$V-A$ Theory for Nucleon Resonances*

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The conserved-axial-vector-current hypothesis and current commutation relations are used to predict that the weak-interaction coupling of nucleon resonances to nucleons is described by a $V-A$ theory.

IT has been shown by one of us[1] that the assumption of a partially conserved axial-vector current (PCAC) or conserved axial-vector current (CAC) with zero-mass pions can be combined with the assumptions of current algebra and analyticity to provide an explanation for the degeneracy of various meson Regge-pole vertices coupled with mesons or currents. These restrictions on the couplings have been shown to apply to the Regge daughter trajectories as well as to the parent trajectories. The extension to daughters is a necessary requirement in order to write simple Veneziano formulas for amplitudes including only mesons and currents as external lines. In addition, these restrictions on the couplings predict directly the form of the couplings governing the decays of various mesons.

It would be of even greater interest if we could obtain predictions regarding the couplings which govern electromagnetic and weak production and decays of the baryons. The $V-A$ theory[2] for the β decay of the neutron is already well established, but the renormalizability of the axial-vector couplings under the assumption of the CAC current commutation relations is still open.[3]

In this paper we extend the techniques mentioned above to the case of axial-vector and vector couplings to baryons. We show that the assumption of CAC with zero-mass pions allows us to predict the form of the couplings to higher-mass baryons and to show that in the CAC limit, the weak production and decay of higher-mass baryons are described by a $V-A$ theory.

We start with the form of the Ward-Takahashi identity for the three-current amplitudes[4]

$$ik_\mu\int d^4xd^4yd^4z\,\exp[i(q\cdot x-q'\cdot y+k\cdot z)]\langle N^a(p')\,|\,T(V_\alpha{}^i(y)A_\beta{}^j(x)A_\mu{}^K(z))\,|\,N^b(p)\rangle$$
$$=\int d^4xd^4yd^4z\,\exp[i(q\cdot x-q'\cdot y+k\cdot z)]\{\langle N^a(p')\,|\,T([A_0{}^K(z),V_\alpha{}^i(y)]A_\beta{}^j(x))\,|\,N^b(p)\rangle\delta(z_0-y_0)$$
$$+\langle N^a(p)\,|\,T([A_0{}^K(z),A_\beta{}^j(x)]V_\alpha{}^i(y))\,|\,N^b(p)\rangle\delta(z_0-x_0)\}\,,\quad(1)$$

where we have assumed the CAC hypothesis

$$\partial_\mu A_\mu(x)=0\,.$$

The superscripts i, j, and K are the isospin indices of the currents, and the subscripts α, β, and μ are the Lorentz indices.

The form factors of the axial-vector current are defined in the usual fashion:

$$\langle N^a(p')\,|\,A_\mu{}^K(0)\,|\,N^b(p)\rangle=[i/(2\pi)^3](\tau^K)_{ab}\bar{u}(p')\times\gamma_5[\gamma_\mu G_A(k^2)+ik_\mu G_P(k^2)]u(p)\,,\quad(2)$$

where $k=p'-p$, so that the CAC condition

$$k_\mu\langle N^a(p')\,|\,A_\mu{}^K(0)\,|\,N^b(p)\rangle=0$$

yields the relation connecting the form factors,

$$G_P(k^2)=(2m/k^2)G_A(k^2)\,.$$

If we proceed to take the limit of Eq. (1) as $k_\mu\to0$, the only contributions to the left-hand side come from the graphs[5] in Fig. 1.

Our result differs from the case considered by Adler[5] in the existence of the contribution coming from the pole of the form factor $G_p(k^2)$. This difference is entirely due to the assumption of CAC and a massless pion in our model. In the $k_\mu\to0$ limit, the contribution to the left-hand side of Eq. (1) therefore has the form

$$-i(m^2/p_0p'_0)^{1/2}\bar{u}(p')[(\tau^K)_{ac}\gamma_5(T_{\alpha,\beta}{}^{i,j}(V,A))_{cb}+(T_{\alpha,\beta}{}^{i,j}(V,A))_{ac}\gamma_5(\tau^K)_{cb}]u(p)G_A(0)\,,\quad(3)$$

where we have defined

$$i(m^2/p_0p'_0)^{1/2}\bar{u}(p')(T_{\alpha,\beta}{}^{i,j}(V,A))_{cb}u(p)=\int d^4xd^4y\,\exp[i(q\cdot x-q'\cdot y)]\langle N^c(p')\,|\times T(V_\alpha{}^i(y)A_\beta{}^j(x))\,|\,N^b(p)\rangle.\quad(4)$$

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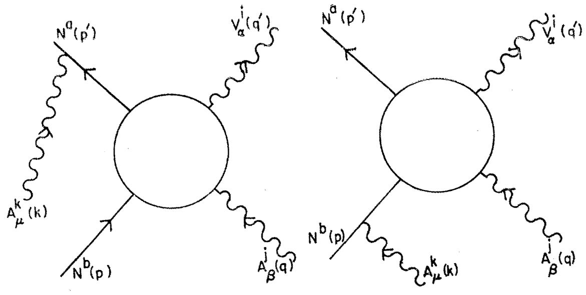

FIG. 1. The Feynman graphs which contribute to the left-hand side of Eq. (1) in the limit $k\to 0$. $V_\alpha{}^i$ and $A_\beta{}^i$ represent the vector and axial-vector currents, respectively.

We next consider the right-hand side of Eq. (1). We assume the validity of the now familiar equal-time commutation relation of the currents

$$[A_0{}^K(z),V_\alpha{}^i(y)]_{z_0=x_0}=i\epsilon_{Kin}A_\alpha{}^n(z)\delta^3(z-y)+i\delta_{Ki}D_{\alpha r}(z)\partial_r\delta^3(z-y)(1-\delta_{\alpha 0}) \quad (5a)$$

and

$$[A_0{}^K(z),A_\beta{}^j(x)]_{z_0=x_0}=i\epsilon_{Kjn}V_\beta{}^n(z)\delta^3(z-x)+i\delta_{Kj}C_{\beta r}(z)\partial_r\delta^3(z-x)(1-\delta_{\beta 0})\,, \quad (5b)$$

where the C_r and D_r designate the Schwinger terms and the summation index r runs from 1 to 3. Since very little is known about the Schwinger terms, we consider the cases in which they do not contribute in Eq. (1). We therefore limit our consideration to the cases where $K\neq i$ and $K\neq j$. In these cases, the right-hand side of Eq. (1) becomes

$$-(m^2/p_0p'_0)^{1/2}\bar{u}(p')[\epsilon_{Kin}(T_{\alpha,\beta}{}^{n,j}(A,A))_{ab}+\epsilon_{Kjm}(T_{\alpha,\beta}{}^{i,m}(V,V))_{ab}]u(p)\,,$$

where $T_{\alpha,\beta}(AA)$ and $T_{\alpha,\beta}(VV)$ are defined by expressions similar to Eq. (4).

Similarly, we can consider a variation of Eq. (1) in which $V_\alpha{}^i$ and $A_\beta{}^j$ are changed to $A_\alpha{}^i$ and $V_\beta{}^j$, respectively. It will also be convenient to consider the combination in which the T product of the left-hand side is changed to

$$T(V_\alpha{}^i(y)A_\beta{}^j(x)A_\mu{}^K(z))+T(A_\alpha{}^i(y)V_\beta{}^j(x)A_\mu{}^K(z))\,. \quad (6)$$

The left-hand side of the equation then becomes

$$-i(m^2/p_0p'_0)^{1/2}\bar{u}(p')\times\{(\tau^K)_{ac}\gamma_5[(T_{\alpha,\beta}{}^{i,j}(V,A))_{cb}+(T_{\alpha,\beta}{}^{i,j}(A,V))_{cb}]+[(T_{\alpha,\beta}{}^{i,j}(V,A))_{ac}+(T_{\alpha,\beta}{}^{i,j}(A,V))_{ac}]\gamma_5(\tau^K)_{cb}\}\times u(p)G_A(0)\,, \quad (3')$$

while the right-hand side becomes

$$-(m^2/p_0p'_0)^{1/2}\bar{u}(p')\times\{\epsilon_{Kin}[(T_{\alpha,\beta}{}^{n,j}(A,A))_{ab}+(T_{\alpha,\beta}{}^{n,j}(V,V))_{ab}]+\epsilon_{Kjm}[(T_{\alpha,\beta}{}^{i,m}(V,V))_{ab}+(T_{\alpha,\beta}{}^{i,m}(A,A))_{ab}]\}u(p)\,. \quad (4')$$

We can show that the quantity (4′) is zero for $K\neq i$ and $K\neq j$. First, let us look at the case where $i\neq j$. We then have the identities

$$\epsilon_{Kin}=\epsilon_{Kij}=-\epsilon_{Kji}=-\epsilon_{Kjm}\,.$$

The expression within the curly brackets of Eq. (4′) therefore becomes

$$\epsilon_{Kij}[(T_{\alpha,\beta}{}^{j,j}(A,A))_{ab}-(T_{\alpha,\beta}{}^{i,i}(A,A))_{ab}+(T_{\alpha,\beta}{}^{j,j}(V,V))_{ab}-(T_{\alpha,\beta}{}^{i,i}(V,V))_{ab}]\,,$$

and since

$$(T_{\alpha,\beta}{}^{j,j}(A,A))_{ab}=(T_{\alpha,\beta}{}^{i,i}(A,A))_{ab}=(T_{\alpha,\beta}{}^{+}(A,A))_{ab}\,,$$

where we define the amplitude T^+ by

$$T_{\alpha,\beta}{}^{i,j}(A,A)=\delta_{ij}T_{\alpha,\beta}{}^{+}(A,A)+i\epsilon_{i,j,l}\tau^lT_{\alpha,\beta}{}^{-}(A,A)\,, \quad (7)$$

it is evident that the quantity in (4′) must vanish for $i\neq j$. Next, we consider the quantity in (4′) when $i=j$. The expression within the curly brackets of (4′) then takes the form

$$\epsilon_{Kin}[(T_{\alpha,\beta}{}^{n,i}(A,A))_{ab}+(T_{\alpha,\beta}{}^{i,n}(A,A))_{ab}+(T_{\alpha,\beta}{}^{n,i}(V,V))_{ab}+(T_{\alpha,\beta}{}^{i,n}(V,V))_{ab}]\,, \quad (8)$$

which can also be seen to vanish if we insert Eq. (7) into (8).

Since the right-hand side of the equation vanishes, we have the following equation for the left-hand side of (3′):

$$\bar{u}(p')\{[(\tau^K)_{ac}\gamma_5\times(T_{\alpha,\beta}{}^{i,j}(V,A))_{cb}+(T_{\alpha,\beta}{}^{i,j}(A,V))_{ac}\gamma_5(\tau^K)_{cb}]+[(\tau^K)_{ac}\gamma_5(T_{\alpha,\beta}{}^{i,j}(A,V))_{cb}+(T_{\alpha,\beta}{}^{i,j}(V,A))_{ac}\times\gamma_5(\tau^K)_{cb}]\}u(p)=0\,, \quad (9)$$

where we have assumed that $G_A(0)\neq 0$ and restricted ourselves to the case where $K\neq i$ and $K\neq j$.

We define the s channel by the variable

$$s=-(p+q)^2\,.$$

If we consider Eq. (8) in terms of the charge states rather than the isospin indices where i and j equal 1, 2, and 3, we find two independent cases in which K equals neither i nor j. We can then rewrite Eq. (9) in terms of the s-channel total isospin amplitudes

$$\bar{u}(p')\{[\gamma_5T_{\alpha,\beta}{}^I(V,A)+T_{\alpha,\beta}{}^I(A,V)\gamma_5]+[\gamma_5T_{\alpha,\beta}{}^I(A,V)+T_{\alpha,\beta}{}^I(V,A)]\}u(p)=0\,, \quad (10)$$

where I represents the total isospin of the s channel.

Following similar arguments we can obtain the identity

$$\bar{u}(p')\{[\gamma_5T_{\alpha,\beta}{}^I(V,V)+T_{\alpha,\beta}{}^I(A,A)\gamma_5]+[\gamma_5T_{\alpha,\beta}{}^I(A,A)+T_{\alpha,\beta}{}^I(V,V)\gamma_5]\}u(p)=0 \quad (11)$$

by starting with an equation similar to (1) in which the T product of the left-hand side is taken as

$$T(V_\alpha{}^i(y)V_\beta{}^j(x)A_\mu{}^K(z))+T(A_\alpha{}^i(y)A_\beta{}^j(x)A_\mu{}^K(z))\,. \quad (12)$$

We can, of course, also start with expressions in which the plus signs in (6) and (12) are replaced by minus signs. Prior to considering the cases involving differences we will discuss some of the implications for coupling strengths implied by the identities in Eqs. (10) and (11). These equations should be valid for any values of the parameters s, t, q^2, and q'^2. In particular they should be true for values of s corresponding to s-channel baryon poles.

In general, the form factors for the process

$$N+V_\mu \to B\,,$$

where the spin of the baryon B, J_B, is greater than $\frac{1}{2}$, can be written in terms of three independent functions

$$\langle B(p')|V_\mu(0)|N(p)\rangle = \frac{1}{(2\pi)^3}\left(\frac{mM_B}{p_0p'_0}\right)^{1/2}\sum_{i=1}^{3}\bar{u}_{\nu,\sigma,\ldots}(p') \times O_{\mu,\nu,\sigma,\ldots}{}^{(i)}u(p)G_{VB}{}^{(i)}(q^2)\,, \quad (13)$$

where $u_{\nu,\sigma,\ldots}(p')$ is the spinor representing the baryon B. For definiteness we take the tensors $O^{(i)}$ as

$$\begin{aligned} O_{\mu,\nu,\sigma,\rho,\ldots}{}^{(1)} &= q_\sigma q_\rho\cdots(P_\mu q_\nu - P\cdot q\delta_{\mu\nu})\,,\\ O_{\mu,\nu,\sigma,\rho,\ldots}{}^{(2)} &= q_\sigma q_\rho\cdots(i\gamma_\mu q_\nu - i\gamma\cdot q\delta_{\mu\nu})\,,\\ \text{and}\quad O_{\mu,\nu,\sigma,\rho,\ldots}{}^{(3)} &= q_\sigma q_\rho\cdots(q_\mu q_\nu - q^2\delta_{\mu\nu})\,, \end{aligned} \quad (14)$$

where $P=p+p'$, and $q=p'-p$, in the case of a normal-parity transition, viz., $\pi_B=(-1)^{J_B-1/2}$. For an abnormal-parity transition we have a common factor γ_5 in (14). In the following parts we discuss mainly the normal-parity transition, but the arguments can be applied as well to the abnormal cases.

Correspondingly, we can also define the form factors of the process $N+A_\mu\to B$ by

$$\langle B(p')|A_\mu(0)|N(p)\rangle = \frac{1}{(2\pi)^3}\left(\frac{mM_B}{p_0p'_0}\right)^{1/2}\sum_{i=1}^{3}\bar{u}_{\nu,\sigma,\ldots}(p') \times\gamma_5 O_{\mu,\nu,\sigma,\ldots}{}^{(i)}u(p)G_{AB}{}^{(i)}(q^2)\,, \quad (15)$$

using the same set of tensors $O^{(i)}$. In the preceding discussion we have omitted the isospin dependence.

If we now consider Eq. (10) for $s\simeq m_B{}^2$, where the baryon resonance B is assumed to dominate the scattering amplitudes, we can insert the forms of the vertices from Eqs. (13) and (15) and the appropriate propagator for the baryon B to obtain the equation

$$\begin{aligned}\sum_{i,j=1}^{3}\bar{u}(p')\{&\gamma_5 O_{\alpha,\nu,\sigma,\ldots}{}^{(i)}P_{\nu,\sigma,\ldots;\nu',\sigma',\ldots}[\gamma_5 O_{\beta,\nu',\sigma',\ldots}{}^{(j)}G_{VB}{}^{(i)*}(q'^2)G_{AB}{}^{(j)}(q^2)\\ &+(-1)^i O_{\beta,\nu',\sigma',\ldots}{}^{(j)}\gamma_5 G_{AB}{}^{(i)*}(q'^2)G_{VB}{}^{(j)}(q^2)] + O_{\alpha,\nu,\sigma,\ldots}{}^{(i)}P_{\nu,\sigma,\ldots,\nu',\sigma',\ldots}[(-1)^i O_{\beta,\nu',\sigma',\ldots}{}^{(j)}G_{AB}{}^{(i)*}(q'^2)G_{VB}{}^{(j)}(q^2)\\ &+\gamma_5 O_{\beta,\nu',\sigma',\ldots}{}^{(j)}\gamma_5 G_{VB}{}^{(i)*}(q'^2)G_{AB}{}^{(j)}(q^2)]\}u(p)=0\,,\end{aligned} \quad (16)$$

where $P_{\nu\sigma\ldots}$ is the numerator of the propagator for the baryon B. The factor $(-1)^i$ comes from the structure of the tensors $O^{(i)}$ which requires the relations

$$\begin{aligned}&\{O^{(2)},\gamma_5\}=0\,,\\ &[O^{(i)},\gamma_5]=0 \quad\text{for}\quad i=1\quad\text{and}\quad 3\,,\end{aligned} \quad (17)$$

and

$$\begin{aligned}\gamma_4[O_{\mu,\nu,\sigma\ldots}{}^{(i)}]^\dagger\gamma_4 &= O_{\mu,\nu,\sigma}{}^{(i)}\,,\\ \gamma_4[\gamma_5 O_{\mu,\nu,\sigma\ldots}{}^{(i)}]^\dagger\gamma_4 &= (-1)^i\gamma_5 O_{\mu,\nu,\sigma\ldots}{}^{(i)}\,,\end{aligned} \quad (18)$$

where $\mu\neq 4$.[6] It is, therefore, obvious that Eq. (16) can be satisfied if

$$G_{VB}{}^{(i)}(q^2) = \pm(-1)^i G_{AB}{}^{(i)}(q^2) \quad (i=1,2,3)\,. \quad (19)$$

An inspection of Eq. (16) further reveals that the independence of the kinematical factors implies that the only solutions to Eq. (16) are the trivial solution where all $G^{(i)}$ vanish and the solutions given in Eq. (19). We have specifically examined the case where $J_B=\frac{3}{2}$ and have found that for this case Eq. (19) is the unique nontrivial solution. These solutions also satisfy the equation analogous to Eq. (16) which arises from Eq. (12).

Next, we discuss the form of Eq. (1) for the cases in which we take the combinations of T products as

$$T(V_\alpha{}^i(y)A_\beta{}^j(x)A_\mu{}^K(z)) - T(A_\alpha{}^i(y)V_\beta{}^j(x)A_\mu{}^K(z)) \quad (6')$$

and

$$T(V_\alpha{}^i(y)V_\beta{}^j(x)A_\mu{}^K(z)) - T(A_\alpha{}^i(y)A_\beta{}^j(x)A_\mu{}^K(z))\,, \quad (11')$$

rather than the combinations in (6) and (11). The left-hand side of Eq. (1) in the case (6′) becomes

$$\begin{aligned}&-i(m^2/p_0p'_0)^{1/2}\bar{u}(p')\\ &\times\{(\tau^K)_{ac}\gamma_5[(T_{\alpha,\beta}{}^{i,j}(V,A))_{cb}-(T_{\alpha,\beta}{}^{i,j}(A,V))_{cb}]\\ &+[(T_{\alpha,\beta}{}^{i,j}(V,A))_{ac}-(T_{\alpha,\beta}{}^{i,j}(A,V))_{ac}]\gamma_5(\tau^K)_{cb}\}\\ &\times u(p)G_A(0)\,.\end{aligned} \quad (20)$$

Here we can write $T^{ij}(V,A)$ and $T^{ij}(A,V)$ in terms of $T^\pm(V,A)$ and $T^\pm(A,V)$ which are defined as Eq. (7). It is not, however, convenient to do so, since the initial and final currents differ and it is necessary to symmetrize V and A in the t channel in order to obtain an actual t-channel isospin definite amplitude.

Instead we consider the following approach. Consider the cases in which the initial and final states, N^b+J^j and N^a+J^i, form pure $I=\frac{3}{2}$ states. Then Eq. (1) gives an equation which includes only $I=\frac{3}{2}$ amplitudes on both the left-hand and right-hand sides. The Schwinger terms are assumed to be isoscalar, which implies that our equation will be free from Schwinger terms. There-

[6] Equation (17) can be used also for the cases in which μ is 4, since we should consider the effect to take complex conjugate of the polarization vector to be multiplied.

fore, we can neglect all Schwinger terms on the right-hand side of Eq. (1), giving us the expression

$$-(m^2/p_0p'_0)^{1/2}\bar{u}(p')\times\{\epsilon_{Kin}[(T_{\alpha,\beta}{}^{n,j}(A,A))_{ab}-(T_{\alpha,\beta}{}^{n,j}(V,V))_{ab}]+\epsilon_{Kjn}[(T_{\alpha,\beta}{}^{i,n}(V,V))_{ab}-(T_{\alpha,\beta}{}^{i,n}(A,A))_{ab}]\}u(p)\,,\quad(21)$$

where (a,i) and (b,j) are restricted to combinations giving $I=\frac{3}{2}$ states and K is restricted to ensure charge conservation.

We have examined all of the possible $I=\frac{3}{2}$ states for the combinations of (a,i) and (b,j) and found that the $I=\frac{3}{2}$ part of Eq. (1) for the combination of T products in (6′) gives

$$\bar{u}(p')\{\gamma_5[T_{\alpha,\beta}{}^{3/2}(V,A)-T_{\alpha,\beta}{}^{3/2}(A,V)]+[T_{\alpha,\beta}{}^{3/2}(V,A)-T_{\alpha,\beta}{}^{3/2}(A,V)]\gamma_5\}u(p)G_A(0)=2\bar{u}(p')[T_{\alpha,\beta}{}^{3/2}(A,A)-T_{\alpha,\beta}{}^{3/2}(V,V)]u(p)\,,\quad(22)$$

which is satisfied by the same solution for the $G^{(i)}$'s given in Eq. (19), provided that $G_A(0)=+1$ for the solutions with (+) sign in (19) and $G_A(0)=-1$ for those with (−) sign in (19). So Eq. (19), the solutions of equations of the type (16) for all cases, can be written as

$$G_{AB}{}^{(i)}(q^2)=(-1)^iG_A(0)G_{VB}{}^{(i)}(q^2)\,,$$

with

$$G_A(0)=\pm1\,.\quad(23)$$

Our result, Eq. (23), may be expected to be accurate to within 20%, since we have completely neglected the pion mass and since the Adler-Weisberger calculation is known to add a 20% correction to the CAC value, $G_A(0)=-1$. The correction due to the CAC violation can presumably be calculated using PCAC and by considering some approximation for the five-point amplitudes which appear in the basic expression, Eq. (1)

The condition $G_A(0)=\pm1$ which we have obtained depends upon our assumption of the commutation relations for the currents and CAC. This is, however, by no means a trivial result as was shown by Blin-Stoyle.[3] It would be interesting to discover whether this relation can be proven in perturbation theory for some Lagrangian model in which CAC and current commutation relations are both satisfied.[3]

In summary, our results predict that the weak production or decay of the higher-mass baryons with $J_B\geq\frac{3}{2}$ and $\pi_B=(-1)^{J_B-1/2}$ must be described by a $V-A$ combination in the sense that

$$\langle N(p')|J_\alpha(0)|B(p)\rangle=\frac{1}{(2\pi)^3}\left(\frac{M_Bm}{p_0p'_0}\right)^{1/2}\sum_{i=1}^{3}\bar{u}(p')\times[1+G_\lambda(0)\gamma_5]O_{\alpha,\nu,\sigma,\ldots}{}^{(i)}u_{\nu,\sigma,\ldots}(p)G_{VB}{}^{(i)}(q^2)\,,\quad(24)$$

noticing Eqs. (17), (18), and (22), where $G_A(0)=\pm1$ and the choice $G_A(0)>0$ has been determined by the analysis of β decay of the neutron.[7] In the case of $\pi_B=(-1)^{J_B+1/2}$, the abnormal-parity transition, Eq. (24), is true if we replace $O^{(i)}$ with $-O^{(i)}\gamma_5$ in (24), which gives exactly the same form apart from an overall sign. It would be most interesting to measure $G_{A\Delta_{33}}{}^{(i)}(q^2)$ by the process $\nu+p\to\mu^-+\Delta^{++}$ to check the relation $G_{A\Delta}{}^{(i)}(q^2)=G_{V\Delta}{}^{(i)}(q^2)$, since this provides a direct test of the current commutation relations since we have fairly accurate results for the values of $G_{V\Delta_{33}}{}^{(i)}(q^2)$.[8]

Finally, we add a few remarks. Quite similar arguments can be applied to the case of $J_B=\frac{1}{2}$, if we use $O^{(i)}$'s as[9]

$$O_\mu{}^{(1)}=\gamma_\mu q^2-q_\mu\gamma\cdot q$$

and

$$O_\mu{}^{(2)}=\sigma_{\mu\nu}q_\nu\,,$$

except for the case of $B=N$, in which $\gamma_5O_\mu{}^{(i)}$ for the axial-vector form factors violates G invariance.

For the nucleon form factors, we find another limitation. In this case Eqs. (10) and (11) cannot be brought into the form of Eq. (16), since our assumption of a massless π meson requires that the point $s=m_N{}^2$ is not a simple pole but is the site of infinitely many branch points. Higher baryons do not encounter this difficulty since they are resonances, and therefore correspond to the poles in the second sheet.

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