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Parton Model and Inelastic Processes with Two Photons*

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It is proposed that highly inelastic hadronic reactions involving two photons (Compton scattering, photoproduction of low-mass μ pairs, hadron-hadron γ-pair production, etc.) be studied to distinguish between two approaches to the parton model: the direct impulse approximation of Bjorken and Paschos, and the field-theoretic method of Drell, Levy, and Yan. Such a distinction may be possible since a field-theoretic justification of the free-parton approximation in such processes cannot be given except in a limited kinematic region.

The idea of pointlike interactions in the impulse approximation is the basic ingredient in Feynman's[1] parton picture. Let us take inelastic lepton-hadron scattering in the Bjorken limit[2] of high energies and large leptonic momentum transfers. In the infinite-momentum center-of-mass frame the proper motion of the pointlike partons in the hadron is slowed down by time dilation. On the other hand, the charge distribution of the hadron is Lorentz-contracted into the shape of a disk. In this frame and in the Bjorken limit, the lepton is predominantly expected to undergo instantaneous and incoherent scattering from those partons which are charged.

The strength of the above-mentioned idea is the simple physical way it can explain[3] the observed[4] scaling in deep-inelastic ep scattering. It lies moreover in the reassuring fact that the assumptions going into this model have been justified within the framework of renormalizable perturbative field theory. This justification has been provided by Drell, Levy, and Yan[5] (DLY) under the requirement that the transverse momenta of the virtual partons be kept under a cutoff generated by the dynamics. If – in the deep-inelastic kinematic region – the leptonic momentum transfer considerably exceeds this cutoff, the impulse approximation can be shown to be valid in an infinite-momentum frame.

In the development of the parton idea the direct impulse approach of Bjorken and Paschos[3] and the field-theoretic method of DLY have evolved in parallel. Although these yield the same results in deep-inelastic ep scattering, we show in this paper that this is not expected to be generally the case for highly inelastic processes involving two photons. Hence, *by testing some of the predictions that Bjorken and Paschos*[6-8] *have made for such processes, one may be able to distinguish between these two approaches to the parton model.* In our work we have considered the following question: Can one justify the assumptions of the parton model for inelastic processes with two photons (with a large momentum transfer between them) in the same manner that DLY justified them for inelastic reactions with one highly virtual photon? In other words, given a canonical renormalizable field theory of strong interactions with a transverse momentum cutoff, can one show that for such processes it is sufficient to consider only the lowest-order electromagnetic diagrams involving the interactions of the photons with a single free-parton line [*vide* Figs. 4(a) and 4(b) of Ref. 3] free of other strong-interaction complications? Our answer is "yes" only for a limited quasiforward region of the entire deep-inelastic kinematic domain and "no" outside this region.

Before proceeding to outline the complete reasoning that led to this conclusion, let us emphasize the most important point first. An affirmative answer to the question posed in the above paragraph is possible when a certain condition is satisfied in the kinematic limits of interest: The diagrams (and except when specified otherwise, we use the term in the sense of old-fashioned perturbation theory) describing the interactions of the photons with the partons have to be exclusively dominated by those in which the two photons interact at the same instant in time (cf. Fig. 1). Of course, this does not automatically rule out the case of spin-$\frac{1}{2}$ partons on the grounds that the seagull diagrams are absent here. The dominant contribution in this case could in principle come from electromagnetic Z graphs [see Figs. 7(b) and 7(d) below] for which the internal line between the γ vertices (together with any intermediate

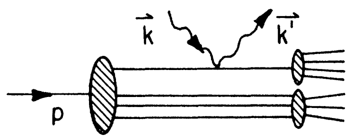

FIG. 1. Instantaneous two-photon interaction with partons.

hadronic interactions) collapses into an instantaneous vertex[9] in the infinite-momentum frame ($P\rightarrow\infty$) of the observer.[10]

There are three reasons why we want to have the two photons interact at the same time:

(1) A constraint of the parton model is that the difference of the longitudinal photon momenta (this is the present equivalent of $q_{\parallel}$ in Ref. 5) cannot go to ∞ as $P\rightarrow\infty$. However, the two γ's are kept exactly or nearly on the mass shell, and all scalar products constructed out of their four-momenta and that of the proton have to be independent of P. These restrictions imply that the longitudinal momentum of each γ has to grow with P (see Sec. B). Consequently, vertices in which a photon dissociates into a forward-moving particle-antiparticle pair (Fig. 2), which then interact with the parton line, cannot be eliminated (these are not included in the parton calculations). On the other hand, the instantaneous two-photon interaction is automatically free of diffraction dissociation. Since the difference in longitudinal momenta can be made to go to zero as $P\rightarrow\infty$, strong-interaction modifications of the instantaneous $\gamma\gamma$ vertex (namely, t-channel exchanges as in Fig. 3) do not contribute to the leading diagrams in this frame as $P\rightarrow\infty$.

(2) The real photon, transversely polarized in the laboratory frame, picks out the transverse components of the corresponding electromagnetic current J_μ. In any frame with infinite longitudinal momentum, this feature will persist. Since the vertices associated with the transverse components are "bad"[5] (i.e., they grow with P in the $P\rightarrow\infty$ frame), the presence of *purely hadronic* vertices with spin-$\frac{1}{2}$ antipartons propagating backwards (Fig. 4) cannot be ruled out. As discussed in Ref. 5, the field-theoretic justification of the parton model fails if such vertices are present. In contrast, the instantaneous 2γ vertices (sea-

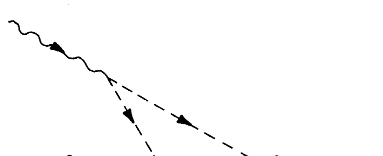

FIG. 2. Photon acting via dissociation into real particle-antiparticle pair.

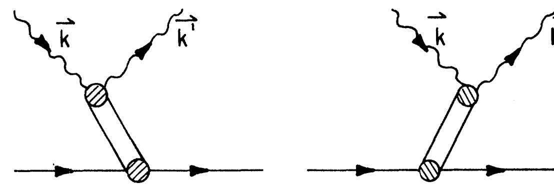

FIG. 3. Strong-interaction modifications of the instantaneous $\gamma\gamma$ vertex via t exchanges.

gulls and Z graphs in spin-0 and spin-$\frac{1}{2}$ cases, respectively) behave as constants as $P\rightarrow\infty$, i.e., the two electromagnetic currents lump into a single local "good"[5] operator.[11]

(3) For noninstantaneous $\gamma\gamma$ interactions, diagrams in addition to the simple Born graphs with a single parton could be important. It is true that correlations of an internal parton line (propagating between the γ vertices) with other partons can be avoided by attributing a large transverse momentum to each γ (see Sec. B). However, the two photons could interact with two different partons [Fig. 5(a)] leading to three groups of particles. Furthermore, even when a single parton is undergoing elastic Compton scattering, the internal line, being far off the energy shell[12] [see Eq. (4) below], will cause nontrivial complications via self-energy insertions due to strong interactions [(Fig. 5(b)]. All these difficulties disappear if the two photons act at the same point.

The question now is whether instantaneous two-photon interactions do in fact dominate in the kinematic region of interest (s, $|t|$, $|u|\gg M_p^{\,2}$). We shall argue that to answer this it is sufficient to study the lowest-order electromagnetic interactions of the photons with a single parton line. In other words, all we have to do is to take the Born graphs for the interaction of a single charged parton with two photons and see if, as $P\rightarrow\infty$ and in the kinematic region of interest, only the instantaneous $\gamma\gamma$ interaction (i.e., seagulls and Z graphs) survives. If that is the case, then our claim is that *for large* $|t|$ no amount of strong-interaction modifications can – at least to any finite order in perturbation theory – enable any other graph to compete with this pointlike interaction. Given the suppression of correlations between the scattered parton and other unscattered ones [*vide* point (3)],

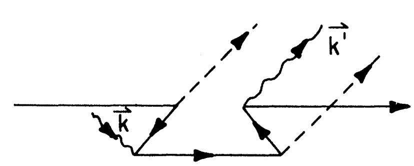

FIG. 4. Purely hadronic vertices with antipartons propagating backwards.

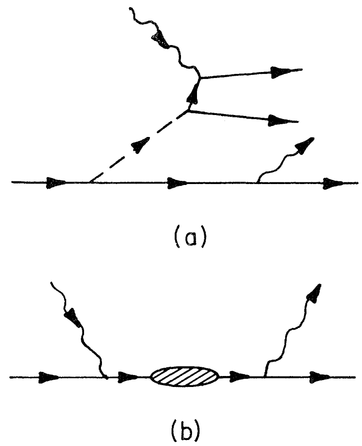

FIG. 5. (a) Photons acting on different partons. (b) Internal self-energy insertion.

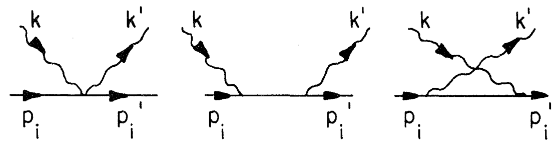

FIG. 6. Feynman diagrams for the Compton scattering of a spin-0 parton.

all noninstantaneous interactions can be classified into four types: (a) vertex corrections, (b) self-energy insertions, (c) non-Born s and u exchanges, and (d) t exchanges. The interactions (a) and (b) simply renormalize the Born amplitude and produce form-factor effects damping the amplitude; (c) cannot cause any enhancement since s- and u-channel resonances are far away; (d) is suppressed because large $|t|$ is involved. Hence the dominance of the instantaneous interaction in the Born graphs is a sufficient condition for the validity of the impulse approximation.

We have at this point established a satisfactory, as well as testable, criterion for the validity of the parton approximation in highly inelastic two-photon processes. Let us see if it holds in inelastic Compton scattering. This specific example has been chosen quite arbitrarily. Our arguments can be readily generalized to the cases of hadronic γ-pair production and low-mass μ-pair photoproduction without affecting the basic conclusion. In the reaction $\gamma p \to \gamma' +$anything, we denote the momenta of γ, p, and γ' by k, p, and k' respectively. The corresponding polarizations of the photons are ϵ and ϵ'. We define

$$s \equiv (k+p)^2, \quad t \equiv (k-k')^2, \quad u \equiv (k'-p)^2,$$

and

$$\nu \equiv \frac{p\cdot(k-k')}{M_p} = \frac{s+u-2M_p{}^2}{2M_p}.$$

The physical region is given by $t+2M_p\,\nu \geq 0$. The parton analysis can be performed in a whole class of frames where

$$p^\mu = (P+M_p{}^2/2P,\, 0,\, 0,\, P),$$
$$q^\mu \equiv (k-k')^\mu = (M_p\nu/P,\, \vec{q}_\perp,\, 0),$$
$$\vec{q}_\perp{}^2 = -t, \quad P\to\infty.$$

The four-momentum of the ith parton can be written in these frames as

$$p_i^\mu = \left(\eta_i P + \frac{\mu_i{}^2+(\vec{k}_\perp^i)^2}{2\eta_i P},\ \ \vec{k}_\perp^i,\ \eta_i P\right).$$

Here $\vec{k}_\perp^i$ is the transverse momentum of the ith parton, η_i is its fraction of the longitudinal momentum P, and μ_i is its mass. We shall also use the notations $s_i \equiv (p_i+k)^2$ and $u_i = (p_i-k')^2$. Although the analysis is carried out in old-fashioned perturbation theory, energy will be nearly conserved for the Compton scattering of a single parton in the asymptotic limit where the parton model assumptions are valid (*vide* Ref. 5). The condition for the elastic scattering of the ith parton implies that $\eta_i = -t/2M_p\nu$.

Consider now the invariant amplitude for the elastic Compton scattering of the ith parton for large $|t|$ and s_i, $|u_i| \gg \mu_i{}^2$. Define $p_i' = p_i + k - k'$.

A. Spin-0 partons. It will be sufficient to consider this case covariantly since the seagull term occurs explicitly as one of the Feynman diagrams (Fig. 6). The general gauge-invariant amplitude is

$$\mathfrak{M}^i = 2\lambda\left(\epsilon\cdot\epsilon' - \frac{\epsilon\cdot k'\epsilon'\cdot k}{k\cdot k'}\right) - \rho\left(\frac{\epsilon\cdot(2p_i+k)\epsilon'\cdot(2p_i'+k')}{s_i-\mu_i{}^2+i\epsilon} + \frac{\epsilon\cdot(2p_i'-k)\epsilon'\cdot(2p_i-k')}{u_i-\mu_i{}^2+i\epsilon} - \frac{2\epsilon'\cdot k\epsilon\cdot k'}{k\cdot k'}\right). \tag{1}$$

The scalar quantities λ and ρ (each is unity in the bare Born theory) simulate strong-interaction effects. In order to avoid a kinematic singularity at $t=0$, we shall take $\lambda-\rho = \sigma k\cdot k'/(k\cdot p_i k'\cdot p_i)^{1/2}$, where λ, ρ, and σ are finite in all physical asymptotic limits. Now

$$\sum_{\epsilon,\epsilon'}|\mathfrak{M}^i|^2 = 8|\lambda|^2 - 16\,\mathrm{Re}(\lambda^*\rho) + 16|\rho|^2 + \frac{16|\rho|^2 t^2\mu_i{}^4}{(s_i-\mu_i{}^2)^2(\mu_i{}^2-u_i)^2} + \frac{32|\rho|^2\mu_i{}^2 t}{(s_i-\mu_i{}^2)(\mu_i{}^2-u_i)} - \frac{16\,\mathrm{Re}(\lambda^*\rho)\mu_i{}^2 t}{(s_i-\mu_i{}^2)(\mu_i{}^2-u_i)}. \tag{2}$$

The first term on the right in Eq. (2) comes from the corresponding term in Eq. (1). When $s_i, |u_i| \gg \mu_i{}^2$, we have

$$\sum_{\epsilon,\epsilon'} |\mathfrak{M}^i|^2 \to 8\,|\lambda|^2 - 16\,\mathrm{Re}(\lambda^*\sigma)\left(\frac{s+u}{\sqrt{-su}}\right) + 16\,|\sigma|^2\left(\frac{s+u}{\sqrt{-su}}\right)^2 + O\left(\frac{\mu_i^2}{s}, \frac{\mu_i^2}{u}\right).$$

We see that when $(s+u)/\sqrt{-su} \to 0$, the differential cross section is dominated by the instantaneous term. Hence in this quasiforward limit the parton-model calculations have a field-theoretic justification for spin-0 partons. Outside this region, σ can be an arbitrary number in the Bjorken limit (although $\lambda \to 1$) and the justification fails.

B. Spin-$\frac{1}{2}$ partons. Now we will compare the calculations done covariantly and in old-fashioned perturbation theory. Any strong-interaction parameters will be ignored here. These are discussed later.[13] Covariantly,

$$\mathfrak{M}^i = \bar{u}(\vec{p}_i{}')\left(\not\epsilon' \frac{\not p_i + \not k + \mu_i}{s_i - \mu_i^2 + i\epsilon}\not\epsilon + \not\epsilon \frac{\not p_i - \not k' + \mu_i}{u_i - \mu_i^2 + i\epsilon}\not\epsilon'\right) u(\vec{p}_i)$$

and

$$\frac{1}{2}\sum_{\epsilon,\epsilon',\,\mathrm{spins}} |\mathfrak{M}^i|^2 = \frac{2}{\mu_i^2}\left[\frac{s_i - \mu_i^2}{\mu_i^2 - u_i} + \frac{\mu_i^2 - u_i}{s_i - \mu_i^2} - 4\mu_i^2\left(\frac{1}{s_i - \mu_i^2} - \frac{1}{u_i^2 - u_i}\right) + 4\mu_i^4\left(\frac{1}{s_i - \mu_i^2} - \frac{1}{\mu_i^2 - u_i}\right)^2\right].$$

Hence, in the limit of our interest,

$$\frac{1}{2}\sum_{\epsilon,\epsilon',\,\mathrm{spins}} |\mathfrak{M}^i|^2 = -\frac{2}{\mu_i^2}\left(\frac{s_i}{u_i} + \frac{u_i}{s_i}\right) + O\left(\frac{\mu_i^2}{s_i}, \frac{\mu_i^2}{u_i}\right) \to -\frac{2}{\mu_i^2}\left(\frac{s}{u} + \frac{u}{s}\right). \tag{3}$$

The old-fashioned perturbation-theoretic diagrams are drawn in Fig. 7. If we consider the contributions to $\mathfrak{M}$ from the Z graphs [Figs. 7(b) and 7(d)] only, we have

$$\mathfrak{M}_Z^i = \bar{u}(\vec{p}_i{}')\left(\not\epsilon' \frac{\gamma_0 E_Q^i + \vec{\gamma}\cdot\vec{Q}_i - \mu_i}{2E_Q^i(E_i + k_0 + E_Q^i - i\epsilon)}\not\epsilon + \not\epsilon \frac{\gamma_0 E_R^i + \vec{\gamma}\cdot\vec{R}_i - \mu_i}{2E_R^i(E_i - k_0' + E_R^i - i\epsilon)}\not\epsilon'\right) u(\vec{p}_i)\,. \tag{4}$$

In Eq. (4), E_i is the energy of the incident ith parton,

$$p_i^\mu = \left(\eta_i P + \frac{\mu_i^2 + (\vec{k}_\perp^i)^2}{2\eta_i P},\ \vec{k}_\perp^i,\ \eta_i P\right),\quad \vec{Q}_i = \vec{p}_i + \vec{k},\quad \vec{R}_i = \vec{p}_i - \vec{k}',\quad E_Q^i = (\vec{Q}_i^2 + \mu_i^2)^{1/2},\quad E_R^i = (\vec{R}_i^2 + \mu_i^2)^{1/2}.$$

We now choose a "true" infinite-momentum frame[10] in which

$$k^\mu = \left(XP + \frac{f^2\vec{q}_\perp^{\,2}}{4XP},\ f\vec{q}_\perp,\ XP - \frac{f^2\vec{q}_\perp^{\,2}}{4XP}\right), \quad \text{as } P \to \infty.$$

Note that in this frame one can take

$$k'^\mu = \left(XP + \frac{f^2\vec{q}_\perp^{\,2} - 4XM_p\nu}{4XP},\ (f-1)\vec{q}_\perp,\ XP - \frac{f^2\vec{q}_\perp^{\,2}}{4XP}\right)$$

and that f and X are related by

$$(2f-1)/X = -2M_p\nu/t. \tag{5}$$

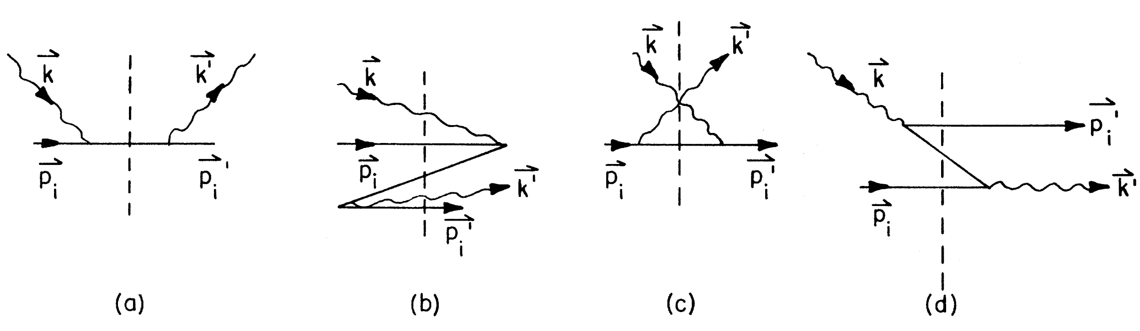

FIG. 7. Old-fashioned diagrams for the Compton scattering of a spin-$\frac{1}{2}$ parton.

The following relations should also be noted:

$$s = M_p{}^2(1+X) + f^2\vec{q}_\perp{}^2/X, \quad u = 2M_p\nu + M_p{}^2(1-X) - f^2\vec{q}_\perp{}^2/X. \tag{6}$$

Substituting $\eta_i = -t/2M_p\nu$ in Eq. (5), we obtain

$$f = \tfrac{1}{2}(1 + X/\eta_i). \tag{7}$$

Now, in our frame,

$$p_i'^\mu = \left(\eta_i P + \frac{\mu_i{}^2 + 2\eta_i M_p \nu + (\vec{k}_\perp^i)^2}{2\eta_i P},\ \ \vec{q}_\perp + \vec{k}_\perp^i,\ \ \eta_i P\right)$$

and writing E_i' for the energy of the scattered ith parton, we have from Eq. (4)

$$\frac{1}{2}\sum_{\epsilon,\epsilon',\text{spins}} |\mathfrak{M}_Z^i|^2 = \frac{1}{\mu_i{}^2}\left(\frac{2(E_i'E_Q^i + \vec{p}_i'\cdot\vec{Q}_i)(E_iE_Q^i + \vec{p}_i\cdot\vec{Q}_i) - p_i\cdot(k-k')[(E_Q^i)^2 + \vec{Q}_i{}^2]}{(E_Q^i)^2(E_i + k_0 + E_Q^i)^2}\right.$$
$$\left. + \frac{2(E_i'E_R^i + \vec{p}_i'\cdot\vec{R}_i)(E_iE_R^i + \vec{p}_i\cdot\vec{R}_i) - p_i\cdot(k-k')[(E_R^i)^2 + \vec{R}^2]}{(E_R^i)^2(E_i - k_0' + E_Q^i)^2} + \frac{2p_i\cdot(k-k')(E_Q^iE_R^i + \vec{Q}_i\cdot\vec{R}_i)}{E_Q^iE_R^i(E_i + k_0 + E_Q^i)(E_i - k_0' + E_R^i)}\right). \tag{8}$$

Ignoring μ_i^2 and $(\vec{k}_\perp^i)^2$ in comparison with s_i, $|u_i|$, and $\vec{q}_\perp{}^2$, in our frame we can write

$$E_i + k_0 \simeq (\eta_i + X)P + \frac{f^2\vec{q}_\perp{}^2}{4XP}, \qquad E_Q^i \simeq (\eta_i + X)P - \frac{f^2\vec{q}_\perp{}^2}{4XP},$$

$$E_i - k_0' \simeq |\eta_i - X|P - \frac{f^2\vec{q}_\perp{}^2 - 4XM_p\nu}{4XP}, \qquad E_R^i \simeq |\eta_i - X|P + \frac{f^2\vec{q}_\perp{}^2}{4XP}.$$

Hence we have from Eq. (8)

$$\frac{1}{2}\sum_{\epsilon,\epsilon',\text{spins}} |\mathfrak{M}_Z^i|^2 \to \frac{2}{\mu_i{}^2}\left(\frac{1}{(1+X/\eta_i)^2} + \frac{1}{(1-X/\eta_i)^2}\right) + O(1/P^2,\text{ mass terms})$$
$$= \frac{1}{2\mu_i{}^2}\left(\frac{1}{f^2} + \frac{1}{(f-1)^2}\right) + O(1/P^2,\text{ mass terms}), \tag{9}$$

where we have used Eq. (7). It is to be noted that the right-hand side of Eq. (9) consists entirely of s- and u-channel contributions from the first two terms of Eq. (8). The s-u interference term does not contribute to the leading order as $P \to \infty$.

Now, for $s \gg M_p{}^2$, the first of Eqs. (6) can be written as[14]

$$s \simeq \frac{f^2\vec{q}_\perp{}^2}{X} = \frac{f^2}{2f-1}2M_p\nu, \tag{10}$$

where we have used Eq. (5) and $t = -\vec{q}_\perp{}^2$. Equation (10) is quadratic in f and the solutions are

$$f_\pm = \frac{s}{2M_p\nu}\left[1 \pm \left(1 - \frac{2M_p\nu}{s}\right)^{1/2}\right].$$

We shall retain either choice and use the upper sign to refer to the frame corresponding to f_+ and the lower sign to mean the one corresponding to f_-. Using now the fact that in the Bjorken limit $2M_p\nu \gg M_p{}^2$, we can replace $2M_p\nu$ by $s+u$ and write

$$f_\pm \simeq \frac{s \pm \sqrt{-us}}{s+u} = \frac{\sqrt{s}}{\sqrt{s} \mp \sqrt{-u}}. \tag{11}$$

Substituting Eq. (11) in Eq. (9), we have

$$\sum_{\epsilon,\epsilon'} |\mathfrak{M}_Z^i|^2 \to \frac{1}{2\mu_i{}^2}\left(\frac{(\sqrt{-u} \mp \sqrt{s})^2}{s} - \frac{(\sqrt{s} \mp \sqrt{-u})^2}{u}\right). \tag{12}$$

If the condition for the exclusive dominance of the Z graphs were valid in this situation, the right-hand sides of Eq. (12) and of Eq. (3) would have agreed. With the choice of f_- and in the limit when $(s+u)/\sqrt{-su} = 2M_p\nu/\sqrt{-su} \to 0$, this is indeed the case. *Hence in this limit the parton results for spin-$\frac{1}{2}$ partons can be given a field-theoretic justification,* although – with the R term in Eq. (3.6) of Ref. 3 now vanishing – there is no way of distinguishing between spin-0 and spin-$\frac{1}{2}$ partons. However, the limit $(s+u)/\sqrt{-su} \to 0$ corresponds only to a quasiforward region of the full deep inelastic kinematics. Outside this region, the right-hand sides of Eq. (12) and of Eq. (3) do not agree and non-Z graphs [Figs. 7(a) and 7(c)] are playing an important role. In fact, when we add the contributions from these diagrams, Eq.

(12) changes to [13,15]

$$\sum_{\epsilon,\epsilon'} |\mathfrak{M}^i|^2 \to \frac{1}{2\mu_i^2}\left(\frac{1}{s}[(\sqrt{-u}\mp\sqrt{s}\,)+(\sqrt{-u}\pm\sqrt{s}\,)]^2 - \frac{1}{u}[(\sqrt{s}\mp\sqrt{-u})+(\sqrt{s}\pm\sqrt{-u})]^2\right), \quad (13)$$

where the first term in each square bracket $[(\sqrt{-u}\mp\sqrt{s}\,)$ and $(\sqrt{s}\mp\sqrt{-u})$, respectively$]$ gives the Z contribution and the second term gives the non-Z contribution. This is then the same as Eq. (3). It is to be noted that the Z term in the frame with f_+ is the non-Z term in the one with f_- and vice versa.

Since graphs with noninstantaneous $\gamma\gamma$ interactions with both spin-0 and spin-$\frac{1}{2}$ partons are present for the general highly inelastic kinematics, vertices and diagrams of the type illustrated in Figs. 2, 4, and 5 cannot be ruled out. However, the impulse approximation and the pointlike interaction between each parton and a photon – the two cornerstones of the parton model – cannot account for such effects. Hence, unless these are suppressed by some mysterious reason, one cannot provide a field-theoretic justification of the parton model with spin-$\frac{1}{2}$ and spin-0 partons for highly inelastic Compton scattering in general, in the manner of Ref. 5. *This means that Eq. (3.6) in Ref. 3 is not necessarily valid in the theory of DLY unless* $(s+u)/\sqrt{-su}\to 0$. Indeed, outside this limited kinematic region, it is not possible to make any meaningful experimental prediction on deep inelastic Compton scattering within a field-theoretic parton model. Similar conclusions can be made about hadron-hadron γ-pair production and the inelastic photoproduction of low-mass muon pairs.

Finally, let us emphasize that in this paper we have not disproved the parton model for highly inelastic two-photon processes. We have simply pointed out the impossibility of providing a field-theoretic (following Drell, Levy, and Yan) justification for it in general. The predictions of Bjorken and Paschos are most interesting. The experimental verification of these predicted results would certainly be evidence in favor of the parton idea. On the other hand, there is a possibility that they would not be borne out by observation except in the quasiforward region where $(s+u)/\sqrt{-su}$ (i.e., $|\vec{k}'|\theta^2/\eta M_p$ in the lab, θ being the scattering angle and η being $|t|/2M_p\nu)\ll 1$. In that case, the success of the parton model in deep-inelastic ep scattering would not necessarily be reduced to a mere accident. The predictions of the parton model for other deep-inelastic processes with just one highly virtual photon would still stand till proved or disproved by direct experiment.

Note added in proof. Recently, Jaffe[16] has shown that the photoproduction of *high*-mass μ pairs can be treated using the parton model.

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[4]R. E. Taylor, in *International Symposium on Electron and Photon Interactions at High Energies, Liverpool, England, 1969,* edited by D. W. Braben and R. E. Rand (Daresbury Nuclear Physics Laboratory, Daresbury, Lancashire, England, 1970), p. 251, and references therein. A more up-to-date report is given by R. E. Taylor, SLAC Report No. SLAC-PUB-740 (unpublished).

[5]S. D. Drell, D. J. Levy, and T.-M. Yan, Phys. Rev. 187, 2159 (1969); Phys. Rev. D 1, 1035 (1970); 1, 1617 (1970); T.-M. Yan and S. D. Drell, *ibid.* 1, 2402 (1970). They consider diagrams order by order in a pseudoscalar meson-baryon field theory.

[6]For inelastic Compton scattering see Ref. 3.

[7]Inelastic photoproduction of μ pairs has been considered in the parton model by J. D. Bjorken and E. Paschos, Phys. Rev. D 1, 1450 (1970).

[8]A parton treatment of the process hadron + hadron $\to 2\gamma$ + anything has been given by E. Paschos, Rockefeller University report (unpublished).

[9]In the language of old-fashioned perturbation theory and the uncertainty principle, Δt – the time lag between the γ vertices (the interval of coherence) – is inversely proportional to the difference of energies in the energy denominator. For a Z graph this energy grows linearly with the momentum of the infinite-momentum frame and $\Delta t\to 0$. Thus, in the infinite-momentum frame, the Z graph is precisely what corresponds to an instantaneous $\gamma\gamma$ interaction. For a regular non-Z diagram, on the other hand, this denominator goes down inversely with the momentum and $\Delta t\to\infty$. For a spin-$\frac{1}{2}$ parton, then, the exclusive dominance of instantaneous two-photon interactions in the $P\to\infty$ frame is synonymous with the survival of only the Z diagrams.

[10]We refer to a "true" infinite-momentum frame, i.e., one in which all Lorentz-invariant quantities are independent of P. A lucid discussion of infinite-momentum frames has been given by S. D. Drell and T.-M. Yan,

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[11]In general, this instantaneous interaction may be nonlocal in space. However, in the kinematic limit of present interest, the configuration-space distance between the points where the two electromagnetic currents interact becomes lightlike. Hence, for our purpose, an instantaneous two-photon interaction is automatically local.

[12]This contrasts with the situation in deep-inelastic *ep* scattering where the scattered parton is very nearly on the energy shell. In this case all self-energy insertions in the intermediate states of the relevant imaginary part of the forward virtual Compton scattering amplitude cancel with corresponding contributions to the renormalization constants of the on-shell intermediate states. See the second paper in Ref. 5, especially the derivation of Eq. (68).

[13]Our conclusions remain unaffected even if we modify the non-Z contributions by strong interactions so long as the Z contributions are left intact, as is required in the parton model.

[14]Note that this is true even if $X\sim s/M_p{}^2$, because of Eq. (5).

[15]In the leading terms there is no s-u interference. The only interference is between the s-channel Z and non-Z diagrams, and similarly between the u-channel Z and non-Z diagrams.

[16]R. Jaffe (unpublished).

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$\Delta I = \frac{1}{2}$ Rule with Fermion Quarks

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It is shown that the Fierz reshuffling symmetry of the $V-A$ interaction combined with current algebra and partial conservation of axial-vector current provides a reasonable explanation for the $\Delta I=\frac{1}{2}$ rule in the framework of the three-triplet model with fermion quarks. This explanation can be distinguished from that based on an *ad hoc* boson quark assumption by studying whether the parity-violating $\Delta Y=0$ nuclear transitions contain a significant $\Delta I=2$ part.

Recently there have been several attempts[1] at explaining the $\Delta I=\frac{1}{2}$ rule for the weak nonleptonic decays by using the Fierz reshuffling property of the $V-A$ interaction and assuming that the quarks are bosons. While the Fierz transformation argument is very suggestive, the association of Bose statistics with spin-$\frac{1}{2}$ objects contradicts the connection between spin and statistics. One is prompted to ask whether it is possible to preserve certain merits of this explanation without giving up Fermi statistics. A possible candidate, which suggests itself in this connection, is the three-triplet model.[2-4] As is well known, almost all the virtues of the symmetric quark model, starting with the successes of the 56-plet of $SU(6)$, can indeed be preserved in the three-triplet model with fermion quarks due to the presence of the second $SU(3)$ degree of freedom. The purpose of this note is to comment that the same is not the case for the $\Delta I=\frac{1}{2}$ rule. One can extend the Fierz reshuffling argument to the three-triplet model with fermion quarks to yield the $\Delta I=\frac{1}{2}$ rule for only a limited set of diagrams (but not all). This, however, is enough to supplement the soft-pion results based on partial conservation of axial-vector current (PCAC) and current algebra, so that together they enable one to provide a reasonable explanation of the $\Delta I=\frac{1}{2}$ rule at least for the hyperon decays.

To present the arguments, we mention briefly the main features of the three-triplet model. The model[5] consists of nine fundamental spin-$\frac{1}{2}$ particles with baryon number $\frac{1}{3}$ labeled by (α, i), where the Greek label runs over the indices S, U, and B, and the Latin label over the indices p, n, and λ. It allows one to define the usual $SU(3)$ group [the one which carries the familiar (I_3 and Y) generators] acting on the index i, and a second $SU(3)$ group called the $SU(3)''$ group acting on the index α. It is assumed that the relevant symmetry for the classification of hadrons is the $SU(3)\times SU(3)''$ group, even though neither $SU(3)$ nor $SU(3)''$ are exact symmetries. In particular, it is presumed that the (α, i)'s transform as the $(\underline{3}, \underline{3})$ representation of the above group, and that the low-lying baryons and mesons are bound states of $((\alpha, i), (\beta, j), (\gamma, k))$ and $((\bar{\alpha}, \bar{i}), (\beta, j))$, respectively, both transforming as $SU(3)''$ singlets.[6] Thus in this picture, the observed baryons (except possibly for those in the 2-BeV region or higher) are built out of one each of S, U, and B and are totally antisym-