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Unitary Symmetry and Duality in Meson-Meson Scattering

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Construction of Veneziano-type amplitudes for the general pseudoscalar meson-meson (*PP*) scattering incorporating unitary symmetry is considered. Emphasis is given to the use of the crossing matrix, both in the *SU*(3) and in the lower symmetry group *SU*(2). All the essential features of the quark-model calculations are reproduced. Eigenvalues of the crossing matrix are seen to correspond to the symmetry properties of the amplitudes, which in turn produce the different signatures of Regge trajectories. Incidentally it may be mentioned that the unwieldy tensorial decomposition in the internal symmetry space has been done away with.

INTRODUCTION

Suppression of exotic resonances in the construction of dual amplitudes has become a problem of prime importance in recent times.[1] It has been observed in many cases that the absence of exotic resonances has strong implications on the dynamical structure of hadrons.[2] In the case of π-π scattering the absence of $I=2$ particles predicts a degenerate ρ-f^0 trajectory. Other similar inferences have been made in the case of $\pi\eta$, $\eta\eta$, $\pi\kappa$, and various other scattering processes. In the usual method of construction of Veneziano-type amplitudes, use is made of the symmetry property of the invariant amplitudes under the interchange of Mandelstam variables.[3] Incidentally, it may be mentioned that sometimes it becomes difficult to obtain such symmetry properties of the amplitudes. Even the proper writing of the invariant structure (in internal-symmetry space) of the amplitudes becomes difficult when higher symmetries than $SU(2)$ are invoked. Several authors have already made some observations regarding the structure of the meson-baryon resonances by combining $SU(3)$ symmetry and duality. In this respect Capps[4,5] has deduced some important consequences about the spectrum of Regge trajectories, which follow from the absence of exotic resonances in the crossed channel. In this paper we have tried to visualize the implications of the inferences made by Capps and others more clearly, through the explicit construction of the amplitudes on the basis of the duality hypothesis. The degeneracy of two octet and singlet trajectories is seen to follow both from meson-meson and meson-baryon scattering.

Recently, a successful attempt has been made by Yahil,[6] from the standpoint of the permutation group, to obtain a method for suppressing the exotic poles in all channels. In his method it is necessary to know the representation of the permutation group on the Mandelstam variables s, t, u, and to use the properties of crossing matrices. It is also worth noticing that in his method one must work with unphysical amplitudes rather than with the physical one.

Mention can also be made of the work of Neville,[7] who has constructed the amplitude for $PP \to PV$ with the help of the quark model for the pseudoscalar and vector mesons. In his approach, the exotic resonances do not occur owing to the proper orientation of the intermediate quark lines in the box diagrams. His amplitudes have also the proper dual nature and signature for all the trajectories mentioned before.

We now describe a method which utilizes only the property of the crossing matrix and no other mathematical and physical assumptions. It is known that the eigenvalues of the crossing matrix can be only ± 1, from which we have constructed eigenamplitudes having the correct symmetry properties. These eigenamplitudes, when supplemented with the ansatz that the exotic channels do not contain any poles, give unique solutions for all the physical amplitudes. Our amplitudes demand the degeneracy of 8_s, 8_a, and singlet trajectories of different signature. In the following we have first considered the case of $SU(2)$ symmetry (e.g., π-π scattering) and then extended it to the case of P-P scattering in the unitary-symmetry scheme. Lastly, we have demonstrated that our $SU(3)$ amplitudes reproduce those of π-π scattering as a particular case. One need not construct the Veneziano structures for the invariant amplitudes and the complicated connections between the invariant and physical amplitudes.

SCATTERING WITH $SU(2)$ SYMMETRY

In π-π scattering the isospin crossing matrix is written as

$$\beta_{st}=\begin{pmatrix}\frac{1}{3} & 1 & \frac{5}{3}\\ \frac{1}{3} & \frac{1}{2} & -\frac{5}{6}\\ \frac{1}{3} & -\frac{1}{2} & \frac{1}{6}\end{pmatrix}.$$

As a first step in our construction of the dual amplitudes, we invoke the physical content of the crossing matrix β, given by the equation

$$\beta_{ts} A_s = A_t \,, \tag{1}$$

from which we want to find linear combinations of the isospin amplitudes A_s (A_t) which will have a simple symmetry property under the (s, t) transformation. In order to accomplish this we construct a similarity transformation S, such that the transformed basis SA_s (SA_t) will have the desired property. Then an immediate consequence of the equation

$$(S\beta_{ts}S^{-1})(SA_s) = SA_t \tag{2}$$

is that $S\beta S^{-1}$ is diagonal. But the eigenvalues of β are found to be $(+1, +1, -1)$, that is, S satisfies an equation of the type

$$S\beta S^{-1} = \begin{pmatrix} 1 & & \\ & 1 & \\ & & -1 \end{pmatrix} = B \text{ (say)}. \tag{3}$$

So,

$$\beta^T S^T = S^T B^T \,. \tag{4}$$

β^T denotes the transpose of β. The explicit structure of S can be obtained from Eq. (4) which suggests that the columns of S^T are nothing but the eigenvectors of β^T. With this simple observation, we find

$$S = \begin{pmatrix} 2 & 9 & -5 \\ 2 & -3 & 7 \\ -4 & 6 & 10 \end{pmatrix} \tag{5}$$

except for a normalizing factor. This expression for S implies that the following combinations of isospin amplitudes,

$$\begin{aligned} &2A_0^s + 9A_1^s - 5A_2^s \,, \\ &2A_0^s - 3A_1^s + 7A_2^s \,, \end{aligned} \tag{6}$$

are symmetric under the (s, t) transformation, corresponding to the eigenvalues $+1$, and that the antisymmetric combination is

$$-4A_0^s + 6A_1^s + 10A_2^s \,, \tag{7}$$

corresponding to eigenvalue -1. The same steps can be followed for the (s, u) crossing matrix and we find the following combinations of the amplitudes with the indicated symmetry properties:

$$\left.\begin{aligned} &2A_0^s - 5A_2^s - 9A_1^s \,, \\ &-2A_0^s - 7A_2^s - 3A_1^s \,, \end{aligned}\right\} \text{ symmetric} \tag{8}$$

$$-4A_0^s + 10A_2^s - 6A_1^s \,, \text{ antisymmetric}. \tag{9}$$

Once Eqs. (6) to (9) are obtained, it is a simple task to construct amplitudes for each isospin channel, which will be in conformity with the high-energy behavior and pole structure (one such amplitude is the well-known Veneziano prescription). Without any loss of generality, we can now write the most general form for the symmetric and antisymmetric amplitudes obtained above, as follows:

$$\begin{aligned} &2A_0 - 3A_1 + 7A_2 = f_1(s, t) + [g_1(s, u) + g_1(t, u)] \,, \\ &2A_0 - 3A_1 - 5A_2 = [h_1(s, u) - h_1(t, u)] \,, \\ &2A_0 + 3A_1 + 7A_2 = f_2(s, u) + [g_2(s, t) + g_2(t, u)] \,, \\ &2A_0 + 3A_1 - 5A_2 = [h_2(s, t) - h_2(u, t)] \,. \end{aligned} \tag{10}$$

In Eqs. (10), $f(x, y)$, $g(x, y)$, $h(x, y)$, etc., are symmetric functions of their arguments. When these equations are coupled with the physical condition that A_2^s does not contain any pole in the s channel, they yield

$$f_1 = f_2 = 0, \quad g_1 = g_2 = h_1 = h_2 \,, \tag{11}$$

along with

$$\begin{aligned} A_2^s &= \tfrac{1}{6} g(t, u) \,, \\ A_1^s &= \tfrac{1}{6} [g(s, t) - g(s, u)] \,, \\ A_0^s &= \tfrac{1}{4}[g(s, t) + g(s, u)] - \tfrac{1}{12} g(t, u) \,. \end{aligned} \tag{12}$$

These are the amplitudes obtained by Shapiro[1] in a completely different manner.

EXTENSION TO THE CASE OF $SU(3)$

In the previous section we have obtained the dual amplitudes for different isospin channels in the case of π-π scattering. The utility of our method can be seen when it is applied to the case of general P-P scattering, where P stands for the octet of pseudoscalar mesons in $SU(3)$ symmetry. The $SU(3)$ crossing matrix for octet-octet scattering has been deduced by De Swart,[8] Cutkosky,[9] and Gourdin.[10] We have found that of Gourdin useful in our analysis. The above-mentioned matrix for $\underline{8} \times \underline{8} \to \underline{8} \times \underline{8}$ can be written as

$$\beta = \begin{pmatrix} \frac{1}{8} & 1 & \frac{27}{8} & \mp 1 & \mp\frac{5}{2} & 0 & 0 & 0 \\ \frac{1}{8} & -\frac{3}{10} & \frac{27}{40} & \mp\frac{1}{2} & \pm 1 & 0 & 0 & 0 \\ \frac{1}{8} & \frac{1}{5} & \frac{7}{40} & \mp\frac{1}{3} & \pm\frac{1}{6} & 0 & 0 & 0 \\ \frac{1}{8} & \mp\frac{1}{2} & \pm\frac{9}{8} & \frac{1}{2} & 0 & 0 & 0 & 0 \\ \frac{1}{8} & \pm\frac{2}{5} & \pm\frac{9}{40} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & & & \\ 0 & 0 & 0 & 0 & 0 & & X & \\ 0 & 0 & 0 & 0 & 0 & & & \end{pmatrix} , \tag{13}$$

where the upper signs are for the (st) matrix and the lower ones correspond to (su); X is a 3×3 matrix which stands for

$$X=\begin{pmatrix} 0 & \frac{1}{15} & 0 \\ 15 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \quad \text{for } (s,t)$$

and

$$X=\begin{pmatrix} 0 & 0 & \frac{1}{15} \\ 0 & -1 & 0 \\ 15 & 0 & 0 \end{pmatrix} \quad \text{for } (s,u).$$

The rows and columns of (13) are labelled according to the following order:

$$A_1,\ A_{8_{ss}},\ A_{27},\ A_{8_{aa}},\ \tfrac{1}{2}(A_{10}+A_{\overline{10}}),\ \tfrac{1}{2}(A_{10}-A_{\overline{10}}),$$
$$\tfrac{1}{2}(A_{8_{sa}}+A_{8_{as}}),\ \tfrac{1}{2}(A_{8_{sa}}-A_{8_{as}}).$$

Here we also have observed that eigenvalues of (13) are (111, −1 −1, 1, −1, −1,) and a matrix similar to S is constructed from the equation

$$B^T S^T = S^T B$$

with $B=\mathrm{diag}\ (1,\ 1,\ 1,\ -1,\ -1,\ 1,\ -1,\ -1)$. It is interesting to note that the crossing matrix breaks up into two submatrices 5×5 and 3×3 and can thus be dealt with separately. Let us first think of the 5×5 one. For this case

$$B=\mathrm{diag}\ (1,\ 1,\ 1,\ -1,\ -1).$$

So,

$$S=\begin{pmatrix} 0 & 0 & 3 & \pm 2 & \pm 1 \\ 0 & 1 & 0 & \mp 1 & \pm 2 \\ 1 & 0 & 0 & \mp 2 & \mp 5 \\ 1 & -5 & 0 & \mp 1 & \pm 5 \\ 0 & 9 & -9 & \pm 5 & \mp 5 \end{pmatrix},$$

which gives us the following eigenamplitudes:

(1) $3A_{27}\pm 2A_{8_a}\pm A_{10}$,

(2) $A_{8_s}\mp A_{8_a}\pm 2A_{10}$,

(3) $A_1\mp 2A_{8_a}\mp 5A_{10}$,

(4) $A_1-5A_{8_s}\mp A_{8_a}\pm 5A_{10}$,

(5) $9A_{8_s}-9A_{27}\pm 5A_{8_a}\mp 5A_{10}$,

where the upper sign [lower sign] corresponds to the (s,t) crossing [(s,u) crossing]. Just as in the case of π-π scattering A_2 was taken as exotic, similarly here $\frac{1}{2}(A_{10}+A_{\overline{10}})$ (henceforth called A_{10}) and A_{27} have been assumed to be so. In the above five eigenamplitudes, the first three are symmetric and the last two antisymmetric. The first three can be recombined and one obtains a symmetric expression containing only A_1, A_{8_s}, and A_{8_a} which does not interest us at this moment, as the amplitudes occurring here are all physical (none is exotic). Two more combinations can be formed [see (4′) and (5′) below] containing both physical and exotic amplitudes and having the proper symmetry properties. We shall now follow the same technique as in the π-π case in writing down the Veneziano form for these eigenamplitudes, which are given below.

(4′) $A_1-5A_{8_s}\mp A_{8_a}\pm A_{10}$, } antisymmetric
(5′) $gA_{8_s}\pm 5A_{8_a}\mp 5A_{10}-9A_{27}$, }

(4″) $A_1-5A_{8_s}\mp A_{8_a}\mp 17A_{10}-6A_{27}$, } symmetric
(5″) $9A_{8_s}\pm 5A_{8_a}\pm 25A_{10}+21A_{27}$. }

The eigenamplitudes being given by (4′)–(5″), the main problem is now to construct the most general type of functions (whose arguments are Mandelstam dynamical variables) which have the correct symmetry property, and have other physical requirements built in. Then, proceeding as before,

(a) $A_1-5A_{8_s}-A_{8_a}-17A_{10}-6A_{27} = f(s,t)+[g(s,u)+g(t,u)]$,

(b) $A_1-5A_{8_s}-A_{8_a}+5A_{10}=[g_1(s,u)-g_1(t,u)]$,

(c) $A_1-5A_{8_s}+A_{8_a}+17A_{10}-6A_{27} = f_1(s,u)+[h(s,t)+h(t,u)]$,

(d) $A_1-5A_{8_s}+A_{8_a}-5A_{10}=[h_1(s,t)-h_1(u,t)]$,

(e) $9A_{8_s}+5A_{8_a}-5A_{10}-9A_{27}=[\alpha(s,u)-\alpha(t,u)]$,

(f) $9A_{8_s}-5A_{8_a}+5A_{10}-9A_{27}=\beta(s,t)-\beta(u,t)$,

(g) $9A_{8_s}+5A_{8_a}+25A_{10}+21A_{27} = \gamma(s,t)+[\alpha_1(s,u)+\alpha_1(t,u)]$,

(h) $9A_{8_s}-5A_{8_a}-25A_{10}+21A_{27} = \delta(s,u)+[\beta_1(s,t)+\beta_1(t,u)]$,

which yields

$$f=f_1=\gamma=\delta=0,$$

$$h=h_1,\quad g=g_1,\quad \alpha=\alpha_1,\quad \beta=\beta_1,$$

and

$$\alpha=-5h,\quad \beta=-5g,$$

along with

$$\begin{aligned} &A_{10}=0,\\ &A_{27}=\tfrac{1}{15}\,\alpha(t,u),\\ &2A_{8_a}=\tfrac{1}{5}[\alpha(s,u)-\alpha(s,t)],\\ &2(A_1-5A_{8_s})=\tfrac{1}{5}[2\alpha(t,u)-\alpha(s,t)-\alpha(s,u)],\\ &18A_{8_s}=\tfrac{1}{5}[5\alpha(s,t)+5\alpha(s,u)-4\alpha(t,u)]. \end{aligned} \tag{14}$$

Exactly the same procedure can be followed for the remaining 3×3 crossing matrix, or we may think that these three amplitudes are unphysical and are not excited, so that we may put them identically equal to zero.

DISCUSSION

Equation (14) gives the final expression for the dual amplitudes for different $SU(3)$ channels when all the trajectories are taken to be degenerate. An interesting observation is that, in spite of this degeneracy, the trajectories occur with different signature in different physical amplitudes, which can be visualized by letting $t\to\infty$, s fixed.

We then have

$$A_{8_a}=\frac{1}{10}\frac{-1+e^{-i\pi\alpha(s)}}{\sin\pi\alpha(s)}\frac{[\alpha(t)]^{\alpha(s)}}{\Gamma(\alpha(s))},$$

$$A_{8_s}=\frac{1}{18}\frac{1+e^{-i\pi\alpha(s)}}{\sin\pi\alpha(s)}\frac{[\alpha(t)]^{\alpha(s)}}{\Gamma(\alpha(s))},$$

$$A_{1}=\frac{8}{45}\frac{1+e^{-i\pi\alpha(s)}}{\sin\pi\alpha(s)}\frac{[\alpha(t)]^{\alpha(s)}}{\Gamma(\alpha(s))}.$$

The $f(t,u)$ term does not contribute as it falls off faster than any power of t.

Finally, we want to show that our $SU(3)$ eigenamplitudes yield the correct dual amplitudes for the different isochannels of the $SU(2)$-symmetry case. We illustrate our method in the case of π-π scattering. Consider the process $\pi^+\pi^+\to\pi^+\pi^+$ which has $I=2$, $I_3=2$ as the quantum numbers of the intermediate channel:

$$|\pi^+\rangle|\pi^+\rangle=\sum_{N,I}C_{I_1I_2}[I,I_3,I_3^1,I_3^2]\times\begin{pmatrix}8 & 8 & N\\ I_10 & I_20 & I0\end{pmatrix}|N,8,8,I^2,I_3,0\rangle.$$

If use is made of the proper Clebsch-Gordan coefficients and isoscalar factors,[10] it is easily seen that

$$|\pi^+\rangle|\pi^+\rangle=|27\rangle,$$

$$A(\pi^+\pi^+\to\pi^+\pi^+)=A_{27}=\tfrac{1}{15}\alpha(t,u),$$

so that the exotic channel in the $SU(2)$-symmetry case does not contain any s-channel pole, as required by the Veneziano model.

In the above discussions, we were primarily interested in the construction of the dual amplitudes for PP scattering with exact $SU(3)$ symmetry. It is rather interesting to note that the same procedure is also effective in describing meson-baryon (MB) scattering. As a prototype of MB scattering we consider here the interaction of octet of pseudoscalar mesons with Sakata-model triplets. The scattering amplitude is written as

$$T=\bar{U}(p')[A(s,t,u)+\gamma QB(s,t,u)]U(p).$$

The invariant amplitudes have the crossing property

$$A(s,t,u)=X^{su}A(u,t,s),$$

$$B(s,t,u)=-X^{su}B(u,t,s),$$

where X^{su} is given by

$$X^{su}=\tfrac{1}{8}\begin{pmatrix}-1 & -6 & 15\\ -3 & 6 & 5\\ 3 & 2 & 3\end{pmatrix}.$$

Proceeding as before, it is easy to obtain

$$S=\begin{pmatrix}1 & 0 & 3\\ 0 & 1 & 1\\ 3 & 2 & -5\end{pmatrix},$$

which leads to

$$A_{15}^s=0,$$
$$A_3^s=-\tfrac{2}{3}A_6^s=-\tfrac{2}{3}a[f(s,t)+f(t,u)]-\tfrac{2}{3}6g(s,u), \quad (15)$$

for the dual structure of the non-spin-flip eigenamplitudes. It is worth mentioning here that in writing the above equations for the eigenamplitudes we have tentatively assumed the degeneracy of the α_3^s and α_6^s which are, respectively, the trajectories of "3" and "6". Similarly, for the case of spin-flip amplitudes, we obtain

$$B_{15}^s=0,$$
$$B_3^s=-\tfrac{2}{3}B_6^s=-\tfrac{2}{3}a'[f(s,t)-f(u,t)]. \quad (16)$$

In Eqs. (15) and (16) we have taken "15" to be an exotic state. The above formulas allow a mixture of octet mesons to be present in the t channel. It can be seen that the structure of the A and B amplitudes obtained is very similar to that for πN scattering written by Gupta *et al*.

CONCLUSION

In the light of the above observations, it is seen that the basic principle underlying our computation is the diagonalization of the crossing matrix. In this method the invariant structure of the amplitude in the internal-symmetry space (which essentially requires the knowledge of projection operators) is not required, and the method is general enough to encompass any higher-symmetry scheme [e.g., $SU(6)$]. Furthermore, the important feature of our method is that we deal with physical amplitudes throughout our calculation.

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Complex Negative-Signature Trajectories and the Pomeranchuk Theorem*

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Within the framework of complex-angular-momentum methods, it is shown that amplitudes which violate the Pomeranchuk theorem require negative-signature trajectories which are of the form $\alpha(t) = 1 \pm \mathrm{const}\sqrt{t} + O(t)$ near $t = 0$. There must be corresponding positive-signature trajectories. The character of the singular surfaces with negative signature is discussed briefly.

I. INTRODUCTION

In previous papers we have discussed Regge pole and branch-point trajectories $\alpha(t)$ which are complex for real $t<0$ due to left-hand cuts in the t plane of these functions.[1] Such trajectories are of interest for the description of diffraction scattering.[2,3] In particular, pole-cut systems with complex trajectories can be used for the construction of rather general and physically meaningful amplitudes which imply different, constant total cross sections for particle and antiparticle scattering.[4,5]

It is the purpose of this paper to show that amplitudes which violate the Pomeranchuk theorem[6] generally require negative-signature trajectories with square-root branch points at $t=0$. They are of the form $\alpha(t) = 1 \pm \mathrm{const}\sqrt{t} + O(t)$. It then follows that there must be corresponding positive-signature trajectories, a fact which can also be proven directly.[7] The character of these singular surfaces of the continued partial-wave amplitude will be discussed briefly.

II. l-PLANE ARGUMENT

We denote the scattering amplitudes for particle and antiparticle scattering by $F(s,t)$ and $\overline{F}(s,t)$, respectively, and we introduce the combinations

$$F_\pm(s,t) = F(s,t) \pm \overline{F}(s,t)\,. \tag{1}$$

Assuming constant asymptotic total cross sections given by σ and $\overline{\sigma}$, we have for $s\to\infty$

$$\mathrm{Im}F_\pm(s,0) \sim s\frac{\sigma \pm \overline{\sigma}}{16\pi}, \tag{2}$$

$$\mathrm{Re}F_-(s,0) \sim -\frac{2}{\pi}s\ln s\frac{\sigma-\overline{\sigma}}{16\pi}, \tag{3}$$

with $\mathrm{Re}F_+(s,0)$ being of the order $s(\ln s)^{-1}$.[2] These properties follow from the familar dispersion relations.[8] From Eq. (3) and the general postulates of dispersion theory or of local field theory, we obtain the bounds

$$(\sigma-\overline{\sigma})^2/4\pi^3 a \leq \sigma_{\mathrm{el}} \leq \sigma \tag{4}$$

for the elastic cross section

$$\sigma_{\mathrm{el}}(s) \sim \frac{16\pi}{s^2}\int_{-s}^{0} dt\,|F(s,t)|^2\,. \tag{5}$$

Here a is a constant defining the maximal relevant angular momentum $L = \frac{1}{2}\sqrt{as}\ln s$ in the s-channel partial-wave expansion of $F(s,t)$ for large values of s.[4,9,6]

Let us consider the continued partial-wave amplitudes $F_\pm(t,\lambda)$. These functions have the usual analytic properties in the complex manifold (t,λ).[2] In particular, they satisfy the continued elastic unitarity condition for $4m^2 \leq t < t_i$ (t_i = first inelastic threshold). This condition forbids certain kinds of singularities. We ask: What are the characteristic fèatures of allowed, isolated singularities of $F_\pm(t,\lambda)$ near $(t,\lambda) = (0,1)$ which are required by the special conditions (2)–(4)? We assume that $F_\pm(t,\lambda)$ has a finite number of such singularities at $\lambda = \alpha_K(t)$, $k = 1,2,\ldots.$ Irrespective of the character of these singularities, each one can contribute a term to the asymptotic expansion of $F_-(s,t)$ for $s\to\infty$ which, except for logarithmic factors, is of the form $s^{\alpha_k(t)}$. Equation (2) requires then that $\alpha_k(0) \leq 1$, with $\alpha_k(0) = 1$ for at least one trajectory