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Baryon Annihilation and Exotic Exchange*

Charles B. Chiu
California Institute of Technology, Pasadena, California 91109
and Institute for Theoretical Physics, State University of New York, Stony Brook, New York 11790

and

Rudolph C. Hwa
Institute for Theoretical Physics, State University of New York, Stony Brook, New York 11790

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Within the framework of duality and quark diagrams, we suggest the importance of connecting the annihilation processes in $B\overline{B}$ collisions to the exchange of an (ϵ) trajectory with $qq\overline{q}\overline{q}$ quark content. It is argued that this ϵ should be an exchange-degenerate pair with natural parity and $I=0$. It is coupled strongly to the $B\overline{B}$ system at $t=0$. For the positive-t region, it is expected to be manifest as a physical vector meson with $G=-1$ at around 1 GeV. Available data do not rule out its existence.

I. INTRODUCTION

In recent years the idea of duality has been proven to be successful in the study of hadron physics. The relationship between the Regge poles in the s and t channels is made particularly transparent by the use of quark diagrams.[1] However, the application of the quark diagrams to the baryon-antibaryon system is hindered by the appearance of four-quark ($qq\overline{q}\overline{q}$) states, which cannot be interpreted in the conventional duality picture based on the usual quark model. It is the purpose of this paper to give a specific interpretation for these four-quark objects in terms of the annihilation process in the s channel and to correlate them with exotic Regge trajectories in the t channel.

Let us first review the status of our present understanding about the exotics. All the well-known mesons have thus far been classified in the $\underline{1}$ and the $\underline{8}$ representation of $SU(3)$. This regularity suggests that these mesons are made out of a $q\overline{q}$ pair. The underlying dynamical reason for having these mesons associated with only a single $q\overline{q}$ pair instead of the exotic multi-$q\overline{q}$ pairs is not clear. In fact, the possibility for the existence of mesons made out of the $qq\overline{q}\overline{q}$ exotic states has been speculated by various authors, for example, in connection with the quark graphs,[2] and also in connection with the explanation of the A_2 splitting.[3] Experimentally, much effort has also been devoted to the search for the I-Y exotic particles, belonging to the $\underline{10}$, $\underline{10}^*$, and $\underline{27}$ representations. No conclusive evidence has been advanced for their existence.[4] On the other hand, in the crossed-channel physical region, there seems to be some indication in favor of the exchange of the I-Y exotic quantum number. The effect observed here is relatively weak and it could alternatively be attributed to the contribution of Regge cuts. It seems that Regge trajectories associated with the $\underline{10}$, $\underline{10}^*$, and $\underline{27}$, if they exist at all, are only weakly coupled to the hadronic system. Furthermore, the lack of evidence for the corresponding physical particles suggests that these trajectories may turn over in the positive-t region before reaching the lowest physical angular momentum, a rather common phenomenon in potential scattering.

The exotic trajectories which we consider in this paper differ from the ones discussed above. Their existence is motivated by the appreciable cross section of baryon-antibaryon annihilation processes, so they must couple strongly to the baryon-antibaryon system at least at $t=0$. They appear in an exchange-degenerate pair, which we shall label[5] collectively by ϵ. Some consequences of our proposal are as follows:

(1) The energy dependence of the baryon annihilation cross section should exhibit a Regge be-

havior with $\sigma_{\text{ann}} \sim s^{-1/2}$.

(2) The annihilation cross sections for $\bar{p}p$ and $\bar{p}n$ should be comparable.

(3) The conventional Regge-pole model for baryon-baryon and baryon-antibaryon scattering should be modified with the inclusion of the ϵ contribution.

(4) There should be a new vector meson in the 1-GeV region, the existence of which is also suggested in Ref. 6 in connection with the explanation of the width of the diffraction peak for NN scattering.

II. THE CONVENTIONAL REGGE-POLE MODEL

We review briefly the conventional Regge-pole description for the nucleon-nucleon and the nucleon-antinucleon total cross sections. The general features of the data are illustrated schematically in Fig. 1. Here one assumes that

$$\begin{aligned}
\sigma(\bar{p}p) &= \sigma_P + \sigma_f + \sigma_\omega + \sigma_{A_2} + \sigma_\rho\,,\\
\sigma(\bar{p}n) &= \sigma_P + \sigma_f + \sigma_\omega - \sigma_{A_2} - \sigma_\rho\,,\\
\sigma(pn) &= \sigma_P + \sigma_f - \sigma_\omega - \sigma_{A_2} + \sigma_\rho\,,
\end{aligned} \tag{1}$$

and

$$\sigma(pp) = \sigma_P + \sigma_f - \sigma_\omega + \sigma_{A_2} - \sigma_\rho\,,$$

where for example the term σ_P stands for the Pomeranchon contribution to the total cross section,

$$\sigma_P = \frac{8\pi\,\mathrm{Im}A_P}{k\sqrt{s}}\,, \tag{2}$$

with k being the c.m. momentum. The symbol A_P stands for the t-channel helicity-nonflip amplitude which is parametrized by

$$A_P = \beta_P s^{\alpha_P}\left(-\frac{e^{-i\pi\alpha_P}+\tau_P}{\sin\pi\alpha_P}\right)\,, \tag{3}$$

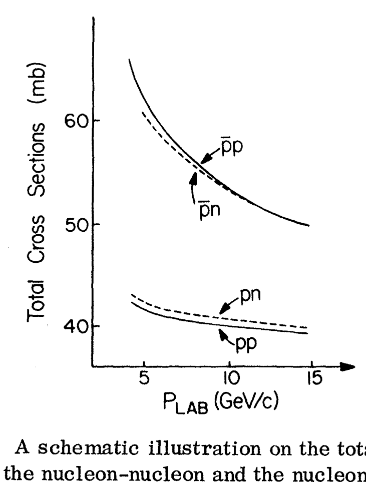

FIG. 1. A schematic illustration on the total cross sections for the nucleon-nucleon and the nucleon-antinucleon scattering. For detailed data, see Ref. 9.

with τ_P being the signature. Other terms in Eq. (1) are given by similar expressions. Since $\alpha_P(0)=1$, the Pomeranchon contribution gives the asymptotic cross section. The other terms with $\alpha(0)<1$ contribute to the energy-dependent part of the cross section. Notice that the sum of the energy-dependent part of the $\bar{p}p$ and $\bar{p}n$ cross sections is much larger than the difference of the same cross sections. Also, the pp and the pn cross sections are relatively flat. These two features together imply that the f and ω contributions ($I_t=0$) dominate over the ρ and the A_2 contributions ($I_t=1$), and that the conditions implied by exchange degeneracy are approximately satisfied, i.e.,

$$\begin{aligned}
&\alpha_f(t)=\alpha_\omega(t), \quad \alpha_\rho(t)=\alpha_{A_2}(t),\\
&\beta_f(t)=\beta_\omega(t), \quad \text{and} \quad \beta_\rho(t)=\beta_{A_2}(t).
\end{aligned} \tag{4}$$

For our discussion below we shall ignore the $I_t=1$ contribution and consider only the $\bar{p}p$ and pp cross sections. For later convenience, we use R to designate f and ω collectively. Thus from Eq. (4) we get

$$\begin{aligned}
&\sigma(pp)=\sigma_P+\sigma_f-\sigma_\omega \simeq \sigma_P,\\
&\text{and}\\
&\sigma(\bar{p}p)=\sigma_P+\sigma_R, \quad \text{where } \sigma_R=\sigma_f+\sigma_\omega.
\end{aligned} \tag{5}$$

The quark diagrams[1] for the Pomeranchon exchange to both pp and $\bar{p}p$ scattering are illustrated in Fig. 2(a). For $\bar{p}p$ scattering with R exchange the diagram is shown in Fig. 2(b). According to the Freund-Harari hypothesis, the Pomeranchon in the t channel is dual to the background contribution in the s channel.[7] The t-channel R trajec-

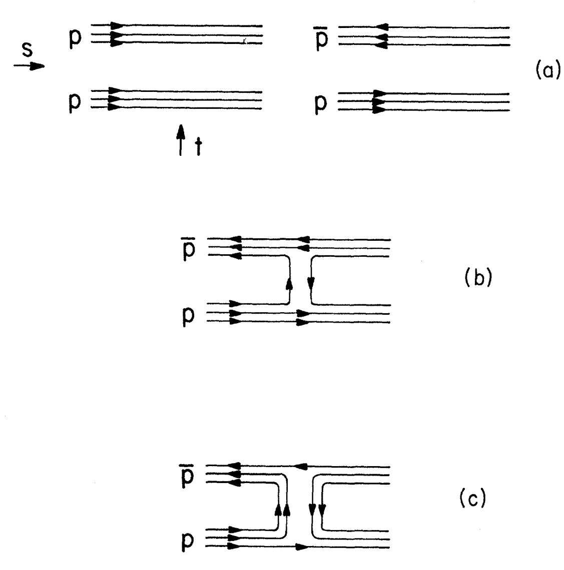

FIG. 2. Quark graphs for (a) the Pomeranchon exchange, (b) the R exchange, and (c) the ϵ exchange.

tory by assumption is made out of $q\bar{q}$ as illustrated in Fig. 2(b). Notice that here the s-channel intermediate state consists of the $qq\bar{q}\bar{q}$ lines. Unlike the situation for the meson-meson scattering, it is well known that for the baryon-antibaryon case, the nonexotic mesons of one channel cannot "bootstrap" the nonexotic mesons in the crossed channel. Also, in a conventional Regge-pole model, according to Eq. (5), the graph such as the one shown in Fig. 2(c) is dismissed altogether.

III. THE ANNIHILATION CROSS SECTION AND EXOTIC EXCHANGE

In this section we present our argument that the diagram such as Fig. 2(c) should not be discarded, in contrary to the usual Regge-pole model mentioned above. This diagram is expected to give an important contribution to the cross section.

To see this, we start with the s-channel unitarity relation for baryon-antibaryon ($B\bar{B}$) scattering

$$\mathrm{Im}A_{ii}=\sum_a \rho_a A_{ia}A^\dagger_{ai}+\sum_p \rho_p A_{ip}A^\dagger_{pi}\,. \tag{6}$$

The intermediate states in the above unitarity sum are divided into two categories: those states (labeled a) without any $B\bar{B}$ pairs and those (p) with $B\bar{B}$ pairs. Thus A_{ia} is the annihilation amplitude while A_{ip} is the elastic or production amplitude. Our task below is to motivate a relationship between the annihilation cross section and the imaginary part of Fig. 2(c).

We start with the annihilation amplitude for the process $B\bar{B}\to MM$, where M signifies a meson.[8] A complete set of quark diagrams for this amplitude is illustrated in Figs. 3(a)–3(c), which we denote, respectively, by F_a, F_b, and F_c. In the dual-resonance model the amplitude F for annihilation into two mesons is a sum of these three terms:

$$F=F_a+F_b+F_c\,. \tag{7}$$

It is evident from Fig. 3(a) that F_a represents the term which has poles in the s and t channels, dual with respect to each other. Similarly, F_b is the su term, and F_c is the tu term.

In computing the two-meson contribution to the first term on the right-hand side of Eq. (6), we have

$$\mathrm{Im}A_{ii}(MM)=\int d^2\Omega_2\rho_2 FF^\dagger\,, \tag{8}$$

where the integration is over all angles of the two-meson system. For the purpose of computing the annihilation cross section, the over-all t of A_{ii} is zero. In the energy region that we shall consider, each F in the integral has significant contribution only in the peripheral regions. Hence, the integral may be divided into two parts: One is over the forward region where the dominant contribution is associated with the t-channel baryon exchanges arising from the terms F_a and F_c; the other is over the backward region where the u-channel baryon exchanges due to F_b and F_c are important. Substituting Eq. (7) into Eq. (8) yields nine bilinear terms in the integrand and, consequently, 18 pieces of integrations in general, counting forward and backward parts separately. Peripheral dominance implies that the t-u interference terms are negligible; they are symbolically

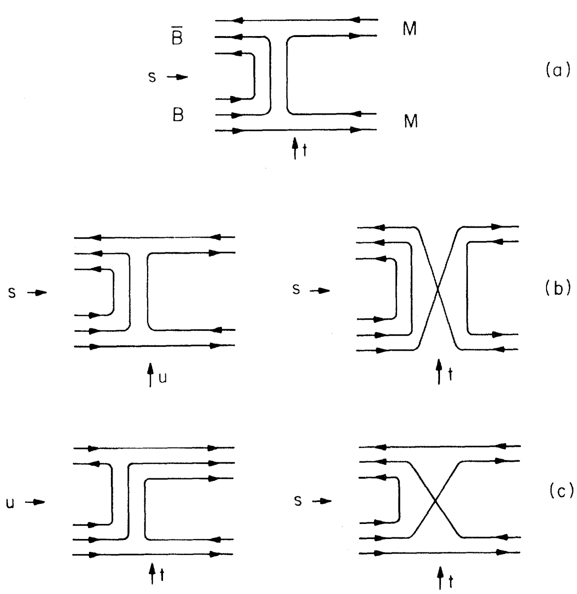

FIG. 3. Quark diagrams for the $\bar{B}B\to MM$ amplitude: (a) st dual diagram F_a, (b) su dual diagrams F_b, and (c) tu dual diagrams F_c.

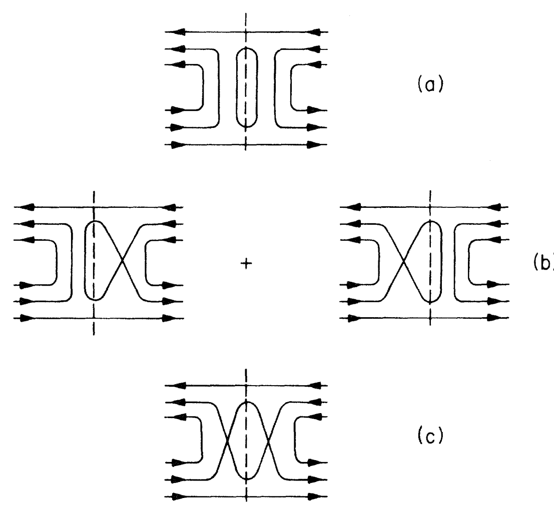

FIG. 4. Quark diagrams for (a) $F_a F^\dagger_a$, (b) $F_a F^\dagger_c+F_c F^\dagger_a$, and (c) $F_c F^\dagger_c$.

$$\int_F (F_a F_b^\dagger + F_b F_c^\dagger + \text{H.c.}) + \int_B (F_a F_b^\dagger + F_a F_c^\dagger + \text{H.c.}),$$

where the labels F and B under the integral signs denote forward and backward regions, respectively. The terms

$$\int_F F_b F_b^\dagger + \int_B F_a F_a^\dagger$$

are small to the second degree, thus even more negligible. The only terms that we need to consider are then

$$I_F \equiv \int_F (F_a F_a^\dagger + F_a F_c^\dagger + F_c F_a^\dagger + F_c F_c^\dagger), \tag{9}$$

$$I_B \equiv \int_B (F_b F_b^\dagger + F_b F_c^\dagger + F_c F_b^\dagger + F_c F_c^\dagger), \tag{10}$$

$$\text{Im} A_{ii}(MM) = I_F + I_B. \tag{11}$$

Using the quark diagrams in Fig. 3, we exhibit the four terms of I_F in Fig. 4 in the same order. Identical diagrams apply also to I_B if the exchange channel of each half of each of the diagrams is interpreted as the u channel.

For annihilation into many mesons the quark diagrams corresponding to F_a, F_b, and F_c are as shown in Figs. 5(a), 5(b), and 5(c), respectively. In the dual model the production of the mesons may be described either by a multiperipheral array, or as decay products of resonances. It is easy to see in the multiperipheral description that when these multimeson annihilation amplitudes are inserted into the unitarity equation, the dominant terms are those in which each pair of multi-Regge links are either both forwardly peaked or both backwardly peaked. Thus the two major pieces of unitarity integrals for each intermediate state of a fixed number of mesons are analogous to Eqs.(9) and (10). Alternatively, these expressions can also be obtained by considering diagrams with only one Regge link between two clusters of particles. Duality does not prefer one over the other mode of description. The four terms in (9) are consecutively illustrated in Fig. 6.

The problem now is to determine which quark diagrams of A_{ii} among those in Fig. 2 have the appropriate imaginary parts corresponding to the various terms in Figs. 4 and 6. We must admit at the very outset that we have no rigorous way of establishing this correspondence, since the application of unitarity to dual diagrams have never been worked out. Another way of expressing the difficulty is that we have no consistent rules to be applied to the quark diagrams when the intermediate state in the unitarity equation is integrated and summed over. We suspect that this is related to the difficulty of unitarizing the dual-resonance amplitude. Fortunately, what we need here is not a quantitative determination of the correspondence, but a qualitative motivation for the existence of the exotic exchange, Fig. 2(c). This we shall be able to do.

Let the $B\overline{B}$ state be divided into two types: those having one $q\bar{q}$ pair annihilation, $|i_1\rangle$, and those having two pairs annihilated, $|i_2\rangle$. We need not consider those states that have no quark annihila-

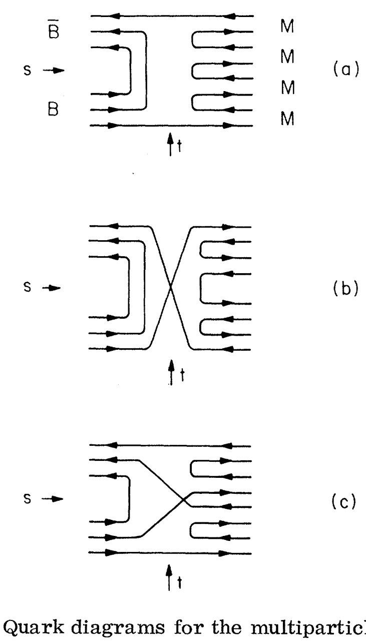

FIG. 5. Quark diagrams for the multiparticle annihilation amplitude: (a) st dual diagram G_a, (b) su dual diagram G_b, and (c) tu dual diagram G_c.

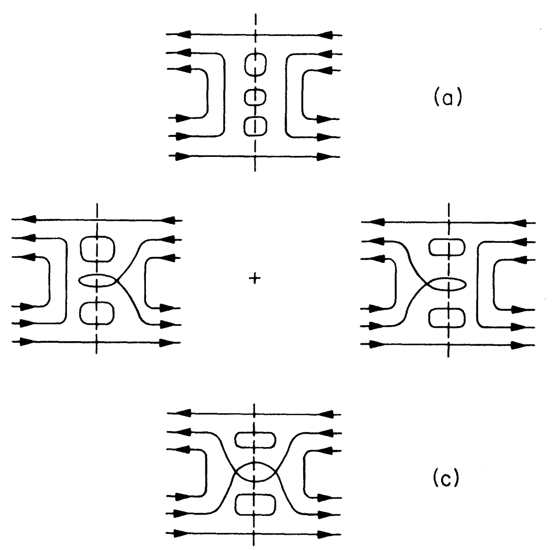

FIG. 6. Quark diagrams for (a) $G_a G_a^\dagger$, (b) $G_a G_c^\dagger + G_c G_a^\dagger$, and (c) $G_a G_c^\dagger$.

tions since we are interested in the annihilation part of the cross section. Clearly, the diagram in Fig. 2(b) corresponds to $\langle i_1|A|i_1\rangle$ while that in Fig. 2(c) corresponds to $\langle i_2|A|i_2\rangle$. Presumably, taking the imaginary parts of these amplitudes does not change the topology of the quark lines. Now, the right-hand side of the unitarity equation is given by the sum of the diagrams of the types shown in Figs. 4 and 6. It is obvious that the diagrams in Figs. 4(a) and 6(a) should contribute to $\mathrm{Im}\langle i_2|A|i_2\rangle$ while those in Figs. 4(c) and 6(c) should contribute to $\mathrm{Im}\langle i_1|A|i_1\rangle$. The ambiguity lies in the interpretation of Figs. 4(b) and 6(b). A cross term between $|i_1\rangle$ and $|i_2\rangle$ cannot directly be identified with a meaningful diagram for $B\overline{B}$ scattering. If one observes that the crisscross lines in these diagrams start and end at the same $B\overline{B}$ states, and thereby interprets these diagrams as contributing to $\mathrm{Im}\langle i_2|A|i_2\rangle$, then two features ensue that are not attractive. First, an $|i_1\rangle$ state is turned into an $|i_2\rangle$ state by the integration over the phase space. Secondly, such a rule is not helpful in understanding the meson scattering problem. Rejecting such an interpretation, we are forced to suppose that these cross terms contribute partly to $\mathrm{Im}\langle i_1|A|i_1\rangle$ and partly to $\mathrm{Im}\langle i_2|A|i_2\rangle$, the relative proportions being unknown.

Let us now attempt a rough estimate of the strength of the exotic exchange. In the interest of clarity we confine our attention here to the two-meson annihilation case, the extension to the multimeson case being straightforward. Moreover, to avoid repetition, we examine explicitly only the forward integral I_F as given by Eq. (9); the procedure for studying the backward integral I_B is similar. For fixed t and large s, the Regge behaviors for F_a and F_c are

$$F_a \sim \beta s^{\alpha-1/2} e^{-i\pi(\alpha-1/2)}, \tag{12}$$

$$F_c \sim \pm \beta s^{\alpha-1/2}. \tag{13}$$

We have included in Eq. (13) the signature factor. Let the baryon be the nucleon, and the meson be the pion. Then the relevant trajectories exchanged are the N_α (even signature) and the Δ (odd signature). The signature of the trajectory affects only the over-all sign of the cross terms $F_a F_c^\dagger + F_c F_a^\dagger$. Evidently, Eqs. (12) and (13) imply

$$F_a F_a^\dagger = \beta^2 s^{2\alpha-1}, \tag{14}$$

$$F_a F_c^\dagger + F_c F_a^\dagger = \beta^2 s^{2\alpha-1}(\pm 2\sin\pi\alpha), \tag{15}$$

$$F_c F_c^\dagger = \beta^2 s^{2\alpha-1}. \tag{16}$$

We see from Eqs. (14) and (16) that the contributions of $F_a F_a^\dagger$ and $F_c F_c^\dagger$ to $\mathrm{Im}\langle i_2|A|i_2\rangle$ and $\mathrm{Im}\langle i_1|A|i_1\rangle$, respectively, are equal and positive. The ambiguous term, Eq. (15), is positive or negative depending upon the value of α. It is known empirically[9] that

$$\alpha_{N_\alpha}(t) \approx -0.39 + 1.01t, \tag{17}$$

$$\alpha_\Delta(t) \approx 0.15 + 0.90t. \tag{18}$$

Thus near $t=0$ where the right-hand side of Eq. (15) is greatest in magnitude, its sign is negative for both signature cases. Positive value commences for $t \lesssim -0.6$ $(\mathrm{GeV}/c)^2$ in the case of N_α exchange, but for $t \lesssim -0.16$ $(\mathrm{GeV}/c)^2$ in the case of Δ exchange. In spite of the strong damping of the residue factor $\beta(t)$ at large $-t$, this oscillatory behavior does provide a suppression effect on the contribution of the cross terms to the integral over t in Eq. (9), although not enough to make it positive in the net. We therefore argue that the contribution of the cross terms, when divided between $\mathrm{Im}\langle i_1|A|i_1\rangle$ and $\mathrm{Im}\langle i_2|A|i_2\rangle$, does not upset the rough equality of the two established above. It is interesting to note that in the meson-meson scattering case the cross term is proportional to $\cos\pi\alpha$ which is very small near $t=0$, and is therefore insignificant in its contribution to the unitarity integral.

The multimeson contribution can be considered similarly. We can then conclude from the above discussion that $\mathrm{Im}\langle i_2|A|i_2\rangle$ contributes to roughly half of the total annihilation cross section. Actually this is an underestimate. This is because there exist additional contributions to $\mathrm{Im}\langle i_2|A|i_2\rangle$ arising from certain terms in the second sum of Eq. (6). Those terms may be represented by diagrams similar to the one in Fig. 6(a) except that one or more of the single closed loops are replaced by double concentric loops. These contributions are positive definite and therefore increase the above estimate.

The experimental annihilation cross section turns out to be quite sizable. In fact, its magnitude is comparable to the difference between the $\overline{p}p$ and pp cross sections. For example, we have,[10] at 3.28 Gev/c, $\sigma_{\mathrm{ann}}(\overline{p}p) = 33.8 \pm 3.2$ mb and $\sigma(\overline{p}p) - \sigma(pp) = 32.2 \pm 2.3$ mb, and at 7.0 GeV/c, $\sigma_{\mathrm{ann}}(\overline{p}p) = 23.6 \pm 3.4$ mb and $\sigma(\overline{p}p) - \sigma(pp) = 17.6 \pm 1.2$ mb. Over half of this annihilation cross section comes from the diagram in Fig. 2(c). A cross section of such a magnitude must be accounted for by a significant t-channel exchange. We therefore propose that the $qq\overline{q}\overline{q}$ object exchanged in the t-channel is to be represented by a Regge pole, labeled ϵ, which is dual to the s-channel annihilation processes. Thus, contrary to the conventional Regge-pole model in which the entire difference between the $\overline{p}p$ and pp cross sections is explained by a pure $q\overline{q}$ exchange, here we assert that a significant portion of this difference should be accounted for by the $qq\overline{q}\overline{q}$

exotic pole exchange. As will be shown below, this exotic pole is expected to be high lying, in contrast to the usual notion of low-lying exotic cuts arising from the elastic iteration of ordinary poles. The departure originates in the completeness of our unitarity sum in which the number of annihilation channels is numerous even at moderate energies, a circumstance where the usual Regge-cut prescription is known to be inadequate.

IV. PROPERTIES OF ϵ

1. *J-parity*. Since the ϵ contributes to the total cross section, it has the J-parity, $P(-1)^J=+1$.

2. *Isospin*. To investigate the isospin of the ϵ, it is useful to look at the quantity

$$\eta=\frac{\sigma_{\text{ann}}(\bar{p}p)}{\sigma_{\text{ann}}(\bar{p}n)}=\frac{1+r}{1-r}\,, \tag{19}$$

where

$$r=\frac{\sigma_{\text{ann}}(I_t=1)}{\sigma_{\text{ann}}(I_t=0)}\equiv\frac{\sigma_{\text{ann}}(\bar{p}p)-\sigma_{\text{ann}}(\bar{p}n)}{\sigma_{\text{ann}}(\bar{p}p)+\sigma_{\text{ann}}(\bar{p}n)}\,. \tag{20}$$

For annihilation at rest,[11] it is found that this ratio η is 1.45 ± 0.07. It follows then that $r\approx0.2$. Consequently, if the notion that the annihilation cross section is governed by ϵ exchange is valid at $\bar{N}N$ threshold, then the isovector part of ϵ must be small. In fact, it should persist in being small even at higher energies for the following reason. The $I_t=1$ channel of $\bar{N}N$ consists of ρ, A_2 and the isovector ϵ. The amplitude for this channel is small because the cross sections for $pn\to np$ and $\bar{p}p\to\bar{n}n$ are both small. The ρ and A_2 contributions are by themselves small as can be inferred independently from the $K^-p\to\bar{K}^0n$ cross section. This is because ϵ does not couple strongly to $\bar{K}K$ as will be discussed below under Sec. IV 8. Thus there is no compelling motivation for introducing an isovector ϵ.[12] Henceforth we shall regard ϵ as an isoscalar object. Since the annihilation cross sections at high energies now receive contributions from the two isoscalar objects ϵ and ω, it is interesting to note that our analysis suggests

$$\sigma_{\text{ann}}(\bar{p}p)\approx\sigma_{\text{ann}}(\bar{p}n) \tag{21}$$

at high energies.

3. *Exchange-degenerate pair*. The absence of the annihilation cross section in the pp channel together with the fact that f and ω are exchange-degenerate implies that the ϵ should be a pair of exchange-degenerate trajectories. We denote the odd signature one by ϵ_1 and the even signature one by ϵ_2. Except for the quark content and a possible difference in $SU(3)$ assignment, the ϵ_1 trajectory has the same quantum number as that of ω and the ϵ_2 that of f. The contribution of ϵ_1 and ϵ_2 to the $\bar{p}p$ elastic amplitude at $t=0$ is given by

$$A_\epsilon\sim-2\beta e^{-i\pi\alpha_\epsilon(0)}s^{\alpha_\epsilon(0)}\,, \tag{22}$$

where $\alpha_\epsilon(0)\approx\frac{1}{2}$. (See Sec. IV 4 below.) Since annihilation cross section is a positive-definite quantity, we must have $\text{Im}\,A_\epsilon>0$. This in turn implies that α_ϵ does not choose nonsense at $\alpha_\epsilon=1$; otherwise, we would have $\beta<0$.[13]

4. *Zero intercept* $\alpha_\epsilon(0)$. The low-energy $\bar{p}p$ differential cross section is approximately saturated by the optical cross section.[10] This implies that the $\bar{p}p$ elastic amplitude is roughly pure imaginary at $t=0$. Now, we know that the contribution from the P is purely imaginary, while that from the R is also nearly so, since $\alpha_R(0)\approx\frac{1}{2}$. Thus the ϵ part of the amplitude must also be imaginary. This agrees with the intuitive notion that the annihilation process is purely absorptive. The phase factor of the ϵ amplitude is therefore

$$-\exp[-i\pi\alpha_\epsilon(0)]\approx i$$

or (23)

$$\alpha_\epsilon(0)\approx\tfrac{1}{2}.$$

The difference between the $\bar{p}p$ and pp total cross sections should be accounted for by α_ω and α_{ϵ_1}. The observed energy dependence of this difference is indeed consistent with the power law $s^{\alpha-1}$, where $\alpha\approx\frac{1}{2}$.[14]

5. *Slope of* $\alpha_\epsilon(t)$. In our present scheme, the exchange of the ϵ in the t channel is dual to the annihilation process in the s channel. From inspection of Fig. 2(c) the latter can alternatively be described by the R trajectories and their daughters in the direct channel. If we assume the approximate validity of local duality, then the slope of $\alpha_\epsilon(t)$ in the negative t region near $t=0$ should be similar to that of the ordinary trajectories.[15] This argument on the slope is not applicable in the positive t region.

6. $SU(3)$ *assignment*. The $qq\bar{q}\bar{q}$ states can be in the $\underline{1}, \underline{8}, \underline{10}, \underline{10}^*$, and $\underline{27}$ representations. However, since the ϵ has the quantum number $I=0$ and $Y=0$, it cannot be in $\underline{10}$ or $\underline{10}^*$. From the $SU(3)$ Clebsch-Gordan coefficients one finds that a $\underline{27}$ for ϵ would lead to $r=\frac{4}{3}$, where r is defined in Eq. (20). Thus Eq. (19) implies that $\eta=-7$. This is ruled out since the ϵ contribution to $\sigma(\bar{p}p)$ and $\sigma(\bar{p}n)$ must be positive definite. Hence ϵ must belong to either $\underline{1}$ or $\underline{8}$.

7. *Quark spin*. For a $qq\bar{q}\bar{q}$ system, there are three possible values for the total quark spins, $j=2$, 1, and 0. Denote the relative angular momentum between qq and $\bar{q}\bar{q}$ by L. We assume that the Regge recurrences correspond to increasing integer values of L. Then for $L=0$, the total angu-

lar momenta J and the parity P of these states are $J^P = 2^+, 1^+$, and 0^+; for $L=1$, they are $J^P = 3^-, 2^-$, 1^-, and 0^-, etc. Consider now the $L=0$ states. The 2^+ state cannot lie on the ϵ trajectory, since the associated 1^- state would have to be nonsense, a situation which violates Sec. IV 3 above. The 1^+ state has the wrong J-parity (see Sec. IV 1). The 0^+ state can, and we suggest that it does, lie on the ϵ trajectory. Now, according to Sec. IV 4, we have $\alpha_\epsilon(0) \approx \frac{1}{2}$, so the point $\alpha_\epsilon = 0$ occurs in the negative t region. There can be no physical particle there. The physical particles on the exchange-degenerate ϵ trajectory are then 1^- at $L=1$, 2^+ at $L=2$, and 3^- at $L=3$, etc. This trajectory is to be associated with the $SU(3)$ octet or singlet. Whether it is in a pure $\underline{1}$ or $\underline{8}$ or a mixture of the two is to be determined by the appearance or absence of the other $SU(3)$ partners. We remark in passing that if it is to be identified with the object needed in Ref. 6, then it must not be in a pure $\underline{8}$. Since ϵ is the leading $qq\bar{q}\bar{q}$ trajectory, the symmetry implied by the quark model must be so badly broken that the trajectories with the triplet quark-spin and other $SU(3)$ multiplets are much lower lying. We do not have a theory for such a breaking.

8. *Decay modes.* We adhere to the rules of Refs. 1 and 2 for the quark diagrams. In particular, the ϵ trajectory couples directly to the $B\bar{B}$ system but not to the nonexotic meson systems. Its coupling to these mesons can be achieved only through a second-order effect, e.g., through a $B\bar{B}$ loop. This is illustrated in Fig. 7. If there is a vector meson on the ϵ trajectory, we expect that it decays weakly into $\pi^+\pi^-\pi^0$ (or $\pi\rho$) and $\pi\gamma$. The relative ratio for these two decay modes depends on the mass of this ϵ_1 vector meson and can be determined in a manner similar to that for the ω.[16]

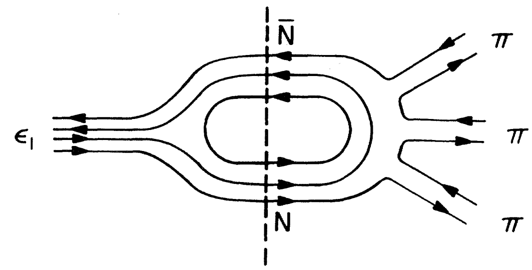

FIG. 7. The coupling of the ϵ_1 meson to three pions via an intermediate $\bar{N}N$ loop.

V. CUTS AND DUALITY

Within the pole approximation our proposal implies that

$$\sigma(pp) = \sigma_P$$

and (24)

$$\sigma(\bar{p}p) = \sigma_P + \sigma_R + \sigma_\epsilon .$$

Thus the difference between the $\bar{p}p$ and pp cross sections is given by $\sigma_R + \sigma_\epsilon$. Since the annihilation cross section we considered is approximately equal to this difference, it appears from Eq. (24) that the R contribution should be at most about half that obtained from the usual Regge-pole model. In reality, our estimate on the R and ϵ contribution is expected to be obscured somewhat by the cut contribution. For example, to the order of the double Reggeon exchange, the amplitude can be symbolically written as

$$\begin{aligned} A = {} & [A_P + A_P \otimes (A_P + A_R + A_\epsilon)] \\ & + [A_R + A_R \otimes (A_P + A_R + A_\epsilon)] \\ & + [A_\epsilon + A_\epsilon \otimes (A_P + A_R + A_\epsilon)] . \end{aligned} \tag{25}$$

The terms on the right-hand side involving the convolution $\otimes$ are cut contributions. Specifically, the term $A_P \otimes A_\epsilon$, for example, is illustrated in Fig. 8 where the two shaded blobs contain the "third" double-spectral functions.

Within our present scheme, in the s-channel physical region near $t=0$, the duality picture can be schematically represented as follows:

$$[A_\epsilon + \text{cuts}]_t \leftrightarrow [A_R + \cdots]_s , \tag{26}$$

$$[A_R + \text{cuts}]_t \leftrightarrow [A_\epsilon + \cdots]_s . \tag{27}$$

In Eq. (26) the t-channel ϵ contribution together with the appropriate cuts is dual to the s-channel Regge trajectories R and their daughters having the $q\bar{q}$ quark content and part of the production cross section. The other energy-dependent piece in the t-channel amplitude given by the left-hand side of Eq. (27) is dual to ϵ plus other contributions in the s channel. We do not expect that the ϵ and its daughters could provide an adequate description for the $qq\bar{q}\bar{q}$ system in the physical s region in spite of the fact that the t-channel ϵ trajectory at $t=0$, being a high-lying J-plane singularity, does control the asymptotic behavior of the s-channel annihilation cross section. In the physical region of the t-channel, near $s=0$, the duality relation is identical to that given by Eqs. (26) and (27) except for the interchange of s and t.

VI. DETECTION OF THE ϵ_1 MESON

As in the case of the $\underline{10}$, $\underline{10}^*$ and $\underline{27}$, where the trajectories are conjectured to turn over before reaching any physical angular momentum states, it is conceivable that the ϵ being made out of $qq\bar{q}\bar{q}$ could eventually also turn over. However, since

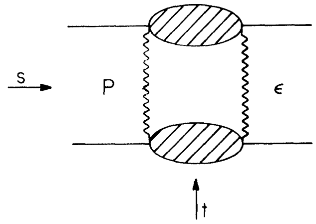

FIG. 8. A diagram for the P-ϵ Regge cut.

$\alpha_\epsilon(0)\approx\frac{1}{2}$ and the local duality picture favors $\alpha_\epsilon(t)$ to have the usual slope in the negative t region near $t=0$, we suggest that it should cross at least the $J=1$ value. If one allows an uncertainty of 1 GeV^2 in extrapolating the trajectory from $\alpha_\epsilon=\frac{1}{2}$ to $\alpha_\epsilon=1$, the trajectory $\alpha_\epsilon(t)$ would cross the $J=1$ point somewhere between 0.7 and 1.2 GeV. This mass range is compatible with what is needed in Ref. 6. Provided that the $N\bar{N}$ vertex function does not have too strong a dependence on the nucleon four-momentum, we anticipate that such a vector meson should be detected. Because of the coupling scheme discussed in Sec. IV 8, this meson can be effectively produced only in the baryon-antibaryon annihilation processes[17] and in backward scattering processes involving baryon exchange. We have looked at some $\pi^+\pi^-\pi^0$ invariant-mass plots from $\bar{p}p$ annihilation. Also, in the reaction[18] $K^-p\to\Lambda^0\pi^+\pi^-\pi^0$, the $\pi^+\pi^-\pi^0$ invariant-mass plot associated with the forward Λ has been compared with that of the backward Λ. Although there is no obvious signal for ϵ_1, those data we have studied are relatively poor in statistics and they do not rule out the possibility for its existence. However, the data do seem to suggest that, if ϵ_1 exists, its coupling to $N\bar{N}$ is no stronger than that of $\eta'(958)$. The situation is more uncertain in the energy region beyond 950 MeV. Future experiments involving a careful study of the 3π invariant-mass plots for the backward – as opposed to the forward – produced baryons in the neighborhood of the 1-GeV region will be a crucial test deciding on the existence of such an object. Also, it might be of interest to study the neutral missing-mass spectrum again near the backward direction for reaction $MB\to B+(\text{missing mass})^0$ in the event that ϵ_1 has some unusual prominent decay modes.

VII. CONCLUSION

The duality picture for the baryon-antibaryon system is investigated. We point out the importance of the special role which annihilation processes play in the $B\bar{B}$ system. We have seen that a significant part of the annihilation processes are dual to the exchange of an exotic ϵ. We have studied the properties of this ϵ and considered its effects on the phenomenology of the total cross sections. Available data on the relevant mass spectrum does not rule out its existence. Future experiments with better statistics are suggested for its detection.

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[1]H. Harari, Phys. Rev. Letters 22, 562 (1969); J. Rosner, *ibid.* 22, 689 (1969).

[2]P. G. O. Freund, R. Waltz, and J. Rosner, Nucl. Phys. B13, 237 (1969).

[3]R. C. Arnold and J. Uretsky, Phys. Rev. Letters 23, 444 (1969); S. F. Tuan, *ibid.* 23, 1198 (1969).

[4]See J. Rosner, "Review of Exotic Mesons," in *Proceedings of the Philadelphia Conference on Meson Spectroscopy, 1970,* edited by C. Baltay and A. Rosenfeld (Columbia Univ. Press, New York, 1970).

[5]The symbol ϵ stands for "annihilation" ('ϵξαϕάνησισ) or "exotic" ('ϵξωτικόσ). We suggest that this designation should supersede the previous label for the "daughter" of ρ, which according to the Particle Data Tables should be referred to as the $\eta_{0^+}(700)$ particle.

[6]C. B. Chiu and R. C. Hwa, Phys. Rev. D (to be published).

[7]P. G. O. Freund, Phys. Rev. Letters 20, 235 (1968); H. Harari, *ibid.* 20, 1395 (1968).

[8]We do not consider here the production of the exotic mesons (ϵ) being proposed in this paper. As will become clear in Sec. IV, their production cross section is expected to be small compared to the total annihilation cross section.

[9]V. Barger and D. Cline, Phys. Rev. Letters 16, 913 (1968).

[10]For the $\bar{p}p$ annihilation data, see T. Ferbel *et al.*, Phys. Rev. 137, B1250 (1965); T. Ferbel *et al.*, Nuovo Cimento 38, 12 (1965). The value for the $\bar{p}p$ annihilation quoted at 7 GeV/c is for annihilation into pions plus the $\bar{n}n+\pi^+\pi^-\cdots$ states. The latter is of the order of 2 mb and is comparable to the strange-particle annihilation cross section. The error quoted is somewhat underestimated. We thank Dr. T. Ferbel for discussions on this point. For the total cross section data, see B. Escoubes *et al.*, Phys. Letters 5, 132 (1963); W. Galbraith *et al.*, Phys. Rev. 138, B913 (1965); and D. V. Bugg *et al.*, *ibid.*

146, 980 (1966).

[11]A. Bettini *et al.*, Nuovo Cimento 47A, 643 (1967).

[12]The reasoning leading us to the conclusion that an isovector ϵ, if existent at all, is low lying or weakly coupled is based on phenomenological observations. This is not surprising from the theoretical point of view. The inelastic process $\bar{p}p \to \bar{n}n$ is in a pure $I_t = 1$ state. Annihilation contributions to its absorptive part does not add coherently, since it lacks the positivity condition which applies only to the imaginary part of the *elastic* amplitude. Indeed, $\mathrm{Im}[A(\bar{p}p \to \bar{n}n)]_{\mathrm{ann}} \propto \frac{1}{2}[\sigma_{\mathrm{ann}}(\bar{p}p) - \sigma_{\mathrm{ann}}(\bar{p}n)]$, which is small as has already been pointed out.

[13]Here we assume that, if α_ϵ were to choose nonsense, β would have a simple zero at $\alpha_\epsilon = 1$. This is so in the simple Veneziano model, for example.

[14]C. B. Chiu, S. Y. Chu, and L. L. Wang, Phys. Rev. 161, 1563 (1967).

[15]C. Schmid, Phys. Letters 20, 689 (1968); C. B. Chiu and A. Kotanski, Nucl. Phys. B7, 615 (1968).

[16]M. Gell-Mann, D. Sharp, and W. G. Wagner, Phys. Rev. Letters 8, 261 (1962).

[17]We note here that resonance production cross section in the multiparticle final states for $\bar{p}p$ annihilation at rest has been explained in terms of the statistical model plus final-state interactions. [See L. Montanet, in Proceedings of the Fifth International Conference on Elementary Particles, Lund, Sweden, 1969, p. 218 (unpublished).] This suggests that the number of events involving ϵ_1 formation in the final states having n pions (typically for $n \geq 5$) is directly correlated to the coupling of $\epsilon_1 \to 3\pi$. Since $\Gamma(\epsilon_1 \to 3\pi) \ll \Gamma(\omega \to 3\pi)$, we expect that the number of ϵ_1 events is suppressed as compared to that of ω. The ϵ_1 events should be much less contaminated if one looks only at the peripheral processes in $\bar{p}p \to \pi^0\epsilon_1$, $\pi^+n \to \epsilon_1 p$, $K^-p \to \Lambda^0\epsilon_1$, etc.

[18]We thank Dr. R. Eisner for providing us with the 3π invariant-mass plot from the reaction $K^-p \to \Lambda\pi^+\pi^-\pi^0$.

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Infinite Set of Quasipotential Equations from the Kadyshevsky Equation*

Robert J. Yaes

Center for Particle Theory, Department of Physics, University of Texas at Austin, Austin, Texas 78712 and Department of Physics, Illinois Institute of Technology, Chicago, Illinois 60616†

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We show that by modifying the propagator in the Kadyshevsky equation, we can obtain an infinite set of quasipotential equations which satisfy both Lorentz covariance and elastic unitarity and of which the Logunov-Tavkhelidze-Blankenbecler-Sugar-Alessandrini-Omnes equation and the Gross equation are special cases. We also show that the perturbation scheme of Chen and Raman, for using the quasipotential equation to obtain approximations to the Bethe-Salpeter equation, can be greatly simplified by the use of resolvent-identity-type arguments.

In potential theory, the off-shell T matrix satisfies the Lippmann-Schwinger equation, the integral form of the Schrödinger equation. Since the free-particle Green's function has the appropriate discontinuity, elastic unitarity is guaranteed by the equation if the potential is real and symmetric.[1]

In the relativistic case, we do not have a simple equation like the Schrödinger equation, so we must resort to the techniques of field theory. However, equations of the same form as the Lippmann-Schwinger equation can prove useful. The most common example is the Bethe-Salpeter equation in the ladder approximation. The terms obtained from iterating this equation correspond to individual Feynman diagrams, the so-called ladder graphs. There is thus a simple immediate connection with field theory. In addition, it can be shown that elastic unitarity is exactly satisfied between the elastic threshold and the threshold for production.[2]

There are, however, serious difficulties associated with the Bethe-Salpeter equation. Since it is a four-dimensional integral equation, it only reduces to a two-dimensional integral equation upon taking a partial-wave projection. In addition, there are the difficulties associated with the indefiniteness of the Lorentz metric, making the equation difficult to deal with except in simple models such as the Wick-Cutkosky model.

In order to circumvent these difficulties, a simpler equation has been proposed by Logunov and Tavkhelidze,[3] by Blankenbecler and Sugar,[4] and by Alessandrini and Omnes.[5] This equation is manifestly covariant, and the Green's function is chosen to have the discontinuity that will insure that the solution satisfies elastic unitarity for a real symmetric "potential." The equation, however, is three-dimensional, and it reduces to a form which is very similar to the Lippmann-Schwinger equation in the center-of-mass system. The equation