Promotion and Threshold Singularities in Massive Quantum Electrodynamics*

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We study the threshold singularities in the J plane for spinor and scalar massive quantum electrodynamics. It is found that they are responsible for the promotion phenomenon discussed recently by Cheng and Wu.

In a recent letter, Cheng and Wu[1] have discovered the phenomenon of "promotion." Stated simply, it says that in massive quantum electrodynamics (QED), an invariant amplitude $A(s,t)$ alters its high-energy behavior as follows:

$$\text{If} \quad A(s,t) \underset{s\to\infty;\ t=0}{\sim} s^a, \quad \text{then } A(s,t) \underset{s\to\infty;\ t=t_0}{\sim} s^{a+1/2},$$

where $t=t_0$ is the crossed-channel threshold and a is some constant. It was also found[1] that such a promotion does *not* happen at three-particle thresholds.

It appears to us that this phenomenon is strikingly reminiscent of threshold poles found by Desai and Newton[2] and by Gribov and Pomeranchuk.[3] The singularities that we refer to are an infinite set of poles which approach the orbital angular momentum value $l=-\frac{1}{2}(3n-5)$ at any n-particle threshold.[3] The proof of their existence requires rather general considerations like analyticity and unitarity.[3,4]

For the case of spin, e.g., $e^+e^-\to\gamma\gamma$, at the two-particle threshold $t=4m_v^2$ (where m_v is the mass of the "photon") the singularities at $l=-\frac{1}{2}$ are raised in the J plane to a maximum of $J=2-\frac{1}{2}=\frac{3}{2}$. At the three-particle threshold these singularities are at $l=-2$ and get shifted to (at most) $J=1$. Thus the (Pomeranchuk) singularity[1] at $J=1$ when $t=0$ is promoted to $J=\frac{3}{2}$ when $t=4m_v^2$, but no promotion occurs at the three-body threshold. This conclusion agrees precisely with that of Cheng and Wu. In this paper we study the problem of promotion in terms of the threshold singularities directly, since to our knowledge they have not been investigated in QED. For illustrative purposes we discuss Compton scattering, rather than pair production as done by Cheng and Wu. The technical details of the computation are much simpler for the former process than for the latter, while the phenomenon of promotion is identical for both (promotion takes place to $J=1$ for Compton scattering, but to $J=\frac{3}{2}$ for pair production). We compute explicitly the parameters of the threshold trajectories and study their motion near threshold for spinor as well as scalar (massive) QED for lowest order in the coupling constant. As a prototype of our effect we discuss spinor and scalar Compton scattering. In what follows we describe the spinor case. Towards the end, we quote the corresponding results for the scalar case.

We find it necessary to work in the LS representation because it is the partial-wave amplitudes $\langle L'S'|T^J|LS\rangle$ which have the well-defined threshold behavior $\sim q^{L+L'}$ as the c.m. three-momentum $q\to 0$. For the scattering of a positive-parity spin-$\frac{1}{2}$ particle from a massive, negative-parity, vector meson, we have six states: three of one parity, i.e., $|L=J+\frac{1}{2},\ S=\frac{3}{2}\rangle$, $|L=J+\frac{1}{2},\ S=\frac{1}{2}\rangle$, $|L=J-\frac{3}{2}, S=\frac{3}{2}\rangle$, and three of the opposite parity. It is only the former set which has the leading singularities near $J=1$ and hence is of interest in the context of this paper. The elastic unitarity condition for T^J (in matrix notation) reads

$$\operatorname{Im}T^J=(q/W)T^{J\dagger}T^J\,, \tag{1}$$

where $W=\sqrt{s}$ is the total energy in the c.m. system. Let $\nu=q^2$ and define $A(\nu,J)=k_J^{-1}T^Jk_J^{-1}$, where

$$k_J=\begin{bmatrix}(\nu/\nu_0)^{J/2+1/4} & 0 & 0\\ 0 & (\nu/\nu_0)^{J/2+1/4} & 0\\ 0 & 0 & (\nu/\nu_0)^{J/2-3/4}\end{bmatrix} \tag{2}$$

and ν_0 is a dimensional constant. With this normalization the elements of $A(\nu,J)$ approach constants at threshold. The unitarity condition (1) then allows us to write

$$A^{-1}(\nu,J)=M(\nu,J)-i(q/W)k_J^2\,, \tag{3}$$

where M has no elastic right-hand cut and thus is expandable in a Taylor series near $\nu=0$. The continuation of Eq. (3) for complex J reads

$$A^{-1}(\nu,J)=M(\nu,J)+(\sqrt{\nu_0}/W)X(\nu,J)\,, \tag{4}$$

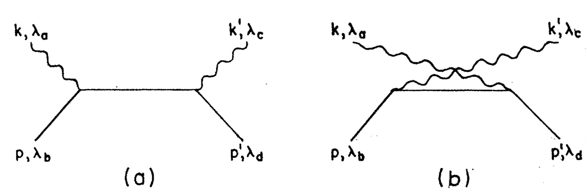

Fig. 1. Born terms for spinor Compton scattering.

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[1] H. Cheng and T. Wu, Phys. Rev. Letters **24**, 759 (1970).

[2] B. Desai and R. Newton, Phys. Rev. **130**, 2109 (1963); R. Newton, *The Complex j Plane* (Benjamin, New York, 1964).

[3] V. Gribov and I. Pomeranchuk, Phys. Rev. Letters **9**, 238 (1962).

[4] B. Desai and B. Sakita, Phys. Rev. **136**, B226 (1964); Y. Srivastava, Nuovo Cimento **37**, 667 (1965). For a discussion in potential theory with spin, see W. Carnahan, J. Math. Phys. **6**, 1709 (1965).

where

$$X(\nu,J)=\begin{bmatrix} \left(\frac{\nu}{\nu_0}\right)^{J+1}\frac{e^{-i\pi(J+1)}}{\sin\pi(J+1)} & 0 & 0 \\ 0 & \left(\frac{\nu}{\nu_0}\right)^{J+1}\frac{e^{-i\pi(J+1)}}{\sin\pi(J+1)} & 0 \\ 0 & 0 & \left(\frac{\nu}{\nu_0}\right)^{J-1}\frac{e^{-i\pi(J-1)}}{\sin\pi(J-1)} \end{bmatrix}. \tag{5}$$

The Regge poles are the solutions of the equation

$$\det[M+(\sqrt{\nu_0}/W)X]=0.$$

We are interested in solutions near $\nu=0$ and $J\rightarrow 1$. It is only the last element, i.e., X_{33}, which survives at threshold (as $J\rightarrow 1$), rendering the problem a single-channel one.

It should be clear then that to compute the Regge trajectories what we need now are the elements of the M matrix (in the LS representation). We shall do the calculation to lowest order in e^2, i.e., the Feynman diagram in Fig. 1(b).

We need not calculate Fig. 1(a), since this only has Kronecker-δ terms in the J plane which vanish at the (unphysical) value $J=1$. The contribution to $A(\nu,J)$ due to Fig. 1(b) is computed in the following tedious manner. From the Feynman amplitude which is given in the tensor basis, we first obtain the 12 independent parity-conserving helicity amplitudes $f_{\lambda\lambda_d:\lambda_a\lambda_b}{}^{\pm}(s,z)$.[5] The next step is to obtain the partial-wave helicity amplitudes $T_{\lambda_c\lambda_d:\lambda_a\lambda_b}{}^{J\pm}(s)$. Then, following Jacob and Wick,[6] we use the transformation

$$|J,\lambda_a\lambda_b\rangle=\sum_{L,S}\left(\frac{2L+1}{2J+1}\right)^{1/2}C_{L,0:\,s,\lambda_a-\lambda_b}{}^{J,\lambda_a-\lambda_b}\times C_{s,\lambda_a:\,s2,-\lambda_b}{}^{s,\lambda_a-\lambda_b}|J,LS\rangle$$

to obtain finally the amplitudes $\langle L'S'|T^J|LS\rangle$. Here another technical detail needs attention. To continue in J, we need to construct the even- and odd-signature amplitudes $A_{\pm}{}^J$. To lowest order in e^2, they are simply the negatives of each other. There is a separate unitarity condition, Eq. (4), for each $A_{\pm}{}^J$. As stated earlier, we only need $\langle J-\frac{3}{2},\frac{3}{2}|A_{\pm}{}^J|J-\frac{3}{2},\frac{3}{2}\rangle$. A rather unpleasant calculation along the above path yields, to lowest order in e^2, for $\nu\rightarrow 0$,

$$\langle J-\tfrac{3}{2},\tfrac{3}{2}|A_{\pm}{}^J|J-\tfrac{3}{2},\tfrac{3}{2}\rangle \xrightarrow{\nu\rightarrow 0} \pm\alpha\left(\frac{mm_v}{\nu_0}\right)\frac{\pi^{1/2}\Gamma(J-\frac{1}{2})}{\Gamma(J)}, \tag{6}$$

where $\nu_0=m_v(2m-m_v)$, $\alpha=e^2/4\pi$, m is the mass of the electron, and m_v is the mass of the vector meson.[7]

Away from threshold the leading singularity in the J plane is a fixed pole at $J=\frac{1}{2}$. We shall now show that promotion occurs at threshold. We obtain the threshold poles near $J=1$ from Eq. (4) by solving an algebraic equation of the form

$$\lambda\ln(\nu/\nu_{\pm})=2n\pi i+O(\lambda^2)\,,\quad n=\pm1,\pm2,\ldots, \tag{7}$$

where $\lambda=J-1$ and

$$\nu_{\pm}\approx\nu_0\exp\left\{\mp\frac{1}{\alpha}\left(1+\frac{m_v}{m}\right)\left[2\left(\frac{m}{m_v}\right)-1\right]^{1/2}\right\}. \tag{8}$$

For $|n|\ll|\ln(\nu/\nu_{\pm})|/2\pi$ and to first order in λ, we obtain the solutions

$$J_n{}^{\pm}(\nu)-1\approx\frac{2n\pi}{|\ln(\nu/\nu_{\pm})|}\left|\frac{-\pi}{|\ln(\nu/\nu_{\pm})|}+i\right|. \tag{9}$$

For a fixed value of ν there are only a finite number of poles to the right of $J=1-\epsilon$, where ϵ is an arbitrary small positive number.[4] Note that it is the parameter ν_0 which determines the realm of validity of the threshold expansion of our amplitude, but it is the parameter $\nu_{\pm}$ which is of most interest to us since it gives information concerning the motion of the threshold poles. Owing to the factor $1/\alpha$ in the exponential, both the even- and odd-signature poles are very near to $J=1$ for $0\lesssim\nu\ll\nu_0$—any motion at all is roughly along the imaginary λ axis.

If the poles behave at all like their potential-theory counterparts (and our u-channel pole is certainly a well-behaved, albeit mildly energy-dependent, effective potential), a more exact calculation would show that as the energy increases, the trajectories move rapidly into the complex λ plane with both Reλ and Imλ tending to ∞. Interesting diagrams showing their behavior in potential scattering can be found in Newton[2] and in Carnahan.[8] Such a calculation is extremely tedious and at present not worth the effort since we feel certain that the result would leave our conclusions unchanged. The behavior of the above threshold poles

[5] M. Gell-Mann, M. Goldberger, F. Low, E. Marx, and F. Zachariasen, Phys. Rev. **133**, B161 (1964).

[6] M. Jacob and G. Wick, Ann. Phys. (N. Y.) **7**, 404 (1959).

[7] We have assumed that $2m$ as well as $2\mu>m_v$. This renders the vertices stable. μ is the mass of a scalar electron to be discussed later.

[8] W. H. Carnahan, Ph.D. thesis, Indiana University, 1964 (unpublished).

is surely not that usually associated with poles destined to produce resonances at higher energies.

An analogous calculation was performed for scalar Compton scattering. Here the signatured partial-wave amplitudes of interest are $\langle J-1,\ 1|A_{\pm}{}^{J}|J-1,\ 1\rangle$. Again, the threshold poles near $J=\frac{1}{2}$ are given by an equation similar to (7) with a new $\nu_{\pm}{}^{(\mathrm{scalar})}$:

$$\nu_{\pm}{}^{(\mathrm{scalar})}\approx m_v(2\mu-m_v)\times\exp\left\{\mp\left(\frac{1}{\alpha}\right)\frac{m_v+\mu}{[m_v(2\mu-m_v)]^{1/2}}\right\}, \quad (10)$$

where μ is the mass of the scalar particle.

Another interesting question is to ask if an infinite number of poles also converge at the pseudothresholds. [A psuedothreshold is at $s=s_2=(m-m_v)^2$, in contrast to the physical threshold, which is at $s=s_1=(m+m_v)^2$.] Here we just make a few brief remarks and hope to return to this question in detail elsewhere. For spinless scattering, it is known[9] that if the amplitude possesses an su double-spectral function, then

$$T_l(s)\xrightarrow[s\to s_2]{} c_l(s-s_2)^l+d_l(s-s_2)^{1/2}, \quad (11)$$

where c_l and d_l are independent of s. Thus, for $l<\frac{1}{2}$ (our region of interest) the behavior at s_2 appears analogous to the one at the physical threshold, s_1. Hence we expect an infinite number of poles to arrive at $J=1$ for spinor-vector scattering and at $J=\frac{1}{2}$ for scalar-vector scattering for $s\to s_2$ as well. The answer to this question may also be found by observing the asymptotic behavior of the scattering amplitudes near the pseudothresholds and noting whether or not promotion occurs.

Finally, since both the leading even- and odd-signature threshold poles remain almost stationary near $J=1$ for small energies, it would seem to be the case that neither set is able to generate physical resonances. So, unless the situation is dramatically changed by higher orders, we find no support for the Cheng-Wu conjecture that such generation may occur.

[9] G. Frye and R. Warnock, Phys. Rev. 130, 478 (1963); E. Abers and V. Teplitz, Nuovo Cimento 39, 739 (1965).

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Broken Chiral and Conformal Symmetry and the Kuo Transformation

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The breaking of conformal and chiral symmetry within the framework of effective Lagrangians is studied, and it is shown that the (8,8) type of symmetry breaking leads to unacceptable values of the pion-pion scattering lengths. A combination of $(3,\bar{3})+(\bar{3},3)$ and (8,8) is then proposed, the particular form being uniquely fixed by the requirement of Kuo transformation. This is then shown to lead to improved scattering lengths for meason-meson scattering processes.

I. INTRODUCTION

SINCE conformal symmetry is a physically interesting generalization of the Poincaré symmetry, it is an extremely attractive idea to investigate the spontaneous breakdown of conformal symmetry to the symmetry of the Poincaré group.[1] Although it is fairly straightforward to formulate effective Lagrange functions which are simultaneously invariant under the conformal group of transformations as well as under a chiral group of higher symmetry like $SU(3)\otimes SU(3)$, there is no unique way of introducing explicit symmetry-breaking terms into the Lagrangian. The suggestion has therefore been made by Isham *et al.*[2] that one should choose a symmetry-breaking term which belongs to an irreducible representation of the combined conformal and chiral groups so that one can take advantage of the existence of a fundamental scalar field $\chi(x)$ of conformal weight -1 and of its nonzero vacuum expectation value. For the case of chiral $SU(3)\otimes SU(3)$, one then has the free-

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