Quasi-Two-Body Production in π^+p Interactions into Four Prongs at 1.95 GeV/c*

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The quasi-two-body reactions $\pi^+p \to \Delta^{++}\rho^0$ and $\pi^+p \to \Delta^{++}\omega^0$ have been extracted from the four- and five-body final states in π^+p interactions at 1.95 GeV/c momentum. The total and differential cross sections for these two reactions have been determined and density-matrix elements of the resonances have been determined as a function of t' (the square of the four-momentum transferred minus the minimum value).

INTRODUCTION

THE following two reactions have been analyzed for quasi-two-body states:

$$\pi^+p \to \pi^+p\pi^+\pi^-, \quad (1)$$

$$\pi^+p \to \pi^+p\pi^+\pi^-\pi^0. \quad (2)$$

Preliminary results from reaction (2) were reported earlier.[1] The final results given here are essentially unchanged, though the resonant-cross-section estimates are appreciably different. The data of this experiment were extracted from 30 000 photographs taken by the BNL 20-in. hydrogen bubble chamber. The chamber was exposed to 1.95-GeV/c π^+ mesons from an internal target in the BNL Alternating Gradient Synchrotron. A two-stage electrostatically separated beam-transport system was used to prepare the beam.

The photographs were scanned for events with four outgoing charged particles with the single requirement that the interaction vertex lie within a specified acceptance region. 3008 events were found in the 80% single scan and the 20% double scan. The single-scan efficiency was found to be 91%. A sample of 2704 of the above-measured events emerged from the kinematics program with successful identifications after two measurement passes. The cross sections given in Table I have been corrected for the residual-measurement failures of less than 10% and for the scan losses. The data of this report come from the 1403 events identified as reaction (1) and 1014 events identified as reaction (2) contained within the 2704-event sample. The cross section for a 105-event sample of the reaction

$$\pi^+p \to \pi^+\pi^+\pi^+\pi^-n$$

is also given in Table I, though no further report of this reaction is included.

Reaction (1) is dominated by associated Δ^{++}-ρ^0 production, which is highly peripheral and suggestive of one-pion exchange. The amount of Δ^{++} production exceeds the ρ^0 production and therefore indicates a moderate amount of a three-body final state. Reaction (2) exhibits prominent structures at the three-pion invariant masses corresponding to the η^0 and ω^0 mesons and at the proton-pion mass of the Δ^{++}. This final state is consistent with being totally dominated by quasi-two-body resonant production without a significant phase-space contribution.

FOUR-BODY FINAL STATE

The associated production of Δ^{++} and ρ^0 is clearly evident in the two-pion, proton-pion mass scatterplot of Fig. 1. Since the scatterplot contains two combinations per event, the Δ^{++}-ρ^0 cluster represents a larger fraction of the events than the plot density alone would indicate. The curve shown on the two-pion mass projection (Fig. 2) of the scatterplot is a composite of the resonance states given in Table I plus reflections and the combinatorial background. The proportions and errors are reflected in the partial cross sections of Table I. The central values of the resonances and their experimental widths are also given in Table I. The proton-pion invariant-mass projection is plotted in Fig. 3. The Δ^{++} resonance is prominent over the alternate combination background. The curve is again a composite of the same reactions in the same propor-

TABLE I. Cross sections and fit parameters.

Reaction	% of state	Cross section (mb)
$\pi^+p\pi^+\pi^-$	...	3.04±0.15
$\pi^+p\pi^+\pi^-\pi^0$	...	2.07±0.15
$\pi^+\pi^+\pi^+\pi^-n$	...	0.18±0.04
$\Delta^{++}\pi^+\pi^-$	54±5	1.64±0.2
$\Delta^{++}\rho$	46±5	1.40±0.2
$\Delta^{++}\eta^0$	11±2	0.23±0.03
$\pi^+p\eta^0$	2±1	0.04±0.02
$\Delta^{++}\omega^0$	69±6	1.41±0.14
$\Delta^{++}\pi^+\pi^-\pi^0$	19±5	0.39±0.05

Resonance	Mass (MeV/c^2)	Width (expt) (MeV/c^2)
Δ^{++} (four body)	1228 ±2	127±10
ρ^0	762 ±3	114±15
Δ^{++} (five body)	1228 ±2	99±10
η^0	549.4±1	8±1
ω^0	792.9±2	30±2

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[1] J. W. Chapman, L. R. Fortney, and E. C. Fowler, in *Lectures in Theoretical Physics*, edited by W. E. Brittin *et al.* (Gordon and Breach, New York, 1968), Vol. VIII-B.

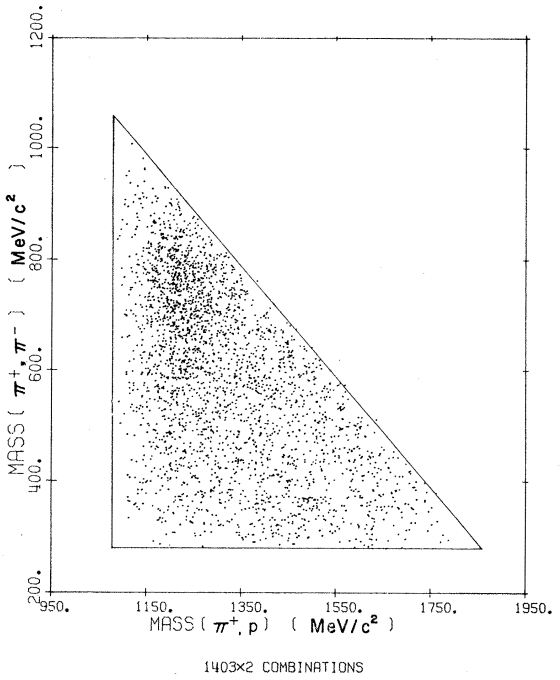

FIG. 1. Two-pion, proton-pion invariant-mass scatterplot for the reaction $\pi^+p \to \pi^+p\pi^+\pi^-$. The boundary curve represents the kinematic limit for events produced by a 1.95-GeV/c momentum beam.

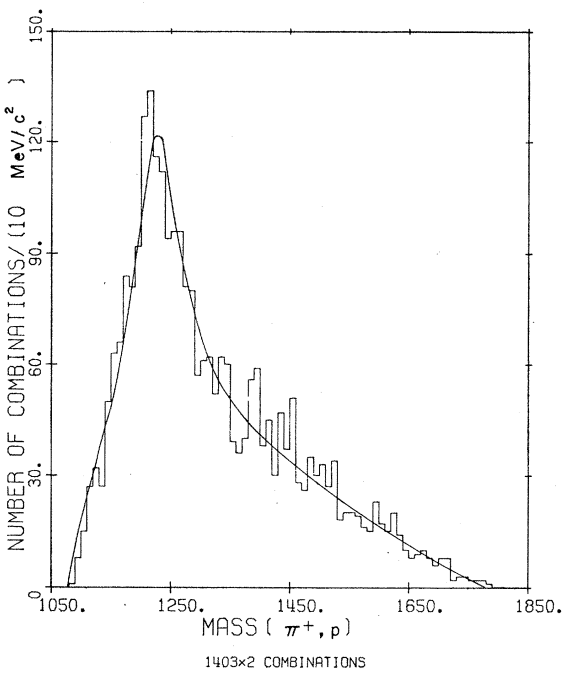

FIG. 3. Proton-pion invariant-mass distribution from the reaction $\pi^+p \to \pi^+p\pi^+\pi^-$. The curve is the best fit to Δ^{++} resonance, ρ^0 reflection, phase space, and combinatorial background in the proportions given in Table I.

tions. The curves were constrained to be proportioned identically because the two histograms were simultaneously least-squares fitted.

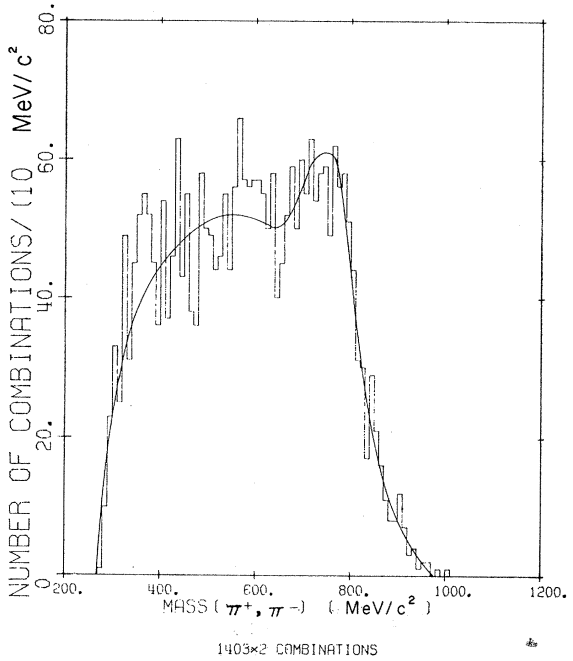

FIG. 2. Two-pion invariant-mass distribution from the reaction $\pi^+p \to \pi^+p\pi^+\pi^-$. The fitted curve is composed of ρ^0 resonance, Δ^{++} reflections, phase space, and combinatorial background in the proportions given in Table I.

The four functional forms for the hypothesized reactions were obtained by a Monte Carlo generation and contain no production dynamics. The fits to the two distributions are of suitable quality, exhibiting χ^2 values of 111 and 176 with 90 degrees of freedom in Figs. 2 and 3, respectively.

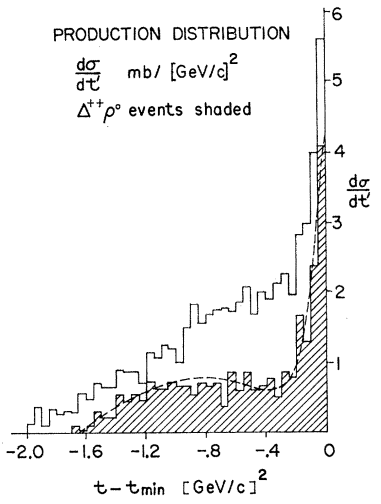

FIG. 4. Two-pion, proton-pion production distribution for $\pi^+p \to \pi^+p\pi^+\pi^-$ events. All combinations appear in the unshaded graph and only those selected as $\Delta^{++}\rho^0$ appear in the shaded plot. 524 events are contained within the shaded histogram of which 46 have a combinatorial ambiguity and are plotted twice with 0.5 weight. The curve is an exponential-plus-background fit which is described in the text.

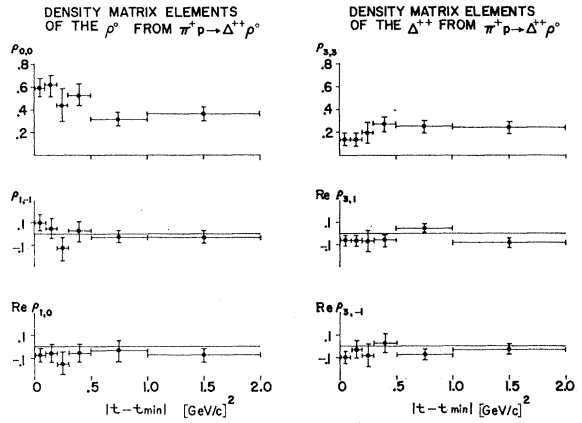

FIG. 5. Density-matrix elements for the ρ^0 and Δ^{++} from $\pi^+p\to\Delta^{++}\rho^0$. The density-matrix elements as obtained from a maximum-likelihood fit to the Gottfried-Jackson (Ref. 2) angular distributions are plotted as a function of t'.

The highly peripheral nature of the reaction is shown in the t' plot of Fig. 4. The shaded region contains those events selected as Δ^{++}-ρ^0 events. The abscissa of the plot, t', is the square of the four-momentum transferred from the π^+ beam to the ρ^0 combination, minus the minimum value possible as calculated for the observed two-pion, proton-pion masses. Each event appears twice in the complete histogram and generally only once in the Δ^{++}-ρ^0 selection. The Δ^{++}-ρ^0 selection contains events within a Δ^{++} band from 1.130 to 1.280 GeV/c^2 and in a ρ^0 band from 0.600 to 0.810 GeV/c^2. The wide ρ^0 cut is necessary to include those ρ^0 events skewed low in mass. This mass skewing is due to the peripheral tendency of the production coupled with the rapidly changing minimum t across the mass spread of the ρ^0. There are 570 combinations which survive this cut, of which 92 are from 46 nonunique events. These two combination events are used in all calculations with a weight of 0.5 associated with each combination. The curve superimposed on the t' distribution is of the form $Ae^{Bt'}+$(a quadratic background). The values of the parameters are

$$A=4.35\pm0.34 \text{ mb}/(\text{GeV}/c)^2,$$
$$B=9.674\pm1.2 \text{ (GeV}/c)^{-2}.$$

No detailed theoretical curves were generated for the data since the information content of the data is restricted to the forward peak. Beyond this peak the nonresonant background masks any structure in the true Δ^{++}-ρ^0 events.

The spin-density matrix elements as defined by Gottfried and Jackson[2] have been calculated for various t' intervals. A maximum-likelihood search for the

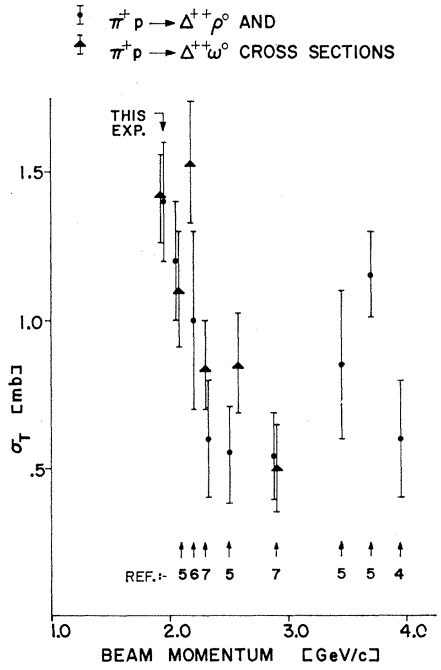

FIG. 6. Total cross section for quasi-two-body production of $\Delta^{++}\rho^0$ and $\Delta^{++}\omega^0$ as a function of laboratory momentum. The numerals labeling each point refer to the reference from which the data were taken.

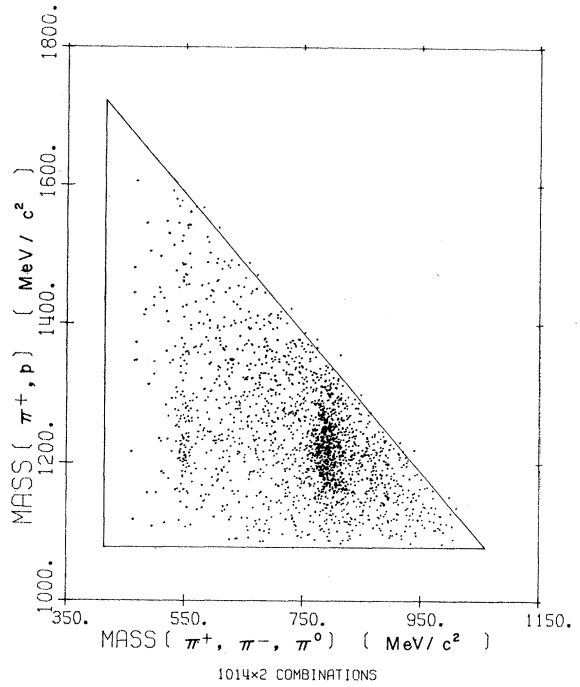

FIG. 7. Three-pion, proton-pion invariant-mass scatterplot for the reaction $\pi^+p\to\pi^+p\pi^+\pi^-\pi^0$. The boundary curve represents the kinematic limit for events produced by a 1.95-GeV/c-momentum beam.

[2] K. Gottfried and J. D. Jackson, Nuovo Cimento **22**, 309 (1964).

parameters of the probability distributions of the form

$$\omega(\theta,\varphi)=(3/4\pi)[\tfrac{1}{2}(1-\rho_{0,0})+\tfrac{1}{2}(3\rho_{0,0}-1)\cos^2\theta-\rho_{1,-1}\sin^2\theta \times\cos 2\varphi-\sqrt{2}\,\mathrm{Re}\rho_{1,0}\sin 2\theta\cos\varphi]$$

for the meson and

$$\omega(\theta,\varphi)=(3/4\pi)[\rho_{3,3}\sin^2\theta+(\tfrac{1}{2}-\rho_{3,3})(\tfrac{1}{3}+\cos^2\theta) -\tfrac{2}{3}\sqrt{3}\,\mathrm{Re}\rho_{3,-1}\sin^2\theta\cos 2\varphi-\tfrac{2}{3}\sqrt{3}\,\mathrm{Re}\rho_{3,1}\sin 2\theta\cos\varphi]$$

for the Δ was performed. The values and errors for the density-matrix elements are plotted in Fig. 5. The results indicate a slow variation of these parameters with energy.[3,4] The cross section for production of Δ^{++}-ρ^0 from this experiment is plotted with data at several other energies[4–7] in Fig. 6. No evidence for any other resonant production was observed in the various three-pion proton–two-pion distributions possible.

FIVE-BODY FINAL STATE

The production of the η^0, ω^0, and Δ^{++} resonances is clearly shown in the three-pion, proton-pion invariant-mass scatterplot of Fig. 7. There are two combinations plotted for each event. The ω^0-meson events are highly concentrated in the proton-pion mass region of the Δ^{++}. It is, however, not possible to determine precisely the fraction of ω^0 and Δ^{++} since the kinematic limits of the π^+p mass from $\pi^+p\omega^0$ is only slightly wider than

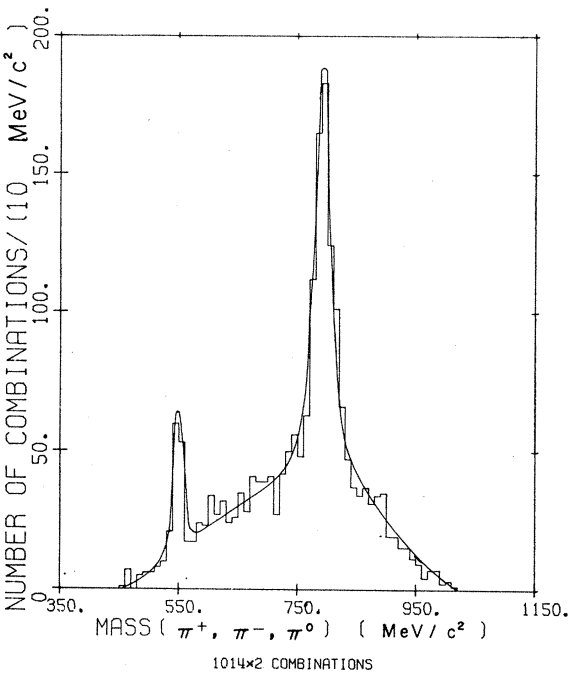

FIG. 8. Three-pion invariant-mass distribution from the reaction $\pi^+p\to\pi^+p\pi^+\pi^-\pi^0$. The curve is a fit to η^0 and ω^0 resonances, Δ^{++} reflections, and a combinatorial background in the proportions given in Table I.

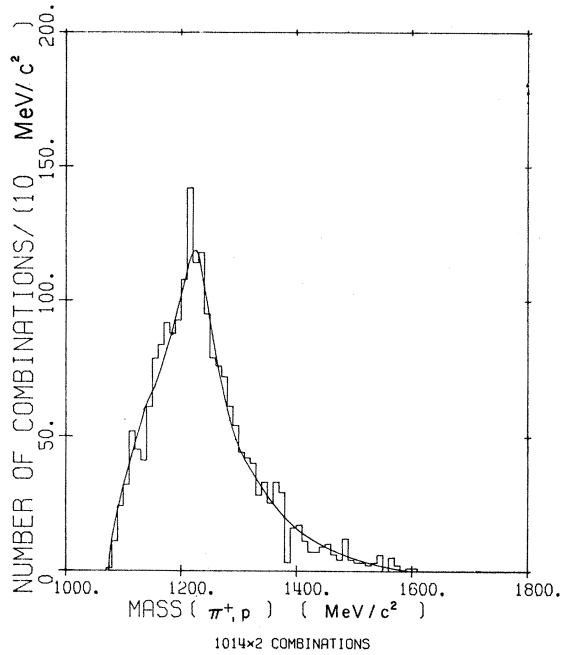

FIG. 9. Proton-pion invariant-mass distribution from the reaction $\pi^+p\to\pi^+p\pi^+\pi^-\pi^0$. The curve is a composite fit of Δ^{++} resonance, η^0 and ω^0 reflections, and a combinatorial background in the proportions given in Table I.

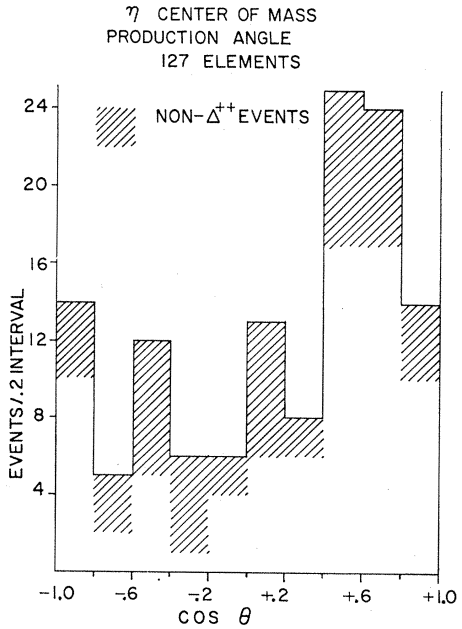

FIG. 10. η^0 production angle distribution. The unshaded histogram represents all η^0 events and the shaded events are those where the π^+p invariant mass is outside the Δ^{++} region.

[3] N. Angelov *et al.*, Joint Institute for Nuclear Research (Dubna) Report No. P1-4657 (unpublished).
[4] Aachen-Berlin-Birmingham-Bonn-Hamburg-London (I.C.)-München Collaboration, Nuovo Cimento **35**, 659 (1965).
[5] S. Meshkov, G. Snow, and G. Yodh, Phys. Rev. Letters **12**, 87 (1964).
[6] F. E. James and H. L. Kraybill, Phys. Rev. **142**, 896 (1966).
[7] D. Roy, Nuovo Cimento **40**, 513 (1965).

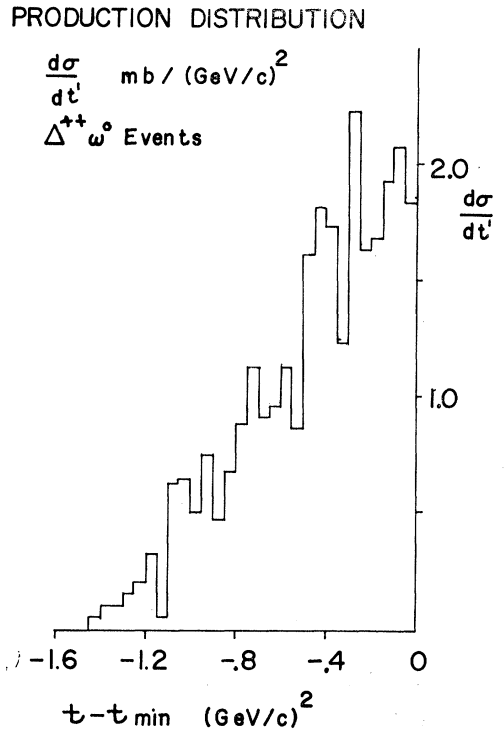

FIG. 11. t' distribution for the $\Delta^{++}\omega^0$ events.

the Δ^{++} mass spread at the energy employed. Only the extreme wings of the π^+p distribution are effective in determining the extent of Δ^{++}-ω^0 associated production. The best fit to the distribution was obtained under the assumption of complete Δ^{++}-ω^0 association for all ω^0 events. The partial cross sections for $\Delta^{++}\omega^0$ and $\pi^+p\omega^0$ of Table I reflect this best-fit condition, but the errors are large as a consequence of the uncertainty. The η^0-meson events also exhibit a strong Δ^{++} association. The estimated percentages of the resonances are illustrated by the fitted curves shown on the three-pion and proton-pion mass distributions of Figs. 8 and 9. The curves are composed of the percentages of resonant states given in Table I plus reflections and the combinatorial background. The resonant masses and their experimental widths are given in Table I. The c.m. production cosine for the η^0 events within a 0.535- to 0.562-GeV/c^2 mass cut are plotted in Fig. 10.

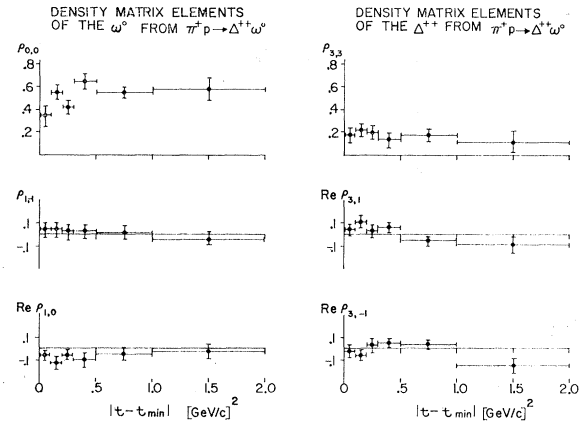

FIG. 12. Density-matrix elements for the Δ^{++} and ω^0 from the reaction $\pi^+p \rightarrow \Delta^{++}\omega^0$. These parameters were evaluated by a maximum-likelihood fit to the Gottfried-Jackson (Ref. 2) angular distribution and are plotted as a function of t'.

The t' distribution for the 571 events selected as Δ^{++}-ω^0 from the three-pion mass region 0.760–0.830 GeV/c^2 and proton-pion mass region 1.140–1.290 GeV/c^2 is plotted in Fig. 11. This sample of events contains 79 events in which each of two combinations appears with a weight of 0.5. Simple one-pion exchange is prohibited by G-parity conservation at the π-ω vertex. ρ-meson exchange is permitted and presumed of primary importance in the production process.

The density-matrix elements for the Δ^{++}-ω^0 events have been computed by a maximum-likelihood search for various t' intervals and are plotted in Fig. 12. The values of the density elements show a mild energy variation when compared to other experiments.[3,4]

CONCLUSIONS

Although the energy of this experiment is far too low to permit a detailed comparison of the data for quasi-two-body production with absorption-modified one-pion-exchange calculations, the general features of the data strongly suggest that the underlying production phenomena are the same from this near-threshold energy up to the highest energies yet observed. Variations in the production characteristics are more likely to occur at these low energies due to direct-channel resonance effects.