$Y_{1abc}=-2(g_{ad}/m_{d}{}^{2})h^{1}{}_{dbc}+F_{ad}h^{3}{}_{bdc}\,, \tag{2.16e}$$

$$Y_{2abc}=-(g_{ad}/m_{d}{}^{2})h^{3}{}_{dbc}-2F_{ad}h^{2}{}_{dbc}\,. \tag{2.16f}$$

In Eq. (2.16c) the index c on the right-hand side is not to be summed.

We first consider the commutator

$$\delta(x^{0}-y^{0})[V^{0}{}_{a}(x),V^{i}{}_{b}(y)]=iC_{abc}V^{i}{}_{c}(y)\delta^{4}(x-y)+c\text{-No. ST}. \tag{2.17}$$

Inserting Eqs. (2.13)–(2.15) into Eq. (2.17), we obtain

$$Z_{3adc}g_{bc}+Z_{1acd}F_{bc}=0\,, \tag{2.18a}$$

$$Z_{2adc}g_{bc}+C_{abc}g_{cd}=0\,, \tag{2.18b}$$

$$Z_{4adc}g_{bc}-Z_{1acd}F_{bc}-Z_{3adc}g_{bc}+C_{abc}F_{cd}=0. \tag{2.18c}$$

Equation (2.18a) arises from the requirement that q-No. ST vanish in the commutator.

We next consider the commutator

$$\delta(x^0-y^0)[V^0{}_a(x),V^0{}_b(y)] = iC_{abc}V^0{}_c(y)\delta^4(x-y)+c\text{-No. ST}. \quad (2.19)$$

Since the left-hand side is to be evaluated only to first order in the coupling constants, while the right-hand side is to be evaluated to zeroth order, Eq. (2.19) may be replaced by

$$\delta(x^0-y^0)\{[V^0{}_{a(1)}(x),V^0{}_{b(1)}(y)] +[V^0{}_{a(1)}(x),V^0{}_{b(2)}(y)]+[V^0{}_{a(2)}(x),V^0{}_{b(1)}(y)]\} = iC_{abc}V^0{}_{c(1)}(y)\delta^4(x-y)+c\text{-No. ST}. \quad (2.20)$$

Again, upon using Eqs. (2.15), we find

$$g_{ac}m_c^{-2}Z_{2bcd}+g_{bc}m_c^{-2}Z_{2acd} +F_{ac}Z_{4bcd}+F_{bc}Z_{4acd}=0, \quad (2.21a)$$

$$g_{ac}m_c^{-2}Y_{1bcd}+g_{bc}m_c^{-2}Y_{1acd} +F_{ac}Y_{2bcd}+F_{bc}Y_{2acd}=0, \quad (2.21b)$$

$$F_{bc}Z_{1adc}-F_{ac}Z_{1bdc}+C_{abc}F_{cd}=0, \quad (2.21c)$$

$$g_{bc}m_c^{-2}Z_{2acd}+F_{ac}Z_{3bcd}-F_{bc}Z_{3acd}+F_{bc}Z_{4acd} -C_{abc}g_{cd}m_d^{-2}=0. \quad (2.21d)$$

In Eq. (2.21d) the repeated index d in the last term is not to be summed. Equations (2.21a) and (2.21b) follow from the requirement that q-No. ST vanish in the current-current commutators. From the above discussion, we note that the conditions imposed by the canonical commutation relations [Eqs. (2.18) and (2.21)] place no restrictions on the $\epsilon_{\mu\nu\alpha\beta}$ couplings in $\mathcal{L}_{(3)}$.

We next turn our attention to the requirements of PCC. From Eqs. (2.8) and (2.12) we have

$$g_{ab}\partial_\mu v^\mu{}_b(x)-F_{ab}(-\Box+\mu_b^2)s_b(x)=0, \quad (2.22)$$

where Eq. (2.22) is to be satisfied only to first order in the coupling constants as stated earlier. Upon using the Lagrangian equations of motion (2.9) and (2.10), we obtain the conditions

$$Z_{1adc}\mu_d^2+Z_{1acd}\mu_c^2=6F_{ab}g^7{}_{bcd}, \quad (2.23a)$$

$$Z_{1adc}+Z_{1acd}=2F_{ab}g^8{}_{bcd}, \quad (2.23b)$$

$$m_d^2(Z_{3acd}-Z_{4acd})=F_{ab}g^1{}_{dcb}, \quad (2.23c)$$

$$Z_{2acd}+Z_{2adc}=-4F_{ab}g^4{}_{bcd}, \quad (2.23d)$$

$$Z_{2acd}m_d^2+Z_{2adc}m_c^2=-2F_{ab}g^3{}_{bcd}, \quad (2.23e)$$

$$Y_{1acd}+Y_{1adc}=4F_{ab}h_{4bcd}. \quad (2.23f)$$

Some of the g^i and h^i are automatically symmetric or antisymmetric in certain indices, as can be seen by examining the $\mathcal{L}_{(3)}$ of Eq. (2.7b). These symmetries are

$$g^{2,6}{}_{abc}=-g^{2,6}{}_{bac}, \quad g^{3,4,8}{}_{abc}=+g^{3,4,8}{}_{acb}, \quad h^{1,2}{}_{abc}=-h^{1,2}{}_{bac}, \quad h^4{}_{abc}=+h^4{}_{acb}, \quad (2.24)$$

while $g^7{}_{abc}$ is symmetric on interchange of any two indices.

III. SOLUTIONS FOR COUPLING CONSTANTS OF THREE-POINT FUNCTIONS

In this section we discuss the solutions of the current-algebra conditions (2.18), (2.21), and (2.23) for the three-point functions. We first remark that these equations are not all independent. Thus Eqs. (2.21a) and (2.21b) are identically satisfied by virtue of the definitions of Z_3, Z_4, Y_1, and Y_2 [Eqs. (2.16)] and the antisymmetry of g^2, g^6, h^1, and h^2 [Eqs. (2.24)]. Equation (2.21c) is automatically satisfied as a consequence of the commutation condition (2.18c) and the PCC conditions [Eqs. (2.23b) and (2.23c)]. Similarly, Eq. (2.21d) follows from Eqs. (2.18b) and (2.23c) and (2.23e). Thus all the conditions arising from the commutators $[V^0{}_a,V^0{}_b]$ are redundant.

In order to solve the remaining equations (2.18) and (2.23), it is convenient to introduce matrices g and F whose components are g_{ab} and F_{ab}, respectively. Equations (2.18) then allow one to determine Z_2 and Z_4 uniquely and relate Z_3 to Z_1:

$$Z_{2adc}=-C_{abe}g_{ed}(g^{-1})_{cb}, \quad (3.1a)$$

$$Z_{4adc}=-C_{abe}F_{ed}(g^{-1})_{cb}, \quad (3.1b)$$

$$Z_{3adc}=-Z_{1aed}F_{be}(g^{-1})_{cb}. \quad (3.1c)$$

Here g^{-1} is the matrix inverse of g. One may now use Eqs. (2.23d) and (2.23e) to determine parts of g^3 and g^4. One finds

$$F_{ab}g^3{}_{bcd}=\tfrac{1}{2}C_{abe}[g_{ed}(g^{-1})_{cb}m_c^2+g_{ec}(g^{-1})_{db}m_d^2], \quad (3.2)$$

$$F_{ab}g^4{}_{bcd}=\tfrac{1}{4}C_{abe}[g_{ed}(g^{-1})_{cb}+g_{ec}(g^{-1})_{db}]. \quad (3.3)$$

Since F_{ab} does not have an inverse, Eqs. (3.2) and (3.3) do not completely determine g^3 and g^4. For the values of the index a for which F_{ab} is diagonal (i.e., $F_{ab}=F_a\delta_{ab}$), the left-hand side of Eq. (3.2) reduces to $F_ag^3{}_{acd}$. One can then determine $g^3{}_{acd}$ and $g^4{}_{acd}$ uniquely for those values of a for which F_a is nonzero. When $F_a=0$, $g^3{}_{acd}$ and $g^4{}_{acd}$ are undetermined. [The right-hand sides of Eqs. (3.2) and (3.3) automatically vanish for these values of a even when $C_{abe}\neq 0$ as a consequence of isospin, hypercharge, and strangeness conservation.] Thus the couplings of the σ and η_V to K^*-K^* and K_A-K_A are undetermined.

As can be seen from the definitions of Z_2 and Z_4 [Eqs. (2.16b) and (2.16d)], Eqs. (3.1a)and (3.1b) allow one to express g^5 and g^6 in terms of g^2. One finds

$$g^5{}_{abc}=m_b^2F_{ea}(g^{-1})_{bf}(g^{-1})_{cd}C_{fde}-2\Lambda_{abc}, \quad (3.4)$$

$$g^6{}_{abc}=\tfrac{1}{2}m_a^2(g^{-1})_{ad}(g^{-1})_{cf}g_{eb}C_{dfe} +\tfrac{1}{2}m_a^2(g^{-1})_{ad}F_{de}g^5{}_{ebc}, \quad (3.5)$$

where

$$\Lambda_{abc}\equiv m_b^2(g^{-1})_{be}F_{ed}g^2{}_{dac} \quad (3.6)$$

is undetermined. For values of a, b, and c where F and g are diagonal, Eqs. (3.4) and (3.5) reduce to

$$g^5{}_{abc}=m_b^2F_a(g_bg_c)^{-1}C_{abc}-2\Lambda_{abc}, \quad (3.7)$$

$$g^6{}_{abc}=\tfrac{1}{2}m_a{}^2(g_ag_bg_c)^{-1}(F_a{}^2m_b{}^2-g_b{}^2)C_{abc} -m_a{}^2F_a(g_a)^{-1}\Lambda_{abc}, \quad (3.8)$$

where Λ_{abc} now takes the form

$$\Lambda_{abc}=m_b{}^2F_b(g_b)^{-1}g^2{}_{bac}. \quad (3.9)$$

If the index b in addition corresponds to a conserved channel (i.e., $F_b=0$) then Λ_{abc} vanishes and $g^5{}_{abc}$ is uniquely determined. If either a or b are conserved, then $g^6{}_{abc}$ is completely determined. If $F_b\neq 0$, then Λ_{abc} is closely related to the $SU(3)$ generalization of λ_A $(\equiv 1+\delta)$, the A_1 anomalous moment.

The remaining current commutator condition, Eq. (3.1c), when combined with the other relations leads to the first Weinberg sum rules. [Recall that Eq. (3.1c) implies the absence of q-No. ST.] This is most easily seen by using Eqs. (3.1) to eliminate Z_2, Z_3, and Z_4 in Eq. (2.21d). Simplifying by Eq. (2.21c) then yields

$$C_{abd}W_{de}+C_{aed}W_{db}=0, \quad (3.10)$$

where

$$W_{ab}\equiv g_{ac}g_{bc}m_c{}^{-2}+F_{ac}F_{bc}. \quad (3.11)$$

Equation (3.10) implies that W_{ab} is an invariant second-rank tensor under the chiral group. For $SU(3)\times SU(3)$ this implies

$$W_{ab}=\lambda\delta_{ab}, \quad a, b=1, \ldots, 8, \bar{1}, \ldots, \bar{8} \quad (3.12)$$

where $\lambda=g_\rho{}^2/m_\rho{}^2$ is arbitrary. (Experimentally, λ has a value close to $2F_\pi{}^2$, though the KSRF (Kewara-bayashi-Suzuki-Riazuddin-Fayyazuddin) relation $g_\rho{}^2=2m_\rho{}^2F_\pi{}^2$ is not deducible without additional assumptions in the hard-pion current-algebra formalism.) Note that though a, b, d, and e in Eq. (3.10) run only over the 16 labels of the current algebra, $1, \cdots, 8, \bar{1}, \ldots, \bar{8}$, the index c in Eq. (3.11) sums over all 18 values $1, \ldots, 9, \bar{1}, \ldots, \bar{9}$. Thus the sum rules for the channels involving mixing read

$$\frac{g_\rho{}^2}{m_\rho{}^2}=\frac{g_{8\omega}{}^2}{m_\omega{}^2}+\frac{g_{8\Phi}{}^2}{m_\Phi{}^2} \quad (3.13a)$$

$$=\frac{g_{\bar{8}D}{}^2}{m_D{}^2}+F_{\bar{8}\eta}{}^2+\frac{g_{\bar{8}E}{}^2}{m_E{}^2}+F_{\bar{8}\eta'}{}^2. \quad (3.13b)$$

For the $U(3)\times U(3)$ chiral group, the labels in Eq. (3.10) also run over the full set of 18 values. One obtains in addition then the relations

$$W_{9a}=0=W_{\bar{9}a}, \quad a=1, \ldots, 8, \bar{1}, \ldots, \bar{8} \quad (3.14)$$

but W_{99} and $W_{\bar{9}\bar{9}}$ remain arbitrary. [Since the group is reducible, Eq. (3.12) need not hold for a and b taking on the additional values $a=b=9$, $\bar{9}$.] Equations (3.14) then lead to

$$\frac{g_{8\omega}g_{9\omega}}{m_\omega{}^2}+\frac{g_{8\Phi}g_{9\Phi}}{m_\Phi{}^2}=0, \quad (3.15)$$

$$\frac{g_{\bar{8}D}g_{\bar{9}D}}{m_D{}^2}+F_{\bar{8}\eta}F_{\bar{9}\eta'}+\frac{g_{\bar{8}E}g_{\bar{9}E}}{m_E{}^2}+F_{\bar{8}\eta}F_{\bar{9}\eta'}=0. \quad (3.16)$$

Aside from yielding the Weinberg sum rules, Eq. (3.1c) relates g^3 to g^1 and g^8. Thus inserting in the definitions of Eqs. (2.16a) and (2.16c) yields

$$g^3{}_{abc}=-\tfrac{1}{2}[m_b{}^2(g^{-1}F)_{bd}g^1{}_{cda}+m_c{}^2(g^{-1}F)_{cd}g^1{}_{bda}] -m_b{}^2m_c{}^2(g^{-1}F)_{bd}(g^{-1}F)_{ce}g^8{}_{ade}. \quad (3.17)$$

In the range of indices where g and F are diagonal this reduces to

$$g^3{}_{abc}=-\tfrac{1}{2}[m_b{}^2F_bg_b{}^{-1}g^1{}_{cba}+m_c{}^2F_cg_c{}^{-1}g^1{}_{bca}] -m_b{}^2m_c{}^2F_bF_c(g_bg_c)^{-1}g^8{}_{abc}. \quad (3.18)$$

Thus, if $F_a=F_b=F_c=0$, then $g^3{}_{abc}=0$, i.e., there is no g^3-type σ-ρ-ρ coupling. For other possibilities, g^3 is generally not zero.

The remaining equations, Eqs. (2.23a)–(2.23c), give information relating g^1, g^8, and g^7 [upon eliminating Z_3 and Z_4 by Eqs. (3.1b) and (3.1c)]. If we consider the range of indices a, b, and c where g and F are diagonal, then for the case where F_a, F_b, and F_c are all nonzero, one finds

$$g^1{}_{abc}=F_am_a{}^2(g_a)^{-1}\{-3(\mu_a)^{-2}g^7{}_{abc}+\tfrac{1}{2}(F_aF_bF_c)^{-1}[2F_b{}^2 -F_a{}^2+\mu_a{}^{-2}(\mu_c{}^2F_c{}^2-\mu_b{}^2F_b{}^2)]C_{abc}\}, \quad (3.19)$$

$$g^8{}_{abc}=\tfrac{3}{2}[(\mu_b)^{-2}+(\mu_c)^{-2}]g^7{}_{abc} +\tfrac{1}{4}(F_aF_bF_c)^{-1}[(\mu_b)^{-2}-(\mu_c)^{-2}] \times(F_b{}^2\mu_b{}^2+F_c{}^2\mu_c{}^2-F_a{}^2\mu_a{}^2)C_{abc}, \quad (3.20)$$

where $g^7{}_{abc}$ is undetermined. [Note that $g^7{}_{abc}$ also cancels out when Eqs. (3.19), (3.20) are inserted into Eq. (3.18).] A subcase of Eqs. (3.19) and (3.20), when two of the subscripts correspond to strange mesons, was derived in the analysis of the K_{l3} decay.[4]

If two of the F_a, F_b, and F_c vanish and one is nonzero, then parity and strangeness conservation imply that $g^i{}_{abc}=0$ for all i. If $F_a=0$ but $F_b\neq 0$ and $F_c\neq 0$, one finds

$$g^1{}_{abc}=m_a{}^2(g_a)^{-1}C_{abc} \quad (3.21)$$

and g^7 and g^8 are arbitrary. (Note that for this case, b and c must be in the same isotopic multiplet.) Other cases may be considered, as well as channels involving mixing, but we shall not pursue the matter further here.

Finally, we should like to note that no assumptions about the nature of the σ commutators, and hence no assumption about the nature of chiral breakdown has been made in any of the above analysis. In Sec. IV we shall see the implications of pole dominance of the σ commutators.

IV. STRUCTURE OF CURRENTS FOR N-POINT FUNCTIONS

In this section we consider the extension of the preceding analysis to N-point processes. As was shown in Papers I and II, an N-point function can be evaluated under the assumption of single-meson saturation by calculating tree and seagull diagrams obtained from an appropriately constructed effective Lagrangian. The effective Lagrangian which represents the N-point function will contain (in addition to $\mathcal{L}_{(3)}$ described in the previous sections) interaction terms which are products of up to N field operators. The Lagrangians considered will be restricted as were the three-point Lagrangians; only those interaction Lagrangians which can be written in the first-order formalism without explicit derivatives will be considered.[9] No chiral or $SU(3)$ symmetry is imposed on the Lagrangian; the coupling constants are determined by imposing the CCR and PCC, as was done for the three-point Lagrangian.

In discussing the N-point functions, we will use a different phase convention for the κ meson than was used in the three-point discussion. This change is made so that the scalar-meson octet can be treated more uniformly in analyzing the couplings. For any combination of three of the nonstrange mesons, for which G parity is defined, only one type of coupling, either an "f-type" (antisymmetric) or "d-type" (symmetric) is allowed and this type is the same for the eighth component as it is for the first three.[10] In fact, for any three mesons, the coupling is f-type if the number of scalar mesons is even and is d-type otherwise.[11] In deciding what types of couplings in are allowed it is useful to define the phase of the strange mesons so that the above rule also holds for them. This choice of phase then results in a κ which is $-i$ times the κ used in the three-point discussion of Sec. II.

The above convention leads to a change in the form of the divergence of the strange-vector current. Thus

$$\partial_\mu V^\mu = -iF_\kappa \mu_\kappa^2 \kappa. \tag{4.1}$$

In Hermitian component form, Eq. (4.1) becomes

$$\begin{aligned}\partial_\mu V^\mu{}_4 &= F_\kappa \mu_\kappa^2 \kappa_5,\\ \partial_\mu V^\mu{}_5 &= -F_\kappa \mu_\kappa^2 \kappa_4,\end{aligned} \tag{4.2}$$

and similarly for the 6 and 7 components.

We can now assign a charge-conjugation signature (CS) to an entire octet. It is equal to λ where

$$C\Omega C^{-1} = \lambda \Omega^T.$$

Ω is the octet written in 3×3 matrix form and C is the charge-conjugation operator. It is also equal to the G parity of the isotopic singlet. The scalar octet, pseudoscalar octet and axial-vector octet all have positive CS. The vector octet has a negative CS.[12] Charge-conjugation invariance is maintained if three fields with total CS[13] positive are coupled with d-type couplings and those with total CS negative are coupled with f-type couplings.

As was the case for the $SU(2)\times SU(2)$ treatment in II, we choose the currents to be quadratic in canonical fields, that is, we require now Eqs. (2.14) and (2.15) to hold exactly (and not just to second order). Physically, this requires that the commutators of currents with scalar fields be dominated by terms linear in the scalar-particle fields:

$$[V^0{}_a(x), s_b(y)]\delta(x^0-y^0) = i(\beta_{abc}s_c + F_{ab})\delta^4(x-y). \tag{4.3}$$

Thus the generalized "σ commutator" is also required to satisfy single-particle saturation. More complicated possibilities, e.g., involving a higher polynomial of fields on the right-hand side of Eq. (4.3), can be considered but we will not do so here. Using Eq. (2.15) with Eq. (4.3) shows that

$$\beta_{abc} = Z_{1abc}. \tag{4.4}$$

Once the quadratic form of V^0, Eq. (2.15), is assumed exact, one can then investigate any additional CCR requirements on the Z's and the Y's defined in Eq. (2.16), since the current algebra can now be computed to all orders. Again we write

$$\begin{aligned}V^\mu &\equiv V^\mu{}_{(1)} + V^\mu{}_{(2)},\\ V^i{}_{(2)} &= 0,\end{aligned} \tag{4.5}$$

where $V_{(1)}$ and $V_{(2)}$ symbolize the linear and quadratic parts, respectively, of the expansion on V in canonical variables. We thus see that the form of the canonical commutation relations gives the following for the complete content of the equal-time CCR. We write (suppressing spatial variables)

$$[V^0{}_{(1)}, V^\mu{}_{(1)}] = c\text{-No.}, \tag{4.6a}$$

$$[V^0{}_{a(1)}, V^0{}_{b(2)}]\delta^\mu{}_0 + [V^0{}_{a(2)}, V^\mu{}_{b(1)}] = iC_{abc}V^\mu{}_{c(1)}\delta^3, \tag{4.6b}$$

$$[V^0{}_{a(2)}, V^0{}_{b(2)}] = iC_{abc}V^0{}_{c(2)}\delta^3. \tag{4.6c}$$

Only the last equation is yet to be satisfied. The resulting algebraic conditions on the Z's and the Y's are given and solved in Appendix A. One finds there that Eq. (4.6c) implies that Y_1 and Y_2 must both vanish and also that

$$Z_{1abc} = (S^{-1})_{bd}A_{ade}S_{ec}, \tag{4.7}$$

where[14]

$$\begin{aligned}A_{abc} &= f_{abc} \quad \text{if } a \text{ is a natural-parity component}\\ &= d_{abc} \quad \text{if } a \text{ and } b \text{ are unnatural-parity components}\\ &= -d_{abc} \quad \text{if } a \text{ and } c \text{ are unnatural-parity components}\end{aligned} \tag{4.8}$$

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[10] We stress that no over-all f or d symmetry is assumed. By an f-type coupling we mean merely one which is restricted to coupling only those components for which f_{abc} is nonzero, while a coupling has f symmetry if the specific $SU(3)$ f_{abc} are used. In other words, an f-type coupling is any antisymmetric, isotopically invariant coupling and a d-type coupling is any symmetric, isotopically invariant coupling.

[11] For example, the $\pi\rho\omega$ system has a d-type coupling. Changing the ω to ρ gives the $\pi\rho\rho$ system. The d-type coupling for three isovectors is zero. Changing the π to η gives the $\eta\rho\rho$ system which also has a d-type coupling.

[12] If a nonet is formed, the ninth meson will be assumed to have the same G parity as the eighth. The same CS parity will be considered a property of the nonet.

[13] The total CS of a product of fields is the product of the individual CS's.

[14] Natural parity refers to the $J^P=0^+, 1^-, 2^+, \ldots$ sequence; unnatural parity refers to the $J^P=0^-, 1^+, 2^-, \ldots$ sequence.

and S is diagonal, except for possible mixing between eighth and ninth components of nonets. The elements of S are arbitrary except that they must be the same within a given isotopic multiplet [for $SU(2)$ invariance]. Comparing this form for Z_1 with Eqs. (4.3) and (4.4) shows that the scalar mesons form a "scaled" (3*,3) + (3,3*) representation of the chiral algebra. The scaling matrix S arises from the fact that the fields of the effective Lagrangian are renormalized fields. Thus the renormalized fields $\tilde{s}_a \equiv S_{ab}s_b$ transform according to the usual (3,3*)+(3,3*) representation and S_{ab} are proportional to the square root of the wave-function renormalization matrix.[15] *Thus the* (3,3*)+(3*,3) *form of chiral breakdown arises from the combined current-algebra and pole-dominance conditions.*

The vanishing of Y_1 and Y_2 allows one to relate $h^1{}_{abc}$ and $h^3{}_{abc}$ in terms of $h^2{}_{abc}$ by Eqs. (2.16e) and (2.16f). Equation (2.23f) implies that $h^4{}_{abc}$ vanishes for values of a such that $F_a \neq 0$. Note that the choice $S=1$ would be consistent with $SU(3)$ symmetric couplings and so the value of S describes, in part, the breakdown of $SU(3)$ invariance.

A second important result deduced in Appendix A is following: *If one assumes pole dominance of the general σ commutator, Eq. (4.3), with the assignment of positive G parity to the scalar isotopic singlets and negative G parity of the scalar isotopic triplet, then the $SU(3)\times SU(3)$ current algebra cannot be satisfied by introducing octets of particles.* If pole dominance of the σ commutator were relaxed, the algebra could be satisfied with octets. If only the G parity of the scalar mesons assignments were changed, however, the formalism would require unphysical conservation relations, such as exact conservation of the axial-vector isotopic singlet and triplet currents. *The pole dominance of the σ commutators is, however, consistent with the introduction of nonets of mesons.* It is not necessary to use a $U(3)\times U(3)$ algebra with nonets of mesons to achieve a consistent solution, but it is sufficient as well as natural.

The three-point PCC conditions of Sec. II give additional restrictions. To determine and express these restrictions simply, it is convenient to write the isotopic singlet currents in different linear combinations than is customary. We define for the vector currents

$$V^\mu{}_A \equiv (\sqrt{\tfrac{2}{3}})V^\mu{}_8 - (\sqrt{\tfrac{1}{3}})V^\mu{}_9\,,\qquad V^\mu{}_B \equiv (\sqrt{\tfrac{1}{3}})V^\mu{}_8 + (\sqrt{\tfrac{2}{3}})V^\mu{}_9\,,\tag{4.9}$$

and similarly for the axial-vector currents. The consequences of this transformation on the algebra and $U(3)$ coupling constants are detailed in Appendix B. The resulting currents are no longer separated according to $SU(3)$ singlet and octet, but are constructed in such a manner that the f and d $U(3)$ invariant coupling constants will not couple the two isotopic singlet states. Since no *a priori* $SU(3)$ invariance is imposed on the formalism, this is a representation which is equally as valid as the usual one. However, as discussed above, the current algebra and the G-parity assignments of the scalar mesons require nonets, and the PCC conditions become diagonal in the representation given by Eq. (4.9). Therefore, this representation will be used henceforth.

The pertinent PCC conditions are given and solved in Appendix C. The results are as follows.

1. The elements of F_{ab} for the eighth and ninth pseudoscalar mesons are determined to within an arbitrary mixing angle, in terms of their masses and the F's and masses of the pion, kaon, and κ meson. The matrix elements of F_{ab} are listed in Eqs. (C23) and (C24)

2. The scaling matrix S for Z_1 is determined to within a choice of solution to a quadratic equation for the pseudoscalar and κ channels. The elements of S and the elements of S^{-1} (for the isotopic singlets) are listed in Eqs. (C26), (C27), and (C21).

3. The product $F_\kappa\mu_\kappa$ is forbidden to lie within a certain range. The requirement is[16]

$$|F_\kappa\mu_\kappa| > |F_K\mu_K| + |F_\pi\mu_\pi| \quad \text{or} \quad |F_\kappa\mu_\kappa| < \big|\,|F_K\mu_K| - |F_\pi\mu_\pi|\,\big|\,.\tag{4.10}$$

4. The scalar-scalar-scalar coupling constant $g^7{}_{abc}$ is determined to within a sign for those channels connecting the κ meson and the pseudoscalar mesons. The expressions are listed in Eqs. (C29) and (C30).

V. EFFECTIVE LAGRANGIAN WHICH PRODUCES QUADRATIC CURRENTS

Section IV described the current-algebra conditions on the three-point coupling constants arising from the requirement that V^0 can be made quadratic in the canonical fields, i.e., that the σ commutators are pole dominated. This condition on V^0 puts constraints on the structure of the N-point Lagrangian since in general a term involving a product of N fields in $\mathcal{L}_I$ could produce a contribution to V^0 of order $N-1$. In this section we determine the form that $\mathcal{L}_I$ must have so that in fact V^0 is restricted to be quadratic.

The solution of this problem can be obtained more simply if the effective Lagrangian is written in second-order formalism, rather than in the first-order formalism. The second-order formalism Lagrangian $\mathcal{L}^2$ is obtained from the first-order formalism Lagrangian $\mathcal{L}^1$ by using the additional equations of motion obtained by varying $\mathcal{L}^1$ with respect to the auxiliary variables (field

[15] S. L. Glashow and S. Weinberg, Phys. Rev. Letters **20**, 224 (1968); I. S. Gerstein, H. J. Schnitzer, and S. Weinberg, Phys. Rev. **175**, 1873 (1968).

[16] This result has previously been obtained by S. L. Glashow and S. Weinberg [Phys. Rev. Letters **20**, 224 (1968)] by assuming that the symmetry-breaking part of the Lagrangian is proportional to local field operators that transform like the (3,3*)+(3*,3) representation of $SU(3)\times SU(3)$. In the present approach the assumptions of current algebra and single-particle dominance of the σ commutators is sufficient to deduce Eqs. (4.10). Similar conclusions have been reached by L. K. Pande, *ibid.* **23**, 353 (1969).

strengths) to eliminate them from $\mathcal{L}^1$. For instance, the equation defining the scalar auxiliary s_μ is

$$s_\mu = \partial_\mu s - \delta\mathcal{L}_I{}^1/\delta s^\mu, \tag{5.1}$$

where $\mathcal{L}_I{}^1$ is the first-order formalism interaction Lagrangian. $\mathcal{L}_I{}^1$ contains no explicit derivatives in the present formalism by construction. Thus, every time Eq. (5.1) is used to eliminate an auxiliary field, a term with at most one derivative acting on any one field is generated, plus possibly terms of higher order in field products which still contain auxiliary fields. Auxiliary fields can be eliminated from $\mathcal{L}^1$ by this program to give an $\mathcal{L}^2$ which contains at most one derivative acting on any one field. Requiring this property is the second-order formalism form of the "smoothness" restriction that $\mathcal{L}_I{}^1$ have no explicit derivatives; there is a one-to-one correspondence between an $\mathcal{L}^2$ satisfying the second-order form of smoothness and the $\mathcal{L}^1$ satisfying the first-order form. The coupling constants of the corresponding three-point $\mathcal{L}$'s turn out to be identical. The correspondence becomes more complex for the higher-order terms.

$\mathcal{L}^1$ and $\mathcal{L}^2$ are numerically equal by construction. Thus

$$\mathcal{L}^2[s;\partial_\mu s;\ldots] = \mathcal{L}^1[s;\partial_\mu s; s_\mu(\partial_\mu s\cdots);\ldots], \tag{5.2}$$

where the explicit dependence on $\partial_\mu s$ in $\mathcal{L}^1$ comes only from the free-field part of the Lagrangian. We have then

$$\frac{\partial\mathcal{L}^2}{\partial(\partial_\mu s)} = \frac{\partial\mathcal{L}^1}{\partial(\partial_\mu s)} + \frac{\partial\mathcal{L}^1}{\partial s_\alpha}\frac{\partial s_\alpha}{\partial(\partial_\mu s)}. \tag{5.3}$$

Now, the free-field part of $\mathcal{L}^1$, Eq. (2.7a), does not depend on derivatives of s_μ, nor does $\mathcal{L}_I{}^1$, by definition. Thus the first-order Lagrange equations read

$$\partial\mathcal{L}^1/\partial s_\mu = 0 \tag{5.4}$$

and hence the useful correspondence

$$\frac{\partial\mathcal{L}^2}{\partial(\partial_\mu s)} = \frac{\partial\mathcal{L}^1}{\partial(\partial_\mu s)} = -s^\mu \tag{5.5}$$

holds. Analogously,

$$\frac{\partial\mathcal{L}^2}{\partial(\partial_\mu v_\nu)} = \frac{\partial\mathcal{L}^1}{\partial(\partial_\mu v_\nu)} = -G^{\mu\nu}. \tag{5.6}$$

Using these results in the Lagrange equations of motion gives (for either $\mathcal{L}^1$ or $\mathcal{L}^2$)

$$\delta\mathcal{L}/\delta s + \partial_\mu s^\mu = 0, \tag{5.7}$$

$$\delta\mathcal{L}/\delta v_\mu + \partial_\rho G^{\rho\mu} = 0. \tag{5.8}$$

These correspondences provide the tools necessary to express the functional dependence of the currents on $\mathcal{L}^2$. (The simplicity of these relations is a consequence of the original choice of limiting the momentum transfer at each vertex by not allowing any explicit derivative couplings in the first-order formalism Lagrangian.) In terms of the auxiliary fields, the assumption that V^0 is quadratic in canonical fields implies that an exact equation for V^μ is (Eqs. 2.15)

$$V^\mu{}_a = (g_{ab}/m_b{}^2)\partial_\nu G^{\nu\mu}{}_b + F_{ab}s^\mu{}_b + Z_{1abc}s^\mu{}_b s_c + Z_{2abc}v_{\nu b}G^{\nu\mu}{}_c + Z_{3abc}s_b\partial_\nu G^{\nu\mu}{}_c + Z_{4abc}\partial_\nu s_b G^{\nu\mu}{}_c. \tag{5.9}$$

(We have already seen in Sec. IV and Y_1 and Y_2 are required to be zero in this formalism.) Using Eqs. (5.5), (5.6), and (5.8) to eliminate the auxiliary fields gives the following result:

$$V^\mu{}_a = -\frac{g_{ab}}{m_b{}^2}\frac{\delta\mathcal{L}}{\delta v_{\mu b}} - F_{ab}\frac{\delta\mathcal{L}}{\delta\partial_\mu s_b} - Z_{1abc}s_c\frac{\delta\mathcal{L}}{\delta\partial_\mu s_b} - Z_{2abc}v_{\rho b}\frac{\delta\mathcal{L}}{\delta\partial_\rho v_{\mu c}} - Z_{3abc}s_b\frac{\delta\mathcal{L}}{\delta v_{\mu c}} - Z_{4abc}\partial_\rho s_b\frac{\delta\mathcal{L}}{\delta\partial_\rho v_{\mu c}}. \tag{5.10}$$

Since $V^\mu{}_a$ is a linear combination of v^μ and $\partial^\mu s$ [Eq. (2.8)], Eq. (5.10) may be viewed as a functional differential equation for the second-order formalism Lagrangian.

It is useful at this point, for the purpose of solving the functional differential equation, to define new variables and introduce a linear combination of v^μ and $\partial^\mu s$ that is "orthogonal" to the linear combination of $\delta/\delta v^\mu$ and $\delta/\delta\partial_\mu s$ that appears in Eq. (5.10). The same procedure was followed in the $SU(2)\times SU(2)$ analysis in II, where a combination of v^μ, s^μ, $G^{\mu\nu}$, and s was found which did not appear in the $SU(2)$ analog to (5.10). The appropriate quantity here is

$$\gamma^\mu{}_a \equiv (\delta_{ab} - F_{ca}W_{cd}{}^{-1}F_{db})\partial^\mu s_b - F_{ca}W_{cd}{}^{-1}g_{db}v^\mu{}_b - Z_{1dab}W_{dc}{}^{-1}s_b V^\mu{}_c, \tag{5.11}$$

where

$$W_{ab} \equiv g_{ac}g_{bc}/m_c{}^2 + F_{ac}F_{bc}. \tag{5.12}$$

[The first Weinberg sum rule, Eqs. (3.12) and (3.14), evaluates W_{ab}. Alternatively in terms of the components A and B of Eq. (4.9) and Appendix B, one may write

$$W_{A,a} = 0 = W_{B,a}, \quad a = 1,\ldots,8,\quad \bar{1},\ldots,\bar{8} \tag{5.13a}$$

$$W_{A,A} = g_\rho{}^2/m_\rho{}^2 + \tfrac{1}{3}\delta_V, \quad W_{B,B} = g_\rho{}^2/m_\rho{}^2 + \tfrac{2}{3}\delta_V, \tag{5.13b}$$

$$W_{A,B} = W_{B,A} = -\tfrac{1}{3}\sqrt{2}\delta_V, \tag{5.13c}$$

where δ_V is undetermined. Similar relationships, with δ_A, hold for the axial-vector components, I and II.] It is also useful to define

$$H_{\mu\nu a} \equiv \partial_\mu v_{\nu a} - \partial_\nu v_{\mu a} + g_{ad}{}^{-1}W_{ed}{}^{-1}C_{ebc}V_{\mu b}V_{\nu c}. \tag{5.14}$$

With this choice of variables, Eq. (5.10) becomes simply

$$V^\mu{}_a = -W_{ab}\frac{\delta\mathcal{L}(V^\mu,\gamma^\mu,H^{\mu\nu},s)}{\delta V_{\mu b}}, \tag{5.15}$$

where use has been made of the three-point current commutator conditions Eqs. (3.1). Equation (5.15) has the obvious solution,

$$\mathcal{L}=-\tfrac{1}{2}V^{\mu}{}_{a}(W^{-1})_{ab}V_{\mu b}+\mathcal{L}'(\gamma,H,s)\,. \tag{5.16}$$

The quadratic parts of $\mathcal{L}'$ are determined so that $\mathcal{L}$ have the required free-field parts. One has then

$$\mathcal{L}=-\tfrac{1}{2}V^{\mu}{}_{a}(W^{-1})_{ab}V_{\mu b}-\tfrac{1}{4}H^{\mu\nu}{}_{a}H_{\mu\nu b} -\tfrac{1}{2}(\delta_{ab}+m_e{}^2g_{ed}{}^{-1}F_{da}g_{ec}{}^{-1}F_{cb})\gamma^{\mu}{}_{a}\gamma_{\mu b} -\tfrac{1}{2}\mu_a{}^2s_a{}^2+\mathcal{L}_I(\gamma,H,s)\,, \tag{5.17}$$

where $\mathcal{L}_I(\gamma,H,s)$ is any Lorentz, isotopic, and octet CS-invariant structure of γ, H, and s that involves products of at least three fields. Some of these terms (all of those containing the components of s_a involving unconserved indices) will be determined by the PCC conditions and will be discussed in Sec. VI. Note that γ and H are primarily derivative-coupling terms. Any term in $\mathcal{L}_I$ containing only these two quantities is determined neither by the current algebra nor by PCC. Thus, even with the ground rule that $\mathcal{L}$ shall have no more than one derivative acting on any one field, the terms in $\mathcal{L}$ representing the highest powers of momentum transfer, i.e., those with the most derivatives, are not determined by the current algebra.

The relations between the functional derivatives of the new and old variables give some insight concerning their meaning. For instance,

$$\frac{\delta\mathcal{L}}{\delta H^{\mu\nu}}=\frac{1}{2}\frac{\delta\mathcal{L}}{\delta\partial^{\mu}v^{\nu}}=-\tfrac{1}{2}G_{\mu\nu} \tag{5.18}$$

by Eq. (5.6). Hence,

$$G_{\mu\nu a}=H_{\mu\nu a}-2\delta\mathcal{L}_I(\gamma,H,s)/\delta H^{\mu\nu}{}_{a}\,. \tag{5.19}$$

Thus $H_{\mu\nu}$ differs from the canonical auxiliary to v^{μ} only by terms in $\mathcal{L}_I$ over and above the minimal terms required to satisfy the current algebra. The definition of γ^{μ} was so chosen that

$$\left[\frac{g_{ab}}{m_b{}^2}\frac{\delta}{\delta v_{\mu b}}+F_{ab}\frac{\delta}{\delta\partial_{\mu}s_b} +s_c\left(Z_{1abc}\frac{\delta}{\delta\partial_{\mu}s_b}+Z_{3abc}\frac{\delta}{\delta v_{\mu b}}\right)\right]\gamma^{\rho}{}_{c}=0\,, \tag{5.20}$$

so that functions of γ^{μ} would not affect Eq. (5.10).

VI. PCC CONDITIONS ON N-POINT LAGRANGIAN

Since Eq. (5.9) is exact [a consequence of the condition that V^0 be quadratic in the canonical fields (pole dominance of the σ commutators)], a simple relation can be obtained for the divergence of V^{μ}. For those currents that are conserved, these conditions merely require, of course, that $\mathcal{L}$ be symmetric under the transformations which they induce. For the unconserved currents, however, the relation produces a functional differential equation which determines the dependence of $\mathcal{L}$ on s_a for those indices corresponding to the unconserved currents. In this section we examine these equations and also show that they are simultaneously integrable. It is thus a straightforward task to construct this s_a dependence of $\mathcal{L}_I$ to any desired order.

The divergence of (5.10) is computed using (5.7) and (5.8) to get

$$\partial_{\mu}V^{\mu}{}_{a}=-F_{ab}\frac{\delta\mathcal{L}}{\delta s_b}-Z_{1abc}\frac{\delta\mathcal{L}}{\delta s_b}s_c-Z_{1abc}\frac{\delta\mathcal{L}}{\delta\partial_{\mu}s_b}\partial_{\mu}s_c -Z_{3abc}\partial_{\mu}s_b\frac{\delta\mathcal{L}}{\delta v_{\mu c}}+Z_{2abc}\partial_{\mu}v_{\nu b}\frac{\delta\mathcal{L}}{\delta\partial_{\mu}v_{\nu c}}+Z_{2abc}v_{\nu b}\frac{\delta\mathcal{L}}{\delta v_{\nu c}} +Z_{4abc}\partial_{\nu}s_b\frac{\delta\mathcal{L}}{\delta v_{\nu c}}. \tag{6.1}$$

The right-hand side must equal $F_{ab}\mu_b{}^2s_b$ by PCC. This equation also is made more tractable by changing to the V, H, γ variables of Eqs. (5.11) and (5.14). One finds

$$F_{ab}\frac{\delta\mathcal{L}}{\delta s_b}=-Z_{1abc}\left(\frac{\delta\mathcal{L}}{\delta s_b}s_c+\frac{\delta\mathcal{L}}{\delta\gamma^{\mu}{}_{b}}\gamma^{\mu}{}_{c}\right) -C_{abc}\left(\frac{\delta\mathcal{L}}{\delta V^{\mu}{}_{b}}V^{\mu}{}_{c}+g_{bd}g^{-1}{}_{ec}\frac{\delta\mathcal{L}}{\delta H^{\mu\nu}{}_{d}}H^{\mu\nu}{}_{e}\right)-F_{ab}\mu_b{}^2s_b. \tag{6.2}$$

If a is a conserved component, i.e., if $F_{ab}=0$, then $Z_{1abc}=f_{ade}S^{-1}{}_{bd}S_{ec}$ by Eq. (4.7). Equation (6.2) becomes for this case

$$0=f_{abc}S^{-1}{}_{db}S_{ce}\left(\frac{\delta\mathcal{L}}{\delta s_d}s_e+\frac{\delta\mathcal{L}}{\delta\gamma^{\mu}{}_{d}}\gamma^{\mu}{}_{e}\right) +f_{abc}\left(\frac{\delta\mathcal{L}}{\delta V^{\mu}{}_{b}}V^{\mu}{}_{c}+g_{bd}g^{-1}{}_{ec}\frac{\delta\mathcal{L}}{\delta H^{\mu\nu}{}_{d}}H^{\mu\nu}{}_{e}\right). \tag{6.3}$$

Equation (6.3) requires only that $\mathcal{L}$ be constructed to be invariant under isotopic and hypercharge rotations, a well-known consequence of conserved currents, and the motivation for originally requiring $\mathcal{L}$ to be constructed to be isotopically invariant.

For the unconserved components, $\mathcal{L}$ is proportional to the chiral symmetry breakdown in the V and H couplings. Since the only V^{μ} term in $\mathcal{L}$ allowed by the quadratic current condition, Eq. (5.16), is chirally symmetric (as a consequence of the first Weinberg sum rule), the $\delta\mathcal{L}/\delta V^{\mu}$ term of Eq. (6.2) vanishes. Thus Eqs. (6.2) and (5.15) are mutually consistent, as a consequence of the absence of q-No. ST.

In the subspace where F_{ab} has an inverse, Eq. (6.2) yields

$$\frac{\delta\mathfrak{L}}{\delta s_a}=-F^{-1}{}_{ar}Z_{1rbc}\left(\frac{\delta\mathfrak{L}}{\delta s_b}s_c+\frac{\delta\mathfrak{L}}{\delta\gamma^\mu{}_b}\gamma^\mu{}_c\right)-F^{-1}{}_{ar}C_{rbc}\times\left(\frac{\delta\mathfrak{L}}{\delta V^\mu{}_b}V^\mu{}_c+g_{bd}g^{-1}{}_{ec}\frac{\delta\mathfrak{L}}{\delta H^{\mu\nu}{}_d}H^{\mu\nu}{}_e\right)-\mu_a{}^2s_a\,,\quad(6.4)$$

where it is understood that all indices of $F^{-1}{}_{ar}$ range only over the unconserved subspace. Equations (6.4) are a set of functional differential equations to determine the dependence of $\mathfrak{L}$ on the unconserved components of s_a. However, for a solution to exist, it is necessary that the integrability condition

$$\frac{\delta^2\mathfrak{L}}{\delta s_b\delta s_a}=\frac{\delta^2\mathfrak{L}}{\delta s_a\delta s_b}\quad(6.5)$$

be satisfied. A verification of Eq. (6.5) is most conveniently carried out by induction. The proof is analogous to that in the $SU(2)\times SU(2)$ case (Appendix A of Paper II) and so we only sketch the details here.

Let $\mathfrak{L}^{(n)}$ be the contribution of all terms to $\mathfrak{L}$ containing n fields. Then Eqs. (6.4) read

$$\frac{\delta\mathfrak{L}^n}{\delta s_a}=-(F^{-1})_{ar}Z_{1rbc}\left(\frac{\delta\mathfrak{L}^{n-1}}{\delta s_b}s_c+\frac{\delta\mathfrak{L}^{n-1}}{\delta\gamma^\mu{}_b}\gamma^\mu{}_c\right)-F^{-1}{}_{ar}C_{rbc}\left(\frac{\delta\mathfrak{L}^{n-1}}{\delta V^\mu{}_b}V^\mu{}_c+g_{bd}g^{-1}{}_{ec}\frac{\delta\mathfrak{L}^{n-1}}{\delta H^{\mu\nu}{}_d}H^{\mu\nu}{}_e\right)\quad(6.6)$$

for $n\geq 3$. Hence

$$\frac{\delta^2\mathfrak{L}^n}{\delta s_b\delta s_a}=-F^{-1}{}_{ar}Z_{1rmn}\left(\frac{\delta^2\mathfrak{L}^{n-1}}{\delta s_m\delta s_b}s_n+\frac{\delta^2\mathfrak{L}^{n-1}}{\delta\gamma^\mu{}_m\delta s_b}\gamma^\mu{}_n\right)-F^{-1}{}_{ar}C_{rmn}\left(\frac{\delta^2\mathfrak{L}^{n-1}}{\delta s_b\delta V^\mu{}_m}V^\mu{}_n+g_{md}g^{-1}{}_{en}\frac{\delta^2\mathfrak{L}^{n-1}}{\delta s_b\delta H^{\mu\nu}{}_d}H^{\mu\nu}{}_e\right)-F^{-1}{}_{ar}Z_{1rmb}\frac{\delta\mathfrak{L}^{n-1}}{\delta s_m}.\quad(6.7)$$

The induction proof proceeds by assuming that $\mathfrak{L}^{n-1}$ is integrable and so an $\mathfrak{L}^{n-1}$ exists obeying

$$\frac{\delta\mathfrak{L}^{n-1}}{\delta s_a\delta s_b}=\frac{\delta\mathfrak{L}^{n-1}}{\delta s_b\delta s_a}.\quad(6.8)$$

Here a and b range over both the conserved and unconserved indices. The second derivatives in Eq. (6.7) involving γ^μ, s, etc., and the term $\delta\mathfrak{L}^{n-1}/\delta s_m$ can be calculated from Eq. (6.6) by replacing $n\to n-1$. One finds by direct calculation then that

$$\frac{\delta^2\mathfrak{L}^n}{\delta s_a\delta s_b}-\frac{\delta^2\mathfrak{L}^n}{\delta s_b\delta s_a}=-F^{-1}{}_{ar}Z_{1rmb}\frac{\delta\mathfrak{L}^{n-1}}{\delta s_m}+F^{-1}{}_{ar}Z_{1rst}F^{-1}{}_{bm}Z_{1mqs}\left(\frac{\delta\mathfrak{L}^{n-2}}{\delta s_q}s_t+\frac{\delta\mathfrak{L}^{n-2}}{\delta\gamma^\mu{}_q}\gamma^\mu{}_t\right)+F^{-1}{}_{ar}C_{rst}F^{-1}{}_{bm}C_{mqs}\frac{\delta\mathfrak{L}^{n-2}}{\delta V^\mu{}_q}V^\mu{}_t+F^{-1}{}_{ar}Z_{2rts}F^{-1}{}_{bm}Z_{2msq}\frac{\delta\mathfrak{L}^{n-2}}{\delta H^{\mu\nu}{}_q}H^{\mu\nu}{}_t-(a\leftrightarrow b)\,.\quad(6.9)$$

Using now the current-algebra conditions (2.21c) and (A5)–(A7) and the full PCC condition (6.2), one may directly verify that the right-hand side of Eq. (6.9) vanishes. This result, plus the fact that $\mathfrak{L}^2$ and $\mathfrak{L}^3$ have been explicitly constructed, completes the proof of the integrability. Note that Eqs. (A6) and (A7) were obtained from the commutators of the quadratic parts of V^0, demonstrating the inherent compatibility of the current-algebra and PCC equations with the assumption of pole dominance of the σ commutators.

The integrability conditions guarantee the existence of a solution of Eqs. (6.4). Thus if we denote by Φ_a and σ_a the unconserved and conserved components, respectively, of the 18 spin-zero mesons s_a, then one may write the $\mathfrak{L}_I$ of Eq. (5.17) as

$$\mathfrak{L}_I(\gamma,H,s)=\mathfrak{L}_{I(1)}(\gamma,H,\Phi,\sigma)+\mathfrak{L}_{I(2)}(\gamma,H,\sigma)\,,\quad(6.10)$$

where $\mathfrak{L}_{I(1)}$ contains at least one factor of Φ and is completely determined by Eqs. (6.4). $\mathfrak{L}_{I(2)}$ is arbitrary and undetermined either by the current-algebra or PCC conditions. In practice, it is straightforward to determine $\mathfrak{L}_{I(1)}$ to any desired order. Thus, if one inserts $\mathfrak{L}_I{}^3$ of Eq. (2.7b) on the right-hand side of Eq. (6.6), one may integrate to obtain $\mathfrak{L}_{I(1)}{}^4$, etc.

VII. SUMMARY AND CONCLUSIONS

In this paper a general solution has been given to the problem of determining hard-meson N-point functions in chiral $SU(3)\times SU(3)$ current algebra within the framework of pole dominance. The current-algebra constraints are realized by constructing an effective Lagrangian, from which the N-point functions may be obtained by calculating the tree and seagull diagrams. The effective Lagrangian is not presumed to have any fundamental significance and is merely a convenient device for realizing the current-current commutation relations (CCR) and partial-current conservation (PCC) conditions. No *a priori* assumptions as to the size or nature of the $SU(3)$ or chiral $SU(3)\times SU(3)$-symmetry breaking are made.

Since the calculations of the previous sections are somewhat lengthy, we give here first a brief summary of the results, in order to obtain some perspective on what has been done. In Sec. II the constraints imposed by the CCR and PCC conditions on the coupling constants of the three-point functions (cubic part of the effective Lagrangian) were obtained. Section III is concerned with the general solution of these algebraic equations. These solutions determine some of the coupling constants completely and allow others to be expressed in terms of a set of undetermined coupling constants. The absence of q-No. ST, when combined with the other current-algebra conditions leads, of course, to the general first Weinberg sum rules. When one goes beyond three-point functions, one encounters a new element, viz., the σ commutators, and it is necessary to make some assumption concerning them. In Sec. IV, it is assumed that $V^0{}_a$ is quadratic in the canonical variables, i.e., that *the σ commutators are pole dominated by the spin-zero mesons* [Eq. (4.3)]. This is within the general spirit of pole dominance of the currents themselves, but implies an additional assumption. Section IV outlines the conditions imposed on the three-point function coupling constants (over and above those obtained in Sec. III) due to this extra condition (details are given in the appendices). The σ-commutator hypothesis can only be achieved for effective Lagrangians having a specific form. This form is calculated in Sec. V [Eq. (5.17)]. Finally, Sec. VI completes the analysis by determining the PCC conditions on the general N-point Lagrangian. These are a set of coupled functional differential equations [Eq. (6.2)] which are shown to be simultaneously integrable. They determine the dependence of the Lagrangian on the spin-zero fields Φ_a whose indices correspond to the unconserved currents; they may be integrated straightforwardly to any desired order.

The complexity of the results are in large part due to the simultaneous imposition of the CCR and PCC conditions. Thus the former are consistent with the assumption of perfect chiral and $SU(3)$ symmetry, while the latter describes the breakdown of these symmetries. (*A priori* it is not clear just how much symmetry breaking the chirally symmetric CCR will allow.) If one were to assume all the currents were conserved, then it is fairly easy, using the methods of this paper, to obtain general results for arbitrary $SU(n)\times SU(n)$ [or $U(n)\times U(n)$] chiral groups. The complications of much of the analysis given here is due to the need to take into account the particular idiosynchrosies of the physical symmetry breakdowns of the $SU(3)\times SU(3)$ case. This can be seen, for example, in the solutions for the three-point coupling constants (Sec. III), the analysis of the PCC conditions for the general N-point functions (Sec. VI), and elsewhere.

The logical tightness of the simultaneous requirements of CCR and PCC is further illustrated by the additional results obtained when one also assumes pole dominance of the σ commutators. Here one finds that if one makes the usual experimental assignment of positive G parity to the scalar isotopic singlets and negative G parity to the scalar isotopic triplets, then the $SU(3)\times SU(3)$ current algebra *can no longer be satisfied by introducing octets of particles*. The simplest consistent choice is to introduce nonents of mesons. The previous success of the field-current identity then suggests that one consider nonents of currents and examine the $U(3)\times U(3)$ algebra, although this of course is not demanded by the formalism. (Even if one were to alter the G-parity assignments, a consistent octet formalism would require unphysical conservation laws, such as conservation of the isotopic singlet and triplet components of the axial-vector currents, and so be unacceptable.) A further important consequence of the pole dominance of the σ commutator is that the scalar mesons must form a "scaled" $(3,3^*)+(3^*,3)$ representation of the chiral algebra [Eqs. (4.7), (4.8), (4.3), and (4.4)]. The scaling parameters of the matrix S (which vary with the isotopic multiplet) are determined in terms of the PCC parameters F_{ab} and scalar masses μ_a [Eqs. (C26) and (C23)], and hence are governed by the magnitude of the lack of current conservation. The scaling parameters are just the wave-function renormalization constants and arise due to the use here of renormalized fields. That is, the formalism uniquely forces a symmetry breakdown corresponding to a $(3,3^*)+(3^*,3)$ term in the interaction.

The previous discussion points to the fact that the type of symmetry breakdown is a direct consequence of the pole-dominance assumption when these are combined with the current-algebra constraints. This suggests the possibility that the correctness of this choice of symmetry breakdown may be due merely to the approximate dynamical validity of pole dominance, rather than possessing any fundamental origin. On the experimental side, the situation appears mixed. Thus in the $SU(2)\times SU(2)$ subspace the $(3,3^*)+(3^*,3)$ symmetry breaking assumption implies that the σ commutator is an isoscalar and hence that the $I=2$ and $I=0$ $\pi\pi$ scattering lengths obey[17] the Weinberg relation[18] $a^0/a^2\simeq -3.5$. A recent experimental determination of this quantity[19] has yielded a value in very close agreement with the theory ($a^0/a^2\simeq -3.3$). On the other hand, matters are less clear in the strange-particle sector. Thus, as pointed out previously,[20] the assumption of a $(3^*,3)+(3,3^*)$-type breakdown leads to a ξ parameter of the K_{l3} decay amplitude close to zero. While the value of ξ is still quite uncertain, recent experiments[21] appear to favor $\xi\lesssim -0.5$. If ξ indeed is large and nega-

[17] The fact that pole dominance of the σ commutator in $SU(2)\times SU(2)$ chiral algebra deduces Weinberg's assumption on the isoscalar nature of the σ commutator was derived in II.

[18] S. Weinberg, Phys. Rev. Letters **17**, 616 (1966).

[19] L. J. Gutay, F. T. Meiere, and J. H. Scharenguivel, Phys. Rev. Letters **23**, 431 (1969).

[20] R. Arnowitt, M. H. Friedman, and P. Nath, Nucl. Phys. **B10**, 578 (1969).

[21] D. Haidt *et al.*, Phys. Letters **29B**, 691 (1969). See also the recent review of experimental results on the K_{l3} form factors by L. M. Chounet, CERN Report No. 70-14 (unpublished).

tive, then it would be difficult to maintain both pole dominance and $(3^*,3)+(3,3^*)$-type chiral breakdown, and one or both assumptions would have to be modified.

We note that the general formalism described in this paper can be applied directly to a number of problems, e.g., $K\pi$ scattering, K_{l4} decay, $\omega\to 3\pi$ decay, K^+-K^0 mass differences, etc. Some of these will be treated in detail in subsequent papers.

Finally, we briefly compare the results obtained here with other hard-meson discussions of the chiral $SU(3)\times SU(3)$ current algebra. Pande[22] had previously obtained expressions for the four-point functions for the case of channels involving strange mesons. The techniques used are similar to those employed here and pole dominance of the σ commutators is also assumed. The three-point functions for all the $SU(3)$ channels have been examined by Gerstein, Schnitzer, and Weinberg[15,23] using the Ward's-identity approach. When pole dominance is imposed, this analysis is equivalent to the three-point discussion of Sec. II. However, these authors do not carry out the analog of the solution of the coupling constant equations (Sec. III). More recently, a very elegant approach to the general N-point function using the Ward's-identity method has been proposed by Zumino.[24]

APPENDIX A

This appendix will consider solutions to the equations

$$[V^0{}_{a(2)}(x),V^0{}_{b(2)}(y)]\delta(x^0-y^0) = iC_{abc}V_{c(2)}\delta^4(x-y)\,, \quad \text{(A1)}$$

where the subscript (2) denotes those terms in the canonical expansion of V^0 which are quadratic in canonical fields. In particular,

$$V^0{}_{a(2)}\equiv -Z_{1acb}s_bs_{0c}+Z_{2abc}v_{ib}G_{0ic} +Z_{3abc}s_b\partial_iG_{0ic}+Z_{4abc}\partial_is_bG_{0ic} +2\epsilon_{ijk}(Y_{1abc}v_{ib}\partial_iv_{kc}+Y_{2abc}\partial_is_b\partial_jv_{kc})\,. \quad \text{(A2)}$$

The algebraic relations implied by Eq. (A1) can be compactly presented in a matrix notation. The second and third indices of a coupling constant will be considered to be matrix indices and will be suppressed in the notation. For instance, the matrix Z_{1a} will have elements

$$(Z_{1a})_{bc}=Z_{1abc}=(Z_{1a}{}^T)_{cb}\,. \quad \text{(A3)}$$

The structure constants C_{abc} can be represented by the matrix C_a, where

$$(C_a)_{bc}=C_{abc} \quad \text{(A4)}$$

and the algebra representation is

$$[C_a,C_b]=-C_{abc}C_c\,. \quad \text{(A5)}$$

[22] L. K. Pande, Phys. Rev. **184**, 1683 (1969).

[23] I. S. Gerstein and H. J. Schnitzer, Phys. Rev. **175**, 1876 (1968).

[24] B. Zumino (private communication) and lectures at the Brandeis Summer Institute, 1970 (unpublished).

In this notation, the algebraic equations implied by Eq. (A1) take the following form for the integrated algebra:

$$[Z_{1a},Z_{1b}]=-C_{abc}Z_{1c}\,, \quad \text{(A6)}$$

$$[Z_{2a},Z_{2b}]=-C_{abc}Z_{2c}\,, \quad \text{(A7)}$$

$$Z_{1a}{}^TZ_{3b}+Z_{3b}Z_{2a}-Z_{1b}{}^TZ_{3a}+Z_{1b}{}^TZ_{4a}=C_{abc}Z_{3c}\,, \quad \text{(A8)}$$

$$Z_{4b}Z_{2a}-Z_{4a}Z_{2b}+Z_{3a}Z_{2b}+Z_{1a}{}^TZ_{4b}=C_{abc}Z_{4c}\,, \quad \text{(A9)}$$

$$Z_{2a}Y_{1b}+Y_{1b}Z_{2a}{}^T-Z_{2b}(Y_{1a}+Y_{1a}{}^T)=-C_{abc}Y_{1c}\,, \quad \text{(A10)}$$

$$-Z_{4a}Y_{1b}+Z_{4b}Y_{1a}+Z_{3a}Y_{1b}+Z_{4b}Y_{1a}{}^T +Z_{1a}{}^TY_{2b}-Y_{2b}Z_{2a}{}^T=C_{abc}Y_{2c}\,. \quad \text{(A11)}$$

The absence of q-No. ST requires in addition the following equations:

$$Z_{1a}{}^TZ_{4b}+Z_{3b}Z_{2a}+a\leftrightarrow b=0\,, \quad \text{(A12)}$$

$$Y_{1b}Z_{2a}{}^T-Z_{2a}Y_{1b}{}^T+a\leftrightarrow b=0\,, \quad \text{(A13)}$$

$$-Y_{2b}Z_{4a}{}^T+Z_{4a}Y_{2b}{}^T+a\leftrightarrow b=0\,, \quad \text{(A14)}$$

$$Z_{3a}Y_{1b}+Z_{1a}{}^TY_{2b}{}^T+a\leftrightarrow b=0\,, \quad \text{(A15)}$$

$$Z_{4a}Y_{1b}{}^T-Y_{2b}Z_{2a}{}^T+a\leftrightarrow b=0\,. \quad \text{(A16)}$$

Since Eqs. (A6) and (A7) are structurally identical to Eq. (A5), an obvious solution to Z_{1a} and Z_{2a} is C_a. There are other solutions, however, which depend on the group structure chosen. At this point, therefore, we specify that C_{abc} will be the structure constants for $U(3)\times U(3)$. The indices will run over 18 values, 1–9 and $\bar{1}$–$\bar{9}$, the bar denoting an unnatural-parity component. One has then

$$C_{ijk}=C_{i\bar{j}\bar{k}}=f_{ijk}\,, \qquad C_{ij\bar{k}}=C_{\bar{i}\bar{j}\bar{k}}=0\,, \quad \text{(A17)}$$

where i, j, and k run over the indices 1–9.

It is useful now to split up the matrix equations (A6)–(A16), into several 9×9 matrix equations, with each matrix representing a specific parity combination. For instance, all of the nonzero elements of Z_{1i} can be represented by the following four 9×9 matrices:

$$(Z_1{}^1{}_i)_{jk}=Z_{1ijk}\,,\quad (Z_1{}^2{}_i)_{jk}=Z_{1i\bar{j}\bar{k}}\,,\quad (Z_1{}^3{}_i)_{jk}=Z_{1\bar{i}\bar{j}k}\,,\quad (Z_1{}^4{}_i)_{jk}=Z_{1\bar{i}j\bar{k}}\,. \quad \text{(A18)}$$

The same convention will be used for all the Z's. We also define the 9×9 matrices[25]

$$(f_i)_{jk}=f_{ijk}\,, \qquad (d_i)_{jk}=d_{ijk}\,, \quad \text{(A19)}$$

with

$$(d_9)_{jk}=(\sqrt{\tfrac{2}{3}})\delta_{jk};\ (f_9)_{jk}=0\,. \quad \text{(A20)}$$

In this notation, Eq. (A6) implies several equations,

[25] We follow the notation of M. Gell-Mann, Physics **1**, 63 (1964).

two of which are

$$[Z_1{}^1{}_i, Z_1{}^1{}_j] = -f_{ijk} Z_1{}^1{}_k, \tag{A21}$$

$$[Z_1{}^2{}_i, Z_1{}^2{}_j] = -f_{ijk} Z_1{}^2{}_k. \tag{A22}$$

A solution for the first eight components of Eq. (A21) is

$$Z_1{}^1{}_i = f_i. \tag{A23}$$

$Z_1{}^1{}_9$ is required by Eq. (A21) to commute with all f_i. Hence it must be zero or diagonal, with the first eight elements equal. However, we want the ninth component to have the same type of coupling as the eighth component, and the same G parity. The only choice which satisfies this requirement is $Z_1{}^1{}_9 = 0$; hence, Eq. (A23) holds for all nine components.

The most general, acceptable solution of Eq. (A21) is then

$$Z_1{}^1{}_i = (S^1)^{-1} f_i S^1. \tag{A24}$$

Likewise, the most general, acceptable solution for $Z_1{}^2{}_i$ is

$$Z_1{}^2{}_i = (S^2)^{-1} f_i S^2, \tag{A25}$$

where S^1 and S^2 are nonsingular matrices. Isoinvariance places severe restrictions on the form of S. From Eqs. (2.21c) and (2.23b) we find that for conserved currents[26]

$$Z_1{}^1{}_i = f_i, \quad i = 1, 2, 3, 8. \tag{A26}$$

Applying Eq. (A24) to Eq. (A26) shows that

$$[f_i, S^1] = 0 \tag{A27}$$

for $i = 1, 2, 3$, and 8. From this and charge conjugation invariance, one can show that S^1 is diagonal, with elements constant throughout each isotopic multiplet, except for possible mixing of isotopic singlet states. The same restrictions apply to S^2.

Note that the solution for Z_2 [in Eq. (A7)] is specified by the three-point algebra to be [see Eq. (3.1a)]

$$Z_{2a} = G^T C_a (G^T)^{-1}, \tag{A28}$$

where $(G)_{ab} \equiv g_{ab}$. Thus for Z_2, the role of S^1 is taken by the vector field-current coupling constants, and the role of S^2 is taken by the axial-vector field-current coupling constants. The invariances initially imposed on g_{ab} are precisely those required of S^1 and S^2. Note that Eq. (A28) gives a solution to Eq. (A7) which thus provides no new information.

Returning to Eq. (A6), we now consider an equation containing $Z_1{}^3$ that is part of Eq. (A6):

$$Z_1{}^2{}_i Z_1{}^3{}_j - Z_1{}^3{}_j Z_1{}^1{}_i = -f_{ijk} Z_1{}^3{}_k. \tag{A29}$$

Using the solutions for $Z_1{}^1$ and $Z_1{}^2$ and defining

$$X^3{}_k \equiv S^2 Z_1{}^3{}_k (S^1)^{-1}, \tag{A30}$$

Eq. (A29) becomes

$$[f_i, X^3{}_j] = -f_{ijk} X^3{}_k. \tag{A31}$$

There are two possible solutions,

$$X^3{}_i = c_3 f_i \tag{A32}$$

or

$$X^3{}_i = c_3 d_i. \tag{A33}$$

Either choice (but not a combination) would conserve G parity, but the two choices result in different G-parity assignments for the particles coupled, namely, the scalar nonets. The correct choice for the Z_2 equation would be given by Eq. (A32), but for $Z_1{}^3$ this choice is not correct. $Z_1{}^3$ couples an axial vector with a pseudoscalar and scalar. For this combination, G parity (and the phase conventions chosen) requires d-type couplings, and Eq. (A33) must be used. Hence

$$Z_1{}^3{}_i = c_3 (S^2)^{-1} d_i S^1 \tag{A34}$$

and likewise one finds

$$Z_1{}^4{}_i = c_4 (S^1)^{-1} d_i S. \tag{A35}$$

There is yet another type of equation contained in Eq. (A6):

$$Z_1{}^4{}_i Z_1{}^3{}_j - Z_1{}^4{}_j Z_1{}^3{}_i = -f_{ijk} Z_1{}^1{}_k. \tag{A36}$$

Using Eqs. (A24), (A25), (A34), and (A35), this becomes

$$c_3 c_4 [d_i, d_j] = -f_{ijk} f_k. \tag{A37}$$

Up to this point, each condition used could have been satisfied by octets of mesons satisfying $SU(3) \times SU(3)$. Such a choice cannot satisfy Eq. (A37). More states are required than the octets afford. A double nonet of currents satisfying $U(3) \times U(3)$ is not necessary to satisfy Eq. (A37), but is sufficient. This condition is a result of the requirement that the general "σ commutator" be pole-dominated, for if this requirement were relaxed, Eq. (A6) would no longer hold. The same phenomenon was observed in the $SU(2) \times SU(2)$ analysis, where it was found that pole dominance of the σ commutator required a scalar singlet state. The close connection between Z_1 and the general "σ commutator" can be seen from Eqs. (4.3) and (4.4). The singlet states can be eliminated, if necessary, preserving the current commutators and divergences, for instance by taking the limit as the masses of the singlet terms approach infinity. This results in a V^0 which is no longer quadratic in canonical fields and a general "σ commutator" which is not pole-dominated.

For a $U(3) \times U(3)$ algebra, Eq. (A37) holds provided that

$$c_3 c_4 = -1. \tag{A38}$$

There is no loss of generality if c_3 is chosen to be 1 and $c_4 = -1$.

We can now put the entire solution of Z_1 into a com-

[26] More precisely, when all three indices of $(Z_1{}^1)_{ijk}$ are in the $I=1$ multiplet one has that S^1 is an orthogonal matrix. By Eqs. (2.16a) and (2.7b), $Z_1{}^1$ for this case corresponds to the $\rho\delta\delta$ coupling, and S^1 can then be eliminated by transforming the $\delta(x)$ field by $\delta_i \to (S^{-1})_{ij}\delta_j$ since the free δ-meson Lagrangian is invariant to an orthogonal transformation.

pact equation of 18×18 matrices,

$$Z_{1a}=S^{-1}A_aS\,,\quad S=\begin{pmatrix}S^1 & 0\\ 0 & S^2\end{pmatrix},\tag{A39}$$

where

$$(A_a)_{bc}=(A_a)_{\bar b\bar c}=f_{abc}\,,\qquad (A_{\bar a})_{\bar b c}=-(A_{\bar a})_{b\bar c}=d_{abc}\,.\tag{A40}$$

The fact that S is diagonal (except for mixing in the eighth and ninth components) with a common value in a given isomultiplet, shows by Eq. (4.3) that the spin-zero mesons form a "scaled" (3*,3)+(3,3*) representation of the chiral algebra. Conditions on the scaling matrix S due to the PCC conditions are given in Appendix C.

The scalar fields of the effective Lagrangian are, of course, renormalized fields. If one defines the unrenormalized fields by

$$\tilde{s}_a\equiv S_{ab}s_b\tag{A41}$$

then Eqs. (4.3) and (4.4) show that $\tilde{s}_a$ transform like a (3*,3)+(3,3*) representation of the algebra. The S_{ab} are directly related to the wave-function renormalization matrix

$$S_{ab}\sim(\sqrt{Z})_{ab}\tag{A42}$$

of Glashow and Weinberg,[15] who postulated a (3*,3)+(3,3*)-symmetry breaking. One sees, therefore, that the *pole dominance of the σ commutators implies results equivalent to a chiral-symmetry-breaking interaction transforming according to the* (3*,3)+(3,3*) *representation of the current algebra.*

Equations (A8), (A9), and (A12) give no new information; they are satisfied by any solution to Eq. (A6) and solutions to the three-point current algebra.

There is another restriction of S due to the three-point commutation relations (2.21c). This equation reduces to

$$A_{bcd}M_{ad}-A_{acd}M_{bd}=C_{abd}M_{dc}\,,\tag{A43}$$

where

$$M_{ab}\equiv F_{ac}S_{bc}\,.\tag{A44}$$

The solution to (A43) is

$$M_{ab}=A_{abe}K_e\,.\tag{A45}$$

By isoinvariance, K_e can have nonzero components only for $e=8$, 9. Since both S and K are undetermined in scale, we can set the scale by requiring that the two components of K be the sine and cosine of some undetermined angle. However, in this formalism, it is more convenient to use certain linear combinations of the 8 and 9 states, namely

$$(A)=(\sqrt{\tfrac{2}{3}})(8)-(\sqrt{\tfrac{1}{3}})(9)\,,\qquad (B)=(\sqrt{\tfrac{1}{3}})(8)+(\sqrt{\tfrac{2}{3}})(9)\,.\tag{A46}$$

The details of this transformation are given in Appendix B. Here, we just mention it to give a motivation for the actual form that we choose to express the linear combination given in Eq. (A45):

$$M_{ab}=A_{abA}\cos\psi+A_{abB}\sin\psi\,.\tag{A47}$$

The angle ψ is determined by PCC conditions, the details of which are given in Appendix C, along with expression for those components of S which are determined.

Note that M_{ab} has the following structure: For unnatural-parity states, it is diagonal, with different elements for different multiplets; for natural-parity states, A_{ab9} is zero, and $A_{ab8}=f_{ab8}$ is nonzero only for the strange components. For these, M_{ab} is not diagonal, but the nonzero elements are $M_{45}=-M_{54}=M_{67}=-M_{76}$. Since S is diagonal, we see that this structure of M_{ab} is consistent with Eq. (A44) and the structure of F_{ab} given in Sec. II. If we had chosen $Z_1{}^3{}_a$ to be f_a instead of d_a in Eq. (A34), A_{abc} would be f_{abc} for all states, and the unnatural-parity states would also have the same structure of M_{ab} as given above for the natural-parity states. The only consistent solution would be for all the axial-vector currents to be conserved, clearly an undesirable consequence. Since the choice of Eq. (A34) was dictated by the G-parity assignments of the scalar mesons, we see that there is a close connection between the type of interaction allowed for the scalar mesons, and the type of partial-current conservation allowed in this formalism.

A similar treatment is used to analyze Y_1 and Y_2. The actual calculations are lengthy and only the results are given here. There are several solutions for Y_1 and Y_2 which satisfy the current-algebra equations, but none which conserve G parity. A few components of Y_1 and Y_2 for which one of the indices are 9 or $\bar{9}$ are not restricted by the current algebra, but are then eliminated by the integrability conditions on the PCC equations. The only acceptable solutions of Y_1 and Y_2 are for all components of both to vanish. This result also depends on the assumption of pole dominance of the "σ commutator" and might be quite different were it relaxed.

The q-No. ST equations give no additional information. The above solutions represent the entire content of the current algebra in this formalism.

APPENDIX B: REPRESENTATION OF $U(3)$ ALGEBRA WHICH DIAGONALIZES ISOTOPIC SINGLET INTERACTIONS

For $U(3)$ nonets, the symmetric invariant coupling d_{abc} connects the two isotopic singlets, since $d_{988}=(\tfrac{2}{3})^{1/2}$. It is mathematically useful to choose linear combinations of the 8 and 9 states such that this does not occur. Matrix equations which must be solved in Appendix C are thereby diagonalized. The resulting linear combinations are also physically interesting, in that they appear to be approximately those combinations chosen by the ω, φ, system. Moreover, this formalism appears to force this combination on the pseudoscalar singlets, as will be seen in Appendix C.

The combinations chosen are those of Eq. (A46). If we let U be a 9×9 matrix such that

$$(U_{ab})=\begin{pmatrix}\sqrt{\frac{2}{3}} & -\sqrt{\frac{1}{3}}\\ \sqrt{\frac{1}{3}} & +\sqrt{\frac{2}{3}}\end{pmatrix},\quad a,b=8,9$$
$$=\delta_{ab},\ \text{either } a \text{ or } b\neq 8,\,9\,, \tag{B1}$$

we can give the transformations of f and d:

$$\tilde{f}_{abc}=U_{aa'}U_{bb'}U_{cc'}f_{a'b'c'}\,, \tag{B2}$$

$$\tilde{d}_{abc}=U_{aa'}U_{bb'}U_{cc'}d_{a'b'c'}\,. \tag{B3}$$

The transformed 9×9 matrices, $\tilde{f}_a$ and $\tilde{d}_a$, satisfy

$$\begin{aligned}[\tilde{f}_a,\tilde{f}_b]&=-\tilde{f}_{abc}\tilde{f}_c\,,\\ [\tilde{f}_a,\tilde{d}_b]&=-\tilde{f}_{abc}\tilde{d}_c\,,\\ [\tilde{d}_a,\tilde{d}_b]&=\tilde{f}_{abc}\tilde{f}_c\,.\end{aligned} \tag{B4}$$

Thus, the algebra of $U(3)$ can be expressed just as well with the $\tilde{f}$'s and $\tilde{d}$'s as with the f's and d's. For instance, the results of Appendix A are equally valid with $\tilde{f}$ and $\tilde{d}$. [In Appendix A, however, it was more convenient to use the standard representation, since the 8×8 matrices form an irreducible representation of $SU(3)$. Having used this representation to find the most general solutions to the current-algebra restrictions, it now becomes convenient to change representation to express the particular solutions that the PCC conditions permit.]

The numerical value of the changed $\tilde{f}$'s and $\tilde{d}$'s are

$$\begin{aligned}&\tilde{f}_{45A}=\tilde{f}_{67A}=\sqrt{\tfrac{1}{2}}\,,\\ &\tilde{f}_{45B}=\tilde{f}_{67B}=\tfrac{1}{2}\,,\\ &\tilde{d}_{11A}=\tilde{d}_{22A}=\tilde{d}_{33A}=0\,,\\ &\tilde{d}_{11B}=\tilde{d}_{22B}=\tilde{d}_{33B}=1\,,\\ &\tilde{d}_{44A}=\tilde{d}_{55A}=\tilde{d}_{66A}=\tilde{d}_{77A}=-\sqrt{\tfrac{1}{2}}\,,\\ &\tilde{d}_{44B}=\tilde{d}_{55B}=\tilde{d}_{66B}=\tilde{d}_{77B}=\tfrac{1}{2}\,,\\ &\tilde{d}_{AAA}=-\sqrt{2}\,,\quad \tilde{d}_{BBB}=1\,,\quad \tilde{d}_{ABB}=\tilde{d}_{AAB}=0\,.\end{aligned} \tag{B5}$$

The $\tilde{f}$'s and $\tilde{d}$'s are, or course, completely antisymmetric and symmetric, respectively. Note that the $\tilde{d}$ coupling does not couple the A and B components. The A and B components then represent the diagonalized isotopic singlet states which are not coupled by $U(3)$-invariant couplings.

APPENDIX C: THREE-POINT PCC RESTRICTIONS ON N-POINT CURRENT-ALGEBRA RESULTS

In Sec. II it was seen that the three-point PCC relations lead to the constraint (2.23a) on Z_{1abc}. In Appendix A, it was shown that the current-algebra constraints when combined with pole dominance of the σ commutators required that Z_1 take the form of Eq. (A39) where the diagonal matrix S is constrained by Eqs. (A44) and (A47). We investigate here the effect of the three-point restriction on the general results of Appendix A. Let us define

$$G_{abc}\equiv 6F_{aa'}F_{bb'}F_{cc'}g^7{}_{a'b'c'}\,. \tag{C1}$$

Then, by substitution into Eq. (2.23a), one has

$$-G_{abc}=M_{bb'}A_{ab'd}M_{dc'}{}^{-1}C_{c'c}+b\leftrightarrow c\,, \tag{C2}$$

where

$$C_{ab}\equiv F_{ac}F_{bc}\mu^2{}_c\,. \tag{C3}$$

The $\bar{8}$ and $\bar{9}$ channels of Eq. (C2) are mixed. To separate them, we use the transformation equations (A46) discussed in Appendix B. With this choice of representation, the matrix $M_{\mu b}$ takes on a simple form with the following elements:

$$\begin{aligned}&M_\pi\equiv M_{\bar{1}\bar{1}}=M_{\bar{2}\bar{2}}=M_{\bar{3}\bar{3}}=\sin\psi\,,\\ &M_K\equiv M_{\bar{4}\bar{4}}=M_{\bar{5}\bar{5}}=M_{\bar{6}\bar{6}}=M_{\bar{7}\bar{7}}\\ &\qquad=-(\sqrt{\tfrac{1}{2}})\cos\psi+\tfrac{1}{2}\sin\psi\,,\\ &M_\kappa\equiv M_{45}=M_{67}=-M_{54}=-M_{76}\\ &\qquad=(\sqrt{\tfrac{1}{2}})\cos\psi+\tfrac{1}{2}\sin\psi\,,\\ &M_{\bar{A}}\equiv M_{\bar{A}\bar{A}}=-\sqrt{2}\cos\psi\,,\\ &M_{\bar{B}}\equiv M_{\bar{B}\bar{B}}=\sin\psi\,.\end{aligned} \tag{C4}$$

The matrix C_{ab} is diagonal in many of its elements:

$$\begin{aligned}&C_\pi\equiv C_{\bar{1}\bar{1}}=C_{\bar{2}\bar{2}}=C_{\bar{3}\bar{3}}=F_\pi{}^2\mu_\pi{}^2\,,\\ &C_K\equiv C_{\bar{4}\bar{4}}=C_{\bar{5}\bar{5}}=C_{\bar{6}\bar{6}}=C_{\bar{7}\bar{7}}=F_K{}^2\mu_K{}^2\,,\\ &C_\kappa\equiv C_{44}=C_{55}=C_{66}=C_{77}=F_\kappa{}^2\mu_\kappa{}^2\,.\end{aligned} \tag{C5}$$

The A and B components of C will be referred to by index.

The quantity G_{abc} is nonzero only for those components for which all indices correspond to unconserved currents. Since by parity conservation, one component must refer to a natural-parity scalar, the only possible nonzero combinations are

$$\begin{aligned}\text{I:}&\quad \pi K\kappa\,,\\ \text{II:}&\quad \bar{A}K\kappa\,,\\ \text{III:}&\quad \bar{B}K\kappa\,.\end{aligned} \tag{C6}$$

From channel I we have (on account of the total symmetry of G_{abc})

$$-G_{\bar{1}4\bar{7}}=\frac{1}{2}\left(\frac{M_K}{M_\kappa}C_\kappa-\frac{M_\kappa}{M_K}C_K\right) \tag{C7}$$

$$=\frac{1}{2}\left(\frac{M_K}{M_\pi}C_\pi-\frac{M_\pi}{M_K}C_K\right) \tag{C8}$$

$$=\frac{1}{2}\left(\frac{M_\pi}{M_\kappa}C_\kappa-\frac{M_\kappa}{M_\pi}C_\pi\right). \tag{C9}$$

Channel II gives

$$-G_{\bar{A}4\bar{5}}=(\sqrt{\tfrac{1}{2}})\left(\frac{M_\kappa}{M_K}C_K-\frac{M_K}{M_\kappa}C_\kappa\right) \tag{C10}$$

$$=(\sqrt{\tfrac{1}{2}})\left(\frac{M_\kappa}{M_{\bar{A}}}C_{\bar{A}\bar{A}}-\frac{M_{\bar{A}}}{M_\kappa}C_\kappa-(\sqrt{\tfrac{1}{2}})\frac{M_\kappa}{M_{\bar{B}}}C_{\bar{B}\bar{A}}\right) \tag{C11}$$

$$=(\sqrt{\tfrac{1}{2}})\left(\frac{M_K}{M_{\bar{A}}}C_{\bar{A}\bar{A}}-\frac{M_{\bar{A}}}{M_K}C_K+(\sqrt{\tfrac{1}{2}})\frac{M_K}{M_{\bar{B}}}C_{\bar{B}\bar{A}}\right) \tag{C12}$$

and channel III gives

$$-G_{\bar{B}4\bar{5}}=\frac{1}{2}\left(\frac{M_K}{M_\kappa}C_\kappa-\frac{M_\kappa}{M_K}C_K\right) \tag{C13}$$

$$=\frac{1}{2}\left(\frac{M_B}{M_\kappa}C_\kappa-\frac{M_\kappa}{M_{\bar{B}}}C_{\bar{B}\bar{B}}+\sqrt{2}\frac{M_\kappa}{M_{\bar{A}}}C_{\bar{A}\bar{B}}\right) \tag{C14}$$

$$=\frac{1}{2}\left(\frac{M_K}{M_{\bar{B}}}C_{\bar{B}\bar{B}}-\frac{M_B}{M_K}C_K+\sqrt{2}\frac{M_K}{M_{\bar{A}}}C_{\bar{A}\bar{B}}\right). \tag{C15}$$

The solutions to these equations are

$$C_{\bar{A}\bar{B}}=C_{\bar{B}\bar{A}}=0\,, \tag{C16}$$

$$C_{\bar{A}\bar{A}}\equiv C_{\bar{A}}=2(C_K+C_\kappa)-C_\pi\,, \tag{C17}$$

$$C_{\bar{B}\bar{B}}\equiv C_{\bar{B}}=C_\pi\,, \tag{C18}$$

$$2G_{\bar{1}4\bar{7}}=-\sqrt{2}G_{\bar{A}4\bar{5}}=2G_{\bar{B}4\bar{5}}=\pm\sqrt{\Delta}\,, \tag{C19}$$

where

$$\Delta\equiv C_\pi^2+C_K^2+C_\kappa^2-2C_\pi C_K-2C_\pi C_\kappa-2C_K C_\kappa$$
$$=(C_K-C_\kappa)^2-C_{\bar{A}}C_{\bar{B}} \tag{C20}$$

and

$$\tan\psi=\sqrt{2}C_\pi(C_\kappa-C_K\pm\Delta^{1/2})^{-1}. \tag{C21}$$

The quantity Δ, defined in Eq. (C20), is unfortunately not well determined experimentally. For one, it depends on C_κ, and the κ has not been definitely established. However, Δ must be positive and hence there results a restriction on the mass of the κ, namely,

$$|F_\kappa\mu_\kappa|>|F_\pi\mu_\pi|+|F_K\mu_K|$$

or

$$|F_\kappa\mu_\kappa|<\big||F_K\mu_K|-|F_\pi\mu_\pi|\big|\,, \tag{C22}$$

a result that was first derived by assuming that the chiral-symmetry breakdown term transforms as[16] (3,3*)+(3*,3).

Equations (C16)–(C22) result from two features in the formalism: (1) the assumption that the general "σ commutator" is pole dominated and (2), the G-parity assignments for the scalar meson octet.

The determination of C_{ab} gives F_{ab} by Eq. (C3) for the singlet pseudoscalars, to within an undetermined mixing angle:

$$\begin{aligned}F_{\bar{A}\bar{8}}&=(C_A)^{1/2}\cos\varphi/\mu_{\bar{8}}\,,\\ F_{\bar{A},\bar{9}}&=-(C_A)^{1/2}\sin\varphi/\mu_{\bar{9}}\,,\\ F_{\bar{B},\bar{8}}&=(C_B)^{1/2}\sin\varphi/\mu_{\bar{8}}=F_\pi\mu_\pi\sin\varphi/\mu_{\bar{8}}\,,\\ F_{\bar{B},\bar{9}}&=(C_B)^{1/2}\cos\varphi/\mu_{\bar{9}}=F_\pi\mu_\pi\cos\varphi/\mu_{\bar{9}}\,.\end{aligned} \tag{C23}$$

The inverse of F_{ab} over these states is

$$\begin{aligned}F_{\bar{8},\bar{A}}^{-1}&=\mu_{\bar{8}}\cos\varphi/(C_A)^{1/2}\,,\\ F_{\bar{9},\bar{A}}^{-1}&=-\mu_{\bar{9}}\sin\varphi/(C_A)^{1/2}\,,\\ F_{\bar{8},\bar{B}}^{-1}&=\mu_{\bar{8}}\sin\varphi/(C_B)^{1/2}\,,\\ F_{\bar{9},\bar{B}}^{-1}&=\mu_{\bar{9}}\cos\varphi/(C_B)^{1/2}\,.\end{aligned} \tag{C24}$$

Note that the particle-state labels are referred to still as $\bar{8}$ and $\bar{9}$. These were just names anyway, since no $U(3)$ or $SU(3)$ symmetry was assumed for these states. They remain just names, which could be changed to A and B if desired. However, no mixing transformation will be performed here on the particle states, or on any coupling constant which couples to a particle.

We now use the equation

$$S_{ab}=M_{ca}F^{-1}{}_{bc} \tag{C25}$$

to compute those elements of the scaling matrix S which are determined:

$$\begin{aligned}S_{44}&=S_{55}=S_{66}=S_{77}=[(\sqrt{\tfrac{1}{2}})\cos\psi+\tfrac{1}{2}\sin\psi]/F_\kappa\,,\\ S_{\bar{1}\bar{1}}&=S_{\bar{2}\bar{2}}=S_{\bar{3}\bar{3}}=\sin\psi/F_\pi\,,\\ S_{\bar{4}\bar{4}}&=S_{\bar{5}\bar{5}}=S_{\bar{6}\bar{6}}=S_{\bar{7}\bar{7}}\\ &=[(-\sqrt{\tfrac{1}{2}})\cos\psi+\tfrac{1}{2}\sin\psi]/F_K\,,\end{aligned} \tag{C26a}$$

and

$$\begin{aligned}S_{\bar{A},\bar{8}}&=-\sqrt{2}\cos\psi\cos\varphi\,\mu_{\bar{8}}/(C_{\bar{A}})^{1/2}\,,\\ S_{\bar{A},\bar{9}}&=\sqrt{2}\cos\psi\sin\varphi\,\mu_{\bar{9}}/(C_{\bar{A}})^{1/2}\,,\\ S_{\bar{B},\bar{8}}&=\sin\psi\sin\varphi\,\mu_{\bar{8}}/(C_{\bar{B}})^{1/2}\,,\\ S_{\bar{B},\bar{9}}&=\sin\psi\cos\varphi\,\mu_{\bar{9}}/(C_{\bar{B}})^{1/2}\,.\end{aligned} \tag{C26b}$$

The components of S_{ab} for $a, b=8, 9$ are not determined. The elements of S^{-1} for the pseudoscalar singlet elements are

$$\begin{aligned}S^{-1}{}_{\bar{8},\bar{A}}&=-\cos\varphi\,(C_{\bar{A}})^{1/2}/(\sqrt{2}\mu_{\bar{8}}\cos\psi)\,,\\ S^{-1}{}_{\bar{9},\bar{A}}&=\sin\varphi\,(C_{\bar{A}})^{1/2}/(\sqrt{2}\mu_{\bar{9}}\cos\psi)\,,\\ S^{-1}{}_{\bar{8},\bar{B}}&=\sin\varphi\,(C_{\bar{B}})^{1/2}/(\mu_{\bar{8}}\sin\psi)\,,\\ S^{-1}{}_{\bar{9},\bar{B}}&=\cos\varphi\,(C_{\bar{B}})^{1/2}/(\mu_{\bar{9}}\sin\psi)\,.\end{aligned} \tag{C27}$$

The coupling constant $g^7{}_{abc}$ is determined in terms of G_{abc} and F_{ab} as follows:

$$6g^7{}_{abc}=F^{-1}{}_{aa'}F^{-1}{}_{bb'}F^{-1}{}_{cc'}G_{a'b'c'}\,, \tag{C28}$$

where the sum is over only unconserved components, i.e., the channels connecting the κ with two pseudoscalars. For these channels,

$$6g^7{}_{\bar{1}5\bar{7}}=\pm\sqrt{\Delta}/2F_\pi F_K F_\kappa \tag{C29}$$

and

$$6g^7{}_{\bar{8}5\bar{5}}=\frac{\pm(\sqrt{\Delta})\mu_8}{F_KF_\kappa}\left[\frac{-\cos\varphi}{\sqrt{(2C_A)}}+\frac{\sin\varphi}{2\sqrt{C_B}}\right],$$
$$6g^7{}_{\bar{9}5\bar{5}}=\frac{\pm(\sqrt{\Delta})\mu_9}{F_KF_\kappa}\left[\frac{\sin\varphi}{\sqrt{(2C_A)}}+\frac{\cos\varphi}{2\sqrt{C_B}}\right]. \quad \text{(C30)}$$

Equation (C29) was first obtained by Pande.[16] One may also easily verify that Eqs. (C26a) and Eq. (C21) are identical to the results of Ref. (23) [Eqs. (19) and (20)] with the notational changes $S_K/S_\pi=\sqrt{(Z_K/Z_\pi)}$ and $S_\kappa/S_\pi=-\sqrt{(Z_\kappa/Z_\pi)}$. Again, unlike Ref. (23), no *a priori* assumption of $(3,3^*)+(3^*,3)$-symmetry breaking has been assumed here.

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Electromagnetic Corrections to Nucleon Transitions of the Vector and Axial-Vector Currents*

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We examine the matrix elements of the divergences of the vector and axial-vector current between nucleon states. These matrix elements are related to the nucleon mass difference and the corrections to the Goldberger-Treiman relation, respectively. For the nucleon mass difference we indicate that for the sign of this quantity to be understood in terms of the electromagnetic interaction requires (i) comparable longitudinal and transverse virtual photon-nucleon cross sections, or (ii) $\sigma_t^p(q^2,\nu)-\sigma_t^n(q^2,\nu)<0$ over a large region of the $(|q^2|,\nu)$ plane, where q^2 is the spacelike virtual-photon mass and ν is the photon energy (this requirement is contraindicated by experimental data at $q^2=0$), or (iii) fixed J-plane poles at $J=0, I=1$ in the virtual Compton amplitude. We also estimate the electromagnetic correction to the Goldberger-Treiman relation, and it is shown to be very small.

I. INTRODUCTION

In this paper we discuss the transition matrix elements $\langle p|V_\mu{}^{(+)}(0)|n\rangle$ and $\langle p|A_\mu{}^{(+)}(0)|n\rangle$ of the vector and axial-vector currents between nucleon states in the presence of the electromagnetic interaction. Our interest in these matrix elements stems from the observation that the matrix elements of the divergence of these currents are related to the nucleon mass difference and the corrections to the Goldberger-Treiman formula. Neither of these quantities is well understood on a theoretical basis.

For the nucleon mass difference we obtain the usual Cottingham formula,[1] assuming that the mass difference is electromagnetic and the interaction is treated to lowest order in $\alpha=1/137$. Assuming that the total cross sections for longitudinally polarized photons or nucleons is suppressed relative to that for transverse polarization, we discuss the extreme difficulty of obtaining the correct sign for $\delta M=M_p-M_n$. Here it is pointed out that if the recently reported[2] qualitative character of the total photon-nucleon cross section $[\sigma(\gamma p)-\sigma(\gamma n)>0$ for physical photons of energy 4–18 GeV] can be extrapolated for virtual photons, then the deep-inelastic region, which is an important region for the nucleon mass shift, will contribute with the wrong sign to δM. We conclude that to have the possibility of understanding the sign of δM in terms of electromagnetism, we must have (i) comparable longitudinal and transverse cross sections, or (ii) $\sigma_t^p(q^2,\nu)-\sigma_t^n(q^2,\nu)<0$ for a large region of the $(|q^2|,\nu)$ plane, or (iii) fixed poles at $J=0$ $I=1$ in the virtual Compton amplitude. The first two of these possibilities can be examined in the forthcoming experiments at SLAC.

We have also examined the radiative corrections to the Goldberger-Treiman formula for π^+ decay. They are estimated to be very small, $\sim\alpha/4\pi$ relative to the observed correction ~ 0.1. In accord with our expectation, the origin of this correction is to be sought in hadron dynamics and not in electromagnetism.

II. VECTOR CURRENT

First we consider the matrix elements of the vector current between proton and neutron states, which has the general form

$$\langle p(p')|V_\mu{}^{(+)}(0)|n(p)\rangle=u(p')\tau^+[\gamma_\mu F_1(t)+i\sigma_{\mu\nu}(p'-p)_\nu F_2(t)+(p'-p)_\mu F_3(t)]u(p). \quad (2.1)$$

The divergence is specified by

$$\langle p(p')|-i\partial_\mu V_\mu{}^{(+)}(0)|n(p)\rangle=u(p')\tau^+[\delta MF_1(t)+tF_3(t)]u(p),$$

where $t=(p'-p)^2$ and $\delta M=M_p-M_n$. If the current is

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