$$\cos\chi \xrightarrow[\nu\to\infty]{} \frac{\Delta^2 - q^2 - \mu^2}{[(\Delta^2 - q^2 + \mu^2)^2 + 4\mu^2 q^2]^{1/2}}. \tag{24}$$

While the discrimination in question was seen to depend not only on Δ^2 and q^2 but also on x, χ on the other hand is seen to be independent of x. Note that when X is a pion, $\cos\chi$ will be equal to unity for essentially the whole of the available phase space (apart from the very small domain where $|\Delta^2 - q^2| \sim 0.02\ \mathrm{GeV}^2$). The quantization axes will now be practically identical for almost all Δ^2 and q^2, and it will be impossible to tell the s channel from the t channel by doing polarization measurements only. Equation (5), however, tells us that even in this situation we may yet have the ϕ-isotropy test as a useful experimental tool.

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*Work supported in part by the U. S. Atomic Energy Commission under Contract No. AT(30-1)-4204.

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PHYSICAL REVIEW D VOLUME 3, NUMBER 11 1 JUNE 1971

Some Consequences of Tensor-Meson Dominance

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(Received 18 January 1971)

Different versions of tensor-meson dominance (TMD) are discussed. Evidence is presented for the need for subtraction terms in the TMD relations in order to remove certain discrepancies in the predictions of tensor-meson pole dominance. Sum rules for coupling constants and results for meson and baryon mass form factors are obtained. Methods of incorporating symmetry breaking are discussed.

INTRODUCTION

The hypothesis that the matrix elements of the stress tensor $\Theta^{\lambda\sigma}$ are dominated by spin-2 mesons has been discussed by several authors.[1-5] In this note we discuss different forms of the tensor-meson-dominance (TMD) hypothesis and show that discrepancies resulting from the simplest form of TMD indicate the necessity for including subtraction terms (and possibly also form factors in the f and f' meson vertices).

We assume the following:

(1) The tensor (spin-2, traceless) part of the matrix elements of $\Theta^{\lambda\sigma}$ are SU_3-symmetric to a good approximation.[6]

(2) The matrix elements of the traceless part of $\Theta^{\lambda\sigma}$ receive important contributions from the f and f' mesons. We shall discuss different versions of this TMD hypothesis.

We shall mainly consider the matrix elements of $\Theta^{\lambda\sigma}$ between pseudoscalar meson states and between spin-$\frac{1}{2}$ baryon states.

(3) The covariantly normalized pseudoscalar-meson states $|M_\alpha(p)\rangle$, with

$$\langle M_\alpha(p_2)|M_\alpha(p_1)\rangle = (2\pi)^3 2p_0\,\delta(\vec{p}_1 - \vec{p}_2), \tag{1}$$

and the covariantly normalized $\frac{1}{2}^+$ baryon states $|B(p)\rangle$, with

$$\langle B_\alpha(p_2)|B_\alpha(p_1)\rangle = (2\pi)^3 (p_0/m)\,\delta(\vec{p}_1 - \vec{p}_2), \tag{2}$$

each form an SU_3 octet to a good approximation. We neglect η-η' mixing; this can be taken into account with a little modification.

The postulate (3), together with the assumption that to a good approximation $\Theta^{00}(x)$ acts as an SU_3 singlet operator plus an SU_3 octet operator between states at rest, leads to a quadratic mass formula for 0^- mesons and a linear mass formula for $\frac{1}{2}^+$ baryons.

The matrix elements of $\Theta^{\lambda\sigma}$ are decomposed as follows:

$$\langle M(p_2)|\Theta^{\lambda\sigma}(0)|M(p_1)\rangle = 2P^\lambda P^\sigma G_1^M(q^2) + (q^2\eta^{\lambda\sigma} - q^\lambda q^\sigma)G_2^M(q^2)\,, \tag{3}$$

where $P=\frac{1}{2}(p_1+p_2)$, $q=(p_1-p_2)$, and $G_1^M(0)=1$; and

$$\langle B(p_2)|\Theta^{\lambda\sigma}(0)|B(p_1)\rangle = \bar{u}(p_2)\left[\tfrac{1}{2}(\gamma^\lambda P^\sigma+\gamma^\sigma P^\lambda)\Gamma_1^B(q^2) + \frac{P^\lambda P^\sigma}{m}\Gamma_2^B(q^2) + \frac{1}{m}(q^2\eta^{\lambda\sigma} - q^\lambda q^\sigma)\Gamma_3^B(q^2)\right]u(p_1) \tag{4a}$$

$$= \bar{u}(p_2)\left[\frac{P^\lambda P^\sigma}{m}\Gamma_T^B + \frac{iq_\rho}{8m}(\sigma^{\lambda\rho}P^\sigma + \sigma^{\sigma\rho}P^\lambda)\Gamma_1^B + \frac{1}{m}(q^2\eta^{\lambda\sigma} - q^\lambda q^\sigma)\Gamma_3^B\right]u(p_1)\,, \tag{4b}$$

where $\Gamma_T \equiv \Gamma_1 + m\Gamma_2$, $i\sigma^{\lambda\rho} \equiv \frac{1}{2}[\gamma^\lambda, \gamma^\rho]$. The form factor Γ_T is normalized to $\Gamma_T(0)=1$ by the baryon mass, and Γ_1 to $\Gamma_1(0)=1$ by the baryon spin, so that $\Gamma_2(0)=0$. Our conventions are $(\gamma\cdot p - m)u(p)=0$, and $p\cdot q = p_0 q_0 - \vec{p}\cdot\vec{q}$ for the scalar product. We define the couplings of the f meson (with field operator $U_f^{\mu\nu}$) as follows:

$$\langle M(k_2)|U_f^{\mu\nu}(0)|M(k_1)\rangle\epsilon_{\mu\nu} = (m_f^2 - q^2)^{-1}\epsilon_{\mu\nu}P^\mu P^\nu \frac{g_{fMM}}{\mu_0}\mathfrak{F}_{fMM}(q^2)\,, \tag{5}$$

$$\langle B(p_2)|U_f^{\mu\nu}(0)|B(p_1)\rangle\epsilon_{\mu\nu} = (m_f^2-q^2)^{-1}\epsilon_{\mu\nu}\bar{u}(p_2)\left\{(\gamma^\mu P^\nu + \gamma^\nu P^\mu)\frac{g_{fBB}^{(1)}}{2m_0}F_{fBB}^{(1)}(q^2) + \frac{P^\mu P^\nu}{m_0}g_{fBB}^{(2)}F_{fBB}^{(2)}(q^2)\right\}, \tag{6}$$

where $\mathfrak{F}_{fMM}(m_f^2) = F_{fBB}^{(1)}(m_f^2) = F_{fBB}^{(2)}(m_f^2) = 1$. A similar definition holds for the f'. $\epsilon^{\mu\nu}$ is the spin-2-meson polarization tensor. μ_0 and m_0 are masses introduced to make g_{fMM} and $g_{fBB}^{(1)}$ dimensionless.[7] A convenient choice is $\mu_0 = m_f$ or $m_{f'}$, $m_0 = m_B$. We define, as in Ref. 3,

$$\langle f(q)|\Theta^{\lambda\sigma}(0)|0\rangle = \epsilon^{\lambda\sigma}\frac{m_f^2}{g_T}\cos\theta_T, \qquad \langle f'(q)|\Theta^{\lambda\sigma}(0)|0\rangle = \epsilon^{\lambda\sigma}\frac{m_{f'}^2}{g_T}\sin\theta_T\,, \tag{7}$$

where θ_T is the f-f' mixing angle. We shall take $\theta_T \approx 30°$.[8]

(4) We shall generalize the TMD hypothesis as written in Ref. 3 to allow for an additional term, which we approximate by a constant,

$$G_1^M(q^2) = a_M + \frac{m_f^2}{m_f^2 - q^2}\frac{\cos\theta_T}{g_T}\frac{g_{fMM}}{2m_f}\mathfrak{F}_{fMM}(q^2) + \frac{m_{f'}^2}{m_{f'}^2-q^2}\frac{\sin\theta_T}{g_T}\frac{g_{f'MM}}{2m_{f'}}\mathfrak{F}_{f'MM}(q^2)\,, \tag{8}$$

$$\Gamma_i^B(q^2) = a_B^{(i)} + \frac{m_f^2}{m_f^2-q^2}\frac{\cos\theta_T}{g_T}\frac{g_{fBB}^{(i)}}{m_B}F_{fBB}^{(i)}(q^2) + \frac{m_{f'}^2}{m_{f'}^2-q^2}\frac{\sin\theta_T}{g_T}\frac{g_{f'BB}^{(i)}}{m_B}F_{f'BB}^{(i)}(q^2)\,, \tag{9}$$

with $i=1,2$. The constants a_M and $a_B^{(i)}$ may be thought of as subtraction constants. Evidence for their presence will be discussed below. In order to obtain numerical results, we shall add the following assumption.

(5) The f' is, to a good approximation, decoupled from states which in a quark model would not contain any strange quarks.[9] This is supported, e.g., by arguments of approximate exchange degeneracy, the observed $f'\pi\pi$ decoupling, and the ϕNN decoupling (which suggests $f'NN$ decoupling).

We shall discuss various special cases of (8) and (9) below.

I. TMD WITH PURE POLE DOMINANCE

This corresponds to assuming $\mathfrak{F}_{fMM}(q^2) = F_{fBB}^{(i)}(q^2) = \cdots = 1$, and $a_M = 0 = a_B^{(i)}$. This immediately leads to the sum rules

$$\frac{g_{fMM}}{2m_f}\cos\theta_T + \frac{g_{f'MM}}{2m_{f'}}\sin\theta_T = g_T = \frac{g_{fBB}^{(1)}}{m_B}\cos\theta_T + \frac{g_{f'BB}^{(1)}}{m_B}\sin\theta_T\,, \tag{10a}$$

$$\frac{g_{fBB}^{(2)}}{m_B}\cos\theta_T + \frac{g_{f'BB}^{(2)}}{m_B}\sin\theta_T = 0. \tag{10b}$$

These results are presumably well known. The results (10a) imply that the couplings

$$\bar{g}_{fMM} \equiv g_{fMM}/2m_f, \quad \bar{g}_{f'MM} \equiv g_{f'MM}/2m_{f'}\,, \quad \bar{g}_{fBB}^{(1)} \equiv g_{fBB}^{(1)}/m_B, \quad \bar{g}_{f'BB}^{(1)} \equiv g_{f'BB}^{(1)}/m_B \tag{11}$$

obey the relations following from SU_3 with f-f' mixing. In particular, one obtains the universality relations

$$\bar{g}_{f\pi\pi} = \bar{g}_{fK\bar{K}} + \bar{g}_{f'K\bar{K}}\tan\theta_T = \bar{g}_{f\eta\eta} + \bar{g}_{f'\eta\eta}\tan\theta_T \tag{12}$$

$$= \bar{g}_{fNN}^{(1)} = \bar{g}_{fNN}^T\,, \tag{13}$$

where $\bar{g}_{fNN}^T \equiv \bar{g}_{fNN}^{(1)} + m\bar{g}_{fNN}^{(2)}$. Using recent data on the observed widths, viz., $\Gamma(f\to\pi\pi)\approx 140$ to 150 MeV, $\Gamma(f'\to K\bar{K})\approx 52$ MeV, and assuming that $g_{f'K\bar{K}}$ and $g_{f'\eta\eta}$ have the same sign as $g_{f\pi\pi}$,[10] we obtain

$$\Gamma(f\to K\bar{K}) \approx 3.4 \text{ to } 3.8 \text{ MeV}, \qquad \Gamma(f'\to\eta\eta)\approx 0.4 \text{ to } 1.6 \text{ MeV}, \tag{14a}$$

$$(g_{fNN}^T)^2/4\pi = (g_{fNN}^{(1)})^2/4\pi \approx 5.6. \tag{14b}$$

The predicted $f \to K\bar{K}$ width is of the same order as the observed width $\Gamma \approx 4.5$ MeV. The limits for $\Gamma(f' \to \eta\eta)$ are also consistent with experiment. On the other hand, the prediction for $g^{(1)}_{fNN}$ disagrees with the estimate quoted from forward πN dispersion relations, $(g^{(1)}_{fNN})^2/4\pi \approx 53$.[11,12] However, the errors in the quoted estimates are liable to be large since the $\pi\pi\, N\bar{N}$ cut contribution has been fitted with just the f-exchange and ϵ-exchange terms.[13] Also, if a subtraction turns out to be required in the dispersion relation for the $A^{(+)}$ amplitude in πN scattering, the estimate of $g^{(1)}_{fNN}$ could be altered considerably. We expect that a more elaborate model will give a considerably smaller estimate of $g^{(1)}_{fNN}$, although probably not as small as (14b).

For the coupling $g^{(2)}_{fNN}$, Renner[5] has noted that the result $g^{(2)}_{fNN}=0$ [which follows from (10b) and $f'NN$ decoupling] has the consequence that the f is decoupled from the $A^{(+)}$ amplitude in πN scattering, which is consistent with the assumption of no subtraction for $A^{(+)}$ made in the analysis of πN dispersion relations.[11,12]

With pure pole dominance of the pion tensor mass form factor $G_1^{\pi}(q^2)$, the rms tensor mass radius r_T^{π} of the pion is predicted to be $\frac{1}{6}(r_T^{\pi})^2 \approx m_f^{-2}$. Recently, the low-momentum-transfer behavior of the 0^--meson mass form factors has been derived by the author, starting with the stress-tensor axial-vector current commutation relations.[14] The results give

$$h \equiv \tfrac{1}{6}(r_T^{\pi})^2 \approx 2m_\rho/(m_\epsilon^{\,2} f_\pi g_{A_1\epsilon\pi}) . \tag{15}$$

If for $g_{A_1\epsilon\pi}$ we use the estimate of the Gilman-Harari model[15] with chiral $SU_2\times SU_2$ sum rules saturated by $0^{\pm}$ and $1^{\pm}$ mesons, $g_{A_1\epsilon\pi}\approx m_{A_1}/f_\pi$, we obtain

$$h \approx \sqrt{2}\, m_\epsilon^{\,-2} \approx \sqrt{2}\, m_\rho^{\,-2} . \tag{16}$$

This is larger than the f-pole dominance result by a factor of 3 to 4. While we do expect corrections to the Gilman-Harari estimate, we expect them to be much smaller than would make (15) agree with $m_f^{\,-2}$.[16] We take this to be a definite indication that there are significant deviations from TMD with pure pole dominance. Further indications of such a deviation and methods of taking these into account will be discussed below.

II. TMD WITH FORM FACTORS

This corresponds to taking $a_M = 0 = a_B^{(i)}$, but not neglecting the q^2 dependence of the fMM, $f'MM$, fBB, and $f'BB$ vertices, as expressed by the form factors $\mathfrak{F}_{fMM}$, $F^{(i)}_{fBB}$, etc. The simplest consequences of (8) and (9), with $a_M=0$ and $a_B{}^{(i)}=0$, are, at $q^2=0$,

$$\bar{g}_{fMM}\mathfrak{F}_{fMM}(0)\cos\theta_T + \bar{g}_{f'MM}\mathfrak{F}_{f'MM}(0)\sin\theta_T = g_T , \tag{17}$$

$$\bar{g}^{(1)}_{fBB}\, F^{(1)}_{fBB}(0)\cos\theta_T + \bar{g}^{(1)}_{f'BB}\, F^{(1)}_{f'BB}(0)\sin\theta_T = g_T , \tag{18}$$

$$\bar{g}^{(2)}_{fBB}\, F^{(2)}_{fBB}(0)\cos\theta_T + \bar{g}^{(2)}_{f'BB}\, F^{(2)}_{f'BB}(0)\sin\theta_T = 0 . \tag{19}$$

These are less restrictive than the results (10). They require that the couplings $\bar{g}_{fMM}$, etc., of the f and f' with the octet 0^- mesons and the octet $\frac{1}{2}^+$ baryons *at zero momentum transfer* obey the same relations as predicted by SU_3 symmetry broken by f-f' mixing. Thus the couplings of the f and f' poles in the MM, MB, and BB forward scattering amplitudes would obey these relations. However, since f or f' exchange should presumably be described by a Regge trajectory rather than a fixed-pole exchange, this prediction cannot be tested without additional assumptions. We make the plausible assumption that $g_{fMM}\,\mathfrak{F}_{fMM}(0)$ is proportional to the coupling of the f trajectory to the 0^- mesons M at zero momentum transfer [and similarly for $g^{(1)}_{fBB}\, F^{(1)}_{fBB}(0)$, etc.]. This would be true, for instance, if $\mathfrak{F}_{f\pi\pi}(0)\approx \mathfrak{F}_{fK\bar{K}}(0)\approx F^{(i)}_{fNN(0)}$,[17] and if the different Regge couplings fMM and fBB vary to about the same extent between $t=0$ and $t=m_f{}^2$.

With this assumption, and the relation $\mathfrak{F}_{f\pi\pi}(q^2) \approx \mathfrak{F}_{fK\bar{K}}(q^2)$ implied by our basic assumption (1), one predicts the following for the Regge residues γ of the f trajectory in forward πN, KN, and NN scattering (at $t=0$)[18]:

$$\gamma^f_{\pi N}(0)/\gamma^f_{KN}(0) \approx g_{f\pi\pi}/g_{fK\bar{K}} , \tag{20}$$

$$\gamma^f_{\pi N}(0) \approx \gamma^f_{NN}(0) . \tag{21}$$

The most plausible Regge-trajectory assignment of the f meson seems to be to the P' trajectory.[19,20] For the couplings of the P' trajectory in forward scattering, the analysis of Barger and collaborators gives[19]

$$\begin{aligned} \gamma^{P'}_{\pi N}(t=0) &= 2.03 \pm 0.08 , \\ \gamma^{P'}_{KN}(t=0) &= 1.77 \pm 0.31 , \\ \gamma^{P'}_{NN}(t=0) &= 3.02 \pm 0.18 . \end{aligned} \tag{22}$$

Consistency of (20) with (22) requires that

$$\begin{aligned} &g_{fK\bar{K}}/g_{f\pi\pi} \approx 0.68 \text{ to } 1 ; \\ &\Gamma(f\to K\bar{K}) \approx 6.5 \text{ to } 14 \text{ MeV} . \end{aligned} \tag{23}$$

The data available on the $f\to K\bar{K}$ decay width suggest a value of the order of 4.5 MeV. This width could still be consistent with the lower limit in (23). Better estimates of $\Gamma(f\to K\bar{K})$ will enable us to test the consistency of (20) with (22). On the other hand, there seems to be a definite disagreement between the prediction (21) and the estimates (22).

Owing to the uncertainty in identifying the f trajectory with the P', it is difficult to draw a definitive conclusion. We take the following point of view. The analysis of forward scattering experiments suggests that for the trajectories important in high-energy forward scattering, the πN and NN residues are in the ratio $\frac{2}{3}$. In the quark model, this has the simple and suggestive interpretation of the universality of the coupling of these trajectories to quarks. We suggest that the f trajectory, whether or not it can be identified with the P', has this property. The result (21) would not be in agreement with this. We take this discrepancy as an indication that it is not valid to assume $a_N^T = 0$; that is, at least the nucleon tensor mass form factor $\Gamma_T^N(q^2)$ cannot be adequately approximated by the f and f' poles alone.

III. TMD WITH FORM FACTORS – SYMMETRY BREAKING

We now examine the results for the fMM couplings if $a_M \approx 0$ is a good approximation (but $a_B^{(1)} \neq 0$). In order to obtain predictions from (17) for the physical f and f' coupling constants defined by the decay modes $f \to M\overline{M}$, it is necessary to extrapolate the form factors $\mathfrak{F}_{fMM}(q^2)$ and $\mathfrak{F}_{f'MM}(q^2)$ from $q^2 = 0$ to $q^2 = m_f{}^2$ and $q^2 = m_{f'}{}^2$, respectively. Our basic assumption (1) implies $\mathfrak{F}_{f\pi\pi}(q^2) \approx \mathfrak{F}_{fK\overline{K}}(q^2)$[17]; then (17) predicts

$$\overline{g}_{f\pi\pi} = \overline{g}_{fK\overline{K}} + \overline{g}_{f'K\overline{K}} \frac{\mathfrak{F}_{f'K\overline{K}}(0)}{\mathfrak{F}_{f\pi\pi}(0)} \tan\theta_T = \cdots . \tag{24}$$

To make a rough estimate of the symmetry breaking in this relation, we assume that $\mathfrak{F}_{fMM}(q^2)$ and $\mathfrak{F}_{f'MM}(q^2)$ do not differ much in shape, so that the difference between $\mathfrak{F}_{f'MM}(0)$ and $\mathfrak{F}_{fMM}(0)$ arises mainly from the f-f' mass difference. This corresponds to a picture in which f-f' mixing and the f-f' mass difference are the primary mechanisms for SU_3 breaking in the f and f' vertices.

We can estimate the coefficient in the second term in (24) if we make the ansatz $G_1^\pi(q^2) \approx (b + cq^2)/(m_f{}^2 - q^2)$, which corresponds to a linear approximation for $\mathfrak{F}_{fMM}(q^2)$. As we shall use this over the relatively large range between $q^2 = 0$ and $q^2 = m_f{}^2$, we shall regard the results only as rough estimates. Using (15), we obtain from (24) the result

$$\overline{g}_{f\pi\pi} = \overline{g}_{fK\overline{K}} + (1+y)^{-1}\overline{g}_{f'K\overline{K}} \tan\theta_T ; \qquad y \equiv \frac{m_{f'}{}^2 - m_f{}^2}{h m_f{}^2}(h - m_f{}^{-2}) . \tag{25}$$

The estimate (16) gives $y \approx 0.32$, which gives

$$\Gamma(f \to K\overline{K}) \approx 5.3 \text{ to } 5.8 \text{ MeV} . \tag{26}$$

Comparing this with (14a) gives an estimate of the effect of symmetry breaking. An accurate measurement of the width for $f \to K\overline{K}$ will make possible a test of our assumptions.

Note that the symmetry breaking is characterized by the parameter y, which is proportional both to the f-f' mass difference and to the deviation of the tensor mass radius from the pole-dominant value. Neglecting the f-f' mass difference would give back the result (12) of pure pole dominance. For any value of the symmetry breaking, (25) preserves the simple relation

$$(\overline{g}_{f\pi\pi} - \overline{g}_{fK\overline{K}})/\overline{g}_{f'K\overline{K}} = (\overline{g}_{f\pi\pi} - \overline{g}_{f\eta\eta})/\overline{g}_{f'\eta\eta} \tag{27}$$

among the physical f and f' coupling constants.

Before proceeding to discuss TMD with subtractions, we mention another consequence of TMD (with $a_M \approx 0$ for $0^\pm$ mesons, or $a_\pi \approx a_\epsilon$), combined with a dynamical assumption. For the $f\epsilon\epsilon$ coupling, we obtain a relation similar to (10a). This gives $g_{f\pi\pi} \approx g_{f\epsilon\epsilon}$, if we assume that the ϵ has only nonstrange quarks (and therefore $g_{f'\epsilon\epsilon} \approx 0$) and $\mathfrak{F}_{f\pi\pi}(q^2) \approx \mathfrak{F}_{f\epsilon\epsilon}(q^2)$. In a Veneziano model for $\pi\epsilon$ scattering with f and A_1 trajectories, Dass and Papageorgiou[21] have obtained the relation

$$g_{A_1\epsilon\pi}{}^2 = \overline{g}_{f\pi\pi}\overline{g}_{f\epsilon\epsilon}/2\alpha' , \tag{28}$$

where α' is the slope of the degenerate π and A_1 trajectories. Combining this with $g_{f\pi\pi} \approx g_{f\epsilon\epsilon}$, and using $m_{A_1}{}^2 - m_\pi{}^2 \approx 2m_\rho{}^2$, we obtain

$$g_{A_1\epsilon\pi} \approx m_{A_1}\overline{g}_{f\pi\pi}/\sqrt{2} \approx 0.9\, m_{A_1} . \tag{29}$$

This agrees with the Gilman-Harari estimate[15] $g_{A_1\epsilon\pi} \approx m_{A_1}/f_\pi \approx m_{A_1}$, to within about 10–15%, if f_π is obtained from the pion decay rate, which gives $f_\pi \approx 135$ MeV.

Note that the above result would be obtained if $a_\pi = a_\epsilon$, even if each subtraction constant is nonzero. The results derived here do not enable a test of whether a subtraction term is required in the meson gravitational form factor. However, confirmation of a discrepancy between (20) and (22) would suggest that $a_M \neq 0$ is required.

IV. TMD WITH A SUBTRACTION IN THE BARYON GRAVITATIONAL VERTEX

We refer to (8) and (9) with $a_M \neq 0$, $a_B^{(i)} \neq 0$ as the subtracted TMD relations. We now discuss the results obtained by assuming $a_M \approx 0$ and $a_B^{(i)} \neq 0$. We write $g_{fNN}^T = v g_{f\pi\pi}$, where v is a constant ($\neq 1$). We assume that the form factors $F_{fNN}^{(i)}(q^2)$ and $F_{f\pi\pi}(q^2)$ have a similar shape.[17] Defining the tensor mass radius r_T^N of the nucleon by $\frac{1}{6}(r_T^N)^2 \equiv \Gamma_T'(0)/\Gamma_T(0)$, we then obtain

$$h_N \equiv \tfrac{1}{6}(r_T^N)^2 = v h_\pi , \quad a_N^T \equiv (1 - v) . \tag{30}$$

The constant v may be estimated by assuming universality of the coupling of the f meson to quarks. We then take

$$\langle\pi(p)|U_f^{\mu\nu}(0)|\pi(p)\rangle=\tfrac{2}{3}\langle\!\langle B(p)|U_f^{\mu\nu}(0)|B(p)\rangle\!\rangle, \qquad (31)$$

where the momentum p is very large compared to the masses,[22] and $|B(p)\rangle\!\rangle$ are baryon states normalized the same way as the meson states – according to (1) rather than (2). Equation (31) implies that $\bar{g}^T_{fNN}=\frac{3}{2}\bar{g}_{f\pi\pi}$, so that $v=\frac{3}{2}$. This gives

$$a_N^T\equiv a_N^{(1)}+ma_N^{(2)}\approx-\tfrac{1}{2},\quad (r_T^N)^2\approx\tfrac{3}{2}(r_T^\pi)^2, \qquad (32a)$$

$$(g^T_{fNN})^2/4\pi\approx 13.5. \qquad (32b)$$

The result (32b) is more than twice as large as (14b), although still much smaller than the quoted experimental estimate of 53.[11,12] We believe that the quoted estimates are in error,[13] and suggest that the universality argument leading to (32b) gives a reasonable estimate of g^T_{fNN}.

The result (32a) suggests a picture in which the low-momentum-transfer behavior of the tensor mass form factors of baryons and mesons reflects primarily the internal structure in terms of some more fundamental entities such as quarks, rather than peripheral-meson-cloud effects, in which the f and f' poles would be more important.

The results for the fMM and $f'MM$ couplings are now the same as in Sec. III above. For the baryon couplings, the sum rule

$$\begin{aligned}&\bar{g}^T_{fNN}\approx\bar{g}^T_{f\Lambda\Lambda}+(1+y_B)^{-1}\bar{g}^T_{f'\Lambda\Lambda}\tan\theta_T\approx\cdots,\\&y_B\equiv\frac{m_{f'}{}^2-m_f{}^2}{h_Nm_f{}^2}(h_N-m_f{}^{-2}),\end{aligned} \qquad (33)$$

would be preserved if the subtraction term $a_B^{(T)}$ is universal, i.e., $a_N^T\approx a_\Lambda^T\approx\cdots$.[23] In obtaining (33), the fBB and $f'BB$ form factors have been extrapolated in a manner similar to that used in obtaining (25). The result (33) would be of interest when data become available on high-energy ΛN scattering.

For the couplings $\bar{g}^{(2)}_{fBB}$, if the decoupling of the f trajectory from the $A^{(+)}$ amplitude in πN scattering[5] is confirmed, this would suggest that $a_N^{(2)}\approx 0$. The subtraction term $a_N^{(2)}$ may be related to the subtraction term in the dispersion relation for the $A^{(+)}$ amplitude in πN scattering; recent analysis of πN scattering has assumed that the high-energy behavior does not require this subtraction term, which corresponds to $a_N^{(2)}\approx 0$. As remarked earlier, however, a reanalysis of the $A^{(+)}$ amplitude seems necessary. The question is still open whether a consistent analysis with no subtractions is possible which will not lead to too large a value for $g^{(1)}_{fNN}$.

V. TMD WITH SUBTRACTIONS IN BOTH THE BARYON AND MESON GRAVITATIONAL VERTICES

We finally discuss the implications of $a_M\neq 0$, $a_B^{(i)}\neq 0$, at least for $i=1$. The former would be suggested if a better measurement of $\Gamma(f\to K\bar{K})$ revealed a discrepancy between (20) and the results (22) from forward scattering. Again writing $\bar{g}^T_{fNN}=v\bar{g}_{f\pi\pi}$, we obtain the following relation between the subtraction constants in the pion and nucleon TMD relations:

$$va_\pi-a_N^T=v-1. \qquad (34)$$

With $v=\frac{3}{2}$, this gives $3a_\pi-2a_N^T=1$.

To obtain further predictions, we reduce the number of parameters to be determined by approximating (8) and (9) as follows:

$$\begin{aligned}G_1^M(q^2)=\tilde{a}_M&+\frac{m_f{}^2}{m_f{}^2-q^2}\frac{\cos\theta_T}{g_T}\bar{g}_{fMM}(0)\\&+\frac{m_{f'}{}^2}{m_{f'}{}^2-q^2}\frac{\sin\theta_T}{g_T}\bar{g}_{f'MM}(0),\end{aligned} \qquad (35)$$

$$\Gamma_i^B(q^2)=\tilde{a}_B^{(i)}+\text{an } f\text{-pole term}+\text{ an } f'\text{-pole term}, \qquad (36)$$

where $\bar{g}_{fMM}(0)=\bar{g}_{fMM}\mathfrak{F}_{fMM}(0)$, etc.

The constants $\tilde{a}_M$ and $\tilde{a}_B^{(1)}$ also obey the sum rule (34). We now obtain

$$(r_T^N)^2=v(r_T^\pi)^2=\tfrac{3}{2}(r_T^\pi)^2, \qquad (37)$$

$$\tilde{a}_\pi=1-m_f{}^2(r_T^\pi)^2/6,\quad \tilde{a}_N^T=1-m_f{}^2(r_T^N)^2/6. \qquad (38)$$

It is interesting that the prediction (37) follows both in the scheme of Sec. IV and the scheme of this section for tensor-meson dominance.

The ratio of the pion and K-meson tensor mass radii is now given by

$$\frac{(r_T^K)^2}{(r_T^\pi)^2}=\frac{g_{fK\bar{K}}(0)}{g_{f\pi\pi}(0)}+\frac{g_{f'K\bar{K}}(0)}{g_{f\pi\pi}(0)}\frac{m_f{}^2}{m_{f'}{}^2}\tan\theta_T. \qquad (39)$$

If $r_T^K\approx r_T^\pi$,[23] this gives the sum rule

$$g_{f\pi\pi}(0)\approx g_{fK\bar{K}}(0)+g_{f'K\bar{K}}(0)\frac{m_f{}^2}{m_{f'}{}^2}\tan\theta_T. \qquad (40)$$

Note that this differs from the sum rule obtained from (17). Again, a good measurement of the $f\to K\bar{K}$ decay width may help distinguish between the two sum rules.

CONCLUSIONS

In conclusion, our results indicate that significant corrections to the f- and f'-pole dominance interpretation of TMD are present. A subtraction term is needed at least in the baryon tensor mass form factor (and possibly also in the meson tensor mass form factor) in order to eliminate discrepancies resulting from the simplest version of TMD.

In dispersion theory, this may be interpreted as showing that the contributions of the high-energy amplitudes (expressed as diffractive scattering or Pomeranchukon exchange) and nonresonant parts of the low-energy amplitudes are relatively more important in the $I=0$ and even-J amplitudes in $\pi\pi$ and $N\bar{N}$ scattering than in the $I=1$, $J=1$ amplitude.[24]

The necessity of an additional term in the tensor mass form factors leads us to speculate that an additional $I=0$ term $\mathcal{T}^{\lambda\sigma}$ is present in the stress tensor $\Theta^{\lambda\sigma}$, and that it has dynamical features different from the part of the stress tensor that can be approximated by the (renormalized) f and f' field operators. We further suggest that this term is a unitary singlet, so that the meson and baryon subtraction terms obey $a_\pi = a_K = a_\eta \equiv a_M$ and $a_N^T = a_\Lambda^T = a_\Sigma^T = a_\Xi^T \equiv a_B^T$. The possibility of a tenth 2^+ meson has been suggested by some authors.[25] However, we prefer not to identify the piece $\mathcal{T}^{\lambda\sigma}$ in the stress tensor with any 2^+ meson. We conjecture further that this piece has matrix elements simply related to the Pomeranchukon coupling. This would require subtraction terms in both the meson and baryon mass form factors. Consequences of this assumption and models in which it may hold will be examined elsewhere.

The possibility of an additional $I=0$ piece in the trace $\Theta_\sigma{}^\sigma$ of the stress tensor has been suggested by Gell-Mann and Carruthers.[26] In recent work, Chang and Freund have given other arguments for such a term.[26] A possibility is that this is just the trace of the term $\mathcal{T}^{\lambda\sigma}$ in the stress tensor.

We hope to discuss elsewhere further results from TMD supplemented by dynamical assumptions.

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[1]H. Pagels, University of North Carolina report (unpublished); P. G. O. Freund, Phys. Letters 2, 136 (1962); D. H. Sharp and W. G. Wagner, Phys. Rev. 131, 1226 (1963); S. H. Patil and Y. P. Yao, *ibid.* 153, 1455 (1966); W. Królikowski, Phys. Letters 24B, 305 (1967).

[2]R. Delbourgo, A. Salam, and J. Strathdee, Nuovo Cimento 49A, 593 (1967).

[3]K. Raman, Brown University reports, 1969 (unpublished); Phys. Rev. D 2, 1577 (1970); J. Math. Phys. (to be published). We have discussed here how a field-source identity relating $\theta^{\lambda\sigma}$ and spin-2-meson field operators may be realized in a field theory and the implications of such an identity.

[4]In recent work, Wess and Zumino [quoted in B. Zumino, lecture notes, CERN report (unpublished)] and C. J. Isham, A. Salam, and J. Strathdee [Phys. Rev. D 3, 867 (1971)] have discussed the f dominance of gravity in a vierbein formalism.

[5]After this work was completed, we received a report by B. Renner [Phys. Letters 33B, 599 (1970)] in which tensor-meson-pole dominance has been discussed, and the significant consequence of the decoupling of the f from the $A^{(+)}$ amplitude in πN scattering has been pointed out. Part of the discussion in Sec. I of our present work overlaps with Renner's paper.

[6]A related assumption has been made by M. Gell-Mann in connection with $SU_3 \times SU_3$ and scale breaking. See M. Gell-Mann, "Symmetry Violation in Hadron Physics," in *Proceedings of the Third Hawaii Topical Conference in Particle Physics* (Western Periodicals, Hollywood, Calif., 1969).

[7]Whether or not the f and f' coupling constants turn out to be proportional to meson and baryon masses seems largely to be a matter of definition of the coupling constants.

[8]The data used here are taken from A. H. Rosenfeld *et al.*, Rev. Mod. Phys. 42, 87 (1970).

[9]E.g., see J. J. Kokkedee, *The Quark Model* (Benjamin, New York, 1969).

[10]This is suggested by SU_3 broken by f-f' mixing, with the Okubo ansatz extended to tensor mesons; e.g., see S. L. Glashow and R. H. Socolow, Phys. Rev. Letters 15, 329 (1965). Also, if $g_{f'K\bar{K}}$ has a different sign from $g_{f\pi\pi}$, one obtains too large a value for $\Gamma(f\to K\bar{K})$. Note that SU_3 with mixing gives $\Gamma(f\to\eta\eta)\approx 0.8$ MeV.

[11]J. Engels, University of Karlsruhe report, 1970 (unpublished); J. Engels and G. Höhler, University of Karlsruhe report, 1970 (unpublished).

[12]P. Achuthan, H.-G. Schaile, and F. Steiner, Nucl. Phys. B24, 398 (1970).

[13]We note especially that in low-energy $I=0$, $\pi\pi$ scattering, the S-wave phase shifts seem to remain large over a wide region. We take this to suggest that nonresonant contributions are important; we believe that the ϵ- and f-pole terms do not provide an adequate parametrization of the t-channel singularities in $A^{(+)}$. We expect that the quoted estimate of $g_{fNN}^{(1)}$ includes the effect of several more complex but important intermediate states.

[14]K. Raman, Wesleyan University report, 1970 (unpublished). See also K. Raman, in Proceedings of the Coral Gables Conference on Fundamental Interactions at High Energy, 1971 (unpublished). It is shown here that both the tensor and scalar mass form factors of 0^- mesons deviate significantly from the predictions of pole dominance.

[15]F. Gilman and H. Harari, Phys. Rev. 165, 1803 (1968).

[16]As noted in Ref. 14, the prediction (15), if equal to $m_f{}^{-2}$, would imply much too large a value of $\Gamma(A_1\to\epsilon\pi)$. The result (15), together with the experimental limit $\Gamma(A_1\to\epsilon\pi) < \Gamma(A_1\to 3\pi) = 95\pm 35$ MeV, requires that $h > \sim 1/(30\mu_\pi{}^2)$, which is still much larger than the pole-dominance prediction. A better experimental estimate of $\Gamma(A_1\to\epsilon\pi)$ would enable a more restrictive statement about the deviation from pole dominance in the mass form factors.

[17]Since most of the intermediate states contributing to the dispersion relations (in the f channel) for the $f\pi\pi$, $fK\bar{K}$, $fN\bar{N}$, etc., form factors are the same, we expect

that $\mathfrak{F}_{f\pi\pi}(q^2) \approx \mathfrak{F}_{fK\bar{K}}(q^2) \approx F_{fNN}(q^2)$ will be a good approximation.

[18]We write the Reggeized f contribution to the $\pi\pi$ amplitude, for instance, as

$$F_{\pi\pi}(s,t) = S(t)(\alpha + \tfrac{1}{2})\beta(t)E_{00}^{\alpha,+}(\cos\theta_T)$$
$$\xrightarrow[s\to\infty]{} S(t)\gamma_{\pi\pi}(t)\Gamma(\alpha+\tfrac{3}{2})\pi^{-1/2}[\Gamma(\alpha+1)]^{-1}s^{\alpha},$$

where $\alpha = \alpha(t)$ is the f trajectory, $S(t) = \frac{1}{2}(1 + e^{-i\pi\alpha})/\sin\pi\alpha$, $\beta(t) = \gamma_{\pi\pi}(t)\alpha(t)(p_t q_t)^{\alpha}$, and $p_t = q_t = \frac{1}{2}(t - 4\mu_\pi^2)^{1/2}$. Note that on taking into account the correct threshold behavior of $\beta(t)$, the dependence on p_t and q_t cancels out, and there is no explicit mass dependence of the residue functions, contrary to the results of Ref. 2. For convenience, we have written the amplitude here with the Gell-Mann ghost-eliminating mechanism. However, the results in the text are not sensitive to the mechanism assumed.

[19]E.g., see the analysis of forward scattering by Barger *et al.*; see V. Barger, review talk in *Proceedings of the Topical Conference on High-Energy Collisions of Hadrons, CERN, 1968* (CERN, Geneva, 1968), and references contained therein. In the analysis of Barger *et al.*, the f lies on the P'.

[20]Recently, the possibility that the f may be on the Pomeranchuk trajectory has been revived; e.g., see Ref. 12 and references quoted therein.

[21]G. Dass and S. Papageorgiou, Nuovo Cimento 64A, 36 (1970). Our definition of $g(A_1\epsilon\pi)$ is half that of these authors, and is the same as that in Ref. 15.

[22]Alternatively, one may postulate that the universality (31) holds in the infinite-momentum frame.

[23]The subtraction terms would obey $a_N^T = a_\Lambda^T = \cdots$ and $a_\pi = a_K = a_\eta$, if they arise from a unitary singlet piece of the stress tensor.

[24]I is the isospin and J is the angular momentum. Note that ρ dominance of the pion form factor gives a reasonable picture at small q^2.

[25]E.g., see H. Munczek *et al.*, Phys. Rev. 145, 1154 (1968).

[26]See M. Gell-Mann, Ref. 6; P. Carruthers, Phys. Rev. D 2, 2265 (1970); L. N. Chang and P. G. O. Freund, Ann. Phys. (N.Y.) 61, 182 (1970).

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Phenomenological Analysis for the Electromagnetic Form Factor of the Pion*

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(Received 22 January 1971)

Using the results from electron-positron colliding-beam experiments, a phenomenological analysis has been made by means of the continuous-dispersion sum rules for the pion electromagnetic form factor. Results such as $\Gamma_\rho = 0.110$ GeV, $a_1(\pi\pi) = 0.028\mu^{-2}$, $|F_\pi(m_\rho^2)|^2 = 48.8$, $r_\pi = 0.62$ F, $\delta\mu$ (the pion mass difference) = 4.3 MeV have been obtained, and sum rules involving amplitudes accessible to $e^+e^- \to \pi^+\pi^-$ and $\pi e \to \pi e$ processes tested.

In this note, we present a phenomenological analysis for the electromagnetic form factor of the pion,[1] $F_\pi(s)$, by means of continuous-dispersion sum rules.[2]

By definition, a phenomenological analysis makes use of available experimental data only, without attempting to understand the underlying dynamics. For the dispersion approach, phenomenological parametrizations in the experimentally unfeasible regions, e.g., near threshold or at an unattainable high energy, are also necessary.

We begin with the following sum rules for $F_\pi(s)$, and its derivative with respect to s, at $s=0$:

$$F_\pi(0) = \frac{s_0^\beta}{\pi}\int_{s_0}^\infty \frac{ds}{s}\,\frac{\cos\pi\beta\,\mathrm{Im}F_\pi(s) + \sin\pi\beta\,\mathrm{Re}F_\pi(s)}{(s-s_0)^\beta}, \tag{1}$$

$$F_\pi'(0) = -\frac{\beta}{s_0}F_\pi(0) + \frac{s_0^\beta}{\pi}\int_{s_0}^\infty \frac{ds}{s^2} \times\frac{\cos\pi\beta\,\mathrm{Im}F_\pi(s) + \sin\pi\beta\,\mathrm{Re}F_\pi(s)}{(s-s_0)^\beta}, \tag{2}$$

where $0 \leqslant \beta < 1$, $s_0 = 4\mu^2$ (μ = pion mass). The sensitive threshold factor $1/(s-s_0)^\beta$ in Eq. (2) can be avoided, if desired, by subtracting off the representation for $F_\pi(0)$ in Eq. (1), yielding (for $0 \leqslant \beta \leqslant 1$)

$$F'(0) = \frac{1-\beta}{s_0}F_\pi(0) - \frac{s_0^{\beta-1}}{\pi}\int_{s_0}^\infty ds\,\frac{(s-s_0)^{1-\beta}}{s^2} \times[\cos\pi\beta\,\mathrm{Im}F_\pi(s) + \sin\pi\beta\,\mathrm{Re}F_\pi(s)]. \tag{2'}$$

Both Eqs. (1) and (2) are valid under the assumption that $F_\pi(s)\xrightarrow[|s|\to\infty]{} 0$. If a definite asymptotic behavior like $1/s$ for $F_\pi(s)$ is used, as in Eq. (5′) below, we will be able to derive two more useful dispersion sum rules[3] (for $0 < \beta \leqslant 1$):

$$\int_{s_0}^\infty ds\,\frac{\cos\pi\beta\,\mathrm{Im}F_\pi(s) + \sin\pi\beta\,\mathrm{Re}F_\pi(s)}{(s-s_0)^\beta} = 0, \tag{3}$$

$$\int_{-\infty}^0 ds\,\frac{\sin\pi\beta\,\mathrm{Re}F_\pi(s)}{[s(s-s_0)]^\beta} = \int_{s_0}^\infty ds\,\frac{\cos\pi\beta\,\mathrm{Im}F_\pi(s) + \sin\pi\beta\,\mathrm{Re}F_\pi(s)}{[s(s-s_0)]^\beta}. \tag{4}$$