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Focusing of Gravitational Radiation by the Galactic Core*

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The possibility is considered that the gravitational radiation observed by Weber has an extragalactic origin and is focused by the galactic core acting as a gravitational lens. While sufficient intensification is possible, too few sources are correctly located for the effect to be important.

Recent work by Weber[1] has indicated the incidence of gravitational radiation on the Earth from the direction of the galactic center. The total energy per pulse corresponds to the conversion of about a solar mass (M_S) at the galactic center. The implied mass loss by the Galaxy of $10-1000 M_S$/yr is the greatest difficulty of Weber's results. A possible escape from this, in keeping with the directionality of the radiation, would be that the galactic core acts as a gravitational lens, focusing gravitational radiation incident upon it from beyond.

In testing this possibility we consider that weak gravitational waves, in the geometrical-optics limit, will follow null geodesics of the background metric. The proof of this is implicitly contained in a discussion by Isaacson[2] of a special case. Thus one can make direct use of the many earlier discussions of the focusing of light by massive bodies.[3] The results used are summarized by a few formulas concerning rays which pass through the external Schwarzschild field of a gravitational "lens" of mass M. If a = angular separation of object and lens, l_0 = distance from observer to object, l_d = distance from observer to lens, I_0 = unfocused intensity of source at observer, I = focused intensity, and G = Newton's constant, then, for $a<A$, we have $I=I_0A/a$, where

$$A=(2/c)[GM(l_0-l_d)/l_0l_d]^{1/2}.$$

We will take the mass of the galactic core to be $M = 10^9 M_S$. For the Earth $l_d = 3\times10^4$ light years. Use of the usual formula for the deflection of null geodesics by a Schwarzschild field shows that the rays will pass a distance 3–5 light years from the galactic center. Recent infrared measurements[4] indicate that the galactic core has radius ~1 light year.

We assume that Weber's apparatus will detect a pulse of radiation of intensity $\geq I_S$, the intensity at the Earth of a pulse of total energy $M_S c^2$ originating at the galactic center. We assume that the sources of all pulses are the conversion to gravitational radiation of stars with mass $\sim M_S$. Under what circumstances will a source outside our Galaxy trigger the apparatus? The unfocused intensity of a source at distance l_0 will be $I_0=I_S(l_d^2/l_0^2)$, and the focused intensity will be $I=I_SAl_d^2/al_0^2$. The apparatus will trigger if $I\geq I_S$. Thus we obtain the criterion

$$al_0^2[l_0/(l_0-l_d)]^{1/2}\leq(4GM/l_dc^2)^{1/2}l_d^2.$$

This restriction on a and l_0 defines the region of space in which a detectable source must lie. Its volume is $\sim10^6$ (light years)^3. (We exclude other galaxies nearer than l_d.) The density of galaxies in the universe[5] is $\sim10^{-19}$ galaxies/(light years)^3, so we will have $\sim10^{-13}$ galaxies in the critical region. We thus conclude that the frequency of extragalactic pulses is negligible.

We have used a very simple model for our gravitational lens. We have ignored the relatively diffuse, but massive, non-spherically-symmetric part of the Galaxy, as well as inhomogeneities in the structure of the core. These, however, would be expected to make the focusing less sharp, making observations even less likely. If one similarly considers the possibility of weaker sources in the spiral arms on the far side of our Galaxy, one is led to the same conclusion.

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Equal-Time Commutators of T^{ij}

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The point-local criterion for the behavior of quantum fields under the variation of an external gravitational field yields canonical momenta which vary with g_{ij}. (Point locality thus produces a variation which differs from those usually considered.) A general expression for equal-time commutators was derived by Schwinger, which is valid only when the canonical momenta do not change with the variation. It is shown that no general expression of that type may be obtained for these variations, since the implicit interchange of the order of taking commutators and variations violates canonical quantum field theory. Thus no general expression for commutators with T^{ij} may be obtained within the framework of the point-local criterion.

I. INTRODUCTION

A physical criterion (point locality) for the behavior of fields under the variation of an external gravitational field has been presented in previous works.[1] The expression for equal-time commutators (ETC's) arising from the quantum action principle[2] was employed there, when $g_{0\mu}$ was the varied field, without any difficulty. However, the point-local criterion yields canonical momenta which vary with g_{ij}. In classical mechanics various types of variations arise, e.g., those which yield Lagrange's equations, those which yield Hamilton's equations, and that of the principle of least action. The point-local criterion produces yet another type of variation. The derivation of the ETC's in Ref. 2 implicitly assumes that the canonical momenta do not vary with the external field. In this paper we show explicitly how the expression for ETC's of Ref. 2 depends upon this assumption. We then demonstrate that one cannot generalize the expression for ETC's to cases where the canonical momenta vary with the external field. Thus, no *general* commutation relations may be obtained for T^{ij} within the framework of the point-locality criterion.

II. FORMALISM WITHOUT AUXILIARY FIELDS

Consider the Lagrangian density

$$\mathcal{L} = \tfrac{1}{2}(\phi_{,\mu}\phi^{,\mu}) + \alpha^\mu \phi_{,\mu}\,, \tag{1}$$

where α^μ is an external field, a comma denotes a derivative, and the usual summation convention is employed. The field equations are

$$A_{,0} = B\,, \tag{2}$$

where

$$A \equiv \phi_{,0} \text{ and } B \equiv -\phi_{,ii} - \alpha^\mu{}_{,\mu}\,. \tag{3}$$

The action principle yields[2]

$$-i\int d^3x'[A(x), \delta\mathcal{L}(x')]_{t=t'} = [\delta' A(x)]_{,0} - \delta' B(x)\,. \tag{4}$$

If α^0 is varied, we obtain

$$\delta\mathcal{L} = \phi_{,0}\delta\alpha^0 \text{ and } \delta' B = -(\delta\alpha^0)_{,0}\,, \tag{5}$$

where we have kept ϕ and $\phi_{,0}$ fixed in $\mathcal{L}$. The variation of B is unambiguous since $\phi_{,0}$ does not appear in it, and it is well known that A should be expressed in terms of the canonical momenta (after which its explicit variation is obtained). Otherwise, an inconsistency will arise.