D. Matrix Elements of Γ_A

From the known action of Γ_A on $\tilde{\psi}$,

$$\Gamma_A\tilde{\psi}_{A_1\cdots A_t}=\tfrac{1}{2}(t+1)\tilde{\psi}_{AA_1\cdots A_t}+(t+1)\lambda_A\tilde{\psi}_{A_1\cdots A_t}+S[-t\theta_{AA_1}\tilde{\psi}_{A_2\cdots A_t}+\tfrac{1}{2}(t-1)\theta_{A_1A_2}\tilde{\psi}_{AA_3\cdots A_t}],$$

one easily verifies that [compare (A12)]

$$\varphi^{\nu\dagger}\Gamma_A\psi_n=(-)^{n+\nu}\sum_{r=0,1,\ldots}G_{\nu n}{}^r(\lambda'\lambda)\times\left[(r-1-t-t')\frac{\lambda_A+\lambda'_A}{1+\lambda\lambda'}+\frac{\partial}{\partial\lambda^A}+\frac{\partial}{\partial\lambda'^A}\right]\{\cdots\}$$

$$=-\left(\frac{\partial}{\partial\lambda^A}+\frac{\partial}{\partial\lambda'^A}\right)\varphi^{\nu\dagger}\psi_n. \quad \text{(A14)}$$

E. Kinematics for Bremsstrahlung

From (2.3), (4.3), and (4.4) we find, with $\mathbf{k}=\mathbf{p}-\mathbf{p}'$,

$$i\lambda V'=[q^2-(\mathbf{q}'-\mathbf{k})^2]/qq', \quad \text{(A15)}$$

$$i\lambda' V=[q'^2-(\mathbf{q}+\mathbf{k})^2]/qq', \quad \text{(A16)}$$

$$V'V=+2(\mathbf{k}+\mathbf{q}-\mathbf{q}')^2/qq', \quad \text{(A17)}$$

$$\lambda'\lambda+1=[(q+q')^2-k^2]/2qq', \quad \text{(A18)}$$

$$\mathcal{E}\lambda=(i/q)(\mu\epsilon_0+\tfrac{1}{2}\boldsymbol{\varepsilon}\cdot\mathbf{p}-\tfrac{1}{2}\boldsymbol{\varepsilon}\cdot\mathbf{p}'), \quad \text{(A19)}$$

$$\mathcal{E}V=(2/q)(\mu\epsilon_0+\boldsymbol{\varepsilon}\cdot\mathbf{q}+\tfrac{1}{2}\boldsymbol{\varepsilon}\cdot\mathbf{p}-\tfrac{1}{2}\boldsymbol{\varepsilon}\cdot\mathbf{p}'). \quad \text{(A20)}$$

PHYSICAL REVIEW D VOLUME 3, NUMBER 2 15 JANUARY 1971

Possible Connection between Chiral Symmetry Breaking and the Nonet of Tensor Trajectories

E. Gal-Ezer, L. Gomberoff, and S. Nussinov
Department of Physics and Astronomy, Tel-Aviv University, Tel-Aviv, Israel
(Received 28 July 1970)

Pole dominance in the complex angular momentum plane is used in order to estimate deviations from a perturbative scheme based on the Gell-Mann–Oakes–Renner model of chiral symmetry breaking. It is seen that the deviation can be very large, leading to a mass pattern which corresponds to almost-conserved $SU(2)\times SU(2)$, even when the Hamiltonian originally deviates considerably from this limit.

I. INTRODUCTION

Various authors have recently emphasized the fact that the successful application of partial conservation of axial-vector current (PCAC) and current algebra reflects an approximate chiral $SU(3)\times SU(3)$ symmetry.[1–4] The axial-vector part of the symmetry is realized via the Goldstone boson mechanism and yields low-energy theorems for massless mesons.

A particularly attractive model for the breaking of $SU(3)\times SU(3)$ is that of Gell-Mann, Oakes, and Renner (GOR).[5] In this model, the energy density has the following form:

$$\theta_{00}(x)=\bar{\theta}_{00}(x)-\epsilon[u_0(x)+cu_8(x)], \quad (1)$$

where $\bar{\theta}_{00}(x)$ is $SU(3)\times SU(3)$ symmetric and u_0 and u_8 belong to a $(3,3^*)+(3^*,3)$ representation of $SU(3)\times SU(3)$. Specifically, this means that u_0 and u_8 belong to two nonets u_i and v_i which satisfy

$$[F_i,u_j(x)]=if_{ijk}u_k(x),\quad [F_i{}^5,u_j(x)]=-id_{ijk}v_k(x),$$
$$[F_i,v_j(x)]=if_{ijk}v_k(x),\quad [F_i{}^5,v_j(x)]=id_{ijk}u_k(x), \quad (2)$$

where F_i $(F_i{}^5)$, $i=1,\ldots,8$, are the vector (axial-vector) generators of $SU(3)\times SU(3)$ and we use generalized d_{ijk} for j, $k=0,\ldots,8$.

Comparing with the general form allowed by the requirement of octet breaking of $SU(3)$, the model (1) consists of (i) neglect of possible $(1,8)+(8,1)$ contributions, and (ii) the requirement that u_0 and u_8 belong to the same $(3,3^*)+(3^*,3)$ representation. (i) and (ii) are suggested by a quark model where the only chiral symmetry breaking occurs via the quark masses.[4]

Originally c was believed to be small,[3] of the order of the ratio of baryon mass differences to baryon masses corresponding to the breaking chain $SU(3)\times SU(3)\to SU(3)\to SU(2)$. Later on, the success of soft-pion theorems led to the adoption of Nambu's suggestion[4] that the small pion mass (rather than the large nucleon mass[6]) is the correct measure for chiral $SU(2)\times SU(2)$

---

[1] R. Dashen, Phys. Rev. **183**, 1245 (1969).
[2] R. Dashen and M. Weinstein, Phys. Rev. **183**, 1261 (1969).
[3] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962); Physics **1**, 63 (1964).
[4] Y. Nambu, Phys. Rev. Letters **4**, 380 (1960).
[5] M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. **175**, 2195 (1968).

[6] This is consistent with the recent calculations of approximately 170-MeV contribution to baryon masses by the $SU(3)\times SU(3)$-breaking Hamiltonian; see J. K. Kim and F. von Hippel, Phys. Rev. Letters **22**, 740 (1969).

symmetry breaking. Indeed the more recent determination[5] yields c close to $-\sqrt{2}$, which is the $SU(2)\times SU(2)$ limit corresponding to zero $\mathfrak{N}$ and $\mathcal{P}$ quark masses and conserved F_i^μ, $F_i^{5\mu}$ for $i=1, 2, 3$.

GOR assumed the following:

(a) Matrix elements of the scalar densities between states of the pseudoscalar octet satisfy $SU(3)$ relations:

$$F_{ijk}(t)=\langle P_i|u_j(0)|P_k\rangle=\alpha(t)\delta_{ij}+\beta(t)d_{ijk}\,,\quad t=(P_i-P_k)^2. \tag{3}$$

(b) PCAC, i.e., smooth extrapolation of the above matrix elements to $P_i=0$ for π, k, and η mesons.

As a consequence they find not only that

$$\text{(c)}\qquad c\simeq\sqrt{2}\frac{m_\pi^2-m_K^2}{m_K^2+\frac{1}{2}m_\pi^2}\simeq-1.25, \tag{4}$$

but also that

$$\langle 0|v_i|P_j\rangle=A\,\delta_{ij} \tag{5}$$

and

$$f_\pi\simeq f_K\simeq f_\eta. \tag{6}$$

(d) The vacuum is approximately $SU(3)$ invariant [in particular, $\langle 0|u_8|0\rangle=0$), so that the $SU(3)$ part of the symmetry is realized "linearly"].

To justify their approximations, GOR note the absence of nearby poles in $F_{ijk}(t)$ due to lack of low-lying 0^+ states. This suggests a smooth t behavior and no significant distortions from the $SU(3)$ limit. As will be seen in Sec. II, the picture presented by GOR is essentially a perturbative expansion around the $SU(3)\times SU(3)$ symmetry limit.

As emphasized by GOR, it is only correct to say that (a)–(d) are mutually consistent when we have (1).

It seems quite difficult (and certainly beyond the limited scope of the present paper) to construct alternative consistent nonperturbative models of chiral symmetry breaking[7] which incorporates the strong-interaction corrections to (a)–(d) above. Our main purpose is to investigate possible "selective dynamical enhancement" of matrix elements $\langle P'|u_0(x)-\sqrt{2}u_8(x)|P\rangle$ [this involves giving up assumption (a)], so that even for c relatively far from $-\sqrt{2}$, i.e., large $SU(2)\times SU(2)$ violation in the "bare Hamiltonian," a small π-meson mass results. The mechanism for that selective enhancement is the pole-dominance scheme in the complex angular momentum plane.[1]

An alternative "derivation" of this dominance scheme, in the case the $SU(3)\times SU(3)$ breaking part of the Hamiltonian in Eq. (1) arises from second-order current×current effects, is given in the Appendix.

II. DETERMINATION OF c FROM MASSES OF PSEUDOSCALAR MESONS

We would like first to prove the formula

$$m_j^2\simeq-\langle j(P)|\epsilon(u_0+cu_8)|j(P)\rangle \tag{7}$$

for covariantly normalized pseudoscalar meson states $|j(p)\rangle$,

$$\langle j(P)|i(P')\rangle=(2\pi)^3 2P_0\delta^3(\mathbf{P}-\mathbf{P}')\delta_{ij}. \tag{8}$$

We can write the Hamiltonian $H=\int d^3x\,\theta_{00}(x)$ as $H=H_S+\epsilon H_8$, where $H_S=\int d^3x[\theta_{00}-\epsilon u_0(x)]$ and $\epsilon H_8=-\epsilon c\int d^3x\,u_8(x)$. First-order perturbation theory gives in general

$$\begin{aligned}E=E_S+\delta E&\simeq\langle i(\sim\mathbf{P})|H|i(\sim\mathbf{P})\rangle\\&=C_0+C_1Y+C_2[\tfrac{1}{4}Y^2-I(I+1)],\end{aligned} \tag{9}$$

where $|i(\sim\mathbf{P})\rangle$ denotes a wave-packet state normalized to one, corresponding to an eigenstate of the $SU(3)$-symmetric part of the Hamiltonian H_S, and hence it belongs to a specific $SU(3)$ representation.

In the limit $\mathbf{P}\to 0$, Eq. (9) reduces to the well-known Gell-Mann–Okubo (GMO) mass formula, in which case, as the above argument suggests, linear masses should in general appear. A notable exception, as emphasized in Ref. 2, is the case of 0^- mesons. Here $\delta E\simeq E$, and there are difficulties with the limit $\mathbf{P}\to 0$, However, in this case $-\epsilon(H_0+cH_8)$ with $H_0=\int d^3x\,u_0(x)$ can be considered as a perturbation to $\bar{H}=\int d^3x\,\bar{\theta}_{00}(x)$ which defines the zero-mass free meson states. We have then again, by analogy with relation (9),

$$E=\bar{E}+\delta E\simeq\langle j(\sim\mathbf{P})|H|j(\sim\mathbf{P})\rangle, \tag{10}$$

where now

$$\begin{aligned}E+\delta E=|\mathbf{P}|+\delta E&=(|\mathbf{P}|^2+m_j^2)^{1/2}\\&=|\mathbf{P}|+m_j^2/2|\mathbf{P}|+\cdots,\end{aligned}$$

which, upon going back to sharp normalized states and the Hamiltonian density, yields Eq. (7):

$$\begin{aligned}m_j^2=2|\mathbf{P}|\,\delta E&\simeq-2|\mathbf{P}|\langle j(\sim\mathbf{P})|\epsilon(H_0+cH_8)|j(\sim\mathbf{P})\rangle\\&=-2|\mathbf{P}|\int d^3x\int d^3P'\int d^3P''\chi^*(\mathbf{P}')\chi(\mathbf{P}'')\\&\qquad\times e^{-ix\cdot(\mathbf{P}'-\mathbf{P}'')}\langle j(\mathbf{P}')|\epsilon(u_0+cu_8)|j(\mathbf{P}'')\rangle.\end{aligned}$$

Doing first the x integration and going to the limit of an infinitely sharp wave packet, we obtain

$$m_j^2\simeq\langle j(\mathbf{P})|-\epsilon(u_0+cu_8)|j(\mathbf{P})\rangle=\epsilon(A+Bcd_{8jj}). \tag{11}$$

In order to obtain, in the $SU(2)\times SU(2)$ limit

---

[7] Several variations of the GOR model have been discussed in the literature. In particular, the possibility of additional spontaneous breaking of $SU(3)$ and its possible connection with a "0^+ kappa Goldstone boson" were considered by S. Glashow and S. Weinberg, Phys. Rev. Letters **20**, 224 (1968); see also P. R. Auvil and N. G. Desphande, Phys. Rev. **183**, 1463 (1969). Attempts to relax the need for K-meson PCAC in deriving c were done by J. Ellis, Nucl. Phys. **B13**, 153 (1969); L. Gomberoff and Z. Grossman, Nuovo Cimento (to be published).

($c=-\sqrt{2}$), $m_\pi^2=0$, we must have[8]

$$A=(\sqrt{\tfrac{2}{3}})B. \quad (12)$$

Using the last relation, we have

$$m_\pi^2=\epsilon\left[(\sqrt{\tfrac{2}{3}})+\frac{c}{\sqrt{2}}\right]B,$$

$$m_K^2=\epsilon\left[(\sqrt{\tfrac{2}{3}})-\frac{c}{2\sqrt{3}}\right]B, \quad (13)$$

$$m_\eta^2=\epsilon\left[(\sqrt{\tfrac{2}{3}})-\frac{c}{\sqrt{3}}\right]B,$$

with the first two relations yielding the GOR result, Eq. (4).[9]

The present derivation of Eq. (7) is actually quite close to the original derivation of GOR which utilized PCAC, namely,

$$\begin{aligned}-\langle i(P')\,|\,\epsilon(u_0+cu_8)\,|\,i(P)\rangle &\simeq -\lim_{P'\to 0}\langle i(P')\,|\,\epsilon(u_0+cu_8)\,|\,i(P)\rangle\\ &=-2f_i i\langle 0\,|\,[F_i^5,\ \epsilon(u_0+cu_8)]\,|\,i(P)\rangle\\ &=2f_i\langle 0\,|\,\partial_\mu F_\mu^{5i}\,|\,i(P)\rangle=m_i^2.\end{aligned} \quad (14)$$

The use of PCAC and the assumption that in the symmetry limit we have eight massless Goldstone bosons are essentially equivalent. The present derivation emphasized the essentially perturbative feature of the GOR picture.

III. SELECTIVE ENHANCEMENT DUE TO DOMINANCE IN COMPLEX ANGULAR MOMENTUM PLANE

In this section we consider possible deviations of $\langle i(P')|u_j(0)|k(P)\rangle$ from $SU(3)$ predictions due to different intercepts of the various nonet Regge trajectories, reflecting t-channel dynamical effects.

Our starting point is the pole approximation in the complex angular momentum plane,[1]

$$-\langle i(P')\,|\,u_j(0)\,|\,k(P)\rangle\simeq\sum_l\frac{\Gamma_l{}^j(t)}{\alpha_l(t)}\gamma_{ilk}(t), \quad (15)$$

where

$$i,k=1,\ldots,8,\quad j=0,\ldots,8,\quad t=(P'-P)^2,$$

and the sum goes over the nonet of tensor meson trajectories. A simple motivation used in Ref. 1 for Eq. (15) is the analog with the pole-dominated form factor

$$-\langle i(P')\,|\,u_j(0)\,|\,K(P)\rangle=\sum_l\frac{\langle 0\,|\,u_j(0)\,|\,R_l\rangle\langle R_l\,|\,\bar{i}(P')K(P)\rangle}{t-m_{Rl}{}^2},$$

where the sum goes over the 0^+ particles. In the present case [Eq. (15)], $1/\alpha_l(t)$ plays the role of a propagator, and $\Gamma_l{}^j(t)$, the Reggeon scalar coupling, is the analog of $\langle 0\,|\,u_j(0)\,|\,R_l\rangle$. Furthermore, we assume that the coupling of the Regge trajectories to the mesons, $\gamma_{ilk}(t)$, and the Reggeon scalar density couplings $\Gamma_l{}^j(t)$, satisfy the $SU(3)$ relations[10]

$$\gamma_{ilk}(t)=\bar{a}(t)\delta_{l0}\delta_{ik}+\bar{b}(t)d_{ilk}, \quad (16a)$$

$$\Gamma_l{}^j(t)=T(t)\delta_{jl}. \quad (16b)$$

Only the validity of Eqs. (16) at $t=0$ will be necessary for our discussion. The assumption of factorized Regge exchange with $SU(3)$-symmetric couplings and $SU(3)$ symmetry breaking occurring only in the different intercepts $\alpha_i(t)$ is usually made in Regge-pole analysis of high-energy meson-baryon scattering, and seems to be experimentally correct.[11] Our assumptions (16) are justified or motivated by this, and will be further discussed in the Appendix.

Substituting Eq. (10) in Eq. (15), we find

$$-\langle i(P)\,|\,u_j(0)\,|\,k(P)\rangle=b(t)d_{ijk}/\alpha_j(t),\quad j=1,\ldots,7. \quad (17)$$

For $j=0$, 8 there are contributions from the mixed f and f' trajectories,

$$\begin{aligned}|f\rangle&=\cos\theta\,|f_8\rangle+\sin\theta\,|f_0\rangle,\\ |f'\rangle&=-\sin\theta\,|f_8\rangle+\cos\theta\,|f_0\rangle.\end{aligned} \quad (18)$$

Equation (16b) implies that we have to consider also the corresponding combinations of the u's

$$-\langle i(P')\,|\cos\theta\,u_8(0)+\sin\theta\,u_0(0)\,|\,k(P)\rangle=\frac{1}{\alpha_f(t)}\{\sin\theta[a(t)+(\sqrt{\tfrac{2}{3}})b(t)]\delta_{ik}+\cos\theta\,b(t)d_{8ik}\}$$

and (19)

$$-\langle i(P')\,|-\sin\theta\,u_8(0)+\cos\theta\,u_0(0)\,|\,k(P)\rangle=\frac{1}{\alpha_{f'}(t)}\{\cos\theta[a(t)+(\sqrt{\tfrac{2}{3}})b(t)]\delta_{ik}-\sin\theta\,b(t)d_{8ik}\}.$$

We will use the canonical "ideal" mixing angle $\sin\theta=\sqrt{\tfrac{2}{3}}$, $\cos\theta=\sqrt{\tfrac{1}{3}}$. This corresponds to the quark

[8] Strictly speaking, we should write $A=A_0+\delta A_1+\cdots$, $B=B_0+\delta B_1+\cdots$, where $\delta=\sqrt{2}+c$ is the parameter which measures the deviation from exact $SU(2)\times SU(2)$ symmetry, with A_0 and B_0 the values for $\delta=0$ satisfying $A_0=(\sqrt{\tfrac{2}{3}})B_0$. In writing relation (12), we have neglected δ corrections and therefore assumed δ (which we set out to determine) to be small, $\delta\ll 1$.

[9] If we neglect mixing between η and η', the GMO mass formula holds for the octet. In this approximation c can actually be anything between -0.9 and -1.3, but if we allow mixing, c is necessarily fixed by the first two relations, Eq. (13), to be $c\simeq-1.25$.

[10] It has been suggested by N. Cabibbo, L. Horwitz, and Y. Ne'eman, Phys. Letters 22, 336 (1968), that the residues at $t=0$ have a universality pattern and span an algebra. From this point of view, it would seem quite natural to assume the $SU(3)$ relations for $\gamma_{ilk}(0)$.

[11] See, e.g., P. D. B. Collins and E. J. Squires, *Regge Poles in Particle Physics, Springer Tracts in Modern Physics* (Springer Verlag, Berlin, 1968), Vol. 45.

composition $f\sim(1/\sqrt{2})(\mathfrak{N}\mathfrak{N}+\mathcal{P}\mathcal{P})$, $f'\sim\bar{\lambda}\lambda$, and is motivated by the need for the λ quark selection rule to suppress $f'\to\pi\pi$, the GMO mass formula, and also by exchange degeneracy and ideal mixing for the vector nonet. Equation (7) then gives

$$m_i^2=\epsilon\frac{1-\sqrt{2}c}{3\alpha_{f'}(0)}\{a(0)+b(0)[(\surd\tfrac{2}{3})-\sqrt{2}d_{8ii}]\}+\epsilon\frac{(\sqrt{2}+c)\sqrt{2}}{3\alpha_f(0)}\left\{a(0)+b(0)\left[(\surd\tfrac{2}{3})+\frac{1}{\sqrt{2}}d_{8ii}\right]\right\}. \quad (20)$$

In the nonet scheme[12] $a(0)=0$, which in addition to ideal mixing is necessary in order to ensure $\gamma_{f'\pi\pi}(0)=0$. Alternatively, $a(0)\simeq a(m_\pi^2)\simeq 0$ can be derived by using Eq. (19) and going, with the aid of PCAC and the algebra of Eq. (2), to $P'=0$ in $\langle\pi(P')|u_0-\sqrt{2}u_8|\pi(P)\rangle$. Equation (20) therefore yields

$$m_\pi^2=\epsilon\frac{\sqrt{2}+c}{\sqrt{3}\alpha_f(0)}b(0),$$
$$m_K^2=\epsilon\left[\frac{\sqrt{2}+c}{2\sqrt{3}\alpha_f(0)}+\frac{1-\sqrt{2}c}{(\surd 6)\alpha_{f'}(0)}\right]b(0), \quad (21)$$

so that we finally find[13]

$$\frac{m_K^2}{m_\pi^2}=\frac{1}{2}+\frac{1-\sqrt{2}c}{2+\sqrt{2}c}\frac{\alpha_f(0)}{\alpha_{f'}(0)}. \quad (22)$$

In the limit $\alpha_f(0)=\alpha_{f'}(0)$ $[=\alpha_{K^{**}}(0)=\alpha_{A_2}(0)]$, Eq. (22) reduces to the original analysis of GOR [see Eq. (13)]. The effect of pole dominance in the complex angular momentum plane is then only to simultaneously enhance all octetlike contributions to masses and to weak and electromagnetic effects. Indeed this fact motivated[1] the introduction of this Reggeized version of the original tadpole mechanism.[14]

Assuming Eqs. (16), we have, when $r=\alpha_f(0)/\alpha_{f'}(0)>1$, "selective-enhancement" contributions which in the quark mnemonic (for nonet couplings) correspond to the fact that $\bar{\lambda}\lambda$ quantum numbers are enhanced relative to $\bar{\mathfrak{N}}\mathfrak{N}$ and $\bar{\mathcal{P}}\mathcal{P}$, leading to an effective Hamiltonian which is closer to the $SU(2)\times SU(2)$-symmetric limit. In estimating r, we use $\alpha_f(0)=0.5\pm0.15$, a value consistent with high-energy fits [as well as the fits $\alpha_\rho(0)$, $\alpha_{A_2}(0)$ which should be equal to $\alpha_f(0)$ by exchange degeneracy and $SU(3)$], and with a linear exchange-degenerate ω-f trajectory.[10] There are no direct high-energy determinations of $\alpha_{f'}(0)$. Assuming a linear f' trajectory with the universal slope $\alpha'=0.9-1.0$ GeV^{-2},[15] we find $\alpha_{f'}(0)\simeq 0$, while $\alpha_{f'}(0)=0.2$ follows from a linear exchange-degenerate ϕ-f' trajectory.[16] Thus $r\geq 2$ and large values for r are not excluded. In this case there is no need for c to be close to $-\sqrt{2}$ in the "true" $SU(3)\times SU(3)$ symmetry-breaking Hamiltonian. Particularly,

$$\alpha_{f'}(0)\simeq 0 \quad (23)$$

ensures $m_\pi^2\simeq 0$ for any c, as long as

$$(1-\sqrt{2}c)/(2+\sqrt{2}c)>0.$$

It is amusing to note that combining (23) with some results of simple dual models,

$$\frac{\alpha_f(0)-\alpha_{f'}(0)}{\alpha'}\simeq m_{f'}^2-m_f^2 \simeq 2(m_{K^{**}}^2-m_{f}^2)\simeq 2(m_K^2-m_\pi^2), \quad (24)$$

and the condition[15] $\alpha_\rho(m_\pi^2)\simeq\alpha_f(m_\pi^2)=\frac{1}{2}$, gives

$$m_\rho^2\simeq 2m_K^2, \quad (25)$$

one of the key relations for the validity of Freund's "mass quantum."[17]

So far, we have considered diagonal matrix elements $\langle i(P)|u|i(P)\rangle$. The general relations (17) and (19) are inconsistent with all possible "PCAC constraints" at $P'=0$ or $P=0$, found by using smoothness and the commutation relations of Eq. (2). Thus, PCAC for $\langle\pi|u_K|K\rangle$ and K-PCAC for $\langle K|u_\pi|K\rangle$ yields $f_\pi/f_K=\alpha_{A_2}(m_K)/\alpha_{K^{**}}(m_K^2)$, but K-PCAC for $\langle K|u_K|\pi\rangle$ and π-PCAC for $\langle\pi|u_\omega|\pi\rangle$ yields $f_\pi/f_K=\alpha_{K^{**}}(m_\pi^2)/\alpha_f(m_\pi^2)$. Assuming $\alpha_f=\alpha_{A_2}$ and linear parallel trajectories, consistency between these relations is achieved only in the limit $\alpha_{A_2}=\alpha_{K^{**}}$ (and then incidentally also $f_K=f_\pi$), i.e., the GOR limit with no "selective enhancement."

The above inconsistency is indeed expected; the selective-enhancement mechanism results from "t-channel dynamics" and the "t channel" is "contracted out" in the soft-pion limit. Our conclusion that K-PCAC must be severely violated (to about 40%) is certainly not ruled out by experiment.

One might also be tempted to use the explicit t dependence of Eq. (17), with $b(t)\approx b(0)$, to predict, for example, $\simeq$30% variation over the region $0<t<m_K^2$ of the matrix element

$$-(i/\sqrt{2})[f^+(t)(m_K^2-m_\pi^2)+tf^-(t)] =\langle K(P)|\partial_\mu V_\mu{}^K|\pi(P')\rangle \sim\langle K(P)|u^K|\pi(P')\rangle\sim 1/\alpha_{K^*}(t).$$

While this value is consistent with the experimental information, it is not clear that it can be trusted. In particular, a nonexistent "ghost pole" at $\alpha(t)=0$ is

[12] S. Okubo, Phys. Letters 5, 165 (1968); S. L. Glashow and R. H. Socolow, Phys. Rev. Letters 15, 329 (1965).

[13] We have not considered the η here, because the nonet scheme for $\langle p_j|u_i|p_k\rangle$ seems to be valid only for $j, k=1\cdots 8$ (in particular, the η is not ideally mixed).

[14] S. Coleman and S. Glashow, Phys. Rev. 134, B671 (1964).

[15] A slope $\alpha'=1/2(m_\rho^2-m_\pi^2)\simeq 0.9$ is implied by the Adler condition $\alpha_\rho(m_\pi^2)=\frac{1}{2}$ for the Veneziano-Lovelace π-π scattering. C. Lovelace, Phys. Rev. Letters 28B, 264 (1968).

[16] K. Kawarabayashi, S. Kitakado, and H. Yabuki, Phys. Letters 28B, 432 (1969).

[17] P. G. O. Freund, Phys. Rev. Letters 23, 449 (1969).

predicted by Eq. (17). We know that some kind of extra "ghost-eliminating factor" must be present in the residue $\gamma_{ijk}(t)$, in Eq. (15), which would modify the t dependence. The only requirement needed in our earlier discussion is the specific demand that the $SU(3)$ relation, Eq. (16), would hold for the residue at $t=0$ (including any ghost-eliminating factor).

IV. CONCLUSIONS AND REMARKS

In this paper we have considered a particular nonperturbative deviation from $SU(3)\times SU(3)$ due to a selective-enhancement mechanism. The enhancement is related to hadron dynamics insofar as it determines the various intercepts $\alpha_i(0)$ of the Regge trajectories, corresponding to different $\bar{q}q$ combinations. The picture presented is incomplete, since we had to rely on perturbative arguments in deriving Eq. (7).[18]

We note that if hadron bindings were such that for all the leading trajectories $\alpha_i(0)<0$, then our results hold as long as $\alpha_{\rho,\omega}(0)>\alpha_{K^*}(0)>\alpha_\phi(0)$. In this case we can have real 0^+ particles [$0^+(\omega)$, $0^+(K^*)$, $0^+(\phi)$, etc.] on the trajectories with $m_{0^+}(\omega)<m_{0^+}(K^*)<m_{0^+}(\phi)$. The couplings of these particles are assumed to obey $SU(3)$. However the (attractive) potential due to $0^+(\omega,\rho)$ will be much more effective in binding, due to its longer range, than $0^+(\phi)$. Thus, systems to which mainly the ϕ couples (i.e., systems with more λ's than u's) will bind more weakly, reproducing again the same general pattern of masses as above.

In the case $\alpha_i>0$ (which is presumably the one realized in nature), the ϕ contribution is enhanced. However, owing to the sign reversal of the enhancement factor $[\alpha(0)]^{-1}$, it corresponds to more repulsion, leading again, as we saw above, to more massive "λ-quark-containing" states.

Since the simple estimates presented in Sec. III suggest a rather small $\alpha_{f'}(0)$, we cannot rule out the possibility that $\alpha_{f'}(0)<0$, and that the analog of the ghost-eliminating factor presented in the ρ, ω, and K^* trajectories is absent. In this case, we have a low-lying $0^+(\phi)$ particle. Such a particle, if it exists, is not expected to couple strongly, because then it could destroy the above-mentioned pattern of masses. Indeed, since its very appearance would be a manifest violation of $SU(3)$, its coupling should be rather small. Such a particle would certainly not be the conjectured low-energy "σ."[19] Having a $\bar{\lambda}\lambda$ composition, it will only weakly affect low-energy $\pi\pi$ dynamics and even if $m_{0^+}(\phi)>2m_\pi$ it could be relatively narrow.[20]

ACKNOWLEDGMENT

It is a pleasure to thank Professor Y. Ne'eman for helpful discussions.

APPENDIX

The dominance scheme in the complex J plane for matrix elements $\langle i|u(0)|j\rangle$, with $u(x)$ a local operator, was originally motivated by the N/D method.[1]

We discuss here an alternative approach which may be relevant in the case when the $SU(3)\times SU(3)$-breaking Hamiltonian $-\epsilon(u_0+cu_8)$ arises as a second-order effect due to a more basic $J_\mu B_\mu$ interaction[21] or several such interactions.[22] In this case the perturbation expression for the ith meson mass would be

$$\delta m_i^2 \simeq m_i^2 = \text{const}\times \int d^4q\, D_{B\mu\nu}(q) M_{\mu\nu}{}^i(\mathbf{q},q_0)\,, \quad \text{(A1)}$$

where $D_{B\mu\nu}$ is the propagator for the B meson and $M_{\mu\nu}{}^i$ is the virtual forward Compton amplitude for the scattering of B with momentum $(q,q^0)=q$ on the meson i with momentum P_i.

An equation similar to (A1) is encountered in calculations of electromagnetic mass differences. Its analysis uses Cottingham's formula[23] and dispersion relations in ν ($\nu=q\cdot p/m=q_0$ for scattering off a massive target in the target's rest frame) at fixed q^2.[24]

In the resulting integral for δm_i^2, the asymptotic region of "large q" is believed to be important and to yield the tadpole part. Roughly speaking, we distinguish two asymptotic regions, the "Regge region" and the "Bjorken region."

In the Regge region ($\nu\to\infty$, $q^2\leq$ const) the following asymptotic expansion is believed to hold for the various invariant amplitudes appearing in $M_{k\nu}{}^i$:

$$\operatorname{Im} t_\lambda{}^i(q^2,\nu)=\sum_l \gamma_{lB(q^2)B(q^2)}(t=0)\gamma_{lP_iP_i}(0)\nu^{\alpha_l(0)-\lambda}\,, \quad \text{(A2)}$$

where the residue was factored into the BB Reggeon and P_iP_i Reggeon couplings, and the sum extends over the nonet of the leading tensor trajectories.

---

[18] An alternative assumption, yielding Eq. (7), is the dominance scheme for θ_{00} as suggested in Ref. 1, together with the additional hypothesis that $\bar{\theta}_{00}$, the $SU(3)\times SU(3)$-symmetric part, satisfies $\langle 0|\bar{\theta}_{00}|i\rangle=0$, where i indicates the various nonet trajectories.

[19] See, e.g., L. Brown and P. Singer, Phys. Rev. Letters **8**, 460 (1962).

[20] In the case $m_{0^+(\phi)}<2m_\pi$ it would be almost stable with only the 2γ decay mode, (like the π^0). Considerations of λ-quark conservation suggest that a favorable place to look for such a particle are decays of strange particles. Conceivably, it could be "masked" by the dominant decay model involving π^0 [such as $K^+\to\pi^+\pi^0$, which may be particularly favorable because the main process is weaker owing to the $\Delta I=\frac{1}{2}$ selection rule, which does not affect $K^+\to\pi^++0^+(\phi)$]. It should be emphasized, however, that there is no theoretical support for the existence of such a particle except for the fact that $\alpha_{f'}(0)\leq 0$ is obtained with a linear f' trajectory having the universal slope of $0.9-1$.

[21] Such an interaction could be the fifth interaction suggested by Y. Ne'eman, Phys. Rev. **134**, B1355 (1964).

[22] A connection between the $SU(3)\times SU(3)$-breaking Hamiltonian and second-order weak and electromagnetic effects was conjectured by N. Cabibbo and L. Maiani, Phys. Rev. D **1**, 707 (1970); and by R. Gatto, G. Sartori, and M. Tonin, Nuovo Cimento Letters **1**, 1 (1969). [A survey of their approach is contained in R. Gatto, Revista Nuovo Cimento **1**, 514 (1969).] For convenience we will assume that the considered interaction is mediated by a boson so that we can discuss a Compton-like amplitude.

[23] W. N. Cottingham, Ann. Phys. (N. Y.) **25**, 424 (1963).

[24] A clear exposition of the various steps involved is given by M. Elitzur and H. Harari, Ann. Phys. (N. Y.) **56**, 81 (1970).

The δm_i^2, generated when (A2) is extrapolated over the whole region $\nu \geq \nu_{\text{thresh}}$,[25] has the form[24]

$$(\delta m_i^2)_{\text{Regge tadpole}} = \sum_l \frac{3}{4\pi\alpha_l(0)} \gamma_{lP_iP_i}(0) \times \left[\int dq^2 D'(q^2) \gamma_{B(q^2)B(q^2)l}(0) \right], \quad \text{(A3)}$$

where an invariant amplitude $t_\lambda{}^i$ with $\lambda=0$ was considered and $D'(q^2)$ depends on $D_{B\mu\nu}$.

Let us compare now (A3) with Eqs. (15) and (16). Assume, in order to compare with Eq. (7), that

$$m_i^2 = \langle P_i | \sum_j a_j u_j(x) | P_i \rangle, \quad \text{(A4)}$$

where

$$a_j = \sum_{\alpha,\beta=1}^{8} c_\alpha c_\beta d_{j\alpha\beta}, \quad \text{(A5)}$$

and the c_α arise from the decomposition of J_μ into the currents of the vector octet

$$J_\mu = \sum_{\alpha=1}^{8} c_\alpha v_\mu{}^\alpha. \quad \text{(A6)}$$

The nonet coupling of the currents into the scalar densities which was assumed in Eqs. (A4) and (A5) (by extending d_{ijk} to nonets) is motivated by the nonet coupling of vector and tensor mesons (or trajectories), by the quark counting implicit in the CHN model,[10] and by the duality diagrams. Equation (A3) coincides then with Eqs. (15) and (16) in the case in which we identify the square bracket of Eq. (13) with $\sum a_j \Gamma_l{}^j(0)$.[26]

The last stage is, however, not very profound since the Regge region does not lead naturally to local $u_i(x)$. These arise in the Bjorken region ($\mathbf{q}$ fixed, $q_0 \to \infty$, i.e., $q^2 \to \infty$, $q^2/\gamma^2 \to 1$). The application of the Bjorken technique[27] together with current algebra yields the following behavior for $M_{\mu\nu}{}^i(q^2,\nu)$ (less its Schwinger terms):

$$M_{00}{}^i(q^2,\nu) \underset{\mathbf{q}\ \text{fixed};\ q_0\to\infty}{\simeq} \langle P_i | \mathcal{H}(0) | P_i \rangle, \quad \text{(A7)}$$

where, taking the particular simple case of second-order weak interaction,[22]

$$\mathcal{H}(x) = [Q_c(t), D_c{}^\dagger(x)] + \text{H.c.} \quad \text{(A8)}$$

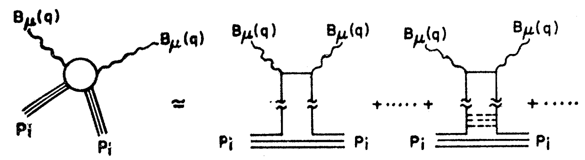

FIG. 1. Diagrams for virtual Compton scattering which are considered in the text. The solid line indicates the parton whose "bare" current interacts with the B_μ boson. The broken line indicates pseudoscalar-meson exchanges which simulate the hadronic interactions of the parton.

$Q_c = \int d^3x\ j_c{}^0(x)$ and $D_c(x) = \partial_\mu j_{c\mu}(x)$ are the charge and divergence of the Cabibbo current. In the framework of the GOR model, $\mathcal{H}(x)$ of Eq. (A8) is readily expressible in terms of the $u_i(x)$. We believe that the completely different theoretical approaches used in the two cases of Bjorken and Regge tadpoles may reflect to some extent out incomplete understanding. Indeed, hopefully, the recent extensive theoretical work on Compton scattering in the "scaling region" of moderately large q^2 and ν, but arbitrary $\omega = q^2/\nu$, could close the gap between the point $\omega \simeq 0$, the Regge region, and the Bjorken point $\omega = \infty$.[28,29]

A picture which was found useful in describing the scaling region is the "parton model." According to this picture, large q^2, ν inelastic electron-hadron scattering [which is related via closure to the $\sum_\mu M_{\mu\mu}{}^i(q^2,\nu)$] occurs as scattering from pointlike constituents of the hadron.[30] A heuristic field-theoretical realization of such a statement[31] involves the summation of the diagram of Fig. 1. The photon interacts here with the bare parton as illustrated by the lack of hadronic interaction, i.e., exchanges (of mesons in this case) across the photon vertex.

We note, however, that the same set of ladder diagrams was used (in a ϕ^3 theory though) to motivate Regge behavior. We therefore believe that independently of the q^2 carried by the photon, the Regge enhancement will always be there since the diagram below the broken line in Fig. 1 does depend on t and ν only and not on q^2. The Bjorken tadpole is the mass counterterm which is necessary in order to renormalize the bare-parton self-energy diagram, which results from the upper part of the diagram when the B loop is closed.[32] Thus the Bjorken tadpole, or the effective local $\mathcal{H}$ it leads to, should also be enhanced by the $1/\alpha_c(0)$ factors with no additional violation of $SU(3)$, which is essentially our conjecture.

---

[25] According to duality this extrapolation will describe the average resonance contributions also in the lower region.

[26] In a simple vector-dominance scheme

$$\gamma_{B(q^2)B(q^2)l}(0) \simeq \sum c_\alpha c_\beta \frac{m_\alpha^2}{m_\alpha^2 - q^2} \frac{m_\beta^2}{m_\beta^2 - q^2} d_{\alpha\beta l},$$

so that small deviations from the exact $SU(3)$ limit, due to the mass differences of the vector mesons [in addition to the major symmetry breaking due to $1/\alpha_l(0)$], might be expected. Our definition of the effective $u_i(x)$ includes this particular aspect of $SU(3)$ breaking.

[27] J. D. Bjorken, Phys. Rev. **148**, 1467 (1966).

[28] A heuristic unifying approach based on a quark parton model was suggested by H. Harari, Phys. Rev. Letters **24**, 286 (1970).

[29] The systematic discussion of light-cone singularities of current commutators [see, e.g., Y. Frishman, Phys. Rev. Letters (to be published)] could be a useful way to unify all these regions.

[30] This in particular implies no form factors in q^2 and the absence of the attendant $SU(3)$ breaking mentioned in Ref. 26.

[31] See S. D. Drell, D. J. Levy, and T. M. Yan, Phys. Rev. Letters **22**, 744 (1969); Phys. Rev. **187**, 2159 (1969).

[32] This is exhibited in a particularly clear way in Gatto (Ref. 22).