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Relationship of Flux Quantization to Charge Quantization and the Electromagnetic Coupling Constant

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The concept of a closed quantized flux loop ("elementary loop") which avoids the implication of magnetic monopoles is investigated, leading to a theory of a charged lepton (muon or electron). In order to reconstruct a continuous magnetic dipole field of a source lepton, it is assumed that the flux loop adopts a statistical distribution of alternative forms characterized by a complex probability amplitude superposition, in a manner somewhat analogous to the superposition of path histories in Feynman's space-time approach to quantum mechanics. Flux quantization results from the equivalence of a line discontinuity of the phase factor of a ψ function of a field lepton (due to its phase multivaluedness by $\pm 2\pi$) to the presence of a line of quantized flux. On the same basis as quantized flux arises from such a singularity of that phase factor, so also an electric field arises when this singularity line is moving. In particular, the source's Coulomb field results from a spinning of the quantized flux loop (about the center of the source) with an angular velocity equal to the *Zitterbewegung* frequency $2mc^2/\hbar$, if the statistical distribution of flux loopforms properly represents the magnetic dipole field of a muon or of an electron. The reconstruction of the magnetic and electric fields of a charged lepton and the comparison of them with the quantized flux hc/e gives a numerical estimate of the electromagnetic interaction constant $e^2/\hbar c$, i.e., an understanding of the relationship between e and $\hbar$. The energy mc^2 and angular momentum $\hbar/2$ may be interpreted as electromagnetic. The theory should work for both muon and electron and is expected to give some insight into the ratio of the masses of these two leptons. A representation of quarks in terms of linked quantized flux loops is suggested to describe a low-lying meson as a linkage of an elementary loop with an antiloop, and a low-lying baryon as three interlinked elementary loops. We are here developing a model approach to problems of structure and conservation laws in particle physics. A more abstract version of a quantized flux theory of particles should be preceded by such an heuristic model.

I. INTRODUCTION

ONE of the puzzles of modern physics is the occurrence of two quantum constants: $\hbar$, indicating the quantization of action, and e, the quantization of electric charge. They are numerically interrelated in the electromagnetic coupling constant, the "fine-structure constant" $e^2/\hbar c$. This relationship has to be considered solely as a relationship between e and $\hbar$: With the advent of special-relativity theory, one eliminates the constant c in $e^2/\hbar c$ by measuring time in units of centimeters. (We shall use, however, the ordinary cgs notation, retaining all the constants so as to bring the discussion of the relationship between the fundamental constants e and $\hbar$ into a familiar language.)

One may ask why, apart from purely historical reasons, one should start with a separate discussion of the issue of the electromagnetic coupling constant here, instead of discussing this issue in relation to all the other coupling constants. There are two reasons for starting the discussion of $e^2/\hbar c$ on its own. (1) Quantum electrodynamics is an exceedingly successful theory: More is known about electrodynamic interactions than about any of the other interactions. This means that a new approach to electromagnetic theory has so many known facts to comply with that any basically wrong new attempt to understand $e^2/\hbar c$ will show its inconsistencies immediately. These constraints permit no important ambiguities in the choice of assumptions for the present approach; once one tries to carry through a theory like this one, based on flux quantization, the assumptions for such a theory are almost uniquely determined with little choice for alternatives.

(2) In studying the theory of the electron and of the muon, we may hopefully tackle their basic properties by electromagnetism alone (and there are good reasons to expect that the weak and the strong interactions might be understood in terms of the same concepts which we are employing in this electromagnetic theory of the electron and of the muon). A theory of the electron has not much hope of being relevant if it cannot give at least a plausible interpretation of the dual existence of the two similar particles, the electron and the muon, and of their essential differences in mass. It is for this reason that from the outset we paid attention to only such formulation of a theory of the lepton which might make it possible to understand this duality. When we develop the theory it will be seen to be more convenient to discuss it first in terms of the muon and then look for the specialization which the electron implies; we have so far, however, only given a crude qualitative discussion of this electron-muon issue.

The problem of e^2 versus $\hbar$ received attention with Sommerfeld's 1916 paper, which marked the first success of linking relativity with quantum theory. In 1925, at the start of quantum mechanics, and then again with Dirac's relativistic theory of the electron, there were expectations that this puzzle of $e^2/\hbar c$ might be solved. In the 1930's, with the inquiry into the basic topics of quantum electrodynamics, there was, in Niels Bohr's words, another hope, but even quantum electrodynamics had failed to shed light on this issue. Heisenberg, during the past few years, has made another attempt towards the resolution of this issue. His starting point is that a coupling of the Maxwell-Lorentz field equations with the Schrödinger-Klein-Gordon or Dirac equation, in a fashion somewhat resembling Hartree's self-consistent field, implies a nonlinear equation because the four-potentials are dependent on $\psi^*\psi$. This interesting starting point of Heisenberg's is similar to what Bohm, Ruderman, Finkelstein (and a little bit myself) and many others tried to follow up in the late 1940's, but only a few results seemed to have been forthcoming in such approaches.

The approach taken to the problem here is heuristic. When we use a quantized magnetic flux loop (an elementary loop) in this theory, we use a space-time description of the forms which this loop may adopt. Such a picture is, for loop motions, on the level of the Bohr theory for electron orbits. We then attach probability amplitudes[1] to the alternative forms of such a loop and superpose these loopforms to represent the structure of a "particle."

We consider an elementary loop, i.e., a quantized flux loop, as the basic unit of particle physics. Whereas a single loop represents an electron or muon, three interlinked loops represent a low-lying baryon, and a loop and an antiloop interlinking represent a low-lying meson. It is impressive to realize how many puzzles of the quark picture get resolved with such an interpretation.

Enormous success has been achieved by abstract approaches to particles physics. Just as a century ago when it was recognized that it is the group which determines the geometry, so later and now we experience the benefit of that recognition in the field of physics, too. That does not absolve us, however, from the need to identify physically the operators and the states entering in the group-theoretical description. It is in that connection and in providing heuristic tools that models are an important counterpart to an abstract theory.

When, as in the present theory, the question is raised as to a structure underlying an electron or a muon (a structure in terms of a probability amplitude distribution of the forms which a quantized flux loop may adopt), that question implies a quest for a spin model of the electron and the muon. The possibility of a spin model of a lepton has sometimes been considered nonexistent. Bopp and Haag have, however, shown that indeed such a model exists and has its representation in the spin-$\frac{1}{2}$ eigenfunctions

$$T_{mn}{}^s=\exp(im\alpha)P_{mn}{}^s(\cos\theta)\exp(in\beta) \qquad (1.1)$$

of the symmetric top (cf. the discussion in Sec. VIII B of the present paper). The authors point out that a molecule in fact does not have such half-integer spin eigenfunctions because its atoms are not rigidly interlocked. The half-integer spin eigenfunctions may, however, directly apply to the probability amplitude distribution of loopforms.

II. OUTLINE

It is not unreasonable, in an attempt to understand quantization of electric charge, to start with the concept of flux quantization. Flux quantization has its basis in the recognition that the phase ϑ of a field lepton's ψ function may be multivalued (changing by $\pm 2\pi$ in going around a quantized flux line) without the function itself being multivalued.[2] This condition of single valuedness of ψ which permits phase multivaluedness by only positive- or negative-integer multiples of 2π is a very familiar quantization condition.

Multivaluedness of the phase ϑ may occur along lines in ordinary three-space and, correspondingly, quantized flux occurs along these lines. Avoiding the unnecessarily complicating concept of magnetic monopoles, we assume that the multivaluedness of ϑ and thus also quantized flux occurs only along lines of the form of closed loops. A quantized flux loop, on the one hand, represents a singularity of the phase factor $e^{i\vartheta}$ of any *field* lepton's wave function; on the other hand, it is assumed that the form and location of that loop is as if it were one of the magnetic dipole field lines of a *source* lepton. In order to construct a meaningful field theory, we assume that one such loop represents only one of a continuous manifold of "loopforms," this manifold somewhat resembling the manifold of magnetic field lines of a source lepton. Accordingly, we propose to formulate the magnetic field of a muon or of an electron entirely in terms of quantized magnetic flux (see Fig. 1).

[1] The footnotes are grouped together under a few general topics. They cover a broad spectrum of references most of which have only an indirect bearing on the present paper. It has been found useful to have the quotations at hand. Papers relating to amplitude superposition: R. P. Feynman, Rev. Mod. Phys. **20**, 367 (1948); Phys. Rev. **76**, 749 (1949); **76**, 769 (1949); **80**, 440 (1950); **84**, 108 (1951); Science **153**, 699 (1966); in *Proceedings of the Twelfth Solvay Institute of Physics Conference* (Interscience, New York, 1961), p. 61; R. P. Feynman and A. R. Hibbs, *Quantum Mechanics and Path Integrals* (McGraw-Hill, New York, 1965); J. R. Klauder, Ann Phys. (N. Y.) **11**, 123 (1960); Lectures, University of Bern, 1962 (unpublished); thesis, Princeton University, 1959 (unpublished); C. Morette-de Witt, Phys. Rev. **81**, 848 (1951); C. W. Misner, Rev. Mod. Phys. **29**, 497 (1953); W. E. Brittin and W. R. Chappell, *ibid.* **34**, 620 (1962); R. P. Feynman, *Quantum Electrodynamics* (Benjamin, New York, 1961); *Theory of Fundamental Processes* (Benjamin, New York, 1961); R. P. Feynman, R. B. Leighton, and M. Sands, *Feynman Lectures in Physics* (Addison-Wesley, Reading, Mass., 1965); F. J. Dyson, Phys. Rev. **75**, 486 (1949); Advanced Quantum Mechanics, lecture notes, Cornell, 1951 and 1954 (unpublished); M. Kac, *Probability and Related Topics in Physical Science* (Interscience, New York, 1959); I. Gel'fand and A. Yaglom, Fortschr. Physik **5**, 517 (1957); S. S. Schweber, J. Math. Phys. **3**, 831 (1962); I. Gel'fand and A. Yaglom, *ibid.* **1**, 48 (1960); L. Streit, Acta Phys. Austriaca Suppl. **II**, 2 (1966); D. ter Haar, *Path Integral Methods in Statistical Mechanics* (to be published); E. R. Speer, *Generalized Feynman Amplitudes* (Princeton U.P., Princeton, 1962); D. Peak and A. Inomata, J. Math Phys. **10**, 1422 (1969); A. M. Arthurs, *Feynman Integrals for Classical Waves* (Wisconsin U.P., Madison, Wisc.); Proc. Cambridge Phil. Soc. **62**, 463 (1966).

[2] The present ϑ is identical with $\frac{1}{2}\theta$, which we used in our spinor review papers; the choice of the notation θ arises from Infeld and van der Waerden's convention of designating θ as the phase of the metric spinor; cf. W. L. Bade and H. Jehle, Rev. Mod. Phys. **25**, 714 (1953); W. C. Parke and H. Jehle, in *Boulder Lectures in Theoretical Physics, 1964*, edited by W. E. Britten *et al.* (Colorado U. P., Boulder, Colo., 1965), Vol. VII A.

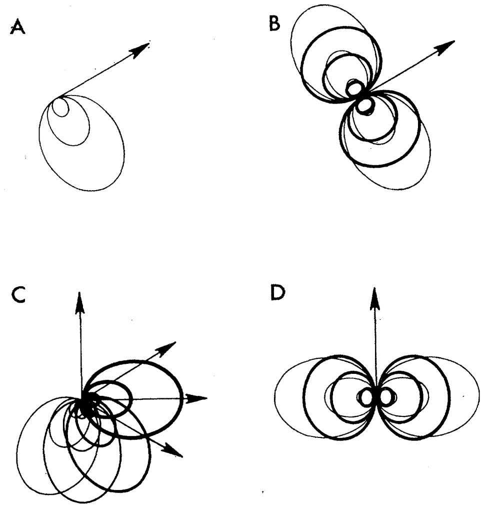

FIG. 1. Superposition of loopforms to result in a magnetic dipole field, (D) pointing in the $+z$ direction, corresponding to a pure quantum state $|\mu_z^+\rangle$. (A) Three generators of loopforms (a one-parameter manifold of different sizes σ), all sharing the same direction of flux orientation $\boldsymbol{\zeta}$ and azimuth α. (B) A sheaf of loopforms corresponding to these generators (A); a sheaf is a two parameter manifold (σ,α) of loopforms all belonging again to the same flux orientation $\boldsymbol{\zeta}$. Differences in thickness of the lines indicate closeness to the reader, to facilitate three-dimensional perception. (C) Generators of type (A), in (C), however, for a one-parameter manifold (ζ_z) of flux orientations $\boldsymbol{\zeta}$ and for a manifold of sizes σ. That is a two-parameter manifold (ζ_z,σ) although in the picture only four different flux orientations $\boldsymbol{\zeta}$, corresponding to four different values of the single parameter ζ_z, are shown. Differences in thickness here represent differences in the magnitudes of the complex probability amplitudes corresponding to the different flux orientations ζ_z, and have nothing to do with the facilitating three-dimensional perception; all these four generators are in the same plane. If the full two-parameter manifold of sheaves of all flux orientations $\boldsymbol{\zeta}$ [not only the generators (C)] and the full manifold of azimuths α is used to construct a field by superposition with the probability amplitudes indicated in (C), the resultant dipole field (D) results. The thicknesses of the lines is here again employed only as a help to three-dimensional visualization.

In such attempts to reconstruct the magnetic field of a muon or an electron from a superposition of alternative loopforms we shall make several assumptions, some of them only to simplify the calculations in a drastic way. We assume that the quantized flux loop, whatever form it adopts, is going through the position of the source lepton. Whereas in an ordinary Maxwell-Lorentz description of the source this is seen to be due to the circumstance that currents and charges are assumed to be confined to the source region, we simply assume here that the flux loop takes on "loopforms" which resemble the classical magnetic field lines of a dipole source. We may leave to a later time the question whether that assumption might be reduced to other more basic assumptions, e.g., in terms of a Lagrangian for loopform amplitudes.

The concept of a manifold of forms ("alternative flux loopforms") which one single elementary loop may adopt, as a representation of the magnetic field of a source lepton, is similar to the concept of a manifold of alternative path histories[1] which is taken to represent a quantum-mechanical path of a particle.

To these alternative loopforms we attach complex probability amplitudes. The difference of the present procedure from Feynman's assignment of probability amplitudes for the path histories is this: For the space-time motion of a particle, Feynman takes the action integral to determine the phase of the contributing amplitudes; for the stationary distribution of alternative loopforms in the present theory, we choose the phases of the amplitudes of a muon as random phased in some specified way and we choose the magnitudes of the complex amplitudes proportional to the square root of the corresponding share of magnetic flux they are to represent on the basis of a magnetic dipole in Maxwell-Lorentz's theory. We should take care not to confuse the phases of the amplitudes with the multivalued phase ϑ (of a field lepton's wave function) which introduces the quantized flux.

Our assumption that all magnetic flux is quantized flux means that the gauge-invariant four-potential $\mathfrak{A}_k$, Eq. (3.4), is set equal to zero. This means that, with the flux-defining singular vector potential $\mathbf{A}$, an electric potential V also arises if quantized flux lines are in motion and if it is assumed that they carry the phase field (i.e., the multivalued ϑ field) in the mean along with them in their motion (for topological reasons this is an obviously necessary assumption). In particular, if the flux loopforms representing a source lepton of magnetic moment $e\hbar/2mc$ all spin around their axes (cf. Figs. 1 and 2) with the angular velocity $2mc^2/\hbar$ of the *Zitterbewegung*, and carry the ϑ field along with the loopforms in their rotational motion, the Coulomb field results without the explicit introduction of an electric charge as a source. This result, so particularly important because it holds for muon and electron alike, is not too astonishing, however. The Dirac electron theory starts with the charge e of the electron and yields a magnetic moment $e\hbar/2mc$; in the present theory the reverse situation applies: Starting with the concept of flux loopforms, provided they reconstruct the magnetic moment $e\hbar/2mc$, we get the Coulomb field equivalent to an electric charge $\pm e$. This charge is $+e$ or $-e$ depending on whether the magnetic moment vector and the spin angular velocity vector of the loopforms are parallel or antiparallel to each other, as indeed it should be. (Charge and current follow from the fields by Maxwell-Lorentz equations.)

In this connection it is to be noted that already several decades ago Dirac remarked that e should be expressed in terms of $\hbar$, not vice versa, because a theory which explains the charge in terms of the square root of Planck's constant yields both signs, $+e$ and $-e$.

If it is attempted to parcel out the quantized flux $\Phi_q = hc/e$, Eq. (3.6), to the alternative loopforms of a *point* dipole source, one will get zero magnetic moment, and infinite magnetic field intensity at the location of that point dipole. The question therefore arises how to avoid these obvious difficulties without relinquishing the basically local character of quantum electrodynamics.

Although this is a formidable problem, it does seem to have an obvious solution. When we use a description of a source lepton in terms of a space-time distribution of loopforms, we necessarily have to attach those loopforms to the *mean* position of the lepton described as a *single* particle. The Pryce-Foldy-Wouthuysen transformation of the Dirac electron to a single-particle representation makes the particle's mean position an operator which is nonlocal of extent $\hbar/mc$ in ordinary position space.

We thereby assume structure for the lepton. A system of loopforms of the form of magnetic field lines of a *point* dipole source do not yet define a characteristic length. It is the length $\hbar/mc$, which characterizes the quasi-nonlocality of the source, which defines the structure of the source lepton.

There are several ways in which this "quasi-nonlocality" might be accounted for in a quantized loopform theory of a lepton. The simplest, and perhaps crudest, heuristic way is to handle the issue as if the source lepton was an extended source of extension $\approx\hbar/mc$ in ordinary position space. This is the approach taken in the present paper.

We may then raise the question: What distribution of the complex probability amplitudes over the space-time configurations of loopforms of quantized flux Φ_q leads to a correct reconstruction of the magnetic, and thereby also electric, field of the source lepton (the title "relationship of flux quantization to charge quantization" refers to that), a field whose electromagnetic energy is mc^2 and electromagnetic angular momentum is $\hbar/2$? This energy then may determine the frequency of the probability amplitude wave.

It is seen in Sec. X that such reconstruction is possible provided $e^2/\hbar c$ is of the right order of magnitude, i.e., $\approx 1/137$. No accurate figures of $e^2/\hbar c$ can yet be calculated because the definition of manifold of loopforms (bundled together into a discreet set of bundles) is only approximately possible with the crude assumptions about quasi-nonlocality used in the present paper.

It should be said that it is one part of the story that an electric Coulomb field appears simply as a consequence of a Bohr (or muon) magneton field and the electron (or muon) spinning frequency. It is then a second part that the reconstruction of these fields and of mc^2 and $\hbar/2$ from quantized flux $\Phi_q=eh/c$ leads to a numerical estimate of the electromagnetic coupling constant.

The relation of muon to electron may be interpreted as random phasedness to phase correlatedness (coherence) of the space-time distribution of loopform amplitudes, corresponding to a spinning of muon loopforms with high angular phase velocity versus a spinning of electron loopforms with slow angular velocity corresponding to a group velocity of muon waves. At a particular region in space, these motions are represented by a high fundamental frequency of muons and a low frequency of electrons corresponding to a beat frequency of muon waves.

We are trying to illustrate this theory in terms of simple manifolds of space-time forms of quantized flux loops. These picturesque aspects have as their purpose only the formulation of a theory in the simplest possible terms. A sophisticated theory of space-time manifolds of "flux loopforms" may involve deeper and more difficult mathematical tools than those used here. We shall also leave it to a later investigation to find out what general equations may govern the (statistical) distribution of the manifold of loopforms over space and time.

In Appendix II we propose an interpretation of mesons and baryons in terms of linked quantized flux loops. According to that view, a quark exhibits its properties only if interlinked with an other quark. The quantum numbers of particle physics seem to relate directly to topological and other properties of linked loops and their probability amplitude distributions. Strong and weak interactions might be qualitatively understood in terms of interactions of quantized flux loops.

It may be appropriate to enumerate the fields we deal with. We started with a wave-mechanical ψ field (I) (of a field lepton or field particle) whose phase ϑ, because of its being single valued only modulo 2π, defines quantized flux loops (cf. Sec. III). This ψ field is considered a semiclassical field which may possess these flux singularities with loopforms; these "alternatives" of fields and loop forms, like the alternatives of Feynman path histories, are then superimposed with complex probability amplitudes to build up a quantum state of the source lepton. All magnetic and electric fields (II) are built up from alternative (spinning) loopforms of these singularities of (I), and all physical quantities should be expressible in terms of that electromagnetic field (II). To define that electromagnetic field, probability amplitude distributions (III) are defined so as to imply electromagnetic fields which satisfy the Maxwell-Lorentz equation. The probability amplitude field (III) may be considered as "carrying" the electromagnetic field (II). As the lepton has spin $\frac{1}{2}$, we assume the carrier probability amplitude field (III) to correspond to spin $s=\frac{1}{2}$ eigenfunctions, of type Eq. (1.1).

We refer for further clarification to the end of Sec. VI. The complex probability amplitude distribution of the loopforms characterizes the internal configuration of the lepton; the fluctuations (*Zitterbewegung*) of this distribution have a structure which is represented by the discrete bundles of spinning loopforms.

This theory tries to give a single-field account of leptons, mesons, and baryons. It therefore does not lend itself to ambiguous assumptions (and it is much more conservative than monopole theories). It is to be compatible with the Maxwell-Lorentz and Dirac theories. It (1) derives the electric Coulomb field from quantized flux. It brings in new features (2) by defining structure and consequently the numbers 207 and 1/137 in the analysis of charged leptons, and (3) by defining the consequences of linkage of two or three loops in low-lying mesons or baryons, for an understanding of the classification of particles and of their interactions.

III. MAGNETIC FIELD FROM QUANTIZED FLUX

Quantization of flux is a concept implicit in a gauge-invariant formulation of electromagnetism.[3] As it is, the expression

$$[\partial_k - i(e/\hbar c)A_k]\psi, \quad A_k = (V, -\mathbf{A}), \quad e = +4.8\times 10^{-10} \text{ cgs}, \tag{3.1}$$

which enters a wave equation of a lepton, a gauge transformation

$$\psi = \psi' e^{+i\varphi}, \quad A_k = A_k' + (\hbar c/e)\partial_k \varphi, \quad k = 0, 1, 2, 3 \tag{3.2}$$

with a gauge variable φ which is a continuous function of time and space, results in a gauge-covariant description of the motion of a lepton. The gauge function φ, apart from being assumed to be continuous, is assumed to be a single-valued function of time and space. Without dispensing with the assumption of single valuedness of the ψ function, we may, instead of the single-valued phase φ, consider a phase ϑ which is single-valued modulo 2π only,[2]

$$\psi = \psi' e^{i\vartheta}. \tag{3.3}$$

This means that $\nabla\vartheta$, considered as a function of ordinary three-space, has singularity lines around which ϑ changes by a positive- or negative-integer multiple of 2π. Blatt called (3.3) appropriately a "pseudo-gauge transformation"; in this paper we also prefer not to call (3.3), with multivalued ϑ, a gauge transformation. Starting with a field lepton's wave function ψ' which is "singularity free" (i.e., whose ϑ' field$=0$) and which moves in a zero A_k' field, the gauge-invariant combination

$$A_k - (\hbar c/e)\partial_k\vartheta = \mathfrak{A}_k = A_k' - (\hbar c/e)\partial_k\vartheta' \tag{3.4}$$

is equal to zero all along, i.e.,

$$\mathfrak{A}_k = 0, \tag{3.5}$$

which implies the fields A_k explicitly written out in (4.2) and (4.3). In other words, from a solution of the wave equation for a field lepton with "nonsingular derivative" of ψ', in a zero field $A_k'=0$, we obtain another solution of the wave equation corresponding to a ψ field with a singularity line and a corresponding A_k field of a quantized flux line.[4]

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The value of the unit of quantized flux is given by

$$-2\pi=\oint\nabla\vartheta\cdot d\mathbf{r}=(e/\hbar c)\oint\sum_{k=1}^{3}A_k dr^k$$

$$=-(e/\hbar c)\int\int \operatorname{curl}\mathbf{A}\cdot d\mathbf{S}=-(e/\hbar c)\Phi_q,$$

$$\Phi_q=hc/e=4.1356\times10^{-7}\ \mathrm{G\ cm}^2. \tag{3.6}$$

F. London,[4] in his papers on superconductivity, recognized the importance of quantized flux which arises in a gauge-invariant theory. Flux quantization was still earlier discussed by Dirac[4] in connection with his hypothesis of magnetic monopoles. London's program on superconductivity has been continued and completed through the work of Onsager, Deaver, and Fairbank, Schafroth, Byers, and Yang, Ginzburg and Landau, Bardeen, Cooper, and Schrieffer, Froehlich, Bloch, Blatt, and many others.[4] Many ideas in this paper stem from F. London.

Further interest in the basic theoretical issues of flux quantization arose through the discussions of Aharonov and Bohm.[4] They raised the question of whether or not a split electron beam channeled around a perfectly localized magnetic field would, in its diffraction pattern, account for that magnetic field (a unit Φ_q causes diffraction with just one fringe unit displacement and might be said to be unobservable in this sense). By carefully considering the all-important role of the potential involved or, alternatively, by using Weisskopf's[4] consideration of gradually switching on the magnetic field, it can be seen that the answer is in the affirmative. Furry and Ramsey[4] and Feynman[4] have further clarified this issue (cf. also the end of Sec. III).

In the case of superconductivity, it is necessary to consider such cooperative effects as the pairing of electrons. In what follows, we shall discuss the quantized flux loop of one lepton. It is not the purpose of this paper to consider the many-body implications of flux quantization.

Our work also departs in two important ways from Dirac's treatment.

(1) In his consideration of singularities, he assumed the flux lines to start at one point and end at another, i.e., to originate from magnetic monopoles. This implies supplementation of electrodynamics by magnetic four-currents and four-potentials. Although Wentzel has shown how to quantize a theory which uses such concepts side by side with the electric four-currents and four-potentials, it seems that the complexity of such a theory might make it very difficult to accommodate and reconstruct the results of relativistic quantum electrodynamics—at any rate without arbitrary assumptions and without a loss of simplicity. Such argumentation is, however, subjective and should not distract us from the merits of monopole theories. In particular, we should point to the interesting monopole theories[4] of Schiff, Schwinger, Peshkin, Zwanziger, Biedenharn, Goldhaber, and many others. In our approach we work, however, without monopoles and consider "elementary loops," i.e., closed flux lines only. The Ruderman-Zwanziger and Barut theories of magnetic pole–anti-pole pairs may be closely related to the present theory.

(2) The kinds of admissible singularities in Dirac's theory are unnecessarily limited: As Dirac does not consider a statistical distribution of singularity lines over a continuous manifold of loopforms, he suggested that the (field lepton's) ψ function should by itself have no discontinuity. Thus, the singularity line of $\nabla\vartheta$ (around which ϑ changes by $\pm2\pi$) should coincide with a nodal line of that ψ function. In that case the factor $e^{i\vartheta}$ does not lead to an infinite derivative of ψ.

Rather than dealing with a particular location of the quantized flux line (the loop attached to a source lepton), we consider a statistical distribution of the forms of such a line. More specifically, we assume that a lepton which is the source of the magnetic field, affecting another lepton, will have one closed flux loop connected with it, which may take on alternative loopforms, forms similar to the Faraday lines of a dipole. Accordingly, each alternative loopform of that flux loop has, associated with it, an infinitesimally small probability amplitude. Thus the total wave function of the field lepton which is under the influence of the magnetic field of the source lepton is also, like that magnetic field, to be constructed by means of a superposition of the ψ functions associated with these various singularity loopforms, each with the aforementioned infinitesimally small probability amplitude. This is done in much the same way as alternative path histories of a particle are superposed to form the description of the quantum-mechanical path of a particle.

Because the amplitude associated with a singularity loop and the amplitude of the corresponding ψ function are infinitesimally small, we may include contributing ψ functions with singularity loops located anywhere—not just along nodal lines of those ψ functions. Though a crude sort of assumption, this may allow us to interpret a continuous magnetic field in terms of a superposition of flux lines in a reasonable manner.

Superposition of alternative loopforms of quantized flux by fractional amounts (with probability amplitudes) also disposes of the frequently raised question in regard to the observability of quantized flux. The effective flux of a lepton is, because of that superposition, only a fractional amount of the quantized flux Φ_q, cf. (9.16) and (9.17). With this qualifying statement in mind, one may consider (3.5) to imply that all magnetic flux is quantized.

The amplitudes associated with the various quantized flux loopforms will have, for the most part, complex phase factors. We emphasize that these should not be confused with the phase ϑ of the contributing ψ function that is weighted with that probability amplitude.

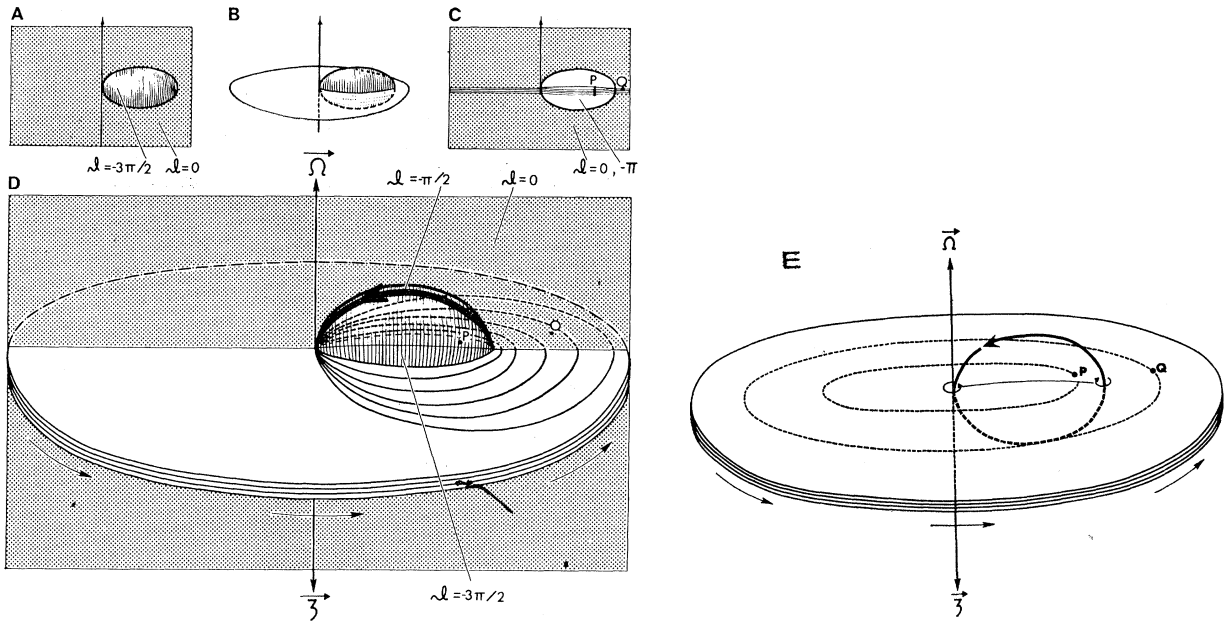

FIG. 2. (A)–(C) and (E) show, as an oval, one of the loopforms of the source lepton. (D) shows half of this loopform, as a heavy drawn line. The multivalued field ϑ is represented in (A)–(E) at a moment at which the loopform's plane (dotted) lies parallel to the picture plane. (As we represent a lepton of negative charge, the upwards-pointing spin axis $\mathbf{\Omega}$ is antiparallel to the downward "flux orientation" axis $\boldsymbol{\zeta}$.) The locus $\vartheta=0$, $0\pm2\pi$, $0\pm4\pi$, etc., may be the dotted part of that loopform plane, the locus $\vartheta=-\pi$, $-\pi\pm2\pi$, $-\pi\pm4\pi$, etc., the undotted part of that plane in (C), i.e., the part inside the loopform. The locus $\vartheta=-\frac{3}{2}\pi$, $-\frac{3}{2}\pi\pm2\pi$, $-\frac{3}{2}\pi\pm4\pi$, etc., may be the front part of the egg-shaped surface, and the locus $\vartheta=-\frac{1}{2}\pi$, $-\frac{1}{2}\pi\pm2\pi$, etc., the rear half of that eggshell, a slice of it appearing in (D). Along with the loopform, the ϑ field spins with the angular velocity $\mathbf{\Omega}$.

Let us consider two field points P and Q lying in the "equatorial" plane, shown in (B)–(D) and (E); this plane is perpendicular to $\boldsymbol{\zeta}$ or $\mathbf{\Omega}$. (D) shows not only the intersections $\vartheta=-\frac{3}{2}\pi$, etc. (solid lines), and $\vartheta=-\frac{1}{2}\pi$, etc. (dashed lines), of that eggshell with the equatorial plane, but also the corresponding intersections of the eggshells of five larger loopforms with that equatorial plane. We represent this equatorial plane as a multisheet (similar to a Riemannian surface). Four of this infinite sequence of sheets are shown in (C)–(E), together with a fifth sheet's start in the form of a slightly upward-bound cut line, a radial line between "aphelion" and "perihelion" of the loopform in (C) and (E). At the center end and at the outer end of this cut line the directions of $\nabla\vartheta$ are given by the small circular arrows, (E). On this multisheet surface we may plot ϑ as a single-valued function. While this multisheet spins, together with the flux loopforms and the attached ϑ function, the points P and Q, both fixed in space, trace out two paths, shown as dotted lines in (E) and as dots in (C). While the point Q, located outside the flux loopform, traces out a circular path on one sheet [e.g., a dotted line in (E), a single dot in (C)], the point P, linked with the loopform, traces out a path, relative to the multisheet, a path which is similar to that of a car climbing out of a basement multilevel garage [e.g., a dotted line in (E), five dots in (C)]. The climb from one sheet to the next, to the next, etc., represents that loopforms' contribution $\partial\vartheta/\partial t$ toward the electric potential.

These pictures represent the space-time behavior of $e^{i\vartheta}$, which is a factor of the wave function of any field lepton which is subject to the source lepton's electromagnetic field.

IV. ELECTRIC FIELD FROM QUANTIZED FLUX

Let us consider the above-mentioned magnetic field curl $\mathbf{A}$ which may result from the multivaluedness of ϑ around a line (see Figs. 2 and 3). This field would be associated with a quantized flux line having a flux of the magnitude Φ_q [Eq. (3.6)]. That is, a flux line

$$\mathfrak{A}_k=0 \tag{4.1}$$

characterizes a field $A_k=(V,\,-\mathbf{A})$,

$$\mathbf{A}=-(\hbar c/e)\nabla\vartheta\,, \tag{4.2}$$

$$V=+(\hbar c/e)\partial\vartheta/\partial ct\,. \tag{4.3}$$

It has been assumed that all singularity lines are closed, i.e., they form loops. The direction of the loopform lines at the location of the source lepton may be called "flux orientation" (formerly named "origin orientation") and be denoted by the unit vector $\boldsymbol{\zeta}$.

The model built from alternative loopforms allows us both to construct the magnetic field of a charged lepton, from a space-time distribution of alternative flux loopforms, and to describe its electric field without introducing explicitly the charge as its source.

In fact, we shall see that if the distribution of alternative loopforms is chosen so that the correct magnetic moment

$$\mu_{\text{eff}}=e\hbar/2mc \tag{4.4}$$

results, and if this distribution of loopforms is assumed to spin about their flux orientations $\boldsymbol{\zeta}$ with angular velocity

$$\Omega=2mc^2/\hbar \tag{4.5}$$

(cf. Sec. V), then the potential V approximates the Coulomb potential. [In (4.4) and (4.5), m is the mass of the muon or of the electron.] In later sections, we shall discuss how the magnetic moment μ_{eff} [Eq. (4.4)] may be obtained from quantized flux Φ_q [(3.6)].

The way in which the electric field arises from the spinning flux loopforms may be seen by considering a very simple model in which it is assumed (I) that the flux orientations $\boldsymbol{\zeta}$ of the alternative loopforms are all the same, viz., that of the magnetic dipole vector $\boldsymbol{\mu}$ (taken to be in the $+z$ direction), and (II) that the source lepton is localizable at a point in position space. The effective magnetic field (defined as being related to the Bohr- or muon-magneton μ_{eff} by the Maxwell-Lorentz equations)

$$\mathbf{B}_{\mathrm{eff}}=3r^{-5}(\boldsymbol{\mu}_{\mathrm{eff}}\cdot\mathbf{r})\mathbf{r}-r^{-3}\boldsymbol{\mu}_{\mathrm{eff}}, \tag{4.6}$$

i.e.,

$$\mathbf{B}_{\mathrm{eff}}=-r^{-3}\boldsymbol{\mu}_{\mathrm{eff}} \tag{4.7}$$

in the equatorial plane, gives rise to a magnetic flux linked with a field point P located a distance r from the dipole,

$$\int_r^\infty B_{\mathrm{eff}}2\pi r dr=(e\hbar/2mc)2\pi/r. \tag{4.8}$$

This corresponds to the fraction

$$F=(e^2/2mc^2)/r \tag{4.9}$$

of the quantized flux (3.6).

In order to avoid topological complications associated with the surfaces $\vartheta=\mathrm{const}$ as the flux loop spins, we assume that the flux loop carries the "field" of the phase ϑ with it. For simplicity, we choose ϑ to be time independent in the rotating coordinate system. When the flux loop makes a complete turn, the phase at P changes by 2π if that loopform extends beyond P; it then contributes to V according to (4.3). The ϑ, multivalued in ordinary three-space, may be represented as a single-valued function on a kind of Riemannian surface lying in the equatorial plane with branch points at the origin and at the point where the loopform again passes through the equatorial plane. There will be infinitely many sheets because ϑ acquires an extra 2π with each revolution (cf. Fig. 2).

Using the angular velocity (4.5), we obtain

$$\begin{aligned}(e/\hbar c)V_{\mathrm{eff}}&=(\partial\vartheta/\partial ct)_{\mathrm{eff}}=(1/c)2\pi F\Omega/2\pi\\&=(1/c)(e^2/2mc^2r)(2mc^2/\hbar)=(e^2/\hbar c)/r.\end{aligned} \tag{4.10}$$

Because of the assumption (I), the electric potential, calculated crudely in this way, is not even spherically symmetrical. Further, because of the assumption (II), an infinite total magnetic flux is implied if the magnetic moment is finite, μ_{eff} [(Eq. (4.4)], i.e., if the potential is V_{eff} [Eq. (4.10)]. Because of the cancellation of m in (4.10), the same Coulomb field results for muon and electron.

We would like to replace the oversimplified assumption (I) by the assumption suggested earlier (cf. Fig. 1), i.e., that the manifold of loopforms from which we construct the magnetic dipole field comprises not only one

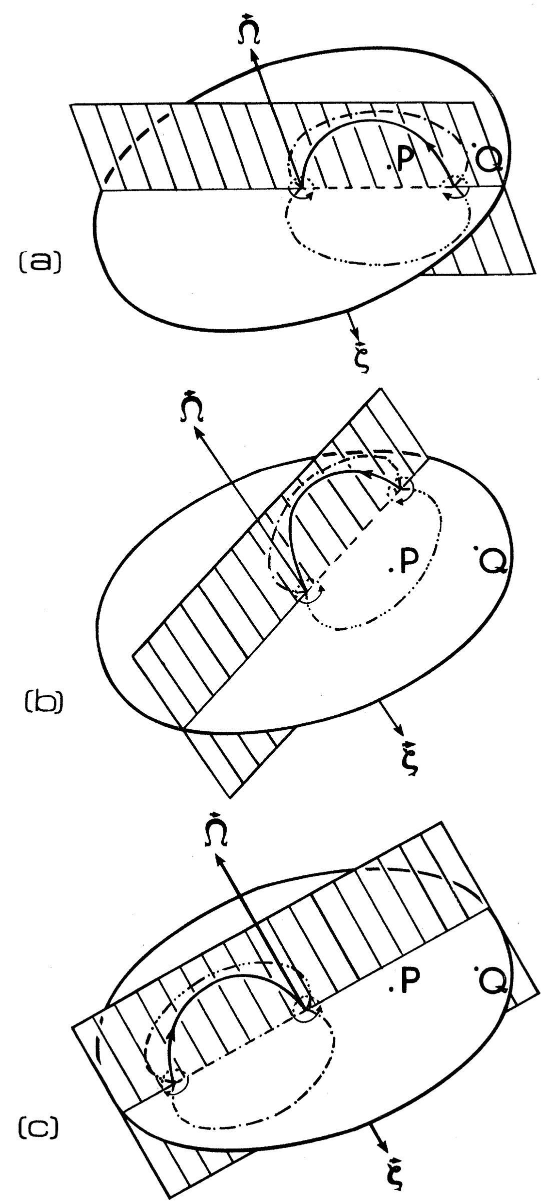

FIG. 3. The intersections of the surfaces $\vartheta=\mathrm{const}$ with the equatorial plane. The plane $\vartheta=0$ (or $\vartheta=0\pm2\pi$, $\pm4\pi$, etc.) is shaded; its intersection (—— ——) is a straight line, and the continuation of this line across the interior of the flux loop, $\vartheta=-2\pi/2$ or $\vartheta=-2\pi/2\pm2\pi$, $\pm4\pi$, etc., has the intersection (—·—·—·—). The surfaces $\vartheta=-\frac{3}{2}\pi$ and $\vartheta=-\frac{1}{2}\pi$ (or $\vartheta=-\frac{3}{2}\pi\pm2\pi$, $\pm4\pi$, ..., and $\vartheta=-\frac{1}{2}\pi\pm2\pi$, $\pm4\pi$, etc.), instead of being egg-shaped as in Figs. 2, happen to be imagined here to be of the form of an apple or tomato surface. (A gauge transformation permits the changeover from one form of such surface to another form.) Their intersections with the equatorial plane are indicated by curves (—···—···—) and (—·—·—·—), respectively. (A)–(C) show the spinning flux loopform with its associated ϑ field at three consecutive moments to illustrate the type of change of ϑ in the course of time. For a point P, ϑ decreases by 2π every revolution; for a point Q, ϑ oscillates back and forth about $\vartheta=0$, to perhaps $\vartheta=-\frac{1}{2}\pi$ on the one side and $\vartheta=-\frac{3}{2}\pi+2\pi=+\frac{1}{2}\pi$ on the other side, staying always on the same "Riemannian" sheet. It may also be noted here already that the expectation value of $\partial\mathbf{A}/\partial ct$, i.e., $(\partial/\partial t)(-\nabla\vartheta)$, has a zero time average for both P and Q. In contrast, the expectation value of $\nabla(\partial\vartheta/\partial ct)$ is not zero, but depends on the spatial distribution of loopforms [cf. Eqs. (5.13), (8.18), and (8.19)].

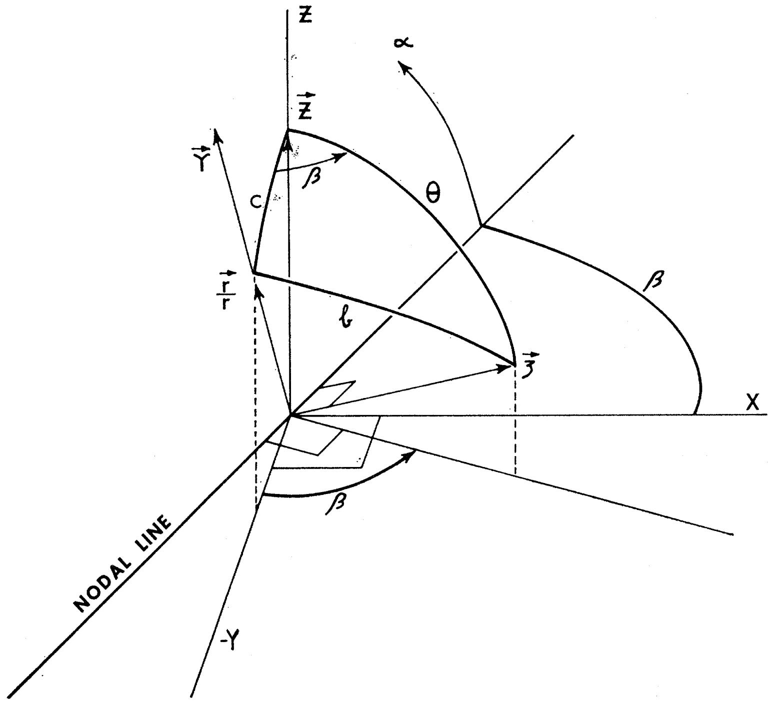

FIG. 4. Euler angles β, θ, α to characterize the position of a sheaf of loopforms of flux orientation $\boldsymbol{\zeta}$, in relation to a field point $P(\mathbf{r})$, which for convenience is laid in the $(-Y, Z)$ plane.

sheaf of loopforms of flux orientation $\boldsymbol{\zeta}$ in the $+z$ direction (two-parametric manifold of size σ and azimuth α), but a manifold of sheaves of different flux orientations $\boldsymbol{\zeta}$ (a manifold of altogether four parameters), with $\boldsymbol{\zeta}$ predominantly in the $+z$ direction. We consider $|\boldsymbol{\zeta})$, i.e., the distribution of amplitudes, which we assume to be similar to that of the projection of a spin state $|\mu_\zeta\rangle$ onto a state $|\mu_z{}^+\rangle$, with $\langle\mu_z{}^+|\mu_\zeta\rangle$ being proportional to $(1+\zeta_z)^{1/2}$; ζ_z is the projection of the unit vector $\boldsymbol{\zeta}$ onto the $+z$ axis (cf. Sec. VIII).

So that we may proceed with the calculation of the electric field in this general case, we consider first the classical expression for the flux associated with an arbitrary field point $P(\mathbf{r})$ at a distance $\mathbf{r}$ from the source lepton, arising from *one* of the sheaves of loopforms of given flux orientation $\boldsymbol{\zeta}$, a sheaf of magnetic moment

$$\boldsymbol{\mu}_\zeta=\mu_\zeta\boldsymbol{\zeta}, \tag{4.11}$$

$$\int_{\mathrm{r}}^{\infty}\int \mathbf{B}\cdot d\mathbf{S}=\oint_{\mathrm{r}} \mathbf{A}\cdot d\mathbf{r}=-\mu_\zeta(2\pi/r)\sin^2(\mathbf{r},\boldsymbol{\zeta})\,. \tag{4.12}$$

The contour integral is to be taken over a circular path lying in a plane perpendicular to $\boldsymbol{\zeta}$ and passing through $P(\mathbf{r})$. This sheaf of loopforms of magnetic moment $\boldsymbol{\mu}_\zeta$ is assumed to spin with the angular velocity

$$\boldsymbol{\Omega}_\zeta=\pm(2mc^2/\hbar)\boldsymbol{\zeta} \tag{4.13}$$

about the flux orientation of the sheaf's magnetic field, every sheaf having the same angular velocity Ω, and to carry the ϑ field with it as it spins. With each revolution, ϑ would increase (or decrease) by 2π at the field point if the total flux Φ_q were linked with $P(\mathbf{r})$. In fact, only the fraction

$$\int_{\mathrm{r}}^{\infty}\int \mathbf{B}\cdot d\mathbf{S}/\Phi_q \tag{4.14}$$

is linked with the point $P(\mathbf{r})$ and thus contributes towards $\partial\vartheta/\partial t$. The average over a period then gives

$$\begin{aligned}(\partial_0\vartheta)_{\mathrm{eff}}&\equiv\langle\partial_0\vartheta\rangle_{\mathrm{av}}\\ &=\mp(2\pi/c)\left(\int_{\mathrm{r}}^{\infty}\int \mathbf{B}\cdot d\mathbf{S}\Big/\Phi_q\right)(\Omega/2\pi)\\ &=(\pm2\pi/c)[\mu_\zeta(2\pi/r)\sin^2(\mathbf{r},\boldsymbol{\zeta})/(2\pi\hbar c/e)]\\ &\qquad\times(2mc^2/2\pi\hbar)\,.\end{aligned} \tag{4.15}$$

It is necessary to average this expression over all flux orientations. This might tentatively be done by replacing the sheaf's μ_ζ by the Bohr or muon magneton (4.4), respectively, and by weighting the flux orientations $\boldsymbol{\zeta}$ with the probabilities $(1+\zeta_z)$ (cf. Sec. VIII).

In order to calculate such an average of $\sin^2(\mathbf{r},\boldsymbol{\zeta})$, cf. Fig. 4, let us consider the spherical triangle spanned out by unit vectors in the direction of the z axis, of $\mathbf{r}$ and of $\boldsymbol{\zeta}$, and name the angle $(\mathbf{r},\boldsymbol{\zeta})=b$, the angle $(\boldsymbol{\zeta},\mathbf{z})=\theta$, $\cos\theta=\zeta_z$, and the angle $(\mathbf{z},\mathbf{r})=c$, $\cos c=r_z/|\mathbf{r}|$. The angle β between the $\mathbf{r}$, $\mathbf{z}$ plane and the $\boldsymbol{\zeta}$, $\mathbf{z}$ plane is important because the probability distribution of $\boldsymbol{\zeta}$ (cf. Sec. VIII) is given by

$$-(1+\zeta_z)(d\zeta_z/2)(d\beta/2\pi)(d\alpha/2\pi). \quad (4.16)$$

Using the notation (8.20) and (8.21) with the choice of the $-y$, z plane so as to contain $\mathbf{r}$, and the new z' axis which is placed in the $\boldsymbol{\zeta}$ direction, β is the Euler angle $\arccos[-\zeta_y(1-\zeta_z^2)^{-1/2}]$; Euler angles (in Goldstein notation) are β, θ, α. With the cosine law

$$\cos b=\cos\theta\cos c+\sin\theta\sin c\cos\beta, \quad (4.17)$$

we obtain

$$\begin{aligned}\langle\sin^2(\mathbf{r},\boldsymbol{\zeta})\rangle_{\text{av}}&=\langle 1-\cos^2 b\rangle_{\text{av}}\\&=\int_0^{2\pi}\int_0^{2\pi}\int_{-1}^{+1}(1-\cos^2\theta\cos^2 c\\&\quad-2\cos\theta\cos c\sin\theta\sin c\cos\beta-\sin^2\theta\sin^2 c\cos^2\beta)\\&\qquad\times(1+\zeta_z)(d\zeta_z/2)(d\beta/2\pi)(d\alpha/2\pi)\\&=\int_{-1}^{+1}\{1-\cos^2\theta\cos^2 c-\tfrac{1}{2}(1-\cos^2\theta)(1-\cos^2 c)\}\\&\qquad\times(1+\zeta_z)(d\zeta_z/2)\\&=\int_{-1}^{+1}\{\tfrac{1}{2}+\tfrac{1}{2}\zeta_z^2+(\tfrac{1}{2}-\tfrac{3}{2}\zeta_z^2)r_z^2/r^2\}d\zeta_z/2\\&=\tfrac{1}{4}[\zeta_z+\tfrac{1}{3}\zeta_z^3+(\zeta_z-3\times\tfrac{1}{3}\zeta_z^3)r_z^2/r^2]_{-1}^{+1}=\tfrac{2}{3}\end{aligned} \quad (4.18)$$

(odd powers of ζ_z in the integrand give no contribution). Accordingly,

$$\begin{aligned}V_{\text{eff}}\equiv\langle V\rangle_{\text{av}}&=(\hbar c/e)\langle\partial_0\vartheta\rangle_{\text{av}}\\&=(\hbar c/e)(\pm 2\pi/c)(e\hbar/2mc)(2\pi/r)\tfrac{2}{3}(e/2\pi\hbar c)\\&\qquad\times(2mc^2/2\pi\hbar)=\pm\tfrac{2}{3}e/r.\end{aligned} \quad (4.19)$$

We thus obtain a spherically symmetric potential which differs from the Coulomb potential by the factor $\frac{2}{3}$.

The present evaluation of (4.15) is done by means of a simple classical magnetic field picture. We proceed now to evaluate this more accurately on the basis of the statistical description of the fields, given in Sec. IX. The reader may defer studying the details of the remainder of Sec. IV, but we report it here because it would be out of place in Sec. IX.

The second factor on the right-hand side of (4.15) should then be understood to mean

$$\left\langle\int_r^\infty\int\mathbf{B}\cdot d\mathbf{S}\Big/\Phi\right\rangle_{\text{av}}. \quad (4.20)$$

The expression which is to be averaged is that fraction of the flux Φ of a bundle of sheaves which is linked with the point $P(\mathbf{r})$ (cf. Figs. 2 and 3). This is the correct expression because for a point for which this fraction is unity (a point on the core surface), ϑ changes by 2π per each spinning period. In order to perform the averaging over the manifold of bundles of sheaves of loopforms, we remember that the contributions towards $\partial_0\vartheta$ are additively composed from contributions of bundles of sheaves. Accordingly, let us consider the contributions from different bundles (in the sense of Sec. IX) as being superposed with probabilities

$$(1+\zeta_z)(d\zeta_z/2)(d\beta/2\pi)(d\alpha/2\pi).$$

If, by (9.12) and (9.16),

$$\begin{aligned}\Phi&=\text{magnetic flux of the bundle}=((\lambda)|\Phi_q|(\lambda))\\&=(2/N)\Phi_q=3\langle\mu_z^+|\Phi_z|\mu_z^+\rangle_{\text{red}}=3\Phi_{\text{eff}}\end{aligned} \quad (4.21)$$

and

$$\begin{aligned}\mu_\zeta&=(2/N)\mu_q=3\langle\mu_z^+|\mu_z|\mu_z^+\rangle_{\text{red}}\\&=3\mu_{\text{eff}}\end{aligned}$$

then that bundle's linkage with a point $P(\mathbf{r})$ is, by (4.12) and (4.21),

$$\begin{aligned}\int_r^\infty\int\mathbf{B}\cdot d\mathbf{S}&=\text{partial bundle flux}\\&=(2/N)\mu_q(2\pi/r)\sin^2(\mathbf{r},\boldsymbol{\zeta})\\&=3\langle\mu_z^+|\mu_z|\mu_z^+\rangle_{\text{red}}(2\pi/r)\sin^2(\mathbf{r},\boldsymbol{\zeta})\\&=3\mu_{\text{eff}}(2\pi/r)\sin^2(\mathbf{r},\boldsymbol{\zeta})\end{aligned} \quad (4.22)$$

if each bundle of sheaves spins about its flux orientation axis (magnetic moment axis) ($\boldsymbol{\zeta}$). The ratio

$$\int_r^\infty\int\mathbf{B}\cdot d\mathbf{S}/\Phi=3\mu_{\text{eff}}(2\pi/r)\sin^2(\mathbf{r},\boldsymbol{\zeta})(N/2\Phi_q) \quad (4.23)$$

is to be averaged over the bundles. Because of the random phasedness, the factor N drops out and the averaging over β, ζ_z is calculated in (4.18). For the potential, we then get

$$\begin{aligned}\langle\partial_0\vartheta\rangle_{\text{av}}&=\pm(2\pi/c)[\tfrac{3}{2}(e\hbar/2mc)(2\pi/r)\langle\sin^2(\mathbf{r},\boldsymbol{\zeta})\rangle_{\text{av}}/\\&\qquad(2\pi\hbar c/e)](2mc^2/2\pi\hbar),\end{aligned} \quad (4.24)$$

$$V_{\text{eff}}=\langle V\rangle_{\text{av}}=\pm e/r. \quad (4.25)$$

Charge and Magnetic Moment

Parallelism of magnetic moment and spin implies, in this theory, and electric field equivalent to a positive charge; antiparallelism implies a field of a negative charge, as indeed it should be.

V. QUASI-NONLOCALITY OF SOURCE

With a point source, two difficulties arise: (a) While the singularities associated with the quantized flux loopforms may be permissible as long as the loopforms are continuously distributed over space, an essential singularity is implied at the location of the point dipole if this is a point through which all loopforms pass. (b) The ratio of the total magnetic flux of the distribution of loopforms to the magnetic dipole moment is ∞ to 1,

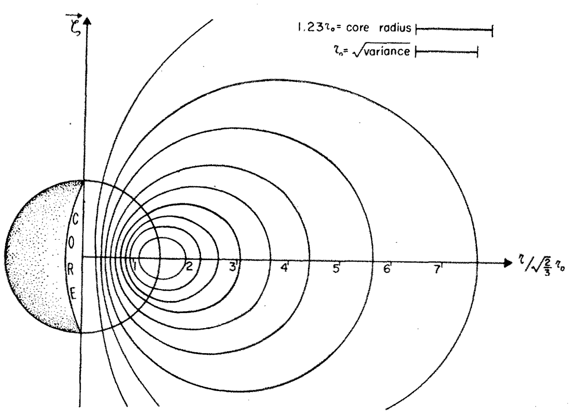

FIG. 5. A source lepton described in terms of alternative loopforms which a quantized magnetic flux loop may adopt. These flux loopforms may resemble the field lines of a magnetic dipole. The concept of a dipole source to which flux loopforms are attached implies, on the one hand, that this source be considered as a single particle. On the other hand, the source should be considered a Dirac particle, which means that a single-particle picture is to be obtained by the nonlocal Pryce-Foldy-Wouthuysen transformation, i.e., that it is nonlocal when viewed in ordinary position space. In a symbolical way, that is expressed in this figure by picturing the source as an extended source, even though such nonlocality, defined by the Pryce-Foldy-Wouthuysen transformation, cannot be truly represented by drawing these field lines in position space. We may call such a source "quasi-nonlocal" because the theory is compatible with local quantum electrodynamics. Accordingly, the extent $\hbar/mc$ of quasi-nonlocality is to be considered a matter of the space-time picture of flux-loopform interpretation, and not a measurable deviation from local quantum electrodynamics.

The spacing of the field lines in Fig. 5 is such that their density is proportional to rB, i.e., the lines represent toroidal surfaces which subdivide the total flux into ten equal parts.

The "first shell," as defined in Figs. 9 and 10, is characterized by $r_{\text{core}} \geqslant r \geqslant 3r_{\text{core}}$, where r shows the size (aphelion distance) of the loopform. The first shell carries about 60% of the total flux.

owing to the infinitesimal smallness of almost all loopforms.

In spite of these difficulties, we obviously do not want to get in conflict with quantum electrodynamics, which is based on the concept of local interaction. We rather solve the dilemma which would arise with an unsophisticated use of a point-source model in the present theory by analyzing the concept of position, remembering the Pryce-Foldy-Wouthuysen (PFW) transformation.[5]

In Sec. IV we calculated the correct isotropic electric Coulomb field as owing to a muon (or Bohr) magneton field spinning with angular velocity $2mc^2/\hbar$. Below we shall give a justification for that choice $\Omega = 2mc^2/\hbar$. We also assumed in that calculation that each loopform spins with $\mathbf{\Omega}$ about its origin orientation axis (i.e., the coreflux axis); for a positively charged lepton, $\mathbf{\Omega}$ and $\boldsymbol{\zeta}$ are parallel, for a negatively charged, antiparallel (see Fig. 3). This alternative essentially characterizes the particle-antiparticle alternative, and may correspond to positive-negative frequencies of the probability amplitude distribution (cf. end of Sec. VIII).

We ought to remember that the picture of a spinning loop in the present theory stands on the same level as the picture of an orbiting electron in a particular stationary orbit in the Bohr atom. The probability amplitude distribution of loopforms, corresponding to (4.16), is smooth in ζ_z, β, and α, and has a time dependence proportional to $e^{i\omega t}$ with one (positive) ω for these loopforms of parallel $\mathbf{\Omega}$, $\boldsymbol{\zeta}$. Because of that, the spinning frequency $\mathbf{\Omega}$ of the statistical distribution of loopform amplitudes is as unobservable a quantity as the orbiting frequency of an electron in one particular Bohr orbit. That concept of spinning of loopforms, which we used in Sec. IV, thus corresponds to a two-component "single-particle" picture in the Foldy-Wouthuysen representation, because this spinning loopform manifold had no admixture from antiparticle contributions (i.e., antiparallel $\mathbf{\Omega}$, $\boldsymbol{\zeta}$). The loopforms may be local in mean position (i.e., the position obtained by a Pryce-Foldy-Wouthuysen transformation from ordinary position). The spin expresses itself in the electric field which it produces from the magnetic field.

The spinning frequency Ω becomes an observable frequency (which defines also the mass) when an actual lepton, localized in some way, is considered, i.e., when

[5] Papers relating to nonlocal aspects of particles: M. H. L. Pryce, Proc. Roy. Soc. (London) **A195**, 62 (1948); **A150**, 166 (1935); L. L. Foldy and S. A. Wouthuysen, Phys. Rev. **78**, 29 (1950); L. L. Foldy, *ibid.* **102**, 568 (1956); **122**, 275 (1961); in *Quantum Theory*, edited by D. R. Bates (Academic, New York, 1962), Vol. III; S. Tani, Progr. Theoret. Phys. (Kyoto) **6**, 267 (1951); K. M. Case, Phys. Rev. **95**, 1323 (1954); T. D. Newton and E. P. Wigner, Rev. Mod. Phys. **21**, 400 (1949); R. J. Finkelstein, Phys. Rev. **75**, 1079 (1949); E. P. Wigner, Ann. Math. **40**, 149 (1939); H. Yukawa, Phys. Rev. **91**, 415 (1953); H. Yukawa and S. Tomonaga, in *Proceedings of the International Conference on Theoretical Physics, Kyoto and Tokyo, September, 1953* (Science Council of Japan, Tokyo, 1954); P. Kristensen and C. Møller, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Medd. **27**, 7 (1952); A. Sankaranarayanan and R. H. Good, Jr., Phys. Rev. **140**, B509 (1965); A. Pais and G. E. Uhlenbeck, *ibid.* **79**, 145 (1950); G. N. Fleming, *ibid.* **137**, B188 (1965); **139**, B963 (1965); R. Ingraham, Nuovo Cimento **24**, 1117 (1962); **27**, 303 (1963); **32**, 323 (1964); **33**, 246 (1964); J. D. Landau, in *N. Bohr and the Development of Physics*, edited by W. Pauli (McGraw-Hill, New York, 1955), p. 52; K. Huang, Am. J. Phys. **20**, 479 (1952); W. Gordon, Z. Physik **50**, 630 (1927); A. O. Barut, H. Kleinert, and S. Malin, Nuovo Cimento **58A**, 835 (1968); Phys. Rev. **156**, 1546 (1967); A. O. Barut and S. Malin, Rev. Mod. Phys. **40**, 632 (1968); S. S. Schweber, *Relativistic Quantum Field Theory* (Harper and Row, New York, 1961); W. C. Parke, thesis, George Washington University, 1967 (unpublished; available from University Microfilms, Ann Arbor, Mich.); H. Jehle and W. C. Parke, Phys. Rev. **137**, B760 (1965); J. J. Giambiagi, Nuovo Cimento **16**, 202 (1960); C. G. Bollini and J. J. Giambiagi, *ibid.* **21**, 107 (1961); R. H. Good, Jr., and M. E. Rose, *ibid.* **24**, 864 (1962); H. Mendlowitz, Phys. Rev. **102**, 528 (1956); Am. J. Phys. **26**, 17 (1958); H. Mendlowitz and K. Case, Phys. Rev. **97**, 33 (1955); V. Bargmann and E. P. Wigner, PNAS **34**, 211 (1948); A. S. Wightman, Rev. Mod. Phys. **34**, 845 (1962); R. Ingraham, in *Lectures in Theoretical Physics*, edited by Wesley E. Brittin, W. B. Downs, and Joanne Downs (Interscience, New York, 1964), Vol. VI, p. 112; E. Schrödinger, Preuss. Akad. Wiss. **24**, 418 (1930); J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill, New York, 1964); C. Møller, *Relativity Theory* (Oxford U.P., London, 1957), Chap. 64; K. Bardacki and R. Acharya, Nuovo Cimento **21**, 820 (1961); H. VanDam and E. Wigner, Phys. Rev. **138**, B1576 (1965); A. S. Wightman and S. S. Schweber, *ibid.* **98**, 812 (1955); R. Ingraham, Nuovo Cimento **32**, 182 (1964); E. M. Henley and W. Thirring, *Elementary Quantum Field Theory* (McGraw-Hill, New York, 1962); R. J. Finkelstein, J. Math. Phys. **1**, 440 (1960); F. A. Kaempffer, *Concepts in Quantum Mechanics* (Academic, New York, 1965), Sec. 20.

negative-frequency terms are admixed to an essentially positive-frequency probability amplitude distribution, as required for a Dirac lepton. This brings in the *Zitterbewegung* which manifests itself in a diverse set of phenomena; the terms bilinear in the probability amplitudes now show this frequency which implies both a spinning angular velocity and fluctuation terms.

As the transformation from PFW representation to ordinary Dirac representation is nonlocal, the source becomes quasi-nonlocal in ordinary position. This was first recognized by Schrödinger in his analysis of the *Zitterbewegung*. We use the word "quasi-nonlocality" to avoid the impression that we would have to deal with a truly nonlocal theory (cf. the analysis by Parke,[5] 1967).

In this quasi-nonlocal description the source lepton might be represented by loopforms in some way approximating the magnetic field lines corresponding to a Gaussian distribution of magnetization (cf. Fig. 5) with a root-mean-square deviation of the order of r_0. Such an approximation of quasi-nonlocality by an "extended source" is very crude indeed, and certainly such approximation should not be misunderstood to mean that we actually postulate an extended source corresponding to a truly nonlocal picture. The *Zitterbewegung* has been shown by Huang to be interpretable as a spinning motion with angular velocity $\Omega=2mc^2/\hbar$ in ordinary position. We may accordingly not be amiss in having postulated this spinning frequency in Eq. (4.5). It was also shown there that the spin angular momentum is characterized by the expectation value (of the component 3 in the direction of spin) for

$$\langle \mathbf{r}\times\dot{\mathbf{r}}\rangle_3=(\hbar/mc)c(1-\cos\Omega t)\,,$$

corresponding to a velocity c and radius $\hbar/mc$ of the *Zitterbewegung* amplitude. Anticipating the Gaussian model of Sec. V C [after Eq. (5.10c) or Eq. (7.1)], Figs. 5 and 6, which sets the core radius as $r=1.23r_0$, we may assume that this core radius value of r corresponds to the *Zitterbewegung* amplitude, which means

$$r_0\approx r/1.23\approx 0.815(\hbar/mc)\,. \tag{5.1a}$$

If, instead of this value, we assume

$$r_0\approx 0.73\hbar/mc\,, \tag{5.1b}$$

we arrive with Eq. (10.3) at the correct numerical value of the electromagnetic interaction constant 1/137. Sometimes, when this factor 0.73 is of no importance, we use the simple approximation $r_0\approx\hbar/mc$.

To be more explicit, it is not only the position of the source but also the location of an entire flux loopform (when viewed in ordinary position representation) which is to be considered as nonlocal. We may illustrate this circumstance in Fig. 10, which shows a loopform; but because of the quasi-nonlocality, it is blown up into a "tube" in the same way as the position of the source is blown up into a "core."

It may finally be remarked that the quasi-nonlocal

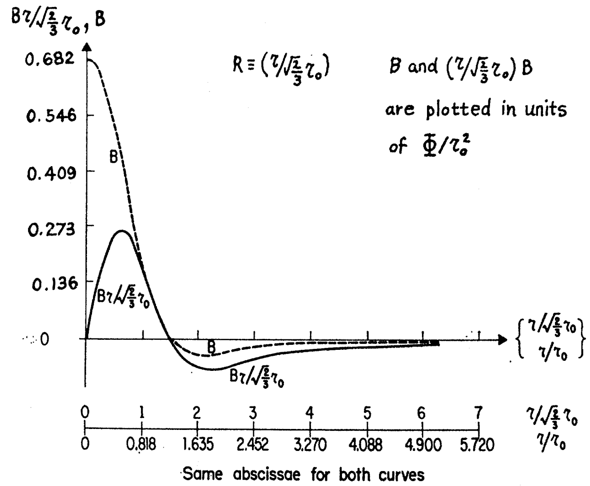

FIG. 6. Assuming the extended source to correspond to a spherically symmetric [cf. Eq. (5.7a] Gaussian distribution of magnetization, we may calculate the magnetic field B (dashed line) in the equatorial plane and the spherically symmetric electric field E (solid line), indicated by RB.

Please note that the size of the core, $r=1.23r_0$, even though shown as differing in size, should be considered of equal size in all illustrations, Figs. 5–16.

description may overcome the difficulties (a) and (b) mentioned in the beginning of this section. In Fig. 6 we have plotted B as a function of r in the equatorial plane, for the extended source approximation.

If the extended-source model may be used to describe the quasi-nonlocality, a spherically symmetric quasi-nonlocal model still gives a spherically symmetric electric field. This is so because a spherically symmetric distribution of magnetization can be obtained by a linear superposition of homogeneous spheres of (constant) magnetization. The magnetic field which results is a linear superposition of these homogeneous-sphere fields. Thus, if it can be shown that the electric field of the homogeneous sphere is spherically symmetric, the resultant will be also. To show this, we recall that the outside magnetic field of a homogeneously magnetized sphere is like the field of a point dipole located at the center of the sphere. The field inside the sphere is constant and parallel. For a field point $P(\mathbf{r})$ inside the sphere, the integral (4.12) becomes

$$\int_{\mathbf{r}}^{\infty}\int \mathbf{B}\cdot d\mathbf{S}=-\int_{\mathbf{r}_\zeta}^{\mathbf{r}}\int \mathbf{B}\cdot d\mathbf{S}\propto r^2\sin^2(\mathbf{r},\zeta)\,, \tag{5.2}$$

where $\mathbf{r}$ is the distance from the origin, and the integration is done over a surface which is perpendicular to the ζ axis at a distance $|\mathbf{r}_\zeta|$ (the projection of $\mathbf{r}$ onto the ζ direction) from the origin. Both (5.2) and (4.12) are proportional to $\sin^2(\mathbf{r},\zeta)$. This means that because of (4.18), both the inside and outside electric fields are spherically symmetric and so is the resultant field of the Gaussian spherically symmetric distribution. The electric field intensity is also plotted in Fig. 6. This is proportional to

$$E_{\text{eff}} \propto RB_{\text{eff}} = B_{\text{eff}} r/(\tfrac{2}{3})^{1/2} r_0, \tag{5.3}$$

and E_{eff} goes over into the Coulomb field $\propto e/r^2$ for large r if B_{eff} goes over into (4.7).

A. Mean Position, Mean Spin

The concept of position of a particle has been analysed[5] by Pryce, Foldy, and Wouthuysen, Newton and Wigner, Huang, Landau, Pais and Uhlenbleck, Wightman, Schweber, Case, Fleming, Yukawa, Tomonaga, Finkelstein, Møller and Kristensen, Ingraham, Sankaranarayanan, and Good, Thirring, Henley, Parke, and many others. (For a casual reading, all subsections A–C may be omitted, here and in later sections.)

This analysis relates to our present theory because we try to attach loopforms to a source lepton in order to reconstruct its electromagnetic field. The transformation from the many-particle to the single-particle picture is achieved by the Pryce-Foldy-Wouthuysen transformation

$$\psi' = e^{iS}\psi, \quad S = -i(\beta\boldsymbol{\alpha}\cdot\mathbf{p}/2p)\arctan(p/mc) \tag{5.4a}$$

which transforms the Dirac equation from the ordinary four-component representation into independent two-component equations, i.e., the FW representation. The position operator, when applied to the FW-transformed ψ' function, has the amazing classical property that

$$\mathbf{V} = d\mathbf{X}/dt = \pm c^2\mathbf{p}/|E| \tag{5.4b}$$

has no *Zitterbewegung*. This mean position operator $\mathbf{X}$ is, in ordinary Dirac representation,

$$\mathbf{X} = \mathbf{x} + (2|E|)^{-1} i\hbar c\beta\boldsymbol{\alpha} - [2|E|^2(|E|+mc^2)]^{-1} \times \hbar c^3[i\beta(\boldsymbol{\alpha}\cdot\mathbf{p})\mathbf{p} + \boldsymbol{\sigma}\times\mathbf{p}|E|/c] + \cdots, \tag{5.4c}$$

where $\mathbf{x}$ is the position operator, i.e., a multiplication with $\mathbf{x}$, in ordinary Dirac representation. In this representation the mean position operator $\mathbf{X}$ is nonlocal of extent $\hbar/mc$, an enormously large amount of nonlocality. The numerical value of $\mathbf{X}-\mathbf{x}$ of course depends on what state is chosen (mean position eigenstate, or a Gaussian distribution over mean position, perhaps a state of optimum localizability for a given span of time, or a stationary state).

It may be remarked that even though we referred here only to nonlocality of position of the source, this nonlocality concept reflects then also on the nonlocal character of the loopforms attached to the source lepton.

If, in a very crude way, it is suggested to represent nonlocality by an extended source picture, it may then be appropriate to use a picture like Figs. 9(a)–9(c), 10, and 11(a) which subdivide ordinary three-space into elementary regions of size $\approx\hbar/mc$. One may then characterize loopforms simply by marking a consecutive sequence of such regions, i.e., representing a loopform by a closed string of beads (like a necklace of popbeads). In Sec. XI we shall discuss such definition of loopforms and of bundles of loopforms.

In a similar way, mean spin is defined and leads to an interpretation of the *Zitterbewegung* in terms of a contribution to the expectation value of ordinary position, a contribution which has an angular frequency

$$2mc^2/\hbar. \tag{5.4d}$$

It is for this reason that we think that the adoption of $2mc^2/\hbar$ rather than $mc^2/\hbar$ for the spinning angular velocity Ω of loopforms is well advised.

B. Quasi-Nonlocality and Spin Frequency

We pointed out that the cancellation of mass in the calculation of the electric field provided for a Coulomb field of the same equivalent charge e for the electron as well as for the muon. As the quantization of the charge e is as fundamental and precise a matter as the quantization of action $\hbar$, we should look into the question of what causes these two kinds of quantization to be precisely related to each other. We recognize that if we assume the amplitudes of loopforms to be chosen so that the muon magneton or Bohr magneton, $e\hbar/2mc$, results, and if we multiply these $e\hbar/2mc$ with the muon's or the electron's spinning angular velocity $2mc^2/\hbar$, respectively, the result is, by Eqs. (4.10), (4.24), or (5.19b), the Coulomb field, independent of the difference in mass m. The magneton is, however, not a fundamental quantity in the present theory; it is [cf. (5.10) and (9.17)]

$$\mu_{\text{eff}} = 3.1 r_0\Phi_{\text{eff}}/4\pi = 3.1 r_0\Phi_q/4\pi(3N/2), \tag{5.5a}$$

i.e., the product of the amount of quasi-nonlocality $r_0 \approx \hbar/mc$ times the quantized flux $\Phi_q = hc/e$ times a reduction factor $(2/3N)$ [cf. (10.3)], i.e.,

$$e\hbar/2mc = \mu_{\text{eff}} \propto (\hbar/mc)(hc/e)(2/3N). \tag{5.5b}$$

The quantum of flux hc/e being a fundamental quantity, we are interested in the mass independence of the ratio of electric charge e in (5.19b) to quantized flux hc/e. Here, the "charge e" is defined as $rV_{\text{eff}} \equiv r\langle V\rangle_{\text{av}}$ at large r:

$$\begin{aligned} e/(hc/e) &\leftarrow r\langle V\rangle_{\text{av}}/(2\pi\hbar c/e) \\ &= (r/2\pi)\langle\partial\vartheta/\partial ct\rangle_{\text{av}} && \text{by (4.3)} \\ &= \pm\left(\frac{1}{c}\frac{e\hbar}{2mc}\bigg/\frac{hc}{e}\right)\frac{2mc^2}{\hbar} && \text{by (4.24)} \\ &= \pm(c^{-1}\mu_{\text{eff}}/\Phi_q)(2mc^2/\hbar) \\ &\propto c^{-1}(\hbar/mc)(2mc^2/\hbar) && \text{by (5.5b),} \end{aligned} \tag{5.5c}$$

(apart from the reduction factor $\frac{2}{3}N$), i.e., a product of the extent of a quasi-nonlocality $\hbar/mc$ and the *Zitterbewegung* spin $2mc^3/\hbar$. As these two factors are, by their definition, inverses of each other, the mass cancels out rigorously.

C. Electromagnetic Field of Extended Source

For large r, the equatorial magnetic field of a dipole of effective moment $\mu_{\rm eff}$ is

$$B_{\rm eff}\to-\mu_{\rm eff}/r^3. \quad (5.6)$$

For a Gaussian distrbution of magnetization,

$$M_{\rm eff}=M_0e^{-R^2}, \quad (5.7a)$$

where

$$M_0=\mu_{\rm eff}(3/2\pi r_0^2)^{3/2}, \quad (5.7b)$$

$$R=r/(\tfrac{2}{3})^{1/2}r_0, \quad (5.7c)$$

we may calculate $B_{\rm eff}$ in the equatorial plane by slicing the area under the Gaussian curve into horizontal layers of radius R', each layer corresponding to a homogeneously magnetized sphere of magnetization

$$dM_{\rm eff}=-M_0 2R'e^{-R'^2}dR', \quad (5.8)$$

so that

$$B_{\rm eff}=M_0\left[-\tfrac{1}{3}4\pi\int_{R'=0}^{R}R^{-3}R'^3 2R'e^{-R'^2}dR' +\tfrac{2}{3}4\pi\int_{R'=R}^{\infty}2R'e^{-R'^2}dR'\right]. \quad (5.9)$$

This gives us the means to evaluate the total magnetic flux $\Phi_{\rm eff}$ and thus the needed relationship (5.10), which expresses the effective magnetic moment $\mu_{\rm eff}$ in terms of the flux $\Phi_{\rm eff}$ and the root mean square r_0 of the Gaussian. The total flux becomes, by numerical integration of (5.9),

$$\Phi_{\rm eff}=4\pi\mu_{\rm eff}/3.1r_0. \quad (5.10)$$

Similarly, we may define μ_q by $\mu_q/\mu_{\rm eff}=\Phi_q/\Phi_{\rm eff}$:

$$\Phi_q=4\pi\mu_q/3.1r_0. \quad (5.10q)$$

Accordingly, by (5.9), (5.7), and (5.10),

$$\begin{aligned}(B_{\rm eff})_{r=0}&=(8\pi/3)M_0\\ &=(8\pi/3)(3.1/4\pi)\Phi_{\rm eff}r_0(3/2\pi)^{3/2}r_0^{-3}\\ &=0.682\Phi_{\rm eff}/r_0^2, \end{aligned}\quad (5.10a)$$

$$(B_{\rm eff})_{r\to\infty}\to-(3.1/4\pi)(r_0/r)^3\Phi_{\rm eff}/r_0^2. \quad (5.10b)$$

Outside the equatorial plane,

$$\begin{aligned}(B_{\rm eff})_{r\to\infty}&\to(3.1/4\pi)\\ &\times[3(r/r_0)^{-5}(\mathbf{z}\cdot\mathbf{r}/r_0)\mathbf{r}/r_0-(r/r_0)^{-3}\mathbf{z}]\Phi_{\rm eff}/r_0^2. \end{aligned}\quad (5.10c)$$

We recorded and plotted $B_{\rm eff}$ in Fig. 6 in units of $\Phi_{\rm eff}/r_0^2$.

Comparing electron and muon, their r_0's (as we shall soon see) as well as their $\mu_{\rm eff}$'s are of the ratio 207 to 1, and thus, by (5.10), their $\Phi_{\rm eff}$'s are equal. Therefore the $B_{\rm eff}$'s at a given r/r_0 are of the ratio 1 to 207^2.

We shall use the same graphs for the effective resultant field (with $\boldsymbol{\mu}_{\rm eff}$ pointing in the $\mathbf{z}$ direction) as well as for the contributing sheaves (whose $\boldsymbol{\mu}$ point in the $\boldsymbol{\zeta}$ directions). In the first case we talk about $\mathbf{B}_{\rm eff}$ and $\Phi_{\rm eff}$, in the second case about contributing $\mathbf{B}$ and Φ.

Figure 5 shows the magnetic field lines of such a Gaussian distribution of magnetic fields, again to be used for $\Phi_{\rm eff}$ with $\boldsymbol{\mu}_{\rm eff}$ pointing in the $\mathbf{z}$ direction, or for the contributing Φ and $\boldsymbol{\mu}$ of sheaves. Figure 5 shows the "core" whose radius we define as reaching from the origin to the zero point of the magnetic field on the equatorial plane; thus its radius becomes $1.23r_0$. Figure 5 shows the symmetry axis and nine toruses in cross section, subdividing space into ten regions, each of which carries 0.1 of the total magnetic flux.

To evaluate the electric field, we begin with the simplified calculation of the field in the equatorial plane, on the basis of Eqs. (4.4)–(4.10), but with $B_{\rm eff}$ taken from (5.9) instead of (4.7), which was used in Eqs. (4.4)–(4.10). We have thus to evaluate

$$\int_r^\infty B_{\rm eff}2\pi r dr=\tfrac{1}{3}4\pi r_0^2\int_R^\infty B_{\rm eff}RdR, \quad (5.11)$$

which is the fraction

$$F=\tfrac{2}{3}r_0^2(e/\hbar c)\int_R^\infty B_{\rm eff}RdR \quad (5.12)$$

of the quantized flux $2\pi\hbar c/e$, (3.6). As the effective vector potential $\mathbf{A}_{\rm eff}$ of a stationary lepton is, in this theory, a time-independent quantity in the average, the electric field is

$$\mathbf{E}_{\rm eff}=-\mathrm{grad}V_{\rm eff}-\partial\mathbf{A}_{\rm eff}/\partial ct=-\mathrm{grad}V_{\rm eff} \quad (5.13)$$

(cf. the caption to Fig. 3 and Fig. 7). In the equatorial plane,

$$\begin{aligned}E_{\rm eff}&=-\partial V_{\rm eff}/\partial r=-(\partial/\partial r)(\hbar c/e)(\partial\vartheta/\partial ct)_{\rm eff}\\ &=-(\hbar/e)(\partial/\partial r)[(2\pi)^{-1}\Omega 2\pi F]\\ &=-(\hbar/e)(2mc^2/\hbar)\tfrac{2}{3}r_0^2(e/\hbar c)(2r_0^2/3)^{-1/2}\\ &\qquad\times(\partial/\partial R)\int_R^\infty B_{\rm eff}RdR\\ &=2(\tfrac{2}{3})^{1/2}B_{\rm eff}Rr_0/(\hbar/mc). \end{aligned}\quad (5.14)$$

We shall now finish this calculation by repeating the arguments which led us to (4.18) and (4.19), to inquire into the spherical symmetry of the electric field. To this effect we consider one of the spheres of radius r', of homogeneous magnetization

$$d\mathbf{M}=\boldsymbol{\zeta}dM, \quad (5.15)$$

contributing to the spherical Gaussian magnetization of flux orientation $\boldsymbol{\zeta}$, analogous to (5.8). With

$$d\mu=dM(4\pi/3)r'^3, \quad (5.16)$$

the magnetic field outside the sphere is as if coming from a point dipole of strength $d\mu$; thus by (4.12),

$$\int_r^\infty\int \mathbf{B}\cdot d\mathbf{S} = -d\mu(2\pi/r)\sin^2(\mathbf{r},\boldsymbol{\zeta})$$
$$= -dM(8\pi^2/3)r'^2(r'/r)\sin^2(\mathbf{r},\boldsymbol{\zeta}). \quad (5.17)$$

For a point inside the sphere,

$$\int_r^\infty\int \mathbf{B}\cdot d\mathbf{S} = -\int_0^r\int \mathbf{B}\cdot d\mathbf{S}$$
$$= -\tfrac{2}{3}4\pi dM[\pi r^2\sin^2(\mathbf{r},\boldsymbol{\zeta})]$$
$$= -dM(8\pi^2/3)r'^2(r/r')^2\sin^2(\mathbf{r},\boldsymbol{\zeta}). \quad (5.18)$$

This means that the angular dependence is in both cases given by $\sin^2(\mathbf{r},\boldsymbol{\zeta})$. The superposition of those magnetized spheres (5.16) (to a Gaussian) will of course then also result in a $\sin^2(\mathbf{r},\boldsymbol{\zeta})$ angular dependence of the flux linked with a point $P(\mathbf{r})$.

Accordingly, we may apply again the consideration of Sec. IV which shows that if the flux loopforms spin around their respective flux orientation axes, the resulting electric field is spherically symmetric. This means now that the electric field E_{eff}, given by (5.14) for the simplified equatorial-plane calculation, is to be multiplied by $\frac{2}{3}$ [remembering (4.18)], to give the magnitude of the spherically symmetrical field:

$$E_{\text{eff}} = \tfrac{2}{3}2(\tfrac{2}{3})^{1/2}(B_{\text{eff}}R)_{\text{equ}}r_0/(\hbar/mc)$$
$$= \tfrac{2}{3}2B_{\text{eff equ}}r/(\hbar/mc). \quad (5.19\text{a})$$

For large r,

$$E_{\text{eff}} \to \tfrac{2}{3}2(e\hbar/2mc)r^{-3}r/(\hbar/mc) = \tfrac{2}{3}e/r^2. \quad (5.19\text{b})$$

In Fig. 6, $(B_{\text{eff}}R)_{\text{equ}} = B_{\text{eff}}r/(\frac{2}{3})^{1/2}r_0$ is plotted in units of $(\Phi_{\text{eff}}/r_0^2)$ with the same ordinate scale as for B_{eff}. Following the end of Sec. IV, we recognize again the disappearance of the factor $\frac{2}{3}$ of (5.19b).

Comparing electron and muon, as mentioned above, m cancels out and (5.19b) gives the same field E_{eff} at a given large r. Thus, at a given r/r_0 the electron's field stands to the muon's field in the ratio of 1 to 207^2 (if, as we shall see, their r_0's are as 207 to 1). This is the same as it was for B_{eff}.

VI. ELECTROMAGNETIC ENERGY AND ANGULAR MOMENTUM

Irrespective of the size (3.6) of the quantized flux, we may note that to a Gaussian distribution of variance r_0^2 and of magnetic moment μ_{eff} (4.4), there corresponds an effective magnetic flux (5.10) (the effective, i.e., "averaged," "reduced" fields being related to the lepton's magnetic moment μ_{eff} and charge e by Maxwell-Lorentz)

$$\Phi_{\text{eff}} = 4\pi\mu_{\text{eff}}/3.1r_0. \quad (6.1)$$

This can be seen by a simple numerical integration. Furthermore,

$$\int\int\int_0^\infty ((B_{\text{eff}})^2+(E_{\text{eff}})^2)(8\pi)^{-1}d^3r$$
$$= \{0.138[(\hbar/mc)/r_0]^3+0.365[(\hbar/mc)/r_0]\}$$
$$\times(e^2/\hbar c)mc^2, \quad (6.2)$$

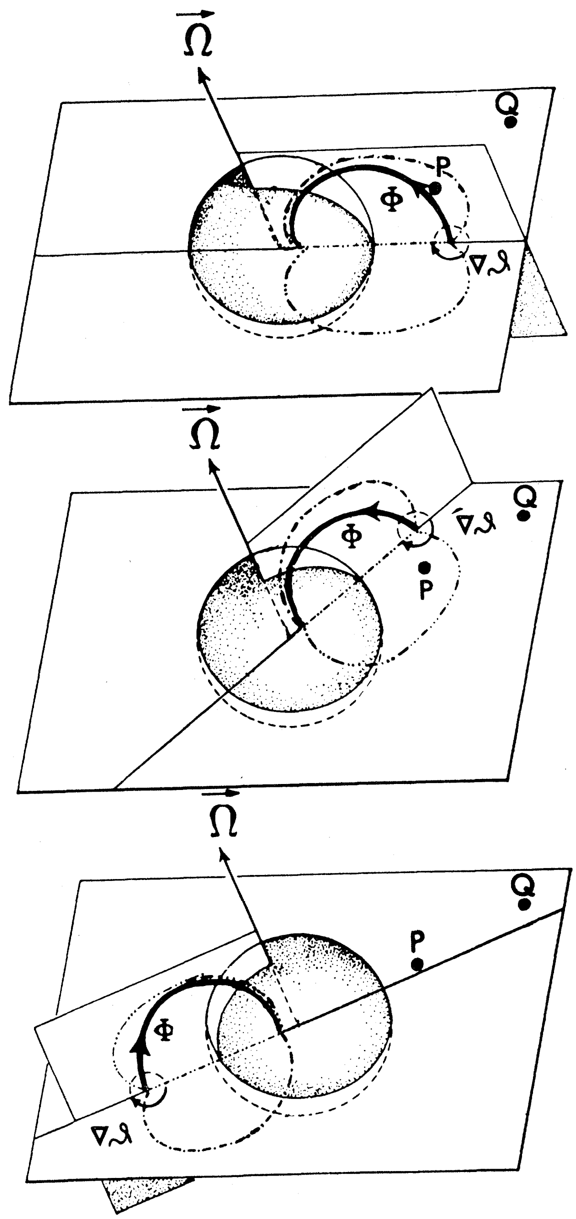

FIG. 7. A spinning loopform for a quasi-nonlocal source (represented as extended source), in analogy to Fig. 3. Half of the flux loop spinning with angular velocity $\boldsymbol{\Omega}$ is visible in this time sequence of three drawings. (Figure 3 was drawn for a point-source model.)

$$\left|\int\int_0^\infty\int \mathbf{r}\times(\mathbf{E}_{\text{eff}}\times\mathbf{B}_{\text{eff}})(4\pi c)^{-1}d^3r\right|$$

$$=0.47[(\hbar/mc)/r_0](e^2/\hbar c)\hbar/2. \quad (6.3)$$

For this theory we require as a condition of consistency for energy and as a consistency for the assumed spin angular momentum $\frac{1}{2}$

$$\int\int_0^\infty\int (B^2+E^2)_{\text{eff}}(8\pi)^{-1}d^3r=mc^2=\hbar\omega, \quad (6.4)$$

$$\left|\int\int_0^\infty\int \mathbf{r}\times(\mathbf{E}\times\mathbf{B})_{\text{eff}}(4\pi c)^{-1}d^3r\right|=\hbar/2. \quad (6.5)$$

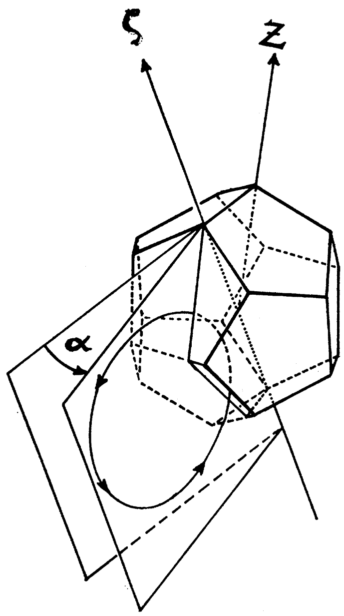

FIG. 8. A flux loopform and a source lepton represented by a pentagon dodecahedron are shown. The parameters ζ and α characterize the flux loopform, ζ being the flux orientation with respect to the direction of resultant magnetic moment or spin (the z axis) and α being the azimuth as indicated. Another parameter, the "size" σ, means, for this loopform, that it reaches to the first shell surrounding the core. The core is here shown as a pentagon dodecahedron with each corner characterizing one flux orientation, to indicate 20 essentially different flux orientations. Neighboring flux orientations are 0.976 rad, i.e., about 1 rad apart, as it should be (whereas a choice of directions to the 12 centers of the pentagons would correspond to neighbor directions differing by 1.425 rad, which is too much). Similarly, one may assume that there are six essentially different azimuth orientations, about 1 per unit rad. This amounts to a manifold of 20×6, i.e., 120 bundles as regards the distribution over ζ and α, accidentally the same number as the count of Fig. 9. The distribution over size σ is governed by the requirement of every bundle carrying the same amount of flux (Fig. 10). Figure 11 then shows the total number of bundles to be about 200; they are characterizing the structuralization of the lepton.

Since, however, r_0 (5.1) is of the order of $\hbar/mc$, (6.2) and (6.3) are more than two orders of magnitude smaller than mc^2 and $\hbar/2$. We certainly do not wish to take the *ad hoc* step of introducing a second field besides **B**, **E** in order to raise the values of (6.2) and (6.3), because we want to understand leptons in terms of quantized flux alone. Neither can we attempt to correct the situation by choosing r_0 some order of magnitude smaller than $\hbar/mc$ in order to get mc^2 from (6.2). We cannot do this because the considerations of Sec. V have determined r_0; also, an adjustment of r_0 to meet (6.2) would still leave (6.3) far behind the requested value $\hbar/2$.

The point is, as we shall see in (9.17) and (9.20) and in Sec. XII, that $(B^2)_{\text{eff}}\gg(B_{\text{eff}})^2$.

We note that the spin $\frac{1}{2}$ is the *assumed* intrinsic spin of the probability amplitude field (1.1), (8.21), which, as it relates to the quantum-mechanical ψ function, defines the expectation values of the electromagnetic quantities. This electromagnetic field is then supposed to reconstruct $\hbar/2$ by (6.5), in order to be consistent.

VII. CONSTRUCTION OF FIELD FROM COMPLEX PROBABILITY AMPLITUDES

This problem of Sec. VI (and its solution) becomes evident at inspection of (6.1). This equation implies that if the quantized flux $\Phi_q=hc/e$ and $r_0\approx\hbar/mc$ were inserted into a similar equation $\Phi_q=4\pi\mu_q/3.1r_0$, the moment μ_q and thus the fields **B** and **E** would be a few hundred times larger than μ_{eff}, $\mathbf{B}_{\text{eff}}$, and $\mathbf{E}_{\text{eff}}$, respectively. This we interpret to mean that the sum Φ_{qz} of the z components of the loopform contributions' quantized flux Φ_q would represent the effective flux Φ_{eff} (and thus B_{eff} and E_{eff}) only if all the complex probability amplitudes which characterize the distribution of Φ_q over different loopforms were in phase. If there is, however, a randomnesss of phase (or differences of phase), the Φ_{eff} may be obtained by a superposition of the contributing loopform amplitudes. The randomness (or difference) of phases is then made responsible for the "reduction" of Φ_{qz} down to Φ_{eff}. This reduction will also be seen to take care of electromagnetic energy and angular momentum.

Coming back to this later, we now should first specify the manifold of loopforms. We recall the four parameters characterizing them: flux orientation ζ of a sheaf of loopforms as shown in Figs. 1(c), 8, and 9; size σ which indicates to what extent the loopform reaches out, i.e., its aphelion distance from the origin (Fig. 5); and azimuth α, i.e., the angle of the loopform plane, measured around the ζ axis (Figs. 8, 9, and 4, also indicated on Figs. 3 and 7).

In his space-time approach to quantum mechanics, Feynman defines the phases of the complex probability amplitudes by the action integrals over the alternative path histories. In the present theory we give random-phased probability amplitudes to the alternative loopforms of muons. A concept of random phasedness can

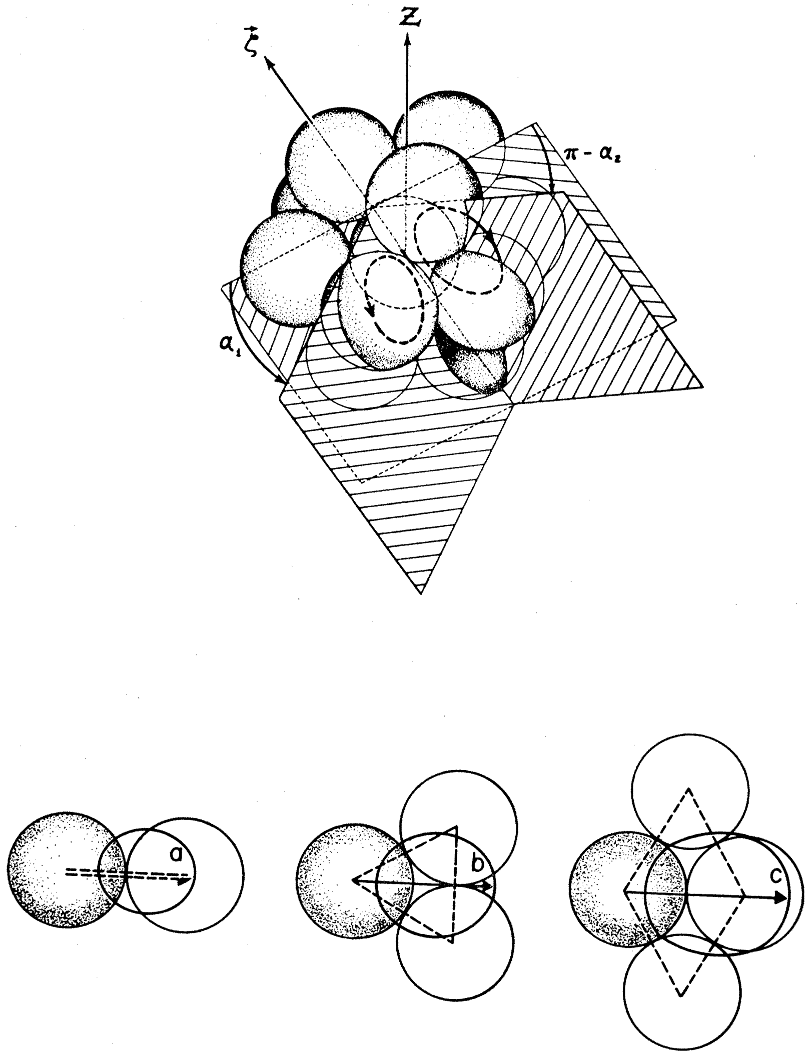

FIG. 9. As an alternative to Fig. 8, statistically independent "bundles" of loopforms [of the first-shell types, (a)–(c)] may be characterized by the sequence of elementary regions through which the loopforms pass. All loopforms are assumed to pass through the "core" [the dashed, hidden sphere on the large drawing, or the dotted-shaded sphere on each of (a)–(c)]. The large drawing shows twelve elementary regions adjacent to the core. This drawing also shows one loopform of type a and one loopform of type b in the form of heavy, mostly dashed, ellipse-shaped loops. They are fully drawn in (a) and (b), and (c) shows another type. We may count 24 loopforms of type a, 48 of type b, and 48 of type c; every one is assumed to carry about the same amount of flux and may thus have equal probability; they share about 60% of the total flux. The large drawing also illustrates flux orientation ζ of the two dashed loopforms and their azimuths α_1 and α_2.

have no meaning in regard to closely neighboring loopforms. It can mean that although such close neighbors have phase-related amplitudes, the phase relatedness gives way to randomness if two loopforms have sufficiently distinct form.

There are several ways to specify what may be considered "sufficiently distinct" loopforms whose amplitudes have random phase differences. We understand by a "bundle of loopforms" a set of neighboring loopforms which are still close enough so that their amplitudes are still in some measure phase related. We might try to subdivide all of a source lepton's flux loopforms into bundles which, if their probability amplitudes have the same absolute value, all carry the same amount of flux [cf. Eq. (8.11)].

Leaving aside, in this paper, the interesting possibility of defining the different bundles as superpositions of generalized spherical harmonics of different s, m, and n, we mention two of the simplest ways of specifying the bundles of loopforms. We designate the radius of the core as

$$r_{\mathrm{core}}=1.23r_0, \tag{7.1}$$

because for a Gaussian distribution of magnetization of variance r_0^2, $1.23r_0$ points to a circle of zero magnetic field in the equatorial plane.

(1) We may bundle loopforms (Fig. 8) together so that one bundle differs in flux orientation from that of a neighbor bundle by about 1 rad. The corners of a pentagon dodecahedron or the centers of the sides of an icosahedron are 0.976 rad apart. These 20 orientations may characterize the bundles' ζ. For each such flux orientation there may be six values of α distinguishing the bundles, which amounts again to about 1 rad. That settles the issue as regards the bundling of loopforms which reach out into the first shell only, i.e., whose aphelion distances are between $r_{\mathrm{core}}\leq r\leq 3r_{\mathrm{core}}$. Also, we

group the loopforms of larger size into similar bundles, each carrying a same amount of flux $\Phi_{(\lambda)}$ (8.11a).

(2) Instead, we may subdivide ordinary 3-space into elementary regions using an array of close-packed spheres, each of radius $1.23r_0$ (Fig. 9). We then specify first-shell distinctly different bundles merely by the sequence of elementary regions which the loopforms pass through [Figs. 9(a)–9(c)]. For the loopforms of larger size, we may group together those loopforms which pass through the same sequence of elementary regions; different but neighboring groups would, however, still have some phase relatedness of their amplitudes. We call a set of several neighboring groups a "bundle" if the bundle's phase relationship to any other bundle is random, which is assumed to mean that all bundles carry the same amount of flux $\Phi_{(\lambda)}$ [cf. (8.11a)].

In either case, in choosing assumptions about the degree of phase relatedness versus random phasedness of loopforms, we essentially choose a unit radian difference in flux orientation ζ or in azimuth α as characterizing statistical independence of the phases of their corresponding probability amplitudes. We thus grouped loopforms together into "bundles" of statistically independent loopforms. This procedure is plausible as regards ζ and α because any other numerical assumption would have been unreasonably arbitrary indeed. As regards the size σ, the grouping into bundles is not so obvious. We assumed that those sizes σ of loopforms which reach into the first shell of elementary regions surrounding the core (cf. Fig. 9) are statistically independent of the loopforms of larger size. Also we assumed that the bundling is to be made so that all bundles carry essentially the same amount of magnetic flux, to characterize the bundling with respect to the parameter σ.

We should like to show now that it is indeed plausible to assume that bundles of statistically independent loopforms all carry essentially the same amount of flux. In the beginning of Sec. XIII it will also be pointed out that this is what is demanded on physical grounds.

To this effect, we recall that quasi-nonlocality implied an "extension" $1.23r_0$ of the position of the source lepton, and similarly of any other definition of structure, i.e., the points of a loopform. A loopform may be thus represented by a tube which one obtains if one moves a sphere of radius $1.23r_0$ with its center on a dipole field line along that dipole field line (this sphere is shown at the location of the core in Fig. 10). We now consider the manifold of similar (congruent) loopforms which may readily be placed into that tube. The figure shows, besides the center line, two other similar lines (dashed lines), one inclined to the left, the other to the right. We may now raise this question: What degree of phase relatedness will this manifold of loopforms, all confined to that tube, be expected to have? Or more simply: What fraction of one statistically independent bundle do these flux loopforms (confined to the tube) represent? We may assume that this fraction is proportional to the product $\delta\alpha\delta^2\zeta$, where $\delta\alpha$ and $\delta^2\zeta$ are the

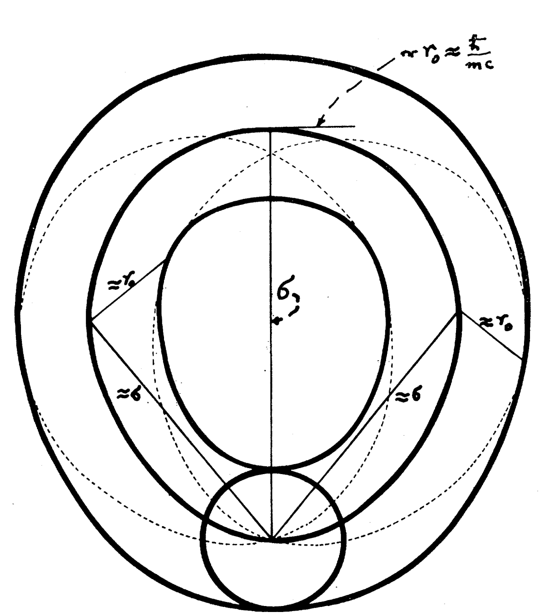

FIG. 10. A flux loopform, considered "quasi-nonlocal." To represent that, a tube of diameter equal to the core diameter is drawn. The question is then raised as to how far the flux loopform may be inclined in the clockwise and in the counterclockwise direction, indicated by the two dashed lines which represent each a flux loopform congruent to the original one (center line) but still confined inside the tube. The angles of clockwise or counterclockwise inclination are of the order of r_0/σ, where $1.23r_0$=radius of the core sphere=tube radius, and σ=size, i.e., about the aphelion distance of the loopform. The manifold of loopforms is not only inclined clockwise or counterclockwise, but spans out a 3-parametric manifold $\delta^2\zeta\delta\alpha$ which is proportional to $(r_0/\sigma)^3 \propto \sigma^{-3} \propto B$ at aphelion $\propto$ flux through the tube. As a statistically independent bundle is characterized by $\delta_0^2\zeta\ \delta_0\alpha=1$ sr$\times$rad, the loopforms fitting into the tube are a fraction $\delta^2\zeta\delta\alpha$ of such a bundle, which fraction is proportional to the flux going through the tube. Thus each statistically independent bundle should carry the same amount of flux. First-shell loopforms may be characterized by an average $\sigma=1$, i.e., an aphelion distance equal to twice the core radius.

intervals corresponding to the permissible inclinations of loopforms as long as they are confined to the tube. As the tube diameter $2.46r_0$ is given, the product $\delta\alpha\delta^2\zeta$ is evidently proportional to σ^{-3} (the size σ being defined as aphelion distance of the loopform). But σ^{-3} is approximately proportional to the magnetic flux carried by that tube. Thus, as regards the σ dependence, we realize that the amount of magnetic flux carried by the tube is a measure for that *fraction* (of one statistically independent bundle) which this magnetic flux tube represents. This means that every statistically independent bundle carries the same amount of flux. That is a crude estimate, but it is significant.

Furthermore, by this definition, each bundle of loopforms of the type Figs. 9(a)–9(c), i.e., each bundle of first shell size, amounts to just about one statistically independent bundle. This definition of correlatedness of loop forms (i.e., of the size of statistically independent bundles) thus encompasses both correlatedness as regards the parameter σ as well as regards the parameters α, ζ. That provides a single basis for the counting of $N \approx 207$ statistically independent bundles of loopforms in Sec. XI and Fig. 11.

The notation for a loopform may be given by $\lambda \equiv \boldsymbol{\zeta}, \alpha, \sigma$; for an elementary bundle by $(\lambda) \equiv (\boldsymbol{\zeta}), (\alpha), (\sigma)$; and the probability amplitude for that bundle of loopforms, referring to a lepton in the spin-up quantum state $|\mu_z{}^+\rangle$, by a normalized

$$\phi_{(\lambda)} \equiv ((\lambda)|\mu_z{}^+\rangle \equiv ((\boldsymbol{\zeta})(\alpha)(\sigma)|\mu_z{}^+\rangle, \quad \sum_{(\lambda)} \phi_{(\lambda)}{}^* \phi_{(\lambda)} = 1. \quad (7.2)$$

The expectation values for the fields $\mathbf{B}$ and $\mathbf{E}$ in terms of those bundle amplitudes $\phi_{(\lambda)}$ are assumed as

$$\mathbf{B}_{\text{eff}} = \langle \mathbf{B} \rangle_{\text{av}} = \langle \sum_{(\lambda)} \phi_{(\lambda)}{}^* (\text{curl}\mathbf{A})_{(\lambda)} \phi_{(\lambda)} \rangle_{\text{av}}, \quad (7.3)$$

$$\mathbf{E}_{\text{eff}} = \langle \mathbf{E} \rangle_{\text{av}} = -\langle \sum_{(\lambda)} \phi_{(\lambda)}{}^* (\text{grad}V)_{(\lambda)} \phi_{(\lambda)} \rangle_{\text{av}} + 0. \quad (7.4)$$

$(\text{curl}\mathbf{A})_{(\lambda)}$ and $(\text{grad}V)_{(\lambda)}$ are bundle averages of the loopforms' $\mathbf{A}$, (4.2), and V, (4.3), averages defined below [(8.18) and (8.19)]. The term with $(\partial\mathbf{A}/\partial t)_{(\lambda)}$ is indicated by the 0 in Eq. (7.4); cf. the caption to Fig. 3 and Eqs. (5.13), (8.18), and (8.19). It may be seen from these expressions that the $\mathbf{B}_{\text{eff}}$ and $\mathbf{E}_{\text{eff}}$ in a region $P(\mathbf{r})$ simply depend on the flux loopforms passing through at that region.

Loopform-bundle amplitude superposition is a concept which implies that for the determination of the effective value of a certain physical quantity, be it $\boldsymbol{\mu}$ or $\mathbf{B}$ or $\mathbf{E}$ or B^2, etc., one considers which bundle manifold contributes toward the desired quantity [e.g., the few bundles contributing toward $\mathbf{B}$ at a certain far-outside region $P(\mathbf{r})$ or, e.g., all bundles contributing toward the field at the core] and thereupon forms the weighted averages (7.3), (7.4), and (9.2)–(9.5).

To be more specific about (7.3) and (7.4), we write those averages in terms of the loopform manifolds (7.2). In order to do that, we digress on the formulations of those bundle amplitudes, considering first only the $(\boldsymbol{\zeta})$ manifold, i.e., a two-parameter manifold of 4π loopform bundles.

VIII. FORMALISM FOR (UNDERTERMINED) SETS OF BUNDLES OF LOOPFORMS

To start with the discussion of the muon (as was stated at the beginning of Sec. VII), we have in mind to make the random-phase relationship of amplitudes of loopform bundles responsible for a reduction from quantized flux Φ_q to effective flux Φ_{eff}. The rationale of the procedure is quite simple: The quantized flux Φ_q is a given quantity, and there is just the one obvious type of quasi-nonlocality which is of extent $\hbar/m_\mu c$ (or $\hbar/m_e c$), respectively. The product of these two quantities is too large by two orders of magnitude for explaining the muon (or Bohr) magneton, respectively. For the description of the electromagnetic field, we have seen that we need probability amplitude superposition of the loopforms. There is thus the obvious assumption before us, i.e., to make randomness of phases responsible for the reduction from Φ_q to Φ_{eff}.

There may be various ways of carrying through such a reduction procedure. We may, at a later time, do that on the basis of superpositions of functions of the type (8.21), but we try to formulate such a reduction procedure now in terms of randomness of the phases of bundles of loopforms, as specified in Sec. VII.

It should be kept in mind, however, that the formalism given here is nothing more than an attempt to illustrate the reduction procedure; definitions and detailed assumptions are made so as to construct one possible, reasonable formalism in terms of which the reduction may be formulated.

We study the expansion of a quantum-mechanical state vector $|\mu_z{}^+\rangle$ into an underdetermined set of loopform-bundle amplitudes $|(\boldsymbol{\zeta}))$ or $|(\boldsymbol{\zeta})(\alpha)(\sigma)) \equiv |(\lambda))$. For an expansion in terms of loopform amplitudes $|\boldsymbol{\zeta})$ or $|\boldsymbol{\zeta}\alpha\sigma) \equiv |\lambda)$, integrals would take the place of sums.

We consider now the expansion of ordinary quantum-mechanical kets of magnetic moment (or spin):

$$\begin{aligned} |\mu_z{}^+\rangle &= \sum_{\mu_\zeta{}'} |\mu_\zeta{}'\rangle\langle\mu_\zeta{}'|\mu_z{}^+\rangle \\ &= |\mu_\zeta{}^+\rangle\langle\mu_\zeta{}^+|\mu_z{}^+\rangle + |\mu_\zeta{}^-\rangle\langle\mu_\zeta{}^-|\mu_z{}^+\rangle \\ &= |\mu_\zeta{}^+\rangle\langle\mu_\zeta{}^+|\mu_z{}^+\rangle + |\mu_{-\zeta}{}^+\rangle\langle\mu_{-\zeta}{}^+|\mu_z{}^+\rangle. \quad (8.1) \end{aligned}$$

From this form, we may obtain a suggestion as to how we might write an (underdetermined) expansion of $|\mu_z{}^+\rangle$ in terms of bundle amplitudes or random-phased amplitudes, respectively, viz.,

$$|\mu_z{}^+\rangle = \sum_{(\zeta)} |(\boldsymbol{\zeta}))((\boldsymbol{\zeta})|\mu_z{}^+\rangle. \quad (8.2)$$

For random-phased amplitudes also, we assume

$$\langle\mu_z{}'|(\boldsymbol{\zeta})_R) = ((\boldsymbol{\zeta})_R|\mu_z{}'\rangle^*, \quad (8.3\text{a})$$

so that also

$$|\mu_z{}^+\rangle = \sum_{(\zeta)} |(\boldsymbol{\zeta})_R)((\boldsymbol{\zeta})_R|\mu_z{}^+\rangle, \quad (8.3\text{b})$$

where $|(\boldsymbol{\zeta}))$ is a two-component spinor with $\mathfrak{N} \approx 4\pi$, i.e., 12 columns, so that

$$\langle\mu_z{}'|(\boldsymbol{\zeta})) = \begin{pmatrix} \cdots\cdots\cdots\cdots \\ \cdots\cdots\cdots\cdots \end{pmatrix}. \quad (8.4)$$

With ζ_z representing the z component of an average $\boldsymbol{\zeta}$ of the bundle $(\boldsymbol{\zeta})$,

$$\langle\mu_z{}^+|(\boldsymbol{\zeta})) = \mathfrak{N}^{-1/2}(1+\zeta_z)^{1/2} \times(\zeta_x - i\zeta_y)^{1/2}/(\zeta_x{}^2+\zeta_y{}^2)^{1/4}, \quad (8.4\text{a})$$

$$\langle\mu_z{}^-|(\boldsymbol{\zeta})) = \mathfrak{N}^{-1/2}(1-\zeta_z)^{1/2} \times(\zeta_x{}^2+\zeta_y{}^2)^{1/4}/(\zeta_x - i\zeta_y)^{1/2}. \quad (8.4\text{b})$$

They are $\mathfrak{N}^{-1/2}(1\pm\zeta_z)^{1/2}$ times a phase factor. It is evident that with (8.4a), (8.4b), and (9.9),

$$\sum_{(\zeta)} \langle \mu_z' | (\boldsymbol{\zeta}))((\boldsymbol{\zeta}) | \mu_z''\rangle = \delta_{\mu_z'\mu_z''}. \tag{8.5}$$

This makes a probability amplitude interpretation possible; (8.4a) and (8.4b) had been chosen so as to imply (8.5). Furthermore,

$$\sum_{\mu_z'}((\boldsymbol{\zeta}) | \mu_z'\rangle\langle\mu_z' | (\boldsymbol{\zeta})) = 2/\mathfrak{N}, \tag{8.6}$$

$$|(\boldsymbol{\zeta})) = |\mu_z^+\rangle\langle\mu_z^+ | (\boldsymbol{\zeta})) + |\mu_z^-\rangle\langle\mu_z^- | (\boldsymbol{\zeta})). \tag{8.7}$$

With

$$\langle\mu_z' | \mu_z | \mu_z''\rangle = \mu\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{8.8}$$

it is possible to set

$$((\boldsymbol{\zeta}_1) | \mu_z | (\boldsymbol{\zeta}_2))$$
$$= \sum_{\mu_z'\mu_z''} ((\boldsymbol{\zeta}_1) | \mu_z'\rangle\langle\mu_z' | \mu_z | \mu_z''\rangle\langle\mu_z'' | (\boldsymbol{\zeta}_2)) \tag{8.9a}$$
$$= \mu[((\boldsymbol{\zeta}_1) | \mu_z^+\rangle\langle\mu_z^+ | (\boldsymbol{\zeta}_2)) - ((\boldsymbol{\zeta}_1) | \mu_z^-\rangle\langle\mu_z^- | (\boldsymbol{\zeta}_2))] \tag{8.9b}$$

and obtain the inverted formula

$$\langle\mu_z' | \mu_z | \mu_z''\rangle = \sum_{(\zeta_1)(\zeta_2)} \langle\mu_z' | (\boldsymbol{\zeta}_1))((\boldsymbol{\zeta}_1) | \mu_z | (\boldsymbol{\zeta}_2))((\boldsymbol{\zeta}_2) | \mu_z''\rangle$$
$$= \mu\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{8.10}$$

Up to this point, we have not dealt with "reduction" but rather have simply converted the ordinary spinor formalism to the new formalism and vice versa, for phase-related amplitudes $|(\boldsymbol{\zeta}))$. Let us assume that these same expansions hold when, instead of the manifold $(\boldsymbol{\zeta})$, the manifold $(\lambda)\equiv(\boldsymbol{\zeta})(\alpha)(\sigma)$ is considered, if we replace $(\boldsymbol{\zeta})$ and $\mathfrak{N}$ by (λ) and N, respectively, where N $(>\mathfrak{N})$ is the larger number of bundles in the manifold (λ).

To reconstruct an appropriate distribution of magnetic field as a function of $\mathbf{r}$, we may choose the magnitudes of the loopform amplitudes as

$$|(\lambda | \mu_z^+\rangle| \approx (1+\zeta_z)^{1/2}\left(\Phi_\lambda \bigg/ \int_{\lambda'} \Phi_{\lambda'}\right)^{1/2}. \tag{8.11a}$$

The term $(1+\zeta_z) = 1+\cos(\boldsymbol{\zeta},\mathbf{z})$ occurs here because the simplest choice for the underdetermined $(\boldsymbol{\zeta}|$ is to set them proportional to $(1+\zeta_z)^{1/2}$ by the analogy of (8.1) with (8.2); the $\langle\mu_\zeta^+|$ are proportional to $(1+\zeta_z)^{1/2}$.

In Sec. VII, along with the Figs. 8, 9, and 10, we have given a definition of "phase relatedness of probability amplitudes of loopforms" and have reformulated that in terms of the concept of "statistically independent bundles of loopforms." We found that such bundles may be characterized as carrying, apart from the factor $(1+\zeta_z)(d\zeta_z/2)$, the same amount of magnetic flux,

$$\Phi_{(\lambda)}/\sum_{(\lambda')} \Phi_{(\lambda')} = N^{-1}, \tag{8.11b}$$

as indicated in (9.8), (9.9);

$$|((\lambda) | \mu_z^+\rangle| = (1+\zeta_z)^{1/2}(\Phi_{(\lambda)}/\sum_{(\lambda')} \Phi_{(\lambda')})^{1/2}$$
$$= (1+\zeta_z)^{1/2}N^{-1/2}. \tag{8.11c}$$

A. Comments on Definition of Bundle Matrix Elements

We specified the underdetermined bundle matrix elements $((\lambda_1) | \mu_z | (\lambda_1))$ and $((\lambda_1) | \mu_z | (\lambda_2))$ from the corresponding quantum-mechanical matrix elements $\langle\mu_z' | \mu_z | \mu_z''\rangle$ which, of course, refer to two quantum states, instead of N bundles. We indicated the conversion both ways in Eqs. (8.9) and (8.10),

$$\langle\ |\ |\ \rangle \to (\ |\ |\ ) \to \langle\ |\ |\ \rangle. \tag{8.12}$$

In the next section it will be shown how the bundle matrix elements $((\lambda_1) | \mu_z | (\lambda_2))$ may be used, by means of random-phased bundle amplitudes $((\lambda_2)_R | \mu_z^+\rangle$, to define averaged reduced quantities

$$((\lambda_1) | \mu_z | (\lambda_1)) \to \langle\mu_z^+ | \mu_z | \mu_z^+\rangle_{\text{red}} = \mu_{\text{eff}}, \tag{8.13}$$

which are identified with the effective values of μ_z for the quantum state $|\mu_z^+\rangle$.

As regards the definition of the bundle matrix elements $((\lambda_1) | \mu_z | (\lambda_2))$, we should be reminded that the definition (8.9a) and (8.9b) was given for a sheaf, i.e., for $((\boldsymbol{\zeta}_1) | \mu_z | (\boldsymbol{\zeta}_2))$. It was adopted in that form so that, with the choice (8.4a) and (8.4b) of the sheaf amplitudes $\langle\mu_z' | (\boldsymbol{\zeta}))$, the appropriate reconstruction of ordinary matrix elements $\langle\mu_z' | \mu_z | \mu_z''\rangle$ (8.10) resulted.

With the definition (8.11a)–(8.11c), of the bundle amplitudes $\langle\mu_z' | (\lambda))$, the possibility is given to extend the definition of sheaf matrix elements $((\boldsymbol{\zeta}_1) | \mu_z | (\boldsymbol{\zeta}_2))$ to bundle matrix elements $((\lambda_1) | \mu_z | (\lambda_2))$. We have simply to imagine that the sheaf generated by the flux loopforms of Fig. 5 is being literally subdivided into bundles which are subsets of sheaves $(\boldsymbol{\zeta})$, characterized by the parameters $(\alpha)(\sigma)$. The subdivision of a sheaf in regard to α is to be uniform. In regard to σ it is given by the second factor in Eq. (8.11c) where the fluxes Φ_λ are the fluxes corresponding to a magnetic dipole field, built up according to the prescription given in Fig. 1, and so as to represent the dipole field (4.6), in other words, so as to satisfy the Maxwell-Lorentz equations.

As a "bundle" is fairly large, in particular as regards the distribution over σ, one might for practical purposes work with 10^{-4} or 10^{-8} fractions of bundles to make approximate use of differential calculus, and use (8.11a).

This procedure, which is simply taking advantage of the extended-source picture of Fig. 5, permits us to define matrix elements not only for such "global" quantities as $((\lambda_1)|\mu_z|(\lambda_2))$ but also for bundle contributions to $V(t,x,y,z)$, $\mathbf{A}(t,x,y,z)$, i.e., to $((\lambda_1)|\mathbf{B}|(\lambda_2))$, etc., as far as contributions from the z component μ_z of the magnetic dipole field are concerned.

In this way we defined the bundle matrix elements, working backwards, i.e., $\langle\ |\ |\ \rangle \rightarrow (\ |\ |\ )$, which is the easiest way to do it and is proper because the Maxwell-Lorentz magnetic dipole field is to result from these flux bundles. We may now proceed to discuss also the forward approach of definition of bundle matrix elements $(\ |\ |\ )$ from the singular flux loopforms.

We propose to specify the bundle matrix elements

$$((\lambda_1)|\mathbf{A}|(\lambda_1)) = -(\hbar c/e)\int_{\lambda\subset(\lambda_1)}((\lambda_1)|\lambda) \times(\lambda|\nabla\vartheta|\lambda)(\lambda|(\lambda_1))d\lambda, \quad (8.14)$$

$$((\lambda_1)|V|(\lambda_1)) = +(\hbar c/e)\int_{\lambda\subset(\lambda_1)}((\lambda_1)|\lambda) \times(\lambda|\partial_0\vartheta|\lambda)(\lambda|(\lambda_1))d\lambda \quad (8.15)$$

[and, similarly, with integration over $\lambda\subset((\lambda_1)\cup(\lambda_2))$, for $((\lambda_1)|\mathbf{A}|(\lambda_2))$, etc.].

Several remarks relating to the multivaluedness of ϑ as function of x, y, and z should be made. There exists such a multivalued function ϑ which differs from loopform λ_1 to loopform λ_2; we tried to illustrate one such function ϑ in Figs. 2, 3, and 7. There does not, however, exist a function ϑ for a manifold of loopforms, neither for a bundle of loopforms nor for the entire lepton, because for every loopform the singularity (and branch) line is located somewhere else. But there exist bundle functions $\nabla\vartheta$ and $\partial_0\vartheta$, constructed from loopform fields $\nabla\vartheta$ and $\partial_0\vartheta$ by (8.14) and (8.15). The individual loopform's field $\nabla\vartheta$ is singular, like an irrotational streaming field with a vortex line; the superposed averaged $\nabla\vartheta$ for a bundle is a smooth rotational field.

We have to make some comments as regards the time derivatives. By $\partial_0\vartheta$ we always mean $(1/c)$ times the time rate of change of ϑ *at a particular location* x, y, z. In a primitive but useful way, we may designate a moving bundle by

$$(\tilde{\lambda}) = (\boldsymbol{\zeta})(\tilde{\alpha})(\sigma). \quad (8.16)$$

This means that the same value, $(\tilde{\lambda})$, i.e., the same $(\tilde{\alpha})$, is associated with one bundle as it spins, i.e., with the bundle of Fig. 3 illustrated there at three subsequent moments: The time rate of change of ϑ is discussed in that figure caption (it would be zero in a coordinate system rotating with a loop bundle).

At the end of Sec. V B we saw that the spinning angular velocity $2mc^2/\hbar$ is a quantity as fundamental as the quasi-nonlocality $\hbar/mc$ for a lepton of mass m, and that $\partial_0\vartheta$ and $\nabla\vartheta$ are interrelated; for a loopform or bundle,

$$\partial_0\vartheta = c^{-1}(\boldsymbol{\Omega}\times\mathbf{r})\cdot\nabla\vartheta. \quad (8.17)$$

Remembering (4.13), which states that the angular velocity of a loopform is in the $\pm$ direction of its flux orientation and is of size $2mc^2/\hbar$, we may consider $\partial_0\vartheta$ as a field which by (8.17) directly follows from the field $\nabla\vartheta$ of a loopform and may be averaged the same way as $\nabla\vartheta$ to obtain $\partial_0\vartheta$ for a bundle of loopforms.

Because of the multivaluedness of a loopform's field $\vartheta(t,x,y,z)$, an important property of the bundle-averaged field $((\lambda_1)|\nabla\vartheta|(\lambda_1))$ results (cf. caption to Fig. 3): Its time derivative, when averaged (averages taken over a period) and when there is stationarity of loopform distribution, becomes zero, and therefore

$$(\partial_0\mathbf{A})_{\text{eff}} = -(\hbar c/e) \times\langle\sum_{(\lambda)}\langle\mu_z{}^+|(\lambda))((\lambda)|\partial_0\nabla\vartheta|(\lambda))((\lambda)|\mu_z{}^+\rangle\rangle_{\text{av}} = 0. \quad (8.18)$$

We apologize for using formula (9.2) with (8.13) already here. On the other hand, however, the above-defined [(8.17) and (8.15)] field $\partial_0\vartheta(t,x,y,z)$ has a gradient (dependent on the distribution of the size of loopforms) which, in the equatorial plane of a loopform bundle [or anywhere for a distribution (4.16) of loopform bundles], points in a radial direction inward or outward for positive or negative leptons,

$$(\nabla V)_{\text{eff}} = +(\hbar c/e) \times\langle\sum_{(\lambda)}\langle\mu_z{}^+|(\lambda))((\lambda)|\nabla\partial_0\vartheta|(\lambda))((\lambda)|\mu_z{}^+\rangle\rangle_{\text{av}} \neq 0. \quad (8.19)$$

Finally, we may specify a more general and important alternative in denoting time dependence of loopforms and bundles of loopforms. Let us start with bundles. In the treatment given by Fig. 3 and Eq. (8.16), we took the attitude that a particular bundle is designated by $(\tilde{\lambda})$, spins and that accordingly ϑ is a function of t, x, y, z. It is, however, to be expected that the temporal behavior, besides a spin about the respective $\boldsymbol{\zeta}$ axes, indicated in Figs. 3 and 7, also consists in a change from one loopform of Figs. 9 to another loopform (cf. Fig. 12, below, and Sec. XIV). We may therefore prefer an alternative description in which the loopform bundles

$$(\lambda) = (\boldsymbol{\zeta})(\alpha)(\sigma) \quad (8.20)$$

designate bundles fixed in space, a description which resembles the gradual switching on and off of light bulbs on an electric sign bulletin (flash board for running news messages), in order to take care of more general motions.

B. Spin Amplitudes $\langle\boldsymbol{\mu}_z{}^+|\boldsymbol{\zeta})$

In analogy to $\langle\mu_z{}^\pm|\mu_\zeta{}^+\rangle$, Eq. (8.1), it was assumed in Eqs. (8.4a) and (8.4b) that the $\boldsymbol{\zeta}$ dependence of the amplitudes $\langle\mu_z{}^\pm|\boldsymbol{\zeta})$, apart from a phase factor, was given by $(1\pm\zeta_z)^{1/2}$. That this is the right choice might also be inferred from the spin model developed by Bopp and Haag[7] (cf. also Gel'fand, Minlos, and Shapiro[7]).

Bopp and Haag recognized that a spin model is not to be formulated in terms of ordinary spherical harmonics because those refer to the motion of a mass point around a center. A spin model is to be formulated in terms of generalized spherical harmonics which are eigenfunctions of the symmetric top, cf. the functions (8.21). A model of loopforms spinning with angular velocity $2mc^2/\hbar$ (8.22) might refer to the latter, certainly not to the former. The eigenfunctions of the spin angular momentum are

$$T^s{}_{mn}(\alpha,\theta,\beta)=e^{+im\alpha}P^s{}_{mn}(\cos\theta)e^{+in\beta}$$
$$s=\tfrac{1}{2},\quad |m|=\tfrac{1}{2},\quad |n|=\tfrac{1}{2}, \tag{8.21}$$

where the Euler angles β, θ, and α are our arccos $(-\zeta_y(\zeta_x{}^2+\zeta_y{}^2)^{-1/2})$, arccos ζ_z, and α, respectively. The $(1\pm\cos\theta)^{1/2}$ occurring in (8.21) appeared in the Eqs. (8.4a) and (8.4b); these expressions had been normalized to give Eq. (8.5) and do not yet contain the dependence of the amplitudes

$$\langle\mu_z{}^+|(\boldsymbol{\zeta})(\alpha)(\sigma))=\langle\mu_z{}^+|(\lambda))$$

on (α) nor on (σ).

The spin $\hbar/2$ requirement of Eq. (6.5) is evident whether one considers it a condition for a loop subject to (8.21) or simply a condition for the resultant structure, i.e., the electron or the muon, cf. the end of Sec. VI. A spinning motion about the flux orientation $\boldsymbol{\zeta}$ has a time dependence of the probability amplitude wave

$$\exp i(\alpha/2-(mc^2/\hbar)t)=\exp\tfrac{1}{2}i(\alpha-(2mc^2/\hbar)t). \tag{8.22}$$

We defined the quantum-mechanical matrix elements as averages of the type of (9.1) (they are written out explicitly in Sec. IX, but are referred to in the text several times in earlier sections). Corresponding averages (8.23) of the probabilities $\langle\,|\,)(\,|\,\rangle$ which fluctuate according to the large number N of bundles,

$$\langle\langle\mu_z{}^+|(\alpha)(\boldsymbol{\zeta})(\sigma))((\alpha)(\boldsymbol{\zeta})(\sigma)|\mu_z{}^+\rangle\rangle_{\mathrm{av}} \tag{8.23}$$

[no summation over $(\lambda)=(\alpha),(\boldsymbol{\zeta}),(\sigma)$], i.e., these averages of the highly fluctuating terms, correspond to the smooth spin $s=\frac{1}{2}$ eigenfunctions. The bundle probability amplitudes may thus be considered as representing the distribution of loopforms and their fluctuation in structuralized bundle language.

We want to discuss the transformation properties of (8.21) or a superposition of them, a sum over positive and negative m, n; we may first consider $m=+\frac{1}{2}$, $n=+\frac{1}{2}$. Let us consider what happens when complex conjugation is carried through. Complex conjugation acting on the field leptons' ψ functions implies [by the definition (3.3) of ϑ]

$$e^{i\vartheta}\longrightarrow e^{-i\vartheta} \tag{8.24}$$

and therefore, by (3.4), an inversion of the flux orientation $\boldsymbol{\zeta}$ and of the accompanying electromagnetic field at the origin (the spinning direction is, as we shall verify, not changed). Such an inversion at the origin is characterized by (cf. Fig. 4)

$$\alpha\longrightarrow-\alpha,\quad \theta\longrightarrow\pi-\theta,\quad \beta\longrightarrow\beta+\pi. \tag{8.25}$$

Complex conjugation acts also on the probability amplitudes, transforming

$$T^s{}_{mn}\longrightarrow T^s{}_{mn}{}^*. \tag{8.26}$$

Applying both (8.25) and (8.26),

$$\sum_{m,n} C_{mn}\exp(+im\alpha)$$
$$\times P^s{}_{mn}(\cos\theta)\exp(+in\beta)\exp(-i\omega t)$$
$$\longrightarrow\sum_{m,n} C_{mn}{}^*\exp(+im\alpha)$$
$$\times P^s{}_{mn}{}^*(-\cos\theta)\exp(-in(\beta+\pi))\exp(+i\omega t). \tag{8.27}$$

Considering for example $m=n=+\frac{1}{2}$, it is clear that this transformation does not change the direction of spinning,

$$\exp i(m\alpha-\omega t)\longrightarrow\exp i(m\alpha+\omega t), \tag{8.28}$$

because by definition of α, the spinning in the direction of increasing α, before the inversion, equals the spinning in the direction of decreasing α after the inversion of the predominant direction of $\boldsymbol{\zeta}$.

As pointed out before, the inversion of flux orientation $\boldsymbol{\zeta}$ by (8.27) with unchanged spin changes the signature of the electric field, i.e., of the charge. This transformation (8.27) is indeed the C transformation which leads from lepton to antilepton. Equation (8.27) shows that a lepton is not in an eigenstate of C.

It is interesting to study the C transformation of the product of lepton and antilepton:

$$C\{\sum_{n'n''} d_{n'n''}P^s{}_{m'n'}(\cos\theta')\exp i(m'\alpha'+n'\beta'-\omega t)P^s{}_{m''n''}{}^*(-\cos\theta'')\exp i(m''\alpha''-n''(\beta''+\pi)+\omega t)\}$$
$$=[\sum_{n'n''} d_{n'n''}{}^*P^s{}_{m'n'}{}^*(-\cos\theta')\exp i(m'\alpha'-n'(\beta'+\pi)+\omega t)P^s{}_{m''n''}(\cos\theta'')\exp i(m''\alpha''+n''(\beta''+2\pi)-\omega t)], \tag{8.29}$$
$$s'=s''=s=\tfrac{1}{2},\quad m'=m''=+\tfrac{1}{2}.$$

Apart from the irrelevant names (primed or double primed) of the variables θ, α, and β and apart from the factor $\exp(in''2\pi)=-1$ with n'' equal to $+\frac{1}{2}$ or $-\frac{1}{2}$, the { } and the [ ] are the same if

$$d_{n'n''}=d_{n''n'}{}^*,$$

e.g., $d_{+\frac{1}{2}+\frac{1}{2}}=1$, all others$=0$, or $d_{-\frac{1}{2}-\frac{1}{2}}=1$, all others$=0$, or $d_{+\frac{1}{2}-\frac{1}{2}}=d_{-\frac{1}{2}+\frac{1}{2}}=1/\sqrt{2}$, all others$=0$. They are the opposite of each other [again apart from the factor $\exp(in''2\pi)=-1$] if $d_{+\frac{1}{2}-\frac{1}{2}}=-d_{-\frac{1}{2}+\frac{1}{2}}=1/\sqrt{2}$, all others$=0$. Thus, as indeed to be expected,

$$\begin{aligned} C|\text{triplet lepton-antilepton}\rangle &= -|\text{triplet lepton-antilepton}\rangle \\ C|\text{singlet lepton-antilepton}\rangle &= +|\text{singlet lepton-antilepton}\rangle. \end{aligned} \tag{8.30}$$

IX. REDUCTION OF FIELD DUE TO AMPLITUDE SUPERPOSITION

We define reduction due to random-phased amplitudes in the following manner: Using Eqs. (8.9a) and (8.9b), we form the matrix elements $((\lambda_1)|\mu_z|(\lambda_2))$ which correspond to given ordinary quantum-mechanical matrix elements $\langle\mu_z'|\mu_z|\mu_z''\rangle$. We then define the reduced $\langle\mu_z'|\mu_z|\mu_z''\rangle_{\text{red}}$ (muon- or Bohr-magneton) as averages, by means of

$$\langle\mu_z'|\mu_z|\mu_z''\rangle_{\text{red}} = \langle \sum_{(\lambda_1)_R(\lambda_2)_R} \langle\mu_z'|(\lambda_1)_R)((\lambda_1)|\mu_{qz}|(\lambda_2))((\lambda_2)_R|\mu_z''\rangle\rangle_{\text{av}}, \tag{9.1}$$

where μ_{qz} represents the value of the unreduced magnetic moment (9.10). In averaging over phases, we only get contributions from $(\lambda_1)_R=(\lambda_2)_R$,

$$\langle\mu_z'|\mu_z|\mu_z''\rangle_{\text{red}} = \langle \sum_{(\lambda)} \langle\mu_z'|(\lambda))((\lambda)|\mu_{qz}|(\lambda))((\lambda)|\mu_z''\rangle\rangle_{\text{av}}. \tag{9.2}$$

Similarly we may define

$$\langle\mu_z'|\mu_z^2|\mu_z''\rangle_{\text{red}} = \langle \sum_{(\lambda)} \langle\mu_z'|(\lambda))((\lambda)|\mu_{qz}^2|(\lambda))((\lambda)|\mu_z''\rangle\rangle_{\text{av}} \tag{9.3}$$

$$= \langle \sum_{(\lambda_1)(\lambda_2)} \langle\mu_z'|(\lambda_1))((\lambda_1)|\mu_{qz}|(\lambda_2)) \times((\lambda_2)|\mu_{qz}|(\lambda_1))((\lambda_1)|\mu_z''\rangle\rangle_{\text{av}}, \tag{9.4}$$

$$(\mu_z^2)_{\text{eff}} \equiv \langle\mu_z^+|\mu_z^2|\mu_z^+\rangle_{\text{red}} \neq \{\langle\mu_z^+|\mu_z|\mu_z^+\rangle_{\text{red}}\}^2 \equiv ((\mu_z)_{\text{eff}})^2. \tag{9.5}$$

The matrix elements of (9.5) are not of the type of common quantum-mechanical matrix elements such as (8.10) and (9.10). In order to show the formation of (9.4) we recall

$$\begin{aligned} \langle\mu_z'|\mu_z^2|\mu_z''\rangle &= \sum_{\mu_z'''} \langle\mu_z'|\mu_z|\mu_z'''\rangle\langle\mu_z'''|\mu_z|\mu_z''\rangle \\ &= \sum_{\mu_z'''\mu_z''''(\zeta)} \langle\mu_z'|\mu_z|\mu_z'''\rangle\langle\mu_z'''|(\zeta)) \times((\zeta)|\mu_z''''\rangle\langle\mu_z''''|\mu_z|\mu_z''\rangle, \end{aligned} \tag{9.6}$$

the latter expression following from (8.5); we may calculate, with the help of (8.9a),

$$\begin{aligned} ((\zeta_1)|\mu_z^2|(\zeta_2)) &= \sum ((\zeta_1)|\mu_z'\rangle\langle\mu_z'|\mu_z|\mu_z'''\rangle\langle\mu_z'''(|\zeta)) \\ &\quad \times((\zeta)|\mu_z''''\rangle\langle\mu_z''''|\mu_z|\mu_z''\rangle\langle\mu_z''|(\zeta_2)) \\ &= \sum_{(\zeta)} ((\zeta_1)|\mu_z|(\zeta))((\zeta)|\mu_z|(\zeta_2)), \end{aligned} \tag{9.7}$$

which leads to (9.4).

We have already commented upon the average formations (9.2)–(9.4) which yield reductions. In those comments we took cognizance of the fact that, if the complex bundle amplitudes were all in phase, a physical quantity μ_z could be simply obtained from the quantized flux Φ_q, parcelled out in loopform bundles. Since the complex bundle amplitudes are not in phase, such a distribution of a random-phased distribution is expected to yield resultant probability amplitudes reduced by a factor $N^{-1/2}$.

We now discuss details of the reduction of linear versus quadratic field quantities. Let us first formulate the problem in terms of the reduction of magnetic moments.

The superposition of dipole fields of a point source as well as those of an extended source, their flux orientations $\boldsymbol{\zeta}$ distributed with probabilities $(1+\zeta_z)$, yields a resultant dipole field in the $+z$ direction, corresponding to a moment (9.14).

The z components of the superposed magnetic moments may simply be added, and so may the z components of the superposed core fields (designated by the projections Φ_z of the core fluxes).

We are now going to obtain the relationship between bundle flux, quantized flux, and effective flux. It is convenient to replace the summations over $(\boldsymbol{\zeta})$ or (λ), necessary in the subsequent calculations, by integrations over $d\zeta_z$. We assume that the same distribution of amplitudes over $\boldsymbol{\zeta}$ or λ holds for all size (outreach) parameters σ and we assume the distribution to be dependent on ζ_z only, independent of α. The square [cf. (8.11)]

$$|\langle\mu_z^+|(\lambda))|^2 = N^{-1}(1+\zeta_z) \tag{9.8}$$

can be interpreted as probability because of its unit norm; cf. (8.5). A summation may be replaced by

$$\sum_{(\lambda)} = N\int_{-1}^{+1} \tfrac{1}{2}d\zeta_z. \tag{9.9}$$

We naturally assume

$$\langle\mu_z'|\mu_{qz}|\mu_z''\rangle = \mu_q\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} \tag{9.10}$$

and obtain thereby, according to (8.9a) and (8.9b),

$$
\begin{aligned}
((\lambda)|\mu_{qz}|(\lambda))
&=\sum_{\mu_z'\mu_z''}((\lambda)|\mu_z'\rangle\langle\mu_z'|\mu_{qz}|\mu_z''\rangle\langle\mu_z''|(\lambda))\\
&=\mu_q[((\lambda)|\mu_z^+\rangle\langle\mu_z^+|(\lambda))-((\lambda)|\mu_z^-\rangle\langle\mu_z^-|(\lambda))]\\
&=\mu_qN^{-1}[(1+\zeta_z)-(1-\zeta_z)]=2\zeta_zN^{-1}\mu_q. \qquad (9.11)
\end{aligned}
$$

Correspondingly, a bundle contributes toward the z component of the coreflux the amount

$$
\begin{aligned}
((\lambda)|\Phi_{qz}|(\lambda))&\equiv((\zeta)(\alpha)(\sigma)|\Phi_{qz}|(\zeta)(\alpha)(\sigma))\\
&=2\zeta_zN^{-1}\Phi_q, \qquad (9.12)\\
((\lambda)|\Phi_q|(\lambda))&=2N^{-1}\Phi_q.
\end{aligned}
$$

This might be interpreted to mean that the quantized flux Φ_q is parcelled out among the N bundles, each having a flux $2N^{-1}\Phi_q$, the factor 2 arising because each of the two quantum states contributes. Equation (9.2) is the "inversion" of (9.11) and (9.12); using the square of (9.10), we get

$$
\begin{aligned}
((\lambda)|\mu_{qz}^2|(\lambda))&=\mu_q^2[|((\lambda)|\mu_z^+\rangle|^2+|((\lambda)|\mu_z^-\rangle|^2]\\
&=\mu_q^2N^{-1}\{1+\zeta_z+1-\zeta_z\}=2N^{-1}\mu_q^2. \qquad (9.13)
\end{aligned}
$$

Accordingly, by (9.2), (9.8), (9.11), and (9.13),

$$
\begin{aligned}
&\langle\mu_z^+|\mu_z|\mu_z^+\rangle_{\mathrm{red}}\\
&\quad=N\int_{-1}^{+1}\tfrac{1}{2}d\zeta_zN^{-1}(1+\zeta_z)2\zeta_zN^{-1}\mu_q=(2/3N)\mu_q, \qquad (9.14)\\
&\langle\mu_z^+|\mu_z^2|\mu_z^+\rangle_{\mathrm{red}}\\
&\quad=N\int_{-1}^{+1}\tfrac{1}{2}d\zeta_zN^{-1}(1+\zeta_z)2N^{-1}\mu_q^2=(2/N)\mu_q^2. \qquad (9.15)
\end{aligned}
$$

As (9.11) leads to (9.14), so (9.12) leads to

$$
\begin{aligned}
\Phi_{\mathrm{eff}}&\equiv\langle\mu_z^+|\Phi_z|\mu_z^+\rangle_{\mathrm{red}}\\
&=(2/3N)\Phi_q=\tfrac{1}{3}((\lambda)|\Phi_q|(\lambda)). \qquad (9.16)
\end{aligned}
$$

The left-hand side is evidently what we called Φ_{eff}, so that

$$\tfrac{3}{2}N=\Phi_q/\Phi_{\mathrm{eff}}=\mu_q/\mu_{\mathrm{eff}}. \qquad (9.17)$$

This result, (9.16) and (9.17), is self-evident, since the average value of ζ_z with probability $(1+\zeta_z)(d\zeta_z/2)$ is equal to $\frac{1}{3}$. Equation (9.17) is giving the relation between bundle flux, quantized flux, and effective flux. Equation (9.16) simply spells out the definition of the bundle flux $((\lambda)|\Phi_q|(\lambda))$. In fact, the "quantum-mechanical matrix element" (9.10) is not a measurable entity. Measurable is the superposed and averaged $\langle\mu_z^+|\mu_z|\mu_z^+\rangle_{\mathrm{red}}$, Eq. (9.14), constructed from (9.11).

The relations (9.12) and (9.16) have an intuitively interesting interpretation:

$$
\begin{aligned}
\text{magnetic bundle flux}&\equiv((\lambda)|\Phi_q|(\lambda))=2N^{-1}\Phi_q\\
&=3\langle\mu_z^+|\Phi_z|\mu_z^+\rangle_{\mathrm{red}}=3\Phi_{\mathrm{eff}}, \qquad (4.21)
\end{aligned}
$$

which we used in Sec. IV as Eq. (4.21).

TABLE I. Contributions of loopform bundles, of flux orientations $\boldsymbol{\zeta}$ shown in Fig. 1(C), towards the effective magnetic moment, if there are $N=207$ bundles. [Instead of those five rows, there are 207 rows (bundles) to be envisaged here.]

(A) $\equiv((\lambda)|\mu_{qz}|(\lambda))=2\zeta_z\mu_qN^{-1}$.
(B) $\equiv|\langle\mu_z^+|(\lambda))|^2=(1+\zeta_z)N^{-1}$.
(C) $\equiv\langle\mu_z^+|(\lambda))((\lambda)|\mu_{zq}|(\lambda))((\lambda)|\mu_z^+\rangle$.
(D) $\equiv\langle\langle\mu_z^+|\mu_z|\mu_z^+\rangle\rangle_{\mathrm{red}}=\sum_{\mathrm{all}\,(\lambda)}\langle\mu_z^+|(\lambda))((\lambda)|\mu_{qz}|(\lambda))((\lambda)|\mu_z^+\rangle$.

ζ_z	(A)	(B)	(C)	(D)
1.0	$0.00966\mu_q$	0.00966	$0.930\times10^{-4}\mu_q$	
0.5	$0.00483\mu_q$	0.00724	$0.350\times10^{-4}\mu_q$	
0	0	0.00483	0	$3.22\times10^{-4}\mu_q$ $=(2/3N)\mu_q$
−0.5	$-0.00483\mu_q$	0.00241	$-0.116\times10^{-4}\mu_q$	
−1.0	$-0.00966\mu_q$	0	0	

As (9.13) leads to (9.15), so

$$((\lambda)|\Phi_{qz}^2|(\lambda))=2N^{-1}\Phi_q^2, \qquad (9.18)$$

$$(\Phi^2)_{\mathrm{eff}}\equiv\langle\mu_z^+|\Phi_z^2|\mu_z^+\rangle_{\mathrm{red}}=(2/N)\Phi_q^2, \qquad (9.19)$$

i.e.,

$$\tfrac{1}{2}N=\Phi_q^2/(\Phi^2)_{\mathrm{eff}}. \qquad (9.20)$$

These calculations, all based on the derivation (8.9) of rounded matrix elements from the ordinary quantum-mechanical matrix elements, gives the following interesting relation. By (9.15) and (9.14),

$$
\begin{aligned}
(\mu^2)_{\mathrm{eff}}&\equiv\langle\mu_z^+|\mu_z^2|\mu_z^+\rangle_{\mathrm{red}}=2N^{-1}\mu_q^2\\
&=(9N/2)\{\langle\mu_z^+|\mu_z|\mu_z^+\rangle_{\mathrm{red}}\}^2\\
&\equiv(9N/2)(\mu_{\mathrm{eff}})^2, \qquad (9.21)
\end{aligned}
$$

and similarly

$$(\Phi^2)_{\mathrm{eff}}=(9N/2)(\Phi_{\mathrm{eff}})^2. \qquad (9.22)$$

In order to illustrate further the magnetic-moment or flux-reduction procedure, i.e., (9.14) or (9.16), we set it into the form of Table I (in which we have anticipated the number $N=120+87=207$ of bundles of loopforms, to be discussed in Sec. XI).

Because we want to estimate the electromagnetic energy and angular momentum, the question comes up how the reduction of fields **B** and **E** and bilinear terms formed from them are to be reduced.

In line with the reduction of magnetic momentum (9.2) one may assume

$$
\begin{aligned}
&\langle\mu_z'|\mathbf{B}|\mu_z''\rangle_{\mathrm{red}}\\
&\quad=\langle\sum_{(\lambda)}\langle\mu_z'|(\lambda))((\lambda)|\mathbf{B}|(\lambda))((\lambda)|\mu_z''\rangle\rangle_{\mathrm{av}}, \qquad (9.23)
\end{aligned}
$$

which is to give the field (4.6), cf. (8.11a) and (8.11b). The definition of $((\lambda)|\mathbf{B}|(\lambda))$ by (8.9a) and (8.9b) is clear cut, but it gives only that **B** contribution which arises from the z component of the bundle's magnetic dipole field; a look at (9.10) makes this evident. It is this contribution which, however, is only of interest for (9.23) because the **B** field from x and y components of magnetic moment cancel out. The full $((\lambda)|\mathbf{B}|(\lambda))$

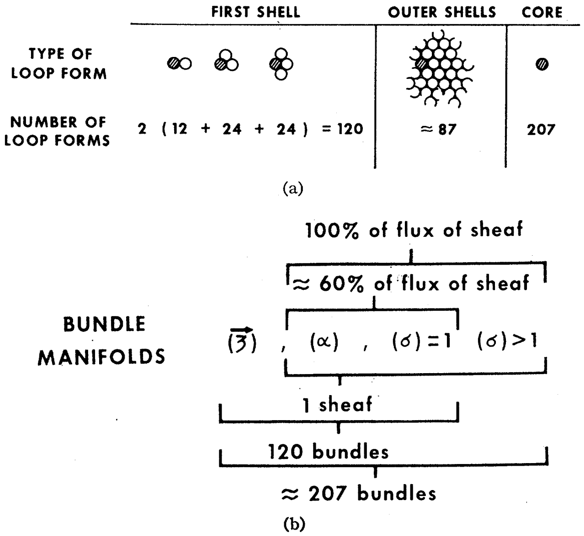

FIG. 11. (a) Types and numbers of loopform bundles. The 120 first-shell bundles are described in Figs. 8 and 9. The estimation of 207–120 = 87 outer-shell bundles is sketched in (b). (b) Bundle manifolds. Estimation of the total number, 207, of bundles of loopforms from the number of first-shell bundles, 120 (cf. Figs. 8–10), by the proportionality $60/100 \approx 120/207$ (cf. Fig. 5).

matrix elements may be considered as $((\lambda)|\mathbf{B}|(\lambda))$ = field of a dipole $2N^{-1}\mu_q$ oriented in $\boldsymbol{\zeta}$ direction, i.e., in agreement with (8.14).

For the electric field calculation we have, in Sec. IV, used the detailed distribution of flux orientation which was a simple procedure because the detailed (reduced) effective magnetic field had to be (4.6); this procedure was actually following the definition (8.15).

When it comes to calculation reduction of the B^2, E^2, $\mathbf{r}\times(\mathbf{E}\times\mathbf{B})$ fields, the straightforward calculations from (8.14) and (8.15) become much more tedious. We may perhaps oversimplify the discussion by simply taking the essence of the previous discussion, i.e., (9.22), and use it in the form

$$(B^2)_{\text{eff}} \approx (9N/2)(B_{\text{eff}})^2. \tag{9.24}$$

Consequently, the averaged quantities in (9.2), (7.3), (7.4), (9.23), and (9.24) are all being reduced in much the same manner compared with the nonreduced expression in (8.10) and the corresponding expression for $\mathbf{B}$, $\mathbf{E}$, and B^2, i.e., all of them by factors of the order of N^{-1}. $\mathbf{B}_{\text{eff}}$ and $\mathbf{E}_{\text{eff}}$ are related to muon- or Bohr-magneton μ_{eff} and charge e by Maxwell-Lorentz equations.

In order to form an average for $\mathbf{B}$ or B^2 at a region $P(r)$, we should sum over those loopform bundles (λ) which actually pass through that region. Only the large size loopforms contribute at distant regions (large r). In parcelling out the total (effective) flux into bundles of equal size, these distant regions are covered by only one or a fraction of one bundle. Consequently, as far as these faraway regions are concerned, there is no reduction and $(B^2)_{\text{eff}} \rightarrow (B_{\text{eff}})^2$. This, indeed, is nicely in accordance with the Maxwell-Lorentz theory which we expect directly to apply to B_{eff}, $(B_{\text{eff}})^2$ at large distances as quasi-nonlocal effects become insignificant at large r where $\mathbf{B}$ changes little over distances of the order of $\hbar/mc$.

As regards the flux which passes through the core, all loopform bundles are contributing to it; this summation over (λ) in (9.23) thus contains all (λ), thus we find that "full reduction" applies.

When we come to estimate the reduction factor applicable to electromagnetic energy and angular momentum, we should be aware of the fact that the major contributions toward these quantities come from the core and from "close-by" regions. Thus, but only approximately, we might start by summing over all (λ) in forming $(B^2)_{\text{eff}}$ which, together with the corresponding $(E^2)_{\text{eff}}(\mathbf{r}\times(\mathbf{E}\times\mathbf{B}))_{\text{eff}}$ formulas, determines energy and angular momentum. As, however, the reduction factors are quite sizeable numbers, such approximation is very crude. Only by taking into account the regional differences in superposition may we achieve a more reliable average reduction for energy and angular momentum [corresponding to (6.1), (6.4), and (6.5)].

In Sec. XII our calculations are done on the basis of the "full" reduction factor. In Sec. XIII we shall discuss the use of the term "regional reduction" for regionally differentiated consideration of amplitude superposition.

X. RECONSTRUCTION OF ELECTROMAGNETIC FIELD AND FINE-STRUCTURE CONSTANT

We reconstructed the magnetic (and thereby also the electric) field of a source lepton by superposition of bundles with random-phased (not in-phase) amplitudes, starting from a quantized flux

$$\Phi_q = 2\pi\hbar c/e \tag{10.1}$$

equally distributed over N bundles. Such an electromagnetic field is equivalent to the field of a muon or Bohr magneton $e\hbar/2mc$, and electric charge e [cf. (5.10) and (5.19a)],

$$\Phi_{\text{eff}} = 4\pi(e\hbar/2mc)/3.1r_0, \tag{10.2}$$

if $e^2/\hbar c$ has the appropriate magnitude.

Indeed, if we know the size of $r_0/(\hbar/mc)$ and the number N, the use of (9.17) with (10.1) and (10.2) yields

$$\tfrac{3}{2}N = \Phi_q/\Phi_{\text{eff}} = 3.1(\hbar c/e^2)[r_0/(\hbar/mc)]. \tag{10.3}$$

as the basic *equation which determines the fine-structure constant.*

With the adoption of $N \approx 207$ (11.1), and r_0 adopted as $0.73\hbar/mc$, (5.1b), one gets the right value for $e^2/\hbar c$. We may be reminded that the factor 3.1 resulted from a simple numerical calculation of the relation between magnetic moment μ_{eff}, magnetic flux Φ_{eff}, and root mean square r_0 of the Gaussian distribution.

As regards r_0, we have indicated its approximate size in Eq. (5.1) for the quasi-nonlocality of the source. This implies that with the angular velocity $2mc^2/\hbar$ of spinning, the outer regions of the core spin with a linear velocity of the order of c, a not unreasonable implication. This, again, as many other features of the present proposal, was anticipated by Slater's model.[6,7]

[6] In recognition of the difficulty (without use of complex probability amplitude superposition) of accommodating mc^2 and $\hbar/2$ as electromagnetic energy and electromagnetic angular momentum, Slater's model [J. C. Slater, Nature (1926)] suggested that the electromagnetic field responsible for the structure of an electron is the field of an electric point charge e and of a magnetic dipole $e\hbar/2mc$, but that this field be considered virtual and thus contributing neither to the energy nor to the angular momentum of the electron. On a circle of radius $r_0=\hbar/2mc$, the electric field intensity is then equal to the magnetic one, and the Poynting vector there fulfills the requirements to make it to represent a velocity (velocity c on a circular path). The field is thus considered to guide a light quantum orbiting with angular velocity $\Omega=c/(\hbar/2mc)=2mc^2/\hbar$. For the two-valued functions of a spinning electron, the lowest eigenvalue of the angular momentum is $\hbar/2$. Using the relativistic relationship, energy = momentum$\times c$, one gets $\hbar/2$ = angular momentum = $r_0\times$momentum, i.e., momentum = mc, and thus energy = mc^2. The latter part of Slater's argument differs slightly from this.

[7] Papers relating to general aspects of this problem and to previous work on it: F. Rohrlich, *Classical Charged Particles* (Addison-Wesley, Reading, Mass., 1965); A. O. Barut, *Electrodynamics and Classical Theory of Fields and Particles* (MacMillan, New York, 1964); M. S. Longuet-Higgins, in *Proceedings of the Thirteenth Symposium in Applied Mathematics, American Mathematical Society*, edited by G. Birkhoff, R. Bellman, and C. C. Lin (American Mathematical Society, Providence, 1962); Proc. Cambridge Phil. Soc. **52**, 234 (1956); **53**, 230 (1956). Classical theories are reviewed, besides in Rohrlich's book, *loc. cit.*, in the article by P. Caldirola, Nuovo Cimento Suppl. **3**, 297 (1956). Topological aspects are discussed by J. A. Wheeler, *Les Houches Notes, 1963* (Gordon & Breach, New York, 1964); *Geometrodynamics* (Academic, New York, 1962). Early attempts at understanding spin before Dirac, and in correspondence terms, are J. Frenkel, Z. Physik **37**, 243 (1926); *Wave Mechanics, Advanced General Theory* (Dover, New York, 1950), p. 321; H. J. Bhabha, and H. C. Corben, Proc. Roy. Soc. (London) **A178**, 273 (1941); H. C. Corben, Nuovo Cimento **20**, 529 (1961); Phys. Rev. **121**, 1833 (1961); **134**, B832 (1964); **145**, 1251 (1966); F. Bopp and R. Haag, Z. Naturforsch. **5a**, 643 (1950); H. Hoenl, Ann. Physik **33**, 565 (1938); H. Hoenl and A. Papapetrou, Z. Physik **112**, 512 (1939); J. W. Weyssenhof, Nature **141**, 328 (1938); P. Nyborg, Nuovo Cimento **23**, 47 (1962); H. C. Corben, *Classical and Quantum Theories of Spinning Particles* (Holden Day, San Francisco, 1968). An interesting model has been given as early as 24 April 1926 by J. C. Slater, Nature **117**, 587 (1926); J. Riess, Ann. Phys. (N. Y.) **57**, 301 (1970); Phys. Rev. D **2**, 647 (1970); R. Haag, Z. Naturforsch. **7a**, 449 (1952); E. P. Wigner, Am. J. Phys. **38**, 1005 (1970); L. DeBroglie, *Introduction to the Vigier Theory* (Elsevier, Amsterdam, 1963); K. Rafanelli and R. Schiller, Phys. Rev. **135**, B279 (1964); W. H. Bostick, *Dialogue on Flux Loops* (Stevens Institute of Technology, Hoboken, N.J., 1968); B. W. Wessel, Z. Naturforsch. **1**, 622 (1946); F. Boop, *ibid.* **3a**, 564 (1948); H. A. Kramers, Physica **1**, 825 (1934); *Quantum Mechanics* (Dover, New York, 1964); *Collected Papers* (North-Holland, Amsterdam, 1956); V. Bargmann, L. Michel, and V. L. Telegdi, Phys. Rev. Letters **2**, 435 (1959); A. E. Ruark (private communication); P. Havas, Phys. Rev. **74**, 456 (1948); J. Riess, Swiss Federal Institute of Technology, thesis, 1968 (unpublished); B. d'Espagnat and J. Prentki, Nucl. Phys. **1**, 33 (1956); R. J. Finkelstein, Phys. Rev. **75**, 1079 (1948); R. J. Finkelstein, R. LeLevier and M. Ruderman, *ibid.* **83**, 326 (1951); R. J. Finkelstein, C. Fronsdal, and P. Kaus, *ibid.* **103**, 1571 (1956); M. H. L. Pryce, Proc. Roy. Soc. (London) **168**, 389 (1938); T. Takabayasi, Prog. Theoret. Phys. Suppl. **21**, 339–82 (1965); Phys. Rev. **139**, B1381 (1965); V. Weisskopf, Rev. Mod. Phys. **21**, 305 (1949); CERN Report No. 62–15, 1962 (unpublished); W. Heisenberg, *Unified Field Theory of Elementary Particles* (Wiley, New York, 1967); P. Roman, Phil. Sci. **V**, 363 (1967); Phys. Rev. **158**, 1377 (1967); V. Bargmann, Rev. Mod. Phys. **34**, 829 (1962); Preuss. Akad. Berlin, 346 (1932); H. B. G. Casimir, *Rotation of a Rigid Body in Quantum Mechanics* (J. B. Wolters, Groningen, 1931); E. P. Wigner, *Group Theory* (Academic, New York, 1959); *Symmetries and Reflections* (Indiana U.P., 1967); M. H. L. Pryce, Proc. Cambridge Phil. Soc. **32**, 614 (1936); A. O. Barut, in *Lectures in Theoretic Physics*, edited by W. E. Britten *et al.* (Gordon and Breach, New York, 1968), Vol. XB, p. 377; L. H. Thomas (personal communication); L. Durand III, *Boulder Lectures* (Interscience, New York, 1962), Vol. IV, p. 524.

XI. ESTIMATE OF NUMBER N OF STATISTICALLY INDEPENDENT BUNDLES OF LOOPFORMS

We now estimate this number of "elementary bundles." This may be done in different ways. As indicated at the start of Sec. VII, the bundle manifold may be characterized by average flux orientation ($\boldsymbol{\zeta}$) of the bundle, average "size" (σ), i.e., average aphelion distance of the bundle of loopforms, and average azimuth (α) (cf. Figs. 1, 8, and 9). It is suggested that although closely similar loopforms ought to have phase-related amplitudes, these phase relationships give way to randomness when loopforms are sufficiently distinct as far as muons are concerned. It is suggestive to consider a difference of $\geqslant 1$ rad in azimuth α as criterion of sufficient difference to imply a random phasedness. Also, a difference $\geqslant 1$ rad in flux orientation $\boldsymbol{\zeta}$ may imply random-phase relationship. So one might subdivide the α, $\boldsymbol{\zeta}$ manifold into 6×20 bundles $(\alpha)(\boldsymbol{\zeta})$ (Fig. 8). The subdivision of "size" σ too, follows on the same principle (Fig. 10). The subdivision of the manifold of flux loopforms $\boldsymbol{\zeta}$, α, σ defines bundles $\{(\boldsymbol{\zeta})(\alpha)(\sigma)\}$. The consideration of Fig. 11(b) would result in a total number $2\pi\times 4\pi\times(100/60)\approx 132$ bundles, or $6\times 12\times(100/60)=120$ bundles if Fig. 8 were interpreted to mean that there are 12 essentially different $\boldsymbol{\zeta}$ directions corresponding to the 12 sides of a pentagon dodecahedron. But the angle between any two neighbors of those 12 directions is 1.425 rad. So we may better choose the 20 corner directions as essentially different, each characterizing a bundle's flux orientation for a given α and σ. These corners are 0.9763 rad$\approx$1 rad apart, as it should be. There result $6\times 20\times(100/60)=200$ bundles in that way of counting the $(\alpha)(\boldsymbol{\zeta})(\sigma)$ manifold of bundles.

Alternatively, we may consider a set of flux loopforms to be defined in the following way: Subdivide ordinary position space by drawing a close-packed set of spheres of radius r_{core} each, one of which represents the core. (These spheres may be called "elementary regions.") Then mark a consecutive set of these spheres in a manner resembling a closed string of beads, one of the beads representing the core. This loop of beads may then represent a set of flux loopforms, the size r_{core} of the beads representing the quasi-nonlocal aspect of the loopform when represented in ordinary position space; cf. Fig. 11(a).

An appropriate analysis of manifolds of loopforms defined by such loops of beads might correspond to some fluctuating form of the loop; the absolute values of the amplitude ("magnitudes") and the phase relationships of the amplitudes of the set of loopforms represented by the loop of beads are appropriately postulated so as to lead to a resultant superposed field which represents the electromagnetic field of the lepton; cf. Eq. (8.11b). Fluctuation effects should also be accounted for. It may be assumed, on the basis of the postulate that every bundle carries the same amount of flux, that the flux passing through an elementary region far from the source is equivalent to less than one elementary bundle.

We proceed now to use this picture in a much simplified manner to define bundles of loopforms. We first consider only loopforms which reach into the first shell, i.e., loopforms of aphelion distance $r_{\text{core}} \leqslant \sigma \leqslant 3r_{\text{core}}$. These loopforms are of the type of Figs. 9(a)–9(c), i.e., connecting the core with one or two or three elementary first-shell regions. There are 12 nearest-neighbor spheres to the core [Fig. 9(a)], 24 nearest-neighbor pairs [Fig. 9(b)], and 24 nearest-neighbor triplets [Fig. 9(c)].

Each of these $12+24+24=60$ loops of beads may be traversed by flux in one or the opposite direction, so that we count 120 essentially different sets of loopforms. [We count also the 12 cases of Fig. 9(a) doubly because they, like the others, should be able to make two alternative contributions to magnetic moment.] As regards distribution of flux over these different types of loops of beads, we assume that the above 120 each carry the same amount of flux and thus may be designated as the 120 elementary bundles of loopforms reaching into the first shell [see Fig. 11(b)].

Consider any one of these bundles. It is composed of a subset of loopforms of the type shown in Fig. 5 [a subset of the sheaf (ζ) of Fig. 5; the full sheaf (ζ) spans all azimuths α and sizes σ]—the subset of aphelion distance $r_c \leq \sigma \leq 3r_c$ and of azimuth confined to 1 rad or 60°. This subset carries 60% of the flux carried by the set of loopforms $r_c \leq \sigma \leq \infty$ and the same azimuth domain. [A numerical integration of the flux, or a counting of the flux lines between $r_c \leq \sigma \leq 3r_c$ shows this figure of 60%. The lines of Fig. 5 are drawn so that their density per unit $d\sigma$ (σ is in the direction perpendicular to ζ; σ is marked as $\mathfrak{r}$ in Figs. 5 and 6; σ is equivalent with the r in the equatorial plane as in Sec. V C) is proportional to σB, the distribution of lines over the angle azimuth α is a constant, independent of size; the lines in Fig. 5 thus represent torus surfaces which subdivide the total flux into ten equal parts.] Consequently, the flux carried by the 120 bundles is 60% of the total linear sum of flux (note that Φ_z at the core region is additive, the $\sum_{(\lambda')}\Phi_{(\lambda')}$ does not appear as resultant anywhere physically). As each bundle is assumed to carry the same amount of flux, there are $120/0.60 \approx 200$ bundles in all.

We make the assumption that a more appropriate or accurate way of counting may have resulted in

$$N \approx 207 \tag{11.1}$$

instead of the above number 200. We assume this because such a result may give us a possibility of representing the difference between muon and electron, as outlined in Sec. XIV and Fig. 12. We illustrate this distribution schematically with $(\sigma)=1$ representing the first shell and $(\sigma)>1$ the remainder shells, in Fig. 11(b) [$(\sigma)=1$ means loopforms of average aphelion size 1 in units of $2\hbar/mc$].

On the one hand, all loopforms which pass through a given sequence of elementary regions should have phase-related amplitudes. This gives an estimate for the minimum size of a bundle, i.e., a bundle of the type Figs. 9(a)–9(c). A bundle could, however, not well be defined larger than these 9(a)–9(c) bundles. On the other hand, for sizes beyond the first shell, $(\sigma)>1$, sets of loopforms which pass through a given sequence of elementary regions, carry less magnetic flux than each of the standard 120 bundles of the first shell; as the "bundles," by definition, each carry the same amount of flux, such sets cannot be considered as full-sized bundles, and we should not, therefore, consider the relative phases of their amplitudes as random, but adopt the argument of Sec. VII and Fig. 10. With Eqs. (5.1), Eq. (10.3) then yields the correct value of the fine-structure constant 1/137.

A better analysis of phase correlation of probability amplitudes of loopforms is obviously necessary both for the calculation of (11.1) and for the definition of phase-related and random-phased motion in Sec. XIV.

XII. CALCULATION OF EFFECTIVE FLUX, ENERGY, AND ANGULAR MOMENTUM FROM NUMBER OF BUNDLES OF STATISTICALLY INDEPENDENT LOOPFORMS

We ask how close we come to those desired results mc^2 and $\hbar/2$ if we choose for $r_0/(\hbar/mc)$ the value 0.73 of Eq. (5.1b); cf. also (10.3) and Ref. 6.

This is done in two steps: (1) In the present section we apply to the densities of electromagnetic field energy and angular momentum everywhere the same reduction factor as should be used in the core region only, whereas (2) in reality the reduction factor goes towards unity for the far-away regions (Sec. XIII).

Neglecting thus, in this section, regional differentiation of the averaging of electromagnetic energy and angular momentum, we use the same full superposition and full reduction everywhere, though it only applies to the core region.

The random phasedness (lack of in-phase relationship) of the complex amplitudes, affecting B^2, E^2, and $\mathbf{E}\times\mathbf{B}$, dictates that we use Eqs. (9.24) on (6.2) and (6.3). Consequently, we get the following averages:

electromagnetic energy

$$\begin{aligned}
&= \tfrac{9}{2}N\{0.138[r_0/(\hbar/mc)]^{-3} \\
&\qquad +0.365[r_0/(\hbar/mc)]^{-1}\}(e^2/\hbar c)mc^2 \\
&= \tfrac{9}{2}\times 207\{0.138/0.73^3 \\
&\qquad +0.365/0.73\}(1/137)mc^2 \\
&= 931\{0.352+0.430\}(1/137)mc^2 = 5.3mc^2,
\end{aligned} \tag{12.1}$$

the first and second term referring to magnetic and electric energy;

$$|\text{angular momentum}| = \tfrac{9}{2}N\{0.47[r_0/(\hbar/mc)]^{-1}\}(e^2/\hbar c)\hbar/2 = 931\{0.47/0.73\}(1/137)\hbar/2 = 4.4\hbar/2. \quad (12.2)$$

The value $\hbar/2$ Eq. (6.5) would be desirable for (12.2) so that the spin $\hbar/2$ of the Bopp-Haag model would be compatible with this electromagnetic term.

The substantial discrepancies from the values mc^2 and $\hbar/2$, respectively, in the above are hardly prohibitive in view of the big difference between average and full reduction factors. A more refined calculation for the values of (12.1) and (12.2) may be made, of course, by estimating the regionally differentiated reduction, instead of replacing it simply by one single (full) reduction as was done here. Using this refined procedure, we find the values 5.3 in (12.1) and 4.4 in (12.2) will decrease quite drastically, although, since there is full reduction at the core, (10.3) does not change.

It may be interesting to note that, in order to reverse the electromagnetic angular momentum, one has to reverse the electric field with respect to the magnetic field, and that is achieved by reverting the sense of the spinning angular velocity with respect to the orientation of the magnetic moment, as indeed it should be.

XIII. REGIONALLY DIFFERENTIATED AMPLITUDE SUPERPOSITION

We will now digress by considering regionally differentiated amplitude superposition and reduction.

At the core and adjacent regions, a large number of bundles of loopforms contribute the magnetic field. Further out, only a few standard bundles contribute to the magnetic field (in accordance with our assumption that each such bundle carries equal amounts of effective magnetic flux). Thus, *as expected*, there will be strong reduction at the core and close to it, but not much reduction further out. The assumption that the Maxwell-Lorentz theory be applicable at large distances from the core, i.e., without quasi-nonlocality and complex amplitude superposition having any appreciable effect, renders this difference in reduction not only expected but *necessary*: At sufficiently large distances, the Maxwell-Lorentz fields $\langle\mathbf{B}\rangle_{\text{av}}$ and $\langle\mathbf{E}\rangle_{\text{av}}$ of Eqs. (7.3) and (7.4) should, as in (6.2) and (6.3), give the energy and angular momentum densities by the Maxwell-Lorentz expressions (6.2) and (6.3). However, this can only be the case if there is no reduction; that is, if at a distant region one or less than one bundle contributes to the magnetic field.

By recalling the way in which reduction comes into effect, we may perhaps gain further insight into the calculation of regionally differentiated reductions. We normalize the complex bundle amplitudes by

$$\sum_{\text{all }(\lambda)} \langle\mu_z{}^+|(\lambda))((\lambda)|\mu_z{}^+\rangle = 1. \quad (8.5)$$

This normalization applies whether the amplitudes are random phased (muon) or phase related (electron); they are not in phase, however. In Eqs. (9.1), (9.2), (9.23), (7.3), and (7.4), we see how the reduced ("average") fields are defined. The reduction is obtained by dropping off-diagonal terms $(\lambda_1)\neq(\lambda_2)$ in (9.1), whereas in the nonreduced expression (8.10) these terms are retained.

Let us consider a certain region $P(\mathbf{r})$ and the subset of the bundles (λ) which contributes toward the fields there. We may determine this subset by considering the spacial coverage of the fields $((\lambda)|\mathbf{B}|(\lambda))$ in (9.23). For example, we might consider $(\text{curl}\mathbf{A})_{(\lambda)}$ in (7.3) and $(\text{grad}V)_{(\lambda)}$ in (7.4). When we sum over (λ) in (9.23), (7.3), and (7.4), we are, consequently, taking a summation over this subset. The reduction expresses itself when we sum only the diagonal terms (λ), i.e., $(\lambda_1)=(\lambda_2)$.

We remarked that as regards the reduction of Φ_q or μ_q to coreflux,

$$\langle\mu_z{}^+|\Phi_z|\mu_z{}^+\rangle_{\text{red}} \quad (9.16)$$

or

$$\langle\mu_z{}^+|\mu_z|\mu_z{}^+\rangle_{\text{red}}, \quad (9.14)$$

respectively, we have to consider superposition of their (additive) z components at the core region. As this superposition involves all flux loopforms, we have for both cases Eq. (10.3) as the relation determining the reduction. The regional reduction of the electromagnetic field is characterized by the remark given above, i.e., that the reduced $\mathbf{B}$ field is again a divergenceless field as was the bundle field $((\lambda)|\mathbf{B}|(\lambda))$.

As regards the reduction of electromagnetic energy and angular momentum, we can, at this time, only present a very crude, qualitative estimate.

The magnetic energy of a magnetic field corresponding to a Gaussian distribution of magnetization,

$$\iiint_0^\infty (B_{\text{eff}})^2(8\pi)^{-1}d^3r = 0.138[r_0/(\hbar/mc)]^{-3}(e^2/\hbar c)mc^2, \quad (6.2)$$

had its numerical factor 0.138 additively arising from contributions of the core (0.109), the first shell (0.026), and the outer shells (0.003).

The corresponding electric field energy

$$\iiint_0^\infty (E_{\text{eff}})^2(8\pi)^{-1}d^3r = 0.365[r_0/(\hbar/mc)]^{-1}(e^2/\hbar c)mc^2 \quad (6.2)$$

comes from the core (0.0718), the first shell (0.1290), the second shell (0.1075), and outer shells (0.0565).

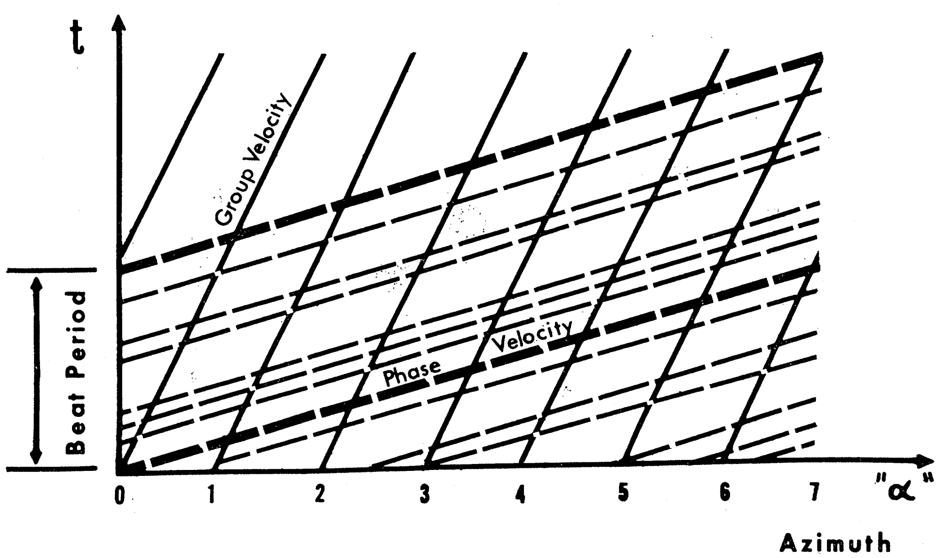

FIG. 12. The motion of probability waves (squares of probability amplitudes) through the manifold of 207 bundles of loopforms $(\lambda)\equiv(\zeta)$, (α), (σ) is supposed to be quasi-ergodic, a wave crest of a probability wave moving periodically through that parameter space λ. In this figure the motion of probability waves, through this four-parametric manifold (λ), is symbolically indicated as a motion through a one-dimensional azimuth space "α" ranging from zero to 207 rad (only 7 rad are drawn in the figure). For the muon there are 207 randomly distributed wave crests (represented in the figure by only seven dashed lines). Each wave crest, considered as a pulse of duration $(2\pi\Omega_\mu)^{-1}$, gives rise to the Fourier term of angular frequency $2\pi\Omega_\mu$. The motion of the wave crests is assumed to be quasi-ergodic with periodicity indicated as "beat period." When a muon is about to decay into an electron, the formerly stationary random time sequence of wave crests may, because of non-linearities in the dynamics of the loop motion, bunch together, create beat periods, and again, because of nonlinear effects, excite probability amplitude waves of frequency equal to that beat frequency (which is 1/207 of the muon frequency, i.e., $2\pi\Omega_\mu/207$ in rad per second). Thus the wave-crest picture of electron waves emerges (full drawn lines) with again 207 crests along the parameter "α" but with smaller angular velocity and larger period. The linear velocities of the loopforms are of the order of c, both in the case of muon and in the case of electron, the muon core being 1/207 of the size of the electron core.

The construction of this diagram results from the following simple considerations. From "α"$=0$, $t=0$, one draws a group velocity line (electron) and a (heavy, dashed, 207 times larger) phase velocity line (muon). For "α"$=0$ the group wave crests are located at $t=0$ and at integer multiples of one beat period. In each beat-period interval there are 207 randomly spaced short-period phase wave crests. These phase wave crests are assumed to pass, in a quasi-ergodic fashion, through the entire "α" space in a time equal to the repeat period of the 207 wave crests, i.e., the beat period. The phase velocity line which passes through "α"$=0$, $t=0$, reaches "α"$=1$ at a time $1/\Omega_\mu$, the beat period is accordingly $207/\Omega_\mu$, and the angular beat frequency is $2\pi\Omega_\mu/207$, the muon waves have in their Fourier distribution angular frequencies $2\pi\Omega_\mu$.

The electromagnetic angular momentum

$$\left|\int\int\int_0^\infty \mathbf{r}\times(\mathbf{E}_{\text{eff}}\times\mathbf{B}_{\text{eff}})(4\pi c)^{-1}d^3r\right| =0.47[r_0/(\hbar/mc)]^{-1}(e^2/\hbar c)\hbar/2 \quad (6.3)$$

comes from the core (0.102), the first shell (0.183), the second shell (0.106), and the outer shells (0.075).

At the core the superposition occurs from all $N=207$ bundles. At the first shell the superposition may be estimated to be from about $207/6\approx35$ bundles because, even though loopform bundles of all sizes (σ) and all flux orientations (ζ) pass through a given region, i.e., one of the spheres of the first shell, only one of the six azimuth regions contributes to a particular given spatial region. This number 35 may be a very crude estimate. Still more crude, but less important, is an assignment of six bundles to regions of the second shell and one bundle to regions of outer shells.

With such numbers, i.e.,

	Total	Core	First shell	Second shell	Outer shell
Number	207	207	35	6	1
Magn. energy	0.138	=0.109	+0.026	+0.003	+0.000
Electr. energy	0.365	=0.0718	+0.1290	+0.1075	+0.0565
Ang. mom.	0.47	=0.102	+0.183	+0.106	+0.075

and with locally differentiated reduction factors $(9N_l/2)$, we get, instead of (12.1),

$$\tfrac{9}{2}\{(207\times0.109+35\times0.026+6\times0.003)[r_0/(\hbar/mc)]^{-3} +(207\times0.072+35\times0.129+6\times0.108+1\times0.056) \times[r_0/(\hbar/mc)]^{-1}\}(e^2/\hbar c)mc^2=[(105.7/0.73^3) +(90.4/0.73)](e^2/\hbar c)mc^2=2.88mc^2, \quad (13.1)$$

where we have used $e^2/\hbar c=1/137$ and $r_0=0.73\hbar/mc$. Similarly, instead of (12.2), we get

$$\tfrac{9}{2}(207\times0.102+35\times0.183+6\times0.106+1\times0.075) \times[r_0/(\hbar/mc)]^{-1}(e^2/\hbar c)\hbar/2=[126.8/0.73] \times(e^2/\hbar c)\hbar/2=1.26\hbar/2. \quad (13.2)$$

These numbers for regionally reduced energy and angular momentum, compared with $5.3mc^2$ and $4.4\hbar/2$, obtained with the oversimplified full reduction of $N=207$ bundles, show that we arrive at a reasonable interpretation of mc^2 and $\hbar/2$ as the electromagnetic energy and angular momentum, even though the numerical calculations are very crude.

XIV. MUON VERSUS ELECTRON

We may attempt to describe a distribution of loopform amplitudes, $\phi\equiv(\lambda|\mu_z{}^+)$, by a Fourier expansion in terms of the loopform parameters $\lambda\equiv\{\zeta,\alpha,\sigma\}$. We shall use $\alpha=\tilde{\alpha}+\Omega t=$ azimuth in the laboratory system rather than $\tilde{\alpha}=$ azimuth in a rotating coordinate system; i.e., we employ the representation resembling a graduals witching on and off of light bulbs on a flash board for running news messages, the bulbs' light intensities characterizing the probabilities, i.e., the bilinear expressions $|\phi|^2$.

We are here discussing the loopform amplitudes which are highly fluctuating (in time and space) quantities, unlike the averages (8.23) which correspond to the smooth spin functions $T^s{}_{mn}$, $s=\frac{1}{2}$.

Figure 12 illustrates the motion of wave crests described in the figure caption; it should be studied here.

At any time, e.g., $t=0$, the probability distribution $|\phi|^2=|(\lambda|\mu_z{}^+)|^2$ is expected to have some 207 wave crests. In the case of the muon, because of the random phasedness of its probability amplitudes, these wave crests are somewhat randomly distributed over that parameter space $\lambda=\{\zeta,\alpha,\sigma\}$. In the case of the electron, because of the phase relatedness of its probability amplitudes, the distribution of 207 wave crests is regular, i.e., periodic in α, one for each of the 207 bundles,

$$(\lambda)\equiv\{(\zeta),(\alpha),(\sigma)\}.$$

Such distributions of wavecrests correspond to the aforementioned distribution of the phases of probability amplitudes which are expected to lead appropriately to the same reduction procedure (Sec. IX) for muon and for electron alike. This is so because, for the electron as well as for the muon, the phases of the probability amplitudes are different at the loci of different bundles (λ); the off-diagonal terms $(\lambda_1)\neq(\lambda_2)$ in (9.1) are therefore expected to contribute nothing in the average.

A wave crest in the four-dimensional parameter space ζ, α, σ may be considered a three-dimensional hypersurface to which there is a field of normals characterizing the propagation of the wave crest. That propagation vector field[8] may, in the case of the muon, be assumed to pass in a kind of quasi-ergodic motion through that parameter space. We consider "quasi-" ergodicity to mean that this motion is defined only in its gross aspects, as it marks out the sequence of discrete $(\zeta)(\alpha)(\sigma)$ bundles (considering the finer specification in terms of the continuum $\zeta\alpha\sigma$ irrelevant as regards the completion of a quasi-ergodic cycle).

As each bundle, as shown in Sec. VII and Fig. 10, "carries" the same amount of flux, this quasi-ergodic motion is appropriately formulated in terms of the manifold of the 207 equally probable bundles $(\lambda)\equiv\{(\zeta)(\alpha)(\sigma)\}$. For numerical calculations it may be easier to use 1/10,000 of a bundle (subdividing each parameter unit into ten subunits). To make another crude simplification, we might introduce a parameter "α" ranging from zero to about 207 rad, by stringing the α intervals of each of the $(\zeta)(\sigma)$ together so that the quasi-ergodic motion of one of those normal vectors (to a wave crest) following its path as a function of time might be visualized as moving linearly (in time) through that interval $0\leq$"α"≤ 207 instead of the complicated random wave motion through the space $\lambda\equiv\{\zeta\alpha\sigma\}$ [cf. Fig. 12, which indicates the wave crests corresponding to that path by dashed lines which may be more irregularly distributed, but with the inclination $\Omega_\mu{}^{-1}=(2m_\mu c^2/\hbar)^{-1}$ shown there].

Figure 12 shows "α" as ranging from zero to seven only, because it would not be easy to draw the whole range zero to 207. The assumption of ergodicity means that the figure is to be considered repetitive in the time direction, with the indicated period; a kind of beat period. When we study the frequency distribution corresponding to such muon waves, we find that the probability amplitudes (as functions of t) have in their Fourier distribution a characteristic angular frequency approximately equal to $2\pi\Omega_\mu=2\pi\times 2m_\mu c^2/\hbar$, corresponding to the width (in time) of each of the pulses [assumed to be approximately equal to $(2\pi)^{-1}$ of the average spacing $1/\Omega_\mu$ of the probability crests], and its overtones. It is evidently the width of the standard pulse (switched-on light bulbs),

$$|\phi(t-\tau)|^2=\sum_\kappa |\psi(\kappa)|^2 \exp i\{\kappa\text{“}\alpha\text{”}-\omega_\kappa(t-\tau)\},$$

$$\kappa_{\mathrm{av}}=\text{small number},\quad \kappa=n2\pi,$$

to which the frequency Ω_μ corresponds. The randomly spaced 207 pulses

$$\chi(\tau)=\sum_{i=1}^{207}\delta(\tau-\tau_i)$$

(a stationary random sequence of pulses, i.e., a Poisson process) give a resulting amplitude in terms of the

[8] H. Hurwitz and M. Kac, Ann. Math. Stat. **15**, 173 (1944); A. Papoulis, *Probability, Random Variables and Stochastic Processes* (McGraw-Hill, New York, 1965); L. Brillouin, *Wave Propagation and Group Velocity* (Academic, New York, 1960); E. Nelson, *Dynamical Theory of Brownian Movement* (Princeton U.P., Princeton, 1967); N. Wiener, *Nonlinear Problems in Random Theory* (MIT, Cambridge, Mass., 1958); A. M. Yaglom, *An Introduction to the Theory of Stationary Random Functions*, translated by R. A. Silverman (Prentice Hall, Englewood Cliffs, N. J., 1962); S. Goldman, *Frequency Analysis, Modulation and Noise* (McGraw-Hill, New York, 1958); D. Middleton, *An Introduction to Statistical Communication Theory* (McGraw-Hill, New York, 1960).

convolution

$$|\phi(t)|^2=\int|\phi(t-\tau)|^2\chi(\tau)d\tau=\sum_i|\phi(t-\tau_i|^2$$

(again with the same characteristic angular frequencies $2\pi\Omega_\mu$ and their overtones in the Fourier distribution).

The actual situation may, however, imply bunching and different sized pulses instead of random-phase distribution of standard pulses, so that the Fourier analysis of $|\phi(t)|^2$ may resemble amplitude modulation, or $\phi(t)$ may resemble beat patterns, with "beat period" equal to the period of quasi-ergodic passage through the 207 bundles, i.e., a "beat period" $=207/\Omega_\mu$ (which corresponds to an angular frequency $2\pi\Omega_\mu/207$, i.e., $\approx 1/207$ of the above angular frequency previously discussed).

There are thus in the case of the muon two (angular) frequencies, $2\pi\Omega_\mu$ and $2\pi\Omega_\mu/207$, the latter may occur in the Fourier analysis of $|\phi(t)|^2$, not in that of $\phi(t)$, and it may have interesting consequences when nonlinearity is assumed to play a role in the dynamics of loopform (nonlinearities may arise when an "interaction" between loops is assumed, e.g., a tendency for loopforms to stay apart from each other or to bunch together). This low "beat period" of the muon $|\phi(t)|^2$ is presumably only a potentiality; its relevance stems from the presence of nonlinear effects like bunching of loopforms. In actuality, as long as the muon remains as a muon, the beat phenomenon might not be present, but during the decay, it may arise as a mixture of beat periods of the order of $207/\Omega_\mu$.

We thus assume that this potentiality of an angular frequency $2\pi\Omega_\mu/207$ of $|\phi_\mu(t)|^2$ may express itself when the muon decays into an electron. We may assume that when random phasedness gives way to phase relatedness (with a sharp single beat frequency) the transition to electron occurs. The thereby induced, phase-related, $\phi_e(t)$ waves are strictly periodic in time and show one single angular frequency $2\pi\Omega_e$ which may be equal to $2\pi\Omega_\mu/207$. Without knowing how the equations of motion for loopforms would have to be written, one may suggest that nonlinear effects cause the muon beat frequency of $|\phi_\mu(t)|^2$ to generate an electron frequency of $\phi_e(t)$ and thus to bring about

$$2\pi\Omega_e=2\pi\Omega_\mu/207\,.$$

In other words we presume that the metastable spinning with the regular muon frequency yields to the other, stable, electron type spinning whose frequency is equal to the beat frequency of the muon.

As we have indicated in the beginning of this section, the electron's ϕ_e distribution, expressed as a function of the parameters $(\lambda)\equiv\{(\zeta),(\alpha),(\sigma)\}$, is to have as many (207) crests of $|\phi_e(\lambda)|^2$ as the muon's $|\phi_\mu(\lambda)|^2$, because these two leptons are alike and subject to the same reduction procedure (Sec. IX).

Accordingly the wave-crest picture of Fig. 12 results, with the "beat period" being the period of quasi-ergodic motion through the 207 bundles. (The electron wave crests assume a velocity in the diagram which may in some sense be interpreted as a muon transition group velocity in relation to the muon crest velocities interpreted as phase velocity.) As we are well advised to consider the linear velocity of spinning loopforms to be of the order of c for muon and electron alike, the electron with its 207 times smaller frequency is 207 times larger than the muon, and its angular spinning velocity Ω_e is 207 times smaller than that of the muon.

As these angular frequencies $2\pi\Omega_\mu$ and $2\pi\Omega_e$ are frequencies of the probability amplitude waves $\phi_\mu(t)$ and $\phi_e(t)$, respectively, we may conclude the mass ratio m_μ to m_e to be as Ω_μ to Ω_e, i.e., about 207 to 1.

It should be noted that even though the reduction formalism of Sec. IX ff. was developed on the assumption of randomphasedness of the amplitudes $\langle\mu_z'|(\lambda)_R)$ in (9.1) and thus directly relates to the muon, this reduction formalism also relates to the electron. Indeed, $((\lambda_1)|\mathbf{B}|(\lambda_2))$ are, as indicated at (8.14) and (8.15), defined as time-independent quantities, whereas the $\langle\mu_z'|(\lambda))$ entering into

$$\langle\sum_{(\lambda_1)(\lambda_2)}\langle\mu_z'|(\lambda_1))((\lambda_1)|\mathbf{B}|(\lambda_2))((\lambda_2)|\mu_z''\rangle\rangle_{\text{av}}$$

bring phase factors into this double sum which thus, in the average, reduces to a diagonal single sum of type (9.2).

It should finally be noted that the introduction of a fundamental length instead of the relationship $r_0\approx\hbar/mc$ (5.1) would not have satisfied a model appropriate for the electron as well as the muon.

XV. RÉSUMÉ

In Sec. III we defined quantized flux, and showed how to avoid magnetic monopoles and how to represent the magnetic field of a lepton (muon or electron) entirely in terms of quantized flux. This was done by assuming that one quantized flux loop belongs to one source lepton but that this flux loop may adopt alternative forms (e.g., the forms of magnetic dipole field lines). Each flux loopform has a complex probability amplitude attached to it in a manner similar to Feynman's reconstruction of a quantum-mechanical path from a superposition of alternative path histories.

In Sec. IV we showed how an electric field follows from flux quantization when the magnetic loopforms are in motion, spinning with angular velocity $2mc^2/\hbar$ around their flux-orientation axes ζ (cf. Figs. 1 and 3). It is seen there that if the magnetic field corresponds to one muon magneton or one Bohr magneton, the resulting electric field is isotropic and of the strength of the Coulomb field for either muon or electron. This simply corresponds to the fact that, by Dirac's theory, the magnetic moment $e\hbar/2mc$ follows from the charge e of an electron.

In Sec. V we started pointing out the inherent difficulties of a point-source model. Recognizing that the attachment of loopforms to a source implies the single-particle picture of that source, described in terms of "mean position," this source is then, by the Pryce-Foldy-Wouthuysen transformation, nonlocal in terms of ordinary position. As it is only this particular kind of representation which makes the source lepton appear to be nonlocal (of an extent $\hbar/mc$), we called this "quasi-nonlocality." It is presumably a very difficult task to properly take account of quasi-nonlocality in a theory of quantized flux loopforms. A grossly simplified model of an extended source is substituted for a rigorous theory (cf. Figs. 5 and 6). For such model again the electric field is isotropic. Appendix II gives expressions for magnetic flux, electromagnetic energy, and electro-magnetic angular momentum for an extended source model.

In Sec. VI it is seen that an unsophisticated approach to the question of effective field, effective energy, and effective angular momentum will not be compatible with the basic facts about muons or electrons. (For example, the product of the quantized flux divided by 4π, and the size of quasi-nonlocality, is two orders of magnitude larger than the actual magnetic moment for either muon or electron.)

Section VII then raises the question of how a complex amplitude superposition may be defined. This section notes the parameters which characterize the flux loopforms (Figs. 8 and 9) and groups the loopforms together into a discrete manifold of bundles of loopforms. It is shown how the electric and magnetic fields may be defined in terms of superposition of bundle fields (7.3) and (7.4).

Section VIII develops the formalism for the description of a quantum state $|\mu_z^+\rangle$ in terms of the (underdetermined) set of complex bundle amplitudes $|(\lambda))$.

Section IX gives the expressions for linear ($\mathbf{B}$) and quadratic ($\mathbf{B}^2$) field quantities, as averages formed from complex probability (bundle) amplitudes. It is discussed what this "reduction" implies for the field close to and far away from the source lepton. Explicit formulas for the reduced magnetic moment are given.

As the preceding section has given a relation between quantized flux and effective flux (in terms of the numbers N of bundles and the extension r_0 of the quasi-nonlocality), it is then recognized in Sec. X that this reconstruction of the magnetic field from quantized flux is compatible with the flux of a muon or Bohr magneton only if $e^2/\hbar c$ has the right order of magnitude. In other words, we arrive at an approximate estimate of the value of $e^2/\hbar c$. To get to a numerical estimate of that, we have to know both N and r_0.

In Sec. XI a count of the number N of "statistically independent bundles" of loopforms is made.

The purpose of Sec. XII is then to show that with the values $N\approx 207$, $r_0\approx 0.73\hbar/mc$, the electromagnetic energy is indeed of the order of magnitude mc^2, and the electromagnetic angular momentum of the order of $\hbar/2$.

Section XIII tries tentatively to take into account that at regions close to the source and regions far from the source the fields result from quite different superpositions of bundles. Regionally differentiated superposition might specify this more detailed aspect of superposition. One might, thereby, arrive at a better evaluation of energy and angular momentum.

Section XIV and Fig. 12 suggest the difference between muon and electron to lie in amplitude distributions being random phased and phase related, respectively. The decay into an electron (and neutrino) may be initiated by phase relatedness of these probability amplitudes setting in, and beat frequencies (of the bilinear probabilities) arising. Nonlinear effects may then cause these muon beat frequencies to excite the low frequencies of the electron's probability amplitudes.

The complex probability amplitude distribution of the loopforms characterizes the internal configuration of the lepton; the fluctuations of this distribution have a structure which is represented by the discrete bundles of loopforms.

Evidently the problem ahead is this: How can the Pryce-Foldy-Wouthuysen type (*Zitterbewegung* caused) quasi-nonlocality be applied in a manner to permit the structuralized *Zitterbewegung* fluctuations to be properly formulated? With the present substitute formulation in terms of an extended source one cannot, of course, expect to obtain accurate numerical results. In two instances we have accordingly made adjustments in order to obtain expected numerical answers: in Sec. XI, Eq. (11.1), for the number $N\approx 207$, and in Sec. V, Eq. (5.1b), for $r_0\approx 0.73\hbar/mc$ in order to get the value 1/137 with Eq. (10.3).

The present theory does not introduce hidden variables. A given quantum state is not constructed by probability superposition from substates. It is formulated in terms of a superposition of probability amplitudes whose structure does not refer to substates but characterizes fluctuations.

We should state what this structural model of leptons is *not* able to do. We could not give a discussion of an accelerated lepton. It differs essentially from a lepton of constant velocity (in the lab system) which may be defined as the Lorentz transform of a lepton at rest. This inherent difficulty in our model may be significant when attempts are made to understand processes of scattering, emission and absorption.

A few more remarks as regards the logical structure of the theory may be in order, in addition to what has already been said at the end of Sec. II. At the present this theory is only capable of approximating the numerical values for 207 and for 1/137. It is certainly not only the computational problem which prevents us at present to get accurate values for these quantities, although the definition of phase correlatedness of loopform amplitudes still needs to be substantially improved. The real problem is that the quasi-nonlocality due to PFW is taken account of, so far, in only a crude way by handling the source as an extended source of approximately Gaussian distribution (Sec. V). Correspondingly, the definition of phase-correlatedness of different loopforms has been given only in semiquantitative terms (Sec. VII). When it is possible to do this in precise quantitative terms, the still more difficult job will have to be tackled: the definition and calculation of the kind of ergodic motion of loopform amplitudes as a function of t, α, ζ, σ.

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APPENDIX I

An analysis of nonlocal theories in QED has been given by Parke [W. C. Parke, thesis, University Microfilms, Ann Arbor, Mich., 1967 (unpublished)]. In this work, the properties of nonlocal fields are investigated and various types distinguished. The most drastic feature of such theories, as is generally recognized, is the lack of a causal relationship between propagating fields and the difficulty of incorporating unitarity (probability interpretation) into such a theory. So far, there is no evidence experimentally for a breakdown of QED at small distances. A careful distinction must be made between a nonlocal field theory and the nonlocal particle properties of relativistic local field theory. This kind of nonlocality arises in an attempt to give a single-particle interpretation to the particle operators. The possibility of pair creation during the observation of a particle gives the single-particle operators their nonlocal character. For example, the interaction of an electron with an external field involves not only the electron wave function at the field point but its value over a region of the size $\hbar/mc$ surrounding that point.

APPENDIX II: REPRESENTATION OF QUARKS IN TERMS OF LINKED QUANTIZED FLUXLOOPS

We wish to consider the choice of appropriate assignments of quantized flux to particles other than electrons and muons. Representing a low-lying meson (meson without orbital angular momentum) as an elementary

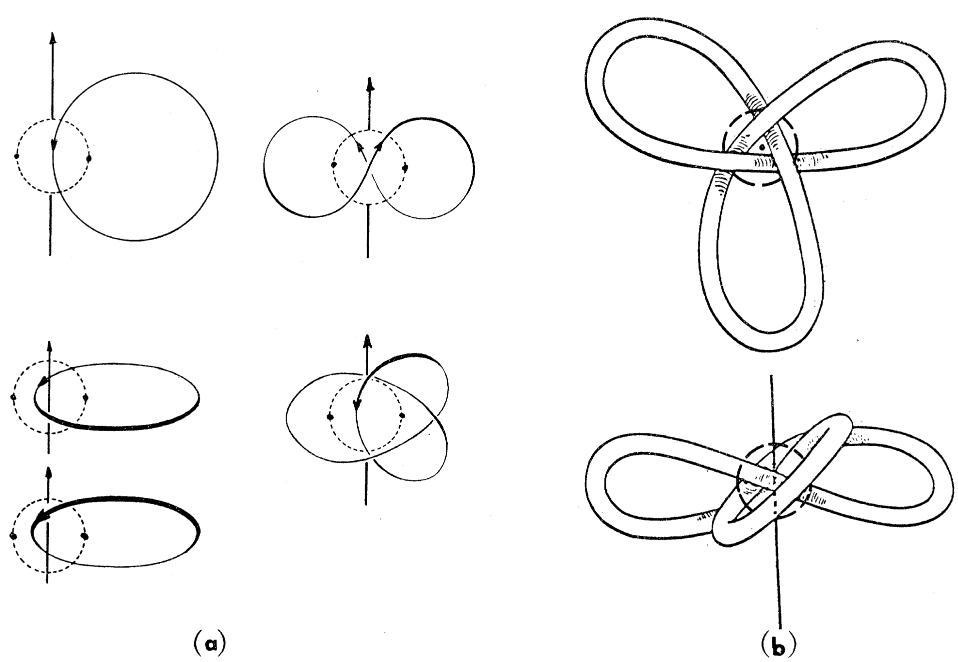

FIG. 13. (a) Suggested topological forms of quantized flux loops representing an $\mathfrak{N}$ quark (single loop), $\mathcal{P}$ quark (figures ∞ loop) and λ (trefoil). (b) A trefoil. An alternative form of a spinning muon which may correspond to a strange muon. The figures show this trefoil in two views, with an axis indicating how it might spin when not linked. A neutrino is considered as an alternative of such a loop with a loop of *same* handedness and spin but opposite orientation, i.e., also opposite coreflux orientation.

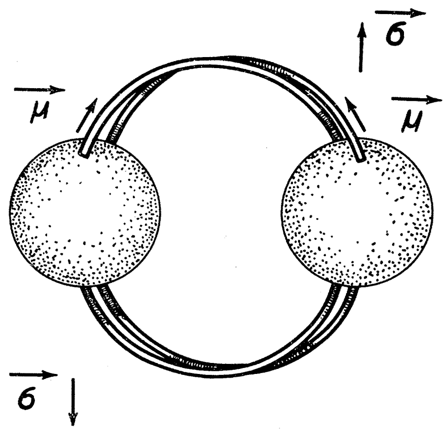

FIG. 14. Annihilation of two, $\overline{\mathcal{N}}\mathcal{N}$, loops belonging to two separate cores (swapping).

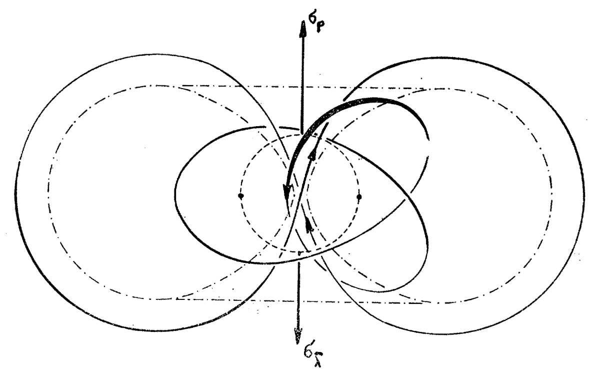

FIG. 15. Suggestion as regards interlinkage of $\mathcal{P}$ quark loop with a $\bar{\lambda}$ quark loop, contributing to a charged K^+ meson. To illustrate the topological (knot-theoretical) relationships in the case of a meson, space is here subdivided by some toroidal surface which surrounds the core equatorial ring (indicated by two dots). The $\mathcal{P}$ (figure ∞) loop is considered to be confined entirely to the exterior of the doughnut, the $\bar{\lambda}$ (cloverleaf) loop is entirely inside the doughnut. The core is a sphere (dashed in this picture) concentric with the doughnut, and of a diameter larger than that of the doughnut's hole. Thus the $\mathcal{P}$ loop may pass through the core twice while the $\bar{\lambda}$ loop passes once through the core. Because of this topological confinement of the $\mathcal{P}$ and $\bar{\lambda}$ loops to two toroidal regions, they may coaxially spin in equal or opposite senses. We might give an explanation for the charge ratio -1 to $+2$ to -1 of $\mathcal{N}$, $\mathcal{P}$, λ, as the effective number of wings. The interlinkage shown in this figure might not be the only possible one permitting independent spinning. The magnetic-moment contribution of a quark is assumed to be proportional to its number of coretraverses.

loop interlinked with an antiloop and a low-lying baryon (baryon without orbital angular momentum) as three interlinked elementary loops, accounts for many of their observed properties. We interpret a quark as an elementary loop only when interlinked with one or two other loops which restrict its independent motion. Hence, we may explain why individual quarks are not observable. We state certain assumptions about their form and topology and discuss their equivalent electric charges.

The space-time depiction of spinning loops which we employ lies on the same level as the particle picture of the Bohr theory. The motion of loops is, however, not to be described in terms of the motion of mass points, but perhaps in terms of the motion of spinning tops. By introducing probability amplitudes for the alternative loopform configurations we again may construct a quantum-mechanical formulation. [A particle, represented by interlinked quark loops is, as in the case of a lepton, to be formulated in terms of probability amplitude functions which represent the magnetic field as α, σ manifolds of loopforms of origin (coreflux) orientation ζ, and of the topological features characterizing $\mathcal{N}$, $\mathcal{P}$ or λ loops, respectively.]

The knot-theoretical linkage properties[9] (which may be referred to as their topology) of flux loops bring many interesting points into the picture. By "linkage" we may denote invariant properties of one or of several interlinked loop(s), topological properties characterizing the imbedding of the loop(s) in ordinary three space. The mathematical characterization of independently spinning flux loops is so complex that it is advisable to start studying them with the help of simple models: rubber catheters available from any medical supply house provide a good way of exploring possible linkage arrangements.

In Fig. 13(a) we illustrate the forms which quarks might take. Simple loops represent $\mathcal{N}$ quarks; figure ∞ loops, $\mathcal{P}$ quarks; and trefoils, λ quarks. Except for some interactions, we assume that the loopforms are unable to slip out of their core attachments (cf. Fig. 5), but maintain the core crossings which characterize the single, figure ∞, and trefoil loops, respectively. We may thus characterize a loop by its intrinsic linkage and interlinkage with the ζ equatorial core circle (cf. Figs. 15 and 16). As regards these λ quarks we note that the right-handed and the left-handed trefoils are topologically distinct loops even if their relationship to the core and to the spin axis were disregarded. Presumably, this "intrinsic linkage" of a λ quark loop represents strangeness. Two λ's of opposite handedness may annihilate each other without flux lines crossing each other; this may explain conservation of strangeness in terms of loop topology (intrinsic linkage).

Annihilation of two simple loops belonging to two separate cores is illustrated in Fig. 14, which will be discussed below.

These loopforms shown in Fig. 13(a) and, interlinked, in Figs. 15 and 16 represent one out of a manifold of

[9] R. H. Fox and R. H. Crowell, *Knot Theory* (Blaisdell-Ginn, New York, 1963) (contains a valuable bibliography); R. H. Fox, *Proceedings of the 1961 Topology Institute, University of Georgia* (Prentice Hall, Englewood Cliffs, N.J., 1962); N. Smythe and C. H. Giffen (private communication); H. Seifert, Acta Mat. **60**, 147 (1933); K. Reidemeister, *Knotentheorie* (Springer, Berlin, 1932; Chelsea, New York, 1961); L. Goeritz, Math. Z. **36**, 647 (1933); P. Alexandroff, *Topology* (Springer, Berlin, 1932; Dover, New York, 1961); E. Artin, Abhandl. Math. Sem. Hamburg **4**, 47, 1925; Am. Scientist **38**, 112 (1950); *Collected Works* (Addison-Wesley, Reading, Mass., 1965); D. Finkelstein, J. Math. Phys. **7**, 1218 (1966); W. S. Massey, *Introduction to Algebraic Topology* (Harcourt, Brace & World, New York, 1967).

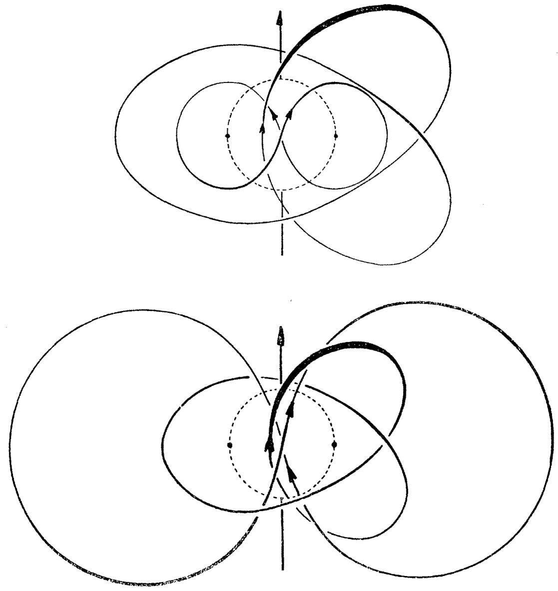

FIG. 16. Alternative loop linkages of the type shown in Fig. 15. The wings of one loop spin in one sense only; this implies approximate proportionality of electric charge and magnetic moment. The core is indicated by the dashed circle, and the core equator (in cross section) by the two dots on that circle. The interlinkages shown in these figures permit the linked loops to spin in opposite sense because they may in fact be thought of as moving entirely inside a doughnut region (which then again overlaps with the core and the core equator), and entirely outside, respectively. The doughnut is not drawn in Fig. 16, but a model of the loops built with rubber catheters and a core equator ring readily establishes this topological property of the linkage. The figures are drawn to indicate that $\mathcal{P}$ may be in the doughnut, $\bar{\lambda}$ outside *or* vice versa; this means that linked quark loops are to be considered as localized objects. This is relevant because, in constructing a baryon in terms of a symmetric function of these loops' amplitudes, besides terms of the kind $\mathcal{P}\uparrow\bar{\lambda}\downarrow$ also terms $\bar{\lambda}\downarrow\mathcal{P}\uparrow$ have to be considered (together with a third quark) without violating the Pauli principle.

topologically identical forms. They define a fiber space, a differentiable manifold, i.e., a vector field which, however, may have some discontinuities as, e.g., at the toroidal interface between two interlinked loops. Fiber-space topology determines the character of such linked loops. The most important properties of such a fiber space may be discussed in terms of the aforementioned loop models.

We have previously mentioned in reference to charge conjugation that the sign of the charge changes when the "orientation" of the flux relative to the spin is reversed, i.e., the direction of the axial flux vector across the core; we use the word "orientation" as it is used in knot theory (instead of "flux orientation" we previously used the term "origin orientation"). As to the problem of the charge associated with the $\mathfrak{N}$, $\mathcal{P}$, λ loops, we surmise that because $\mathcal{P}$ has two wings, $\mathcal{P}$ might have twice the charge of $\mathfrak{N}$, which has one wing. For the λ quark [Fig. 13(a)] one of its wings contributes in one way, the other two in the opposite way, to the electric field; the effective charge would be assumed to be proportional to the net number (one minus two) of wings. We assume that loops spin approximately like rigid bodies.

The actual calculation of the effective electric charge field originating from a set of interlinked loops is, however, not simple. We do not want to rely on detailed assumptions about interlinked loopforms, but rather make a very few simplified assumptions to substantiate the model. Let us refer to linked loops of the linkage shown in Figs. 15 and 16.

Linear addition of the electromagnetic fields of several quarks' loop contributions corresponds to a particle composed of several quarks having a probability amplitude which is the product of its quark contributions. Disregarding *Zitterbewegung* contributions towards the quark loops' probability amplitudes, the instantaneous field may be given by expressions of the following type where the left-hand sides are understood to be μ_z contributions to the reduced field $\mathbf{B}$, in Heisenberg representation

$$
\begin{aligned}
\langle\mu_z'|\mathbf{B}(\mathbf{r})|\mu_z''\rangle
&= \sum_{(\lambda')^{\bar{q}1}} \sum_{(\lambda'')^{\bar{q}1}} \sum_{(\lambda')^{q1}} \sum_{(\lambda'')^{q1}} \langle\mu_z'|(\lambda')^{\bar{q}1}{}_R) \\
&\quad\times\exp(+i\omega_{\bar{q}1}t)\langle\mu_z'|(\lambda')_{q1R})\exp(+i\omega_{q1}t) \\
&\quad\times\{((\lambda')^{\bar{q}1}|\mathbf{B}(\mathbf{r})|(\lambda'')^{\bar{q}1})+((\lambda')_{q1}|\mathbf{B}(\mathbf{r})|(\lambda'')_{q1})\} \\
&\quad\times((\lambda'')_{q1R}|\mu_z''\rangle\exp(-i\omega_{q1}t) \\
&\quad\times((\lambda'')^{\bar{q}1}{}_R|\mu_z''\rangle\exp(-i\omega_{\bar{q}1}t)
\end{aligned}
\tag{A1}
$$

for a meson and similarly

$$
\begin{aligned}
&=\sum\sum\sum\sum\sum\sum\langle|)\exp(+i\omega_{q_1}t)\langle|)\exp(+i\omega_{q_2}t)\langle|) \\
&\quad\times\exp(+i\omega_{q_3}t)\{(|\mathbf{B}_1|)+(|\mathbf{B}_2|)+(|\mathbf{B}_3|)\}(|\rangle \\
&\quad\times\exp(-i\omega_{q_3}t)(|\rangle\exp(-i\omega_{q_2}t)(|\rangle\exp(-i\omega_{q_1}t)
\end{aligned}
\tag{A2}
$$

for a baryon. Remembering the representation of quarks as covariant, the antiquarks as contravariant $SU(3)$ vectors, the negative frequency $\omega_{\bar{q}_1}$ of the $SU(3)$ contravariant vector amplitude $((\lambda'')^{\bar{q}1}{}_R|\mu_z''\rangle$ may be considered equivalent to the positive frequency $\omega_{\text{meson}}-|\omega_{\bar{q}_1}|$ of a covariant antisymmetric tensor amplitude

$$((\lambda'')_{q_2q_3R}|\mu_z''\rangle=\epsilon_{\bar{q}_1q_2q_3}((\lambda'')^{\bar{q}1}{}_R|\mu_z''\rangle \tag{A3}$$

with a fundamental anti-symmetric tensor $\epsilon_{\bar{q}_1q_2q_3}$ of frequency ω_{meson}; we may thus have $-\omega_{\bar{q}_1}=\omega_{q_1}=\frac{1}{3}\omega_{\text{meson}}$ for the meson; $\omega_{q_1}=\omega_{q_2}=\omega_{q_3}=\frac{1}{3}\omega_{\text{baryon}}$ for the baryon. Let us look in more detail into the composition of several quark loops. A quark or antiquark is represented by a dichotomic variable as regards its spin. Composition of dichotomic variables of several quarks proceeds by ordinary addition as their spin-$\frac{1}{2}$ vector addition amounts to parallel-antiparallel addition. In the picture of loopforms this implies that the several loops representing a particle have to be considered to perform coaxial spinning, and that holds for every coreflux orientation ζ, one of which is represented in Fig. 15.

Indeed this seems, as regards linkage, the only possible motion of such loops. For coaxially spinning interlinked loops, their attachment to the core (dashed circle in Fig. 15) may be characterized by a core equatorial ring (indicated by two dots), i.e., a circular ring (in a plane perpendicular to the spinning axis ζ) which holds the loops together.

In contemplating the forms taken by $\mathcal{P}$ or λ quarks, or by a set of linked quarks, we presume that the magnetic flux loopforms adopt configurations as if they were mutually repulsive, the flux lines representing one of the alternative forms of the set of linked loops may be pictured as staying as distant apart from each other as possible; but, at the core region, that distance is limited by the core size, which is of the order of $\hbar/mc$ (Figs. 15 and 16). [For this reason it may be that there is no strange quark of the $\mathcal{P}$ type, Fig. 13(a), a quark with two wings and two coretraverses but of the intrinsic linkage of a λ quark, Fig. 13(a); its form could be imagined by pushing the left wing of the λ quark, Fig. 13(a), into the core.]

It should be noted that charge conservation is implicit in this theory which is based on flux quantization because the electromagnetic field is defined in a gauge-invariant way [cf. Eqs. (3.4) and (3.5)]: Invariance with respect to gauge transformations [generated by continuous single-valued gauge functions φ (3.2), i.e., not pseudo-gauge-transformations ϑ (3.3)], applied to the effective fields of a set of interacting particles, implies the conservation of their charge. Given the charge of the electron, muon, this might account for the integrity of the electric charges of all particles produced by interactions: As we will indicate below, the form of the neutrino implies zero charge. Therefore the $\pi\rightarrow\mu\nu$ decay implies an electric charge of the pion equal to that of the muon (or electron). Similarly, as the charge of the neutron is assumed again to be proportional to its effective number of loopwings, i.e., $-1-1+2=0$ for $\mathfrak{N}\mathfrak{N}\mathcal{P}$ quarks, the charge is zero. Therefore the $\mathfrak{N}\rightarrow\mathcal{P}e^-\bar{\nu}$ implies that the proton's charge $=-$electron charge. It should also be noted that the parcelling out of charge e to the two mesonic or three baryonic loops is solely subject to this total charge conservation law and that it might even be premature to consider the charges of quarks as fixed entities.

We now want to comment on the fractionality of the electric charge of quarks. The highly successful analysis of magnetic moments in $SU(6)$ assumes proportionality of the magnetic moment contribution of a quark to the product of electric charge contribution of a quark times the signature of the spin; it explains the ratio $-\frac{2}{3}$ of magnetic moment of the neutron to that of the proton. As our calculation of effective electric field on the basis of $V=+(\hbar c/e)\partial\vartheta/\partial ct$ for a spinning loop results in a corresponding proportionality relation (the effective electric charge is proportional to the magnetic moment times the spinning angular velocity), the above $SU(6)$ result may be interpreted with loops of the type of Fig. 13(a) on the basis of the equality of the number of coretraverses (which gives the magnetic moment) and the number of effective wings (which is responsible for the effective electric field, i.e., charge).

Realizing that for symmetric spin functions of nucleons (resultant spin up) the magnetic moments are $3e\hbar/2m_{\text{nucleon}}c$ and $-2e\hbar/2m_{\text{nucleon}}c$ for proton and neutron, respectively, while for the loops given in Fig. 13(a), by $SU(6)$, the average number of coretraverses is $+3$ and -2 for protron and neutron, respectively, we find that the magnetic contribution of one coretraverse is $e\hbar/2m_{\text{nucleon}}c$, i.e., -1, $+2$, -1 times of that amount for $\mathfrak{N}$, $\mathcal{P}$, λ quarks, respectively. These magnetic moment contributions result from our theory if one gives all the quark loops of a nucleon a common core extension of about $\hbar/m_{\text{nucleon}}c$; the fractional electric charges of the quarks result from our theory if one adopts for each quark a spinning angular velocity Ω of the size $\frac{1}{3}$ $(2m_{\text{nucleon}}c^2/\hbar)$ because the effective electric charge of a loop is proportional to the number of coretraverses or wings times the quantized flux (hc/e) times the reduction factor times the core extension $(\hbar/m_{\text{nucleon}}c)$ times the fractional frequency

$$(2m_{\text{quark}}c^2/\hbar)=\tfrac{1}{3}(2m_{\text{nucleon}}c^2/\hbar)\,.$$

It is here assumed that the interaction between linked quark loops is small enough to permit a simple $SU(6)$ product combination as spin functions (there is no orbital wave function for low-lying baryons). Thus the time factors get simply multiplied with each other so that the frequencies of the quark amplitudes add up in the case of nucleons. In line with the discussion of meson and baryon probability amplitudes earlier in this Appendix, for mesons the frequency of a quark loop is again $\frac{1}{3}$ of the frequency for a meson resulting in a spinning angular velocity $\frac{1}{3}(2m_{\text{meson}}c^2/\hbar)$.

The above frequency assignments may not only account for the fractional electric charges of quarks; they may also permit us, on the one hand, to understand quark annihilation as being conditional on cancellation of frequencies, $\omega_{\bar{q}_1}+\omega_{q_1}=0$, i.e., to distinguish antiloop from loop by the signature of its frequency and thus explain loop conservation (and presumably, lepton and baryon number conservation) and, on the other hand, to understand the meson masses as corresponding to $\omega_{\text{meson}}=\omega_{q_2q_3}+\omega_{q_1}=\frac{2}{3}\omega_{\text{meson}}+\frac{1}{3}\omega_{\text{meson}}$.

In any case, representing a quark as a linked elementary loop renders meaningless the concept of an individual independent quark. When a loop unlinks, within the constraints of the conservation laws, it behaves as a muon or whatever else it comes to be.

The concept of parity transformation is brought into a new perspective here, because a space inversion has well-defined implications on the intrinsic linkage and on the interlinkage of flux loops. A P transformation applied to the magnetic field $\mathbf{B}(\mathbf{r})$ of a statistical distribution of linked flux loops does not change the orientation of the axial-vector **B** but places it at the P transformed position $-\mathbf{r}$ and thus may also change the topology of the linked loops. The flux orientation, given by $\boldsymbol{\zeta}$, is not changed. We assume that if we invert the handedness of a loop and of the linkage of loops, we obtain a change of the signature of strangeness (S conjugation which means a P transformation applied to the topological form of the loop or linked loops). If such a P transformation is applied to the handedness (strangeness) in addition to complex conjugation discussed in Sec. VIII, the result is what is commonly denoted as C conjugation or particle-antiparticle conjugation. The transition from particle to antiparticle or loop to antiloop is thus expected to imply also a change of the time dependence $\exp(-i\omega t)$ to $\exp(+i\omega t)$ of the probability amplitude distribution. The latter assumption may permit one to understand the conservation of number of loops minus number of antiloops which is important for baryon and lepton conservation laws. We might note that a loop is defined by its attachment to the core [Figs. 13(a) and 13(b)], by its intrinsic topology corresponding to strangeness [Figs. 13(a) and 13(b)], by its interlinkage with other loops of the same particle, and by its probability amplitude distribution over flux orientation $\boldsymbol{\zeta}$, azimuth α, and time t, which determines its magnetic moment, spin (and thus charge), and loop-antiloop character. Indeed, the magnetic-moment direction is given by the predominant orientation of $\boldsymbol{\zeta}$, given that the spin depends on the alternative $\exp i(\frac{1}{2}\alpha\mp\omega t)$, the $\mp$ thus determines the signature $\pm$ of the electric charge; the change of charge and strangeness then characterizes the antiloop. The wave function of an annihilating particle-antiparticle pair has then a time dependence $\exp(-i\omega t)\exp(+i\omega t)$, which is interesting.

We may denote by $\bar{\lambda}$ the quark which, compared with λ, is of opposite charge and frequency, has a complex conjugate probability amplitude distribution, and is also of opposite handedness. A λ and a $\bar{\lambda}$, not interlinked with each other, may then annihilate each other by mergers of their flux loops without the need of flux lines "cutting across" each other. This states, from the point of view of linkage, a necessary condition for a fast process. A fuller discussion is given below. This merger is initiated at two contact points of the trefoil pair.

A suggestion for a possible linkage of loops is illustrated in Fig. 16. The gist of this figure is to indicate that either parallel or antiparallel spinning is possible for these interlinked loops if one of them is confined to the region inside the doughnut, the other to the outside region (and analogously to three toroidal regions for baryons). As the loops of a particle are assigned to different spacial regions, there will be no conflict from the Pauli principle with symmetric combinations of probability amplitudes of loops representing baryons. This is very important. Without violating the Pauli principle, a symmetric function of these quarks (which in contrast to quarks considered as particles, may be considered as "localized" objects) may be formed, permitting baryons then to be constructed as symmetric quark functions in the customary $SU(6)$ fashion. No orbital wave function enters into the description of low-lying baryons.

It is not impossible that there might be loop linkages other than those shown in Fig. 15, but they would be more irregular and not so clear to justify or to evaluate. The picture of a quark as a loop implies quite different ways of description from a quark commonly considered as a particle. The distribution of probability amplitudes of a loop attached to a core is given by expressions similar to those of a solid body, i.e., by generalized spherical harmonics rather than by ordinary spherical harmonics which characterize the distribution of the probability amplitudes of a particle moving about a center of attraction. It also seems appropriate to attribute only spin angular momentum to a linked $\bar{q}q$ pair of loops and consider a higher meson's orbital angular momentum in terms of two or more $\bar{q}q$ orbiting about each other, or rather, to reinterpret the higher meson spins as being due to higher spins of the quarks instead of being due to any involvement of orbital angular momentum. If ϕ_r denotes the probability amplitude of an interlinked loop-antiloop and ϕ_l denotes that of a mirror linked structure, then $(\phi_r+\phi_l)/\sqrt{2}$ and $(\phi_r-\phi_l)/\sqrt{2}$, respectively, might account for the different parities of higher-lying states. These might be reasonable predictions.

Neutrinos

The main text of this paper on electrons and muons did not handle the problem of the neutrino. We may ask: What loop (a single loop, as we are dealing with a lepton) may characterize an uncharged particle of spin $\frac{1}{2}$ and of definite helicity?

A λ loop might be considered [see Fig. 13(b)]. Its handedness is intrinsic and does not depend on any particular core attachment. In accordance with our earlier recognition that a muon or electron is given by a superposition of alternative loopforms, we now suggest that the neutrino amplitude may be given by the zero-charge superposition:

$$\nu=(\lambda+\lambda^0)/\sqrt{2}. \tag{A4}$$

We denote by λ^0 a λ of opposite equivalent electrical charge because of opposite flux orientation, not opposite handedness. The mirrored image of such a neutrino would correspond to the antineutrino $\bar{\nu}$.

Complex as its structure may appear, this hypothesized neutrino generates an electric field consisting of but small fluctuations so that the associated electromagnetic energy corresponds to a small mass only. The decay of π^+ into μ^+ and ν or the inverse reaction whose sequential steps of linkage are easier to study are both compatible with the linkage requirements, if the above structural assumptions about neutrinos are made. The reaction implies that flux lines "cut across" each other once (cf. below the discussion of weak interaction). We note in passing, that the decay $\pi^- \longrightarrow \mu^- \bar{\nu}$ is of opposite handedness as the π^+ decay.

Electron-neutrinos and muon-neutrinos may be said to arise from some phase-related and completely random-phased probability amplitude distributions, respectively, as it was with the electrons and muons. This circumstance may be responsible for the two types of neutrinos involved in muon decay. For the balance of this paper we shall consider only muon-type, i.e., random-phased, probability amplitudes to represent quarks, mesons, and baryons. It should be noted that, whereas a single random-phased loop decays into a phase-related electron loop, linked loops (i.e., quarks) are expected to disturb each other so much that their amplitudes remain random phased.

As regards the decay of a muon into an electron, it should be noted that it is understandable why this proceeds with the participation of a muon-neutrino and an electron-neutrino, not merely with a photon. In order for that process to occur, the probability amplitudes product of $\mu^- \bar{\nu}_\mu$ and that of $e^- \bar{\nu}_e$ should match when the amplitude products' Fourier distributions over ω and over k are considered, and also when the *internal* distributions due to structure are considered. The participation of both types of neutrinos may make it possible to meet this condition because the frequency and wave-number distributions of the μ^- and $\bar{\nu}_\mu$ are similar (but opposite signature of frequency) and so are the distributions of e^- and $\bar{\nu}_e$, which makes matching possible.

We have already pointed out, regarding loopforms, that the $\mathcal{N}$ quark resembles a muon loop, while linked with other loops, it corresponds to fractional charge, however. The λ quark loop might exist on its own, not interlinked, so it might be worth while to look for two strange muons (or electrons?), but we cannot yet predict their properties. This might be another reasonable prediction. In any case, the cross section for associated production of a strange lepton pair is expected to be very small mainly because the formation of a pair of trefoils would have to be simultaneously initiated at two contact points of the forming trefoil pair.

Photons

Fluctuations or oscillations of a source due to a perturbation may lead to the disconnection of a magnetic flux loop from the core, the loop moving outward with velocity c, like the magnetic field lines of an oscillator. However, since our theory of electrons and muons does not treat fluctuating or accelerated sources, it is better not to discuss interactions with photons and other particles in any detail at this point, except from the point of view of conservation laws.

Combination of Elementary Loops

We shall distinguish between different combinations (which result in diverse interactions), considering two loops.

(A) *Linkage* of two loops in one core. The loops resemble (in the case of an $\bar{\mathcal{N}}\mathcal{N}$ pair) two members of a chain with linkage occurring in the common core. It is assumed that all quarks belonging to the same core are coaxial, i.e., spinning with common axes, for any $\bar{q}q$ or qqq. We denote the linkage with a notation (which also indicates parallel or antiparallel spin of the two quarks), by simply placing the quark symbols next to each other, e.g.,

$$\pi^+ = \bar{\mathcal{N}}\downarrow\mathcal{P}\uparrow\cdots.$$

Coaxially arranged loops corresponding to linked quarks may even be able to spin in opposite directions if their sizes fit each other (Figs. 15 and 16). Their common axis may adopt various origin orientations ζ as previously discussed in the case of leptons. The symbol $\bar{\mathcal{N}}\downarrow\mathcal{P}\uparrow$ means accordingly a superposition of alternative forms which the *pair* of loops may adopt. As the $\bar{q}q$ loops are assigned to different regions of space (cf. Figs. 15 and 16) and similarly for qqq loops, an $\bar{\mathcal{N}}\downarrow\mathcal{P}\uparrow$ and a $\mathcal{P}\uparrow\bar{\mathcal{N}}\downarrow$ are two disticnct pairs, and there is no conflict with the Pauli principle, from symmetric qqq combinations.

(B) Extending this superposition concept, we may form superposition of pairs of loopforms from two different pairs of loops, belonging to the same core, e.g., additive superposition of the two terms $\bar{\mathcal{P}}\mathcal{P}$ and $\bar{\mathcal{N}}\mathcal{N}$ ("*superposition of linked pairs of loops*"):

$$\pi^0 = (\bar{\mathcal{P}}\mathcal{P} - \bar{\mathcal{N}}\mathcal{N})/\sqrt{2}$$

or

$$\pi^0 = (\bar{\mathcal{P}}\uparrow\mathcal{P}\downarrow - \bar{\mathcal{P}}\downarrow\mathcal{P}\uparrow - \bar{\mathcal{N}}\uparrow\mathcal{N}\downarrow + \bar{\mathcal{N}}\downarrow\mathcal{N}\uparrow)/2, \qquad \text{(A5)}$$
$$\pi^- = (\bar{\mathcal{P}}\uparrow\mathcal{N}\downarrow - \bar{\mathcal{P}}\downarrow\mathcal{N}\uparrow)/\sqrt{2}.$$

(C) *Interactions "without change of loops"* may be of a type in which there is no change in the number and types and core attachment of loops.

(D) *Actual interactions with a change* in the number or types of loops afford us an opportunity to discuss the loop interpretation of the conservation of spin, charge and strangeness. In the course of such interactions linkage may change.

Actual Interactions between Elementary Loops

In every actual interaction of type (D) between two elementary loops, we have to consider the *topology* of the process, whether it consists of loops attached to the same or to different cores. First, we will discuss the case of two loops attached to the *same core*, and their interactions under the assumption that spin angular momentum or charge may be exchanged among simultaneously participant reactants.

A simple type of interaction is the annihilation of two loops. This may occur when they have opposite orientation and merge with each other (e.g., and $\bar{\mathfrak{N}}$ with $\mathfrak{N}$ or a $\bar{\mathcal{P}}$ with $\mathcal{P}$ or a $\bar{\lambda}$ with λ). Opposite orientation, for a $\bar{q}q$ combination, implies parallel spin. Such a process is possible only if the resultant spin can be transferred to another of the reactants (quarks). Besides, such reactions are impeded if the two merging loops which are brought together into one core, have the same handedness (strangeness S) because to complete the zip-up annihilation process the annihilating pair of flux loops would have to cut across each other, which is assumed to be a slow (weak) process. A meson merging with a baryon may thus annihilate the meson's $\bar{q}$ with a q from the baryon, a *strong* interaction. Of course, the core merging is dependent on a really close encounter. The second-order process of the virtual-meson exchange of that type would again be a strong interaction. Strong interaction might not always imply a complete $\bar{\mathcal{P}}\mathcal{P}$ or $\bar{\mathfrak{N}}\mathfrak{N}$ annihilation and creation with each meson exchange.

When two cores merge, with a quark and its corresponding antiquark coming together with antiparallel orientation and paralled spin, these two loop's motions may lock in phase and thus get a chance for very rapid annihilation.

This chance for rapid annihilation because of synchronized loops occurring in the (short-range) interaction of merging cores may be compared with the interactions, cf. Fig. 14, of loops belonging to separate, *different cores*. Those annihilations may be much slower because there is no synchronized spinning of the two loops, interacting about essentially the same axis. We may thus get a qualitative clue as to the difference between strong and "electromagnetic" interactions, Fig. 14, even though both are in the present theory understood to be of electromagnetic origin.

For the merging of two loops, when two cores merge in a strong interaction, it is not only of importance to know the topologies of the merging loops (i.e., their intrinsic linkages) and their interlinkages, but also to know the two loopform distributions in terms of generalized spherical harmonics. The operator representing the scalar product of angular momentum and magnetic moment, which characterizes the electric field, plays a predominant role as do the angular momentum operators.

Many variations of such annihilation schemes may be found, e.g., the decay of

$$\pi^0=\tfrac{1}{2}(\bar{\mathcal{P}}\uparrow\mathcal{P}\downarrow-\bar{\mathcal{P}}\downarrow\mathcal{P}\uparrow-\bar{\mathfrak{N}}\uparrow\mathfrak{N}\downarrow+\bar{\mathfrak{N}}\downarrow\mathfrak{N}\uparrow)$$

into $\gamma+\gamma$, where we assume the additive terms $\bar{\mathcal{P}}\uparrow\mathcal{P}\downarrow$ and $\bar{\mathcal{P}}\downarrow\mathcal{P}\uparrow$ to annihilate, i.e., $\bar{\mathcal{P}}\uparrow$ with $\mathcal{P}\uparrow$ and simultaneously $\mathcal{P}\downarrow$ with $\bar{\mathcal{P}}\downarrow$, the two processes cancelling their spins.

So far we have not yet paid attention to the other loops which may be interlinked with the reacting loops, in one core or in the other core, during an interaction. In either type of interaction for shared cores or for separate cores, the completion of the zipping up is apparently prevented by the presence of those other loops which are in the way and which are not directly involved in the interaction. We may inquire as to what then happens to the interacting loops. The question thus arises how loop annihilation may become possible when that implies crossing of some of the other flux loops.

Let us consider a meson $(q_1\bar{q}_2)$ and a baryon $(q_3q_4q_5)$ in strong interaction. We may consider as a first step that their cores merge; while that occurs, the combined system meson-baryon makes a transition, denoted by b (before) $\rightarrow a$ (after). Using the long expressions for $\mathbf{B}(\mathbf{r})$ given in this Appendix, the values

$$\sum\sum\langle a|\mu_z'\rangle\langle\mu_z'|\mathbf{B}(\mathbf{r})|\mu_z''\rangle\langle\mu_z''|b\rangle$$

for the instantaneous $\mathbf{B}(\mathbf{r})$ contain terms of the time dependence

$$\exp i[\omega_{q_1}+\omega_{\bar{q}_2}+\omega_{q_3}+\omega_{q_4}+\omega_{q_5}]_a t \times\exp -i[(\omega_{q_1}+\omega_{\bar{q}_2})+(\omega_{q_3}+\omega_{q_4}+\omega_{q_5})]_b t,$$

i.e., of nonvanishing frequency. The instantaneous value for $\mathbf{B}$ at a particular point $\mathbf{r}$ may then pass through zero frequently. We assume that at the moments when the resultant instantaneous field $\mathbf{B}(\mathbf{r})$ of the combined system passes through zero at a point $\mathbf{r}$, loopforms may readily cross there; that happens at any point, frequently. It means that loops may readily cross in that first step of a strong-interaction process.

The second step of that process is the merging of a loop with an antiloop, e.g., $\bar{q}_2$ of the meson with q_3 of the baryon. Such annihilation, or its inverse, the creation of a pair of loops, is assumed to be conditional on the frequency of $\bar{q}_2q_3$ after the first step (a=after first step), i.e., $(\omega_{\bar{q}_2}+\omega_{q_3})_a$ to be zero as it should lead to zero frequency after that final step, i.e., $(\omega_{\bar{q}_2}+\omega_{q_3})_f=0$ (f=final state). To such a $\bar{q}_2q_3$ pair thus corresponds zero frequency $\langle f|\mathbf{B}(\mathbf{r})|a\rangle$, which implies that the field $\mathbf{B}$ does not pass through zero value in the manner it did above; thus the loops corresponding to $\bar{q}_2$, q_3 do not readily cross in the annihilation process or its inverse.

The annihilation or creation of a $\bar{q}q$ pair is thus dependent on the topological compatibility which we discussed earlier, i.e., on their strangenesses adding up to zero.

Considering changes of linkage, which amounts to a quantized flux line "cutting across" itself or cutting across another line, it appears that such changes in the intrinsic linkage of a loop (e.g., of a λ quark), or changes in the interlinkage of a loop with other loops, occur only slowly and represent *weak* interactions. The production or annihilation of a neutrino or changes in strangeness represent typical examples.

Now we may briefly consider interactions involving two elementary loops associated with two *different cores*. Such interactions may again lead to changes in the numbers of loops, either by annihilation or its reverse. With some probability, the loops may have coinciding sections, each loop reaching over to the core of the other. For annihilation, the magnetic moments of the two loops now need to be parallel. In the event that their spins are antiparallel, charge- and not only spin-conservation is achieved. Flux loop zip-up reactions of this type may be called a "swapping" of loops (Fig. 14). This is an annihilation process which does not depend on close encounter; its probability depends on an inverse power of the separation; this process is usually called "electromagnetic" interaction.

Whereas, in the case of electrons and muons, detailed field calculations have become possible, similar calculations seem to be unattainable with the present means for the linked loops of mesons and baryons. For them, this theory gives some insight into the classification of particles, their conservation laws and interactions.

While we have presented only sketched remarks, the study of the elementary loops, their linkages, their association into mesons and baryons, and most of all, the reaction mechanisms should prove encouraging. We should expect new insights into the nature of the various types of strong interactions, electromagnetic and weak interactions. These interactions might all together fall under the theory outlined here.