168, 1731 (1968).

[12]In the treatment of self-consistency equations in Ref. 2, the absence of states of baryon number 2 is assumed *a priori* and used to deduce the equality of the baryonic interactions of corresponding even- and odd-parity meson states.

[13]One of the first models in which baryonic interactions are forced to satisfy a commutation relation similar to Eq. (4.9) is that of J. C. Polkinghorne, Ann. Phys. (N.Y.) 34, 153 (1965). In the Polkinghorne model, which involves vector mesons and baryons of one parity, any group representation is satisfactory for the baryons.

[14]This point has been emphasized by J. L. Rosner, Phys. Rev. Letters 21, 950 (1968); 21, 1468(E) (1968).

[15]The nature of this proof shows that the two-quark solution works for the same reason that it satisfies the "duality diagram" test of Harari and Rosner. See H. Harari, Phys. Rev. Letters 22, 562 (1969); J. L. Rosner, *ibid.* 22, 689 (1969).

[16]Examples of such superpositions are the virtual (resonance) states calculated in the $SU(6)_W$-potential model of R. H. Capps, Phys. Rev. 158, 1433 (1967); 165, 1899 (1968).

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PHYSICAL REVIEW D VOLUME 3, NUMBER 12 15 JUNE 1971

Persistence of the "Photon" in Conformal-Dual Models*

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The spin-1 zero-mass "photon" of the (conformal) Veneziano model persists in the spectra of all "new" (conformal) dual models of the Bardakci-Halpern type. In these generalized models, however, the "photon" does not, in general, decouple through statistics.

The conformal Veneziano[1] model (all Ward identities[2] working) involves unit leading-trajectory intercept, and hence a spin-1 zero-mass "photon" Γ. As far as we know, this particle is not related to the real photon (of electromagnetism). In any case, Γ decouples from the model via Bose statistics, so it is no problem. Recently, Bardakci and the author introduced a new class of dual-conformal models[3] that includes spin. In each of the simple examples we discussed, Γ appeared again in the spectrum. What we want to show here is that this feature is completely general: For any model of our type, Γ persists in the spectrum. Moreover, it does not in general decouple through statistics; for example, in the additive models, it couples to baryon-antibaryon pairs. For models with spin-orbit forces, we cannot determine the coupling until after the gauge states are removed.

We begin by describing our models in general terms.[4] We assume we have found a set of conformal generators J_m, constructed as Fourier components of a density $J(\theta)$:

$$J_m = \frac{1}{2\pi}\int_0^{2\pi} d\theta\, e^{-im\theta} J(\theta), \tag{1}$$

where the density is itself constructed of sums of bilinears in local fields and currents, which we denote by $\{\pi(\theta)\}$ and $\{V(\theta)\}$, respectively. The $\{V(\theta)\}$ carry Lorentz and internal-symmetry labels, while $\{\pi(\theta)\}$ in general carries Lorentz and fifth (etc.) operator labels. For reference, we record the projective subalgebra of the J_m,

$$[J_0, J_{\pm 1}] = \mp J_{\pm 1}, \tag{2a}$$

$$[J_{+1}, J_{-1}] = 2J_0, \tag{2b}$$

together with the equation for mass-shell states $|\Psi\rangle$,

$$J_0|\Psi\rangle = |\Psi\rangle\ . \tag{3}$$

We can exhibit the dependence of J_m on the external 4-momentum p^μ ($\mu = 0, 1, 2, 3$) in the form

$$J_m = -ap^2\delta_{m,0} + p\cdot A_m + B_m, \tag{4}$$

where the (assumed known) number a and the operators A_m^μ, B_m are p^μ-independent (though the operators may depend on fifth operators). In the sector where fifth quantum numbers are zero, we have a vacuum state $|0\rangle$ such that[5]

$$A_m^\mu|0\rangle = 0, \quad m \geq 0 \tag{5a}$$

$$B_m|0\rangle = 0, \quad m \geq -1\ . \tag{5b}$$

Equations (1)–(5) complete our statement of the generalized models; it includes, of course, the ordinary Veneziano model. Now we want to show that any such system contains a "photon" Γ. The particle will occur in the sector with zero fifth (etc.) quantum numbers, so we will confine ourselves to this case.

Consider Eq. (2a). By differentiating with respect to p^μ, we obtain

$$[J_0, A^\mu_{-1}] + [A^\mu_0, p\cdot A_{-1}] + [A^\mu_0, B_{-1}] = A^\mu_{-1}. \qquad (6)$$

Differentiating again by p^ν gives

$$[A^\mu_0, A^\nu_{-1}] + [A^\nu_0, A^\mu_{-1}] = 0. \qquad (7)$$

Equation (7) tells us that the symbol

$$M^{\mu\nu}_{-1} \equiv [A^\mu_0, A^\nu_{-1}] \qquad (8)$$

is antisymmetric in (μ, ν). Taking this together with (6), on the vacuum, we establish the identity

$$J_0 A^\mu_{-1}|0\rangle = (1 - ap^2)A^\mu_{-1}|0\rangle + p_\nu M^{\nu\mu}_{-1}|0\rangle . \qquad (9)$$

This statement already suggests quite strongly that there exists a spin-1 zero-mass state whose wave function at $p^\mu = 0$ is just $A^\mu_{-1}|0\rangle$. Motivated by this, we shall now attempt to construct such a state as a solution to Eq. (3).

Turning our attention then to Eq. (3), we seek a state of the form

$$|\Gamma\rangle = \alpha_0(p^2) A^\mu_{-1}|0\rangle + \cdots , \qquad (10)$$

where $\alpha_0(p^2)$ is the unknown weight of $A^\mu_{-1}|0\rangle$, and the dots indicate a linear combination of the complete set of states generated by the action of J_0 on $A^\mu_{-1}|0\rangle$. In general, this is an expansion in terms of all states with spin 1 in the first sector. In practice, this is a set of about six states,[6] which we however will denote quite generally by

$$\{a^\mu_{(i)}\}, \quad i = 0, 1, \ldots, n; \quad a^\mu_{(0)} = A^\mu_{-1},$$
$$\{p_\nu M^{\nu\mu}_{(i)}\}, \quad i = 0, 1, \ldots, m; \quad p_\nu M^{\nu\mu}_{(0)} = p_\nu M^{\nu\mu}_{-1} \qquad (11)$$

on the vacuum. We have suppressed the subscript (−1) with the understanding that from now on we are in the first sector. In our quark models, the states (11) are the only ones that appear, but we remark that our proof goes through similarly if we assume the existence of higher-indexed operators, such as, e.g., $p_\lambda p_\sigma p_\kappa M^{\lambda\sigma\kappa\mu}$, etc.

In general, we need all these states in $|\Gamma\rangle$ because they are mixed under the action of J_0. To exhibit this explicitly, we assume we have calculated the coefficient matrices L, N, $\bar{L}$, and $\bar{N}$ in

$$J_0 a^\mu_{(i)}|0\rangle = \sum_{j=0}^{n} L_{ij} a^\mu_{(j)}|0\rangle + \sum_{j=0}^{m} N_{ij} p_\nu M^{\nu\mu}_{(j)}|0\rangle, \qquad (12a)$$

$$J_0 p_\nu M^{\nu\mu}_{(j)} = \sum_{j=0}^{m} \bar{L}_{ij} p_\nu M^{\nu\mu}_{(j)}|0\rangle + \sum_{j=0}^{n} \bar{N}_{ij} a^\mu_{(j)}|0\rangle . \qquad (12b)$$

We already know some properties of these matrices: From (9), we read off that

$$L_{0j} = (-ap^2 + 1)\delta_{0,j} ,$$
$$N_{0j} = \delta_{0,j} . \qquad (13)$$

Further, from simple counting of powers of p^μ, and the form of J_0,

$$L_{ii} = -ap^2 + \lambda_i \quad (\lambda_0 = 1),$$
$$\bar{L}_{ii} = -ap^2 + \bar{\lambda}_i , \qquad (14)$$
$$\bar{N}_{ij} = p^2 \bar{n}_{ij} ,$$

where λ_i, $\bar{\lambda}_i$, and $\bar{n}_{ij}$ are independent of p^μ. All other matrix elements are independent of p^μ. This is all we shall need to know of the coefficient matrices.

Now we write a general $|\Gamma\rangle$ as

$$|\Gamma\rangle = \sum_{i=0}^{n} \alpha_i(p^2) a^\mu_{(i)} + \sum_{i=0}^{m} \beta_i(p^2) p_\nu M^{\nu\mu}_{(i)}|0\rangle \qquad (15)$$

in terms of the unknown functions α_i, β_i. Substituting into (3), using (12), and identifying (operator) coefficients, we arrive at the following $n+m+2$ linear equations restricting the $n+m+3$ unknowns $\alpha_i \beta_i$ and p^2:

$$\alpha L + p^2 \beta \bar{n} = \alpha , \qquad (16a)$$

$$\alpha N + \beta \bar{L} = \beta , \qquad (16b)$$

where for simplicity we have adopted a matrix notation in which α and β are now row vectors. Now we construct solutions at $p^2 = 0$: At $p^2 = 0$, an evident solution to (16a), using (13) and (14), is

$$\alpha = [\alpha_0, 0, 0, 0, \ldots]. \qquad (17)$$

This leaves us with the system (16b), being $m+1$ equations in $m+1$ unknowns. There is a unique solution

$$\beta = \alpha N (1 - \bar{L})^{-1}, \qquad (18)$$

unless $\det[1 - \bar{L}] = 0$. This possible exception, a "dynamical accident," does not occur in any of the models we have studied, but, for completeness, we note that even then, the photon will persist. Assume that, in fact, $\det[1 - \bar{L}] = 0$. What is happening of course is that the naive intuition of Eq. (9) is breaking down, in that the normalization of the $M^{\mu\nu}$ terms has become infinite relative to the initial guess A^μ_{-1}. So, staying at $p^2 = 0$, we take the trivial solution to (16a), namely, $\alpha = 0$, in which case (16b) is just ($\bar{\bar{L}} \equiv 1 - \bar{L}$)

$$\beta \bar{\bar{L}} = 0, \quad \det[\bar{\bar{L}}] = 0. \qquad (19)$$

This guarantees us at least one solution Γ whose wave function is entirely of $M^{\mu\nu}_{(i)}$ type.

In summary then, we have shown that any system of the form Eqs. (1)–(5) contains a "photon" Γ, and, barring the "dynamical accident," the wave function of Γ is indeed

$$|\Gamma\rangle = A^\mu_{-1}|0\rangle + O(p). \qquad (20)$$

We find this interesting. For example, in the most general case of physical interest, that of broken $SU(3)$ and $SU(2)$, one would introduce the broken-symmetry assumption by assuming that

$$A^{\mu}_{-1}=C_0\pi^{\mu}_{-1}+C_9V^{\mu 9}_{-1}+C_8V^{\mu 8}_{-1}+C_3V^{\mu 3}_{-1}\,. \tag{21}$$

Then, after all the coefficients in the algebra are determined, we can be sure that Γ will exist and transform as in (21). In this sense, Γ is the "pivot" for symmetries and symmetry breaking in the theory.

Since Γ persists in the spectrum, it becomes important to know whether it couples into the theory. In the old "orbital" Veneziano model, its coupling to two ground states of momentum k_1 and k_2 is

$${}_q\langle 0|\epsilon\cdot\pi_{+1}e^{\sqrt{2}Q(1)\cdot k_2}|0\rangle_{k_1}\sim\epsilon\cdot(k_1-k_2)\,, \tag{22}$$

where $q=-(k_1+k_2)$ is the "photon" momentum and ϵ^{μ} its polarization vector (with $\epsilon\cdot q=0$). Because the coupling is antisymmetric with respect to the two ground states, it vanishes after Bose symmetrization.

On the other hand, in the additive models, we calculate the Γ coupling into a baryon-antibaryon pair as

$$\begin{aligned}{}_q\langle 0|\epsilon\cdot\pi_{+1}e^{\sqrt{2}Q(1)\cdot k_2}\Psi_{l_1}(1)\Psi_{l_2}(1)\Psi_{l_3}(1)b^{\dagger}_{m_1}b^{\dagger}_{m_2}b^{\dagger}_{m_3}|0\rangle_{k_1}\sim\ &\epsilon\cdot(k_1-k_2)(\delta_{l_1m_1}\delta_{l_2m_2}\delta_{l_3m_3}+\delta_{l_3m_1}\delta_{l_1m_2}\delta_{l_2m_3}\\ &+\delta_{l_2m_1}\delta_{l_3m_2}\delta_{l_1m_3}-\delta_{l_2m_1}\delta_{l_1m_2}\delta_{l_3m_3}\\ &-\delta_{l_3m_1}\delta_{l_2m_2}\delta_{l_1m_3}-\delta_{l_1m_1}\delta_{l_3m_2}\delta_{l_2m_3})\end{aligned} \tag{23}$$

for Fermi quarks, and all signs + for Bose quarks. In either case, the term in brackets is symmetric with respect to $\{l_1l_2l_3\}\leftrightarrow\{m_1m_2m_3\}$, so the overall coupling is antisymmetric. Hence, Γ couples to (Fermi) baryons, and, in general, couples to a conserved ninth (baryon-number) current. This is a disturbing aspect of the additive models, but in any case these models do not have realistic spectra. In the "realistic" models with spin-orbit forces, the problem of gauge states prevents a quick answer: Γ is a real state [satisfying $(K_l-K_0)|\Gamma\rangle=0,\ l\geq 0$], but no other state is so simple. The individual (K-degenerate) contributions to the baryon states do couple to Γ, but until we isolate "real" baryons, we cannot state their couplings.

One further remark is worth making. In Ref. 3, we also found a "photon" in a model where only the first J-identity was working (and all the K-identities), i.e., for which the mass-shell equation was of the form $J_0|\Psi\rangle=-n|\Psi\rangle,\ n=0,1,\ldots$. We can show that this also is completely general,[7] i.e., Γ always appears for $n=0$: In this case, one need remember that there is another commuting conformal group

$$\begin{aligned}&K_m=-a'p^2+p\cdot A'_m+B'_m\,,\\ &[K_l,J_m]=0\,,\quad J_0+K_0=N_0-p^2\,,\end{aligned} \tag{24}$$

where N_0 is the "sector operator." In the first sector ($N_0=1$), the mass-shell equation (for $n=0$) is then equivalent to

$$(K_0+p^2)|\Psi\rangle=|\Psi\rangle\,. \tag{25}$$

By the methods above, we establish the persistence of Γ again; this time its wave function begins with $A^{\mu\prime}_{-1}$.

Finally a conjecture: It seems quite plausible that the "photon" may transcend even these generalized models, in the sense that Γ may follow directly from conformal invariance. Such a theorem is evidently true for our models, but proving it generally requires a knowledge of the most general form of a conformal system.

Note added in proof. Our argument here is for a class of stable resonance models. Perhaps the most intriguing possibility is that in a unitary model, conformal invariance will continue to force a "singularity" at spin 1, zero mass (Pomeranchuk trajectory, cut?).

We would like to thank Dr. M. A. Virasoro and Professor K. Bardakci for helpful conversations.

*Research supported by the Air Force Office of Scientific Research, Office of Aerospace Research, United States Air Force, under Contract No. F44620-70-C-0028.

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[4]We are excluding models with terms linear in the currents, such as that introduced in Ref. 3 for symmetry breaking: $\Delta J_0=\lambda J^8_0+\lambda^2$, $\Delta J_{\pm 1}=\lambda J^8_{\pm}$. These models, containing in general an arbitrary parameter λ, do not satisfy all the Ward identities. This is another example of the general theorem that conformal invariance fixes all the parameters in the theory except a slope.

[5]It is with Eqs. (5) that we exclude the models mentioned in Ref. 4.

[6]For example, in the general case of broken $SU(3)$ and

$SU(2)$ the set would be A_{-1}^{μ}, $V_{-1}^{\mu 9}$, $V_{-1}^{\mu 8}$, $V_{-1}^{\mu 3}$, $T_{-1}^{\mu\nu 8}$, $T_{-1}^{\mu\nu 3}$, where A_{-1}^{μ} is a linear combination of all the other vectors including π_{-1}^{μ}, as in Eq. (21).

[7]Here we are speaking only of the models with nonzero spin-orbit forces.

PHYSICAL REVIEW D VOLUME 3, NUMBER 12 15 JUNE 1971

Asymptotic Behavior and the Possibility of Duality without Ghosts in Feynman-Diagram Models*

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Elastic scattering of two spinless particles with equal or unequal masses is considered in Feynman-diagram models of the Van Hove type with the usual couplings and with arbitrary spectra of masses, spins, and coupling constants, including infinitely large masses and spins. It is shown that if the amplitude defined by analytic continuation of the sum of all lowest-order diagrams with s-channel poles has no unphysical or u-channel singularities, then for fixed t this amplitude is not bounded by any power of s as $|s| \to \infty$ in any infinitely multiply connected domain which excludes only neighborhoods of the poles. Since a dual Born term with no u-channel poles can be represented entirely by the sum of s-channel pole diagrams (or by the sum of t-channel pole diagrams), this bad asymptotic behavior cannot be canceled in the Born approximation in a dual multiparticle theory. Regge asymptotic behavior is also supposed to be associated with duality, and consequently one sees that Feynman-diagram models of the Van Hove type cannot exhibit duality. The result is independent of the order of summation of diagrams. It depends crucially on the requirement that coupling constants be real.

I. INTRODUCTION

It is well known that it is possible to construct Feynman-diagram models[1-4] with simple couplings which exhibit Regge asymptotic behavior in one channel. At first, one might hope that by introducing infinitely many particles and adjusting all the masses and coupling constants, a model of the Van Hove type[1-4] possessing Regge behavior in two channels could be found. The existence of dual-resonance models[5,6] would tend to support this hope if implicit poles in t were allowed in the sum of s-channel pole diagrams. One might even hope that some model of the Van Hove type could fully incorporate duality, in which case the infinite sum of s-channel pole diagrams would be equal to a similar sum of t-channel pole diagrams. However, we will prove that the asymptotic behavior of any such generalized Van Hove model (with the usual couplings[7]) is unacceptable from the point of view of duality. The factors which are primarily responsible for this negative result are (1) *the absence of ghosts* and (2) *the simple couplings of the arbitrarily high-spin particles which are present in the model*. We prove that if the sum of all lowest-order diagrams with s-channel poles has no implicit unphysical or u-channel singularities, then for fixed t, the amplitude defined by this sum is not bounded by any power of s as $|s| \to \infty$ in any infinitely multiply connected domain which excludes only neighborhoods of the poles. No assumption is made about the domain of convergence of the sum of diagrams other than its existence.

From the point of view of applications of the Van Hove model, the result indicates that outside the usually accepted region of applicability the model is quite badly behaved.

From the point of view of duality the result is mildly unfortunate because of its generality. It implies that no dual multiparticle theory with trajectories and daughters of any shape or spacing can correspond to a ghost-free theory with the simple and conventional couplings[7] of the model. Of course, the results of this paper do not imply that all dual multiparticle theories must have ghosts. Thus, there is no contradiction between the present conclusions and the general belief that the Veneziano model with unit intercept is free of ghosts.

Although duality and positivity do come into conflict within the fairly general framework treated here, the results which are obtained do not rule out the possibility of a ghost-free dual Feynman-diagram model associated with a somewhat more complicated theory such as a theory with nonminimum derivative couplings. Abarbanel[8] has con-