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[11]In general, this instantaneous interaction may be nonlocal in space. However, in the kinematic limit of present interest, the configuration-space distance between the points where the two electromagnetic currents interact becomes lightlike. Hence, for our purpose, an instantaneous two-photon interaction is automatically local.

[12]This contrasts with the situation in deep-inelastic *ep* scattering where the scattered parton is very nearly on the energy shell. In this case all self-energy insertions in the intermediate states of the relevant imaginary part of the forward virtual Compton scattering amplitude cancel with corresponding contributions to the renormalization constants of the on-shell intermediate states. See the second paper in Ref. 5, especially the derivation of Eq. (68).

[13]Our conclusions remain unaffected even if we modify the non-Z contributions by strong interactions so long as the Z contributions are left intact, as is required in the parton model.

[14]Note that this is true even if $X \sim s/M_p^2$, because of Eq. (5).

[15]In the leading terms there is no s-u interference. The only interference is between the s-channel Z and non-Z diagrams, and similarly between the u-channel Z and non-Z diagrams.

[16]R. Jaffe (unpublished).

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$\Delta I = \frac{1}{2}$ Rule with Fermion Quarks

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It is shown that the Fierz reshuffling symmetry of the $V-A$ interaction combined with current algebra and partial conservation of axial-vector current provides a reasonable explanation for the $\Delta I = \frac{1}{2}$ rule in the framework of the three-triplet model with fermion quarks. This explanation can be distinguished from that based on an *ad hoc* boson quark assumption by studying whether the parity-violating $\Delta Y = 0$ nuclear transitions contain a significant $\Delta I = 2$ part.

Recently there have been several attempts[1] at explaining the $\Delta I = \frac{1}{2}$ rule for the weak nonleptonic decays by using the Fierz reshuffling property of the $V-A$ interaction and assuming that the quarks are bosons. While the Fierz transformation argument is very suggestive, the association of Bose statistics with spin-$\frac{1}{2}$ objects contradicts the connection between spin and statistics. One is prompted to ask whether it is possible to preserve certain merits of this explanation without giving up Fermi statistics. A possible candidate, which suggests itself in this connection, is the three-triplet model.[2-4] As is well known, almost all the virtues of the symmetric quark model, starting with the successes of the 56-plet of $SU(6)$, can indeed be preserved in the three-triplet model with fermion quarks due to the presence of the second $SU(3)$ degree of freedom. The purpose of this note is to comment that the same is not the case for the $\Delta I = \frac{1}{2}$ rule. One can extend the Fierz reshuffling argument to the three-triplet model with fermion quarks to yield the $\Delta I = \frac{1}{2}$ rule for only a limited set of diagrams (but not all). This, however, is enough to supplement the soft-pion results based on partial conservation of axial-vector current (PCAC) and current algebra, so that together they enable one to provide a reasonable explanation of the $\Delta I = \frac{1}{2}$ rule at least for the hyperon decays.

To present the arguments, we mention briefly the main features of the three-triplet model. The model[5] consists of nine fundamental spin-$\frac{1}{2}$ particles with baryon number $\frac{1}{3}$ labeled by (α, i), where the Greek label runs over the indices S, U, and B, and the Latin label over the indices p, n, and λ. It allows one to define the usual $SU(3)$ group [the one which carries the familiar (I_3 and Y) generators] acting on the index i, and a second $SU(3)$ group called the $SU(3)''$ group acting on the index α. It is assumed that the relevant symmetry for the classification of hadrons is the $SU(3) \times SU(3)''$ group, even though neither $SU(3)$ nor $SU(3)''$ are exact symmetries. In particular, it is presumed that the (α, i)'s transform as the $(\underline{3}, \underline{3})$ representation of the above group, and that the low-lying baryons and mesons are bound states of $((\alpha, i), (\beta, j), (\gamma, k))$ and $((\bar{\alpha}, \bar{i}), (\beta, j))$, respectively, both transforming as $SU(3)''$ singlets.[6] Thus in this picture, the observed baryons (except possibly for those in the 2-BeV region or higher) are built out of one each of S, U, and B and are totally antisymmetric in the indices (α, β, γ).

The hadronic part of the weak-interaction current may be chosen (in this model) to preserve the $SU(3)$ structure of the familiar Cabibbo current. Thus, in general, it would have the form

$$J_\mu = \sum_{\alpha,\beta=S,U,B} C_{\alpha\beta}\bar{\psi}_{(\alpha,i)}\gamma_\mu(1+\gamma_5)[(\lambda_1+i\lambda_2)_{ij}\cos\theta + (\lambda_4+i\lambda_5)_{ij}\sin\theta]\psi_{(\beta,j)}. \qquad (1)$$

The coefficients $C_{\alpha\beta}$ determine the $SU(3)''$ structure of the current and are chosen[7] so that every term on the right-hand side of (1) carries +1 unit of electric charge. Assuming the usual current-current form of the weak interaction, the $|\Delta S|=1$ nonleptonic interaction H_W' contains in general $\Delta I=\frac{1}{2}$ and $\frac{3}{2}$ [as well as $SU(3)$ octet and 27-plet] parts. It may be written as a linear combination of terms of the form

$$\{\bar{\psi}_{(\alpha,i)}(x)\gamma_\mu(1+\gamma_5)\psi_{(\beta,j)}(x)\}\{\bar{\psi}_{(\gamma,k)}(x)\gamma_\mu(1+\gamma_5)\psi_{(\delta,l)}(x)\}$$
$$=\{\bar{\psi}_{(\gamma,k)}(x)\gamma_\mu(1+\gamma_5)\psi_{(\beta,j)}(x)\}\{\bar{\psi}_{(\alpha,i)}(x)\gamma_\mu(1+\gamma_5)\psi_{(\delta,l)}(x)\}, \qquad (2)$$

which holds for q-number spinor fields satisfying anticommutation relations (corresponding to fermions). Thus the above form is symmetric under the interchange

$$(\alpha, i) \leftrightarrow (\gamma, k). \qquad (3)$$

Following the spirit of usual quark-model calculations, if $\bar{\psi}_{(\alpha,i)}$ and $\bar{\psi}_{(\gamma,k)}$ lead to the creation of two quarks both of which are constituents of the same low-lying baryon, only the part of (2) that is antisymmetric under the interchange $(\alpha \leftrightarrow \gamma)$ will contribute to the said process, since the low-lying baryons are $SU(3)''$ singlets. In turn [by (3)], only the part of (2) which is antisymmetric in the $SU(3)$ indices $(i \leftrightarrow k)$ will contribute to this process. For $\Delta S=1$ transitions, let $\psi_{(\beta,j)}$ annihilate a λ quark. In this case the antisymmetry in $(i \leftrightarrow k)$ forces the quarks created by $\bar{\psi}_{(\alpha,i)}$ and $\bar{\psi}_{(\gamma,k)}$ to be in the $I=0$ state, which implies the $\Delta I=\frac{1}{2}$ rule. It is easy to see that the said transition also satisfies the $SU(3)$ octet property. The above argument implies, in particular, that matrix elements of type $\langle B_2|H_W'|B_1\rangle$ (where B_1 and B_2 are two low-lying baryon states) will satisfy the $SU(3)$ octet property and the $\Delta I=\frac{1}{2}$ rule to the extent that we may neglect quark-antiquark excitations in B_1 and B_2. On the other hand, if $\bar{\psi}_{(\alpha,i)}$ and $\bar{\psi}_{(\gamma,k)}$ do not lead to creation of quarks in the same baryon [as may be the case for a meson→meson or a baryon→(baryon+meson) transition via H_W'], there is no reason in general to expect antisymmetry under $(\alpha \leftrightarrow \gamma)$; hence in these cases the $\Delta I=\frac{1}{2}$ rule cannot be concluded.[8] By contrast, the derivation of the $\Delta I=\frac{1}{2}$ rule based on boson quarks[1] is not subject to the above limitations.

We now discuss how the above result, obtainable in the three-triplet model, supplements the soft-pion results based on current algebra[9] and PCAC, as follows. First of all, as is well known, the latter leads to a pure $\Delta I=\frac{1}{2}$ rule[10] for $K\to 3\pi$ and $K\to 2\pi$ decays in the limit of soft pions. The $\Delta I=\frac{3}{2}$ amplitude is expected to extrapolate smoothly from the soft pion to the physical point as there are no poles in this case, and thus is expected to be suppressed compared to the $\Delta I=\frac{1}{2}$ amplitude. Thus current algebra already provides at least some rationale for the $\Delta I=\frac{1}{2}$ rule for K decays. This is not so, however, for the weak hyperon decays of the type $B_i\to B_f+\pi_j$, where B_i and B_f are members of the $\frac{1}{2}^+$ baryon octet and π_j are the pions of different charges. For these, current algebra leads to the following results in the limit of soft pions.

(a) The S-wave amplitudes are given by matrix elements of the form $\langle B_f|[F_j,H_W^{\mathrm{PC}}]|B_i\rangle$, where F_j is the jth vector charge (corresponding to the pion of type j) and H_W^{PC} is the parity-conserving part of H_W'. Even though the above matrix elements contain in general both $\Delta I=\frac{1}{2}$ and $\frac{3}{2}$ transitions, it has been pointed out by Suzuki[11] and Sugawara[12] that they happen not to cause any difficulty from the experimental point of view since experimentally $S(\Sigma_+^+)\simeq 0$. Thus, the only shortcoming perhaps is that current algebra does not explain why $S(\Sigma_+^+)\simeq 0$, which in the current-algebra approach involves a pure 27-plet $\Delta I=\frac{3}{2}$ transition. It is also worth noting that the S-wave amplitudes are expected to extrapolate smoothly from the soft pion to the physical point, at least insofar as it may be judged on the basis of baryon-pole diagrams.[13]

(b) At the soft-pion point, the P-wave amplitudes are given (apart from surface terms) by matrix elements of the form $\langle B_f|[F_j, H_W^{\mathrm{PV}}]|B_i\rangle$, which vanish in the $SU(3)$ limit[14] (H_W^{PV} is the parity-violating part of H_W). It is known that the P-wave amplitudes extrapolate badly from the soft pion to the physical point, especially due to the baryon-pole terms.[13] A careful consideration[15] of these pole contributions leads to the result that the physical P-wave amplitudes involve the weak vertex essentially through matrix elements of the forms $\langle B'|H_W^{\mathrm{PC}}|B_i\rangle$ and $\langle B_f|H_W^{\mathrm{PC}}|B''\rangle$, where B' and B'' are intermediate baryons belonging to the $\frac{1}{2}^+$ octet, as well as to other low-lying multiplets such as the decuplet and the Roper octet, etc. Thus the isospin property of the P-wave amplitudes will be determined primarily by that of the above matrix elements. *Current algebra alone does not shed any light on this question.* However, the arguments following (3) imply that the matrix elements of the type $\langle B'|H_W^{\mathrm{PC}}|B_i\rangle$, etc. do satisfy the $\Delta I=\frac{1}{2}$ rule and the $SU(3)$ octet property. This then explains the $\Delta I = \frac{1}{2}$ rule for the P-wave decays.

One has the same situation for the matrix elements of interest for S-wave decays, since $[F_j, H_W^{\text{PC}}]$ can be expressed in terms of the $SU(3)$ and $SU(2)$ partners of the corresponding pieces of H_W^{PC}. [Note that F_j (with $j = 1,2,3$) are among the generators of $SU(3)$ and $SU(2)$.] The octet property for the S-wave decays not only implies the $\Delta I = \frac{1}{2}$ rule and the Lee-Sugawara sum rule [in the $SU(3)$ limit], but also explains (in the current-algebra framework) why $S(\Sigma_+^+) \simeq 0$.

We next comment on the $\Delta Y = 0$ parity-violating nonleptonic interaction, which will manifest itself in nuclear transitions. These could perhaps be discussed in terms of the properties of the two-nucleon potential, which arises due to various meson exchanges ($\pi, \eta, \rho, \omega, \phi$,...etc.). For the parity-violating transitions, CP invariance forbids π^0 and η exchanges and requires the $\pi^\pm$ exchanges to satisfy $\Delta I = 1$. The Fierz reshuffling argument does not eliminate the latter; it is, however, small, being proportional to $\sin^2\theta$. On the other hand, the current-algebra treatment (in the sense in which it was applied to hyperon decays) does not apply to the other exchanges (ρ, ω, ϕ,...etc.), for which one should expect both $\Delta I = 0$ and 2 (proportional to $\cos^2\theta$). This is in contrast to the result of Ref. 1 which requires dominantly[16] the $\Delta I = 0$ property.

To conclude, we have argued that the three-triplet model[17] with fermion quarks can fruitfully supplement the soft-pion results based on current algebra and PCAC, so that together they provide a satisfactory explanation of the $\Delta I = \frac{1}{2}$ rule at least for the hyperon decays. The stated explanation can be distinguished from that of Ref. 1, based on boson quarks, by studying the isospin properties of the parity-violating nuclear transitions.

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[1]R. P. Feynman, M. Kislinger, and F. Ravndal, Caltech. Report No. CALT68-279 (unpublished). While we noticed the remark made in this paper recently through the above reference, the same observation was made earlier by C. H. Llewellyn Smith, Ann. Phys. (N.Y.) 53, 544 (1969), and by T. Goto, O. Hara, and S. Ishida, Progr. Theoret. Phys. (Kyoto) 43, 845 (1970).

[2]M. Y. Han and Y. Nambu, Phys. Rev. 139, B1006 (1964).

[3]N. Cabibbo, L. Maiani, and G. Preparata, Phys. Letters 25B, 132 (1967).

[4]See a recent paper by J. C. Pati and C. H. Woo, Phys. Rev. D 3, 1173 (1971), for the physical distinction between the three-triplet models proposed in Refs. 2 and 3 and certain experimental consequences of either model.

[5]For simplicity, we present our arguments in the model of Ref. 3 only. However, similar arguments can also be provided in the model of Ref. 2.

[6]A possible dynamical mechanism that favors low-lying levels to be $SU(3)''$ singlets has been proposed by Han and Nambu, Ref. 2.

[7]This specification is necessary if the Gell-Mann–Nishijima formula has the modified form $Q = I_3 + \frac{1}{2}Y + \frac{1}{3}C$, where $C = 1$, 1, and -2 for the S, U, and B triplets, respectively. We note that J_μ must necessarily have at least a $SU(3)''$ singlet piece in it (for which $C_{\alpha\beta} = \delta_{\alpha\beta}$) in order to allow the semileptonic decays of low-lying $SU(3)''$ singlet states. For further remarks on the structure of $C_{\alpha\beta}$ see Ref. 9.

[8]It is easy to check that the further restriction of the weak-interaction current being $SU(3)''$ singlets allows one to deduce the $\Delta I = \frac{1}{2}$ rule for some additional types of diagrams, but still not for all diagrams.

[9]There are several possibilities of choosing the coefficients $C_{\alpha\beta}$ in the current J_μ given by (1), which would preserve the current commutation relations and hence the usual applications of current algebra. The simplest choice, for example, is that $C_{\alpha\beta} = \delta_{\alpha\beta}$, which implies that J_μ transforms as an $SU(3)''$ singlet. This allows one to preserve not only the commutation relations of the quark model, but also PCAC with pions forming an $SU(3)''$ singlet. For other possible choices, which preserve the commutation relations, see S. Okubo, Phys. Rev. 179, 1629 (1969).

[10]M. Suzuki, Phys. Rev. 144, 1154 (1966).

[11]M. Suzuki, Phys. Rev. Letters 15, 986 (1965).

[12]H. Sugawara, Phys. Rev. Letters 15, 870 (1965).

[13]Note that the meson-pole diagrams involving the $\bar{K}^* \to \pi$ transition (for S wave) and the $\bar{K} \to \pi$ transition (for P wave) do not extrapolate smoothly. However, in the first place their net contribution seems to be weaker than that of the baryon-pole terms, but more importantly (for the arguments of the present paper) they satisfy the $\Delta I = \frac{1}{2}$ rule and the octet property by the argument of Suzuki (see Ref. 10).

[14]Even if one allows for $SU(3)$ breaking in these matrix elements, the argument presented in this paper for the $\Delta I = \frac{1}{2}$ rule is not affected.

[15]L. S. Brown and C. M. Sommerfield, Phys. Rev. Letters 16, 751 (1966); A. Kumar and J. C. Pati, *ibid.* 18, 1230 (1967); C. Itzykson and M. Jacob, Nuovo Cimento 48A, 655 (1967).

[16]Note that the argument of Ref. 1 eliminates the $\Delta I = 2$ part for boson quarks, but not the $\Delta I = 1$ part which, however, is small, being proportional to $\sin^2\theta$.

[17]As is well known, for many considerations the paraquark model proposed by O. W. Greenberg [Phys. Rev. Letters 13, 598 (1969)] provides a suitable alternative to the three-triplet model. We found that with the most obvious choice of currents the parafermion model does not guarantee the $\Delta I = \frac{1}{2}$ rule even for the limited set of diagrams discussed in this paper.