Comments and Addenda

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Okubo's Second-Class Currents in K_{l2} and K_{l3} Decays*

Kyungsik Kang and Robert Olshansky
Department of Physics, Brown University, Providence, Rhode Island 02912
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The modification of the weak-interaction theory recently proposed by Okubo is compared with the current-current weak-interaction theory generalized to include local scalar or tensor interactions for K_{l2} and K_{l3} decays. In contrast to the latter, the existence of Okubo's tensor or scalar densities is difficult to verify from K_{l2} and K_{l3} experiments unless one has a reliable theoretical model for the pure $V-A$ vertices. In particular, we discuss various possible explanations for the K_{l3} parameters.

In a recent Letter,[1] Okubo proposed a simple way of introducing second-class currents[2] by incorporating sources other than vector densities and provided an explanation on the K_{l3} parameters. The effective weak-interaction Hamiltonian in his case is given, in terms of a charged intermediate-vector-boson field W_μ, by

$$H=g[h_\mu(x)+l_\mu(x)]W_\mu(x)+\text{H.c.}, \qquad (1)$$

where $l_\mu(x)$ is the leptonic current and $h_\mu(x)$ is the effective hadronic current given by

$$h_\mu(x)=J_\mu(x)+2\partial_\mu T_{\mu\nu}(x)-\partial_\mu S(x). \qquad (2)$$

Here $J_\mu(x)$ represents the usual hadronic current and $T_{\mu\nu}(x)$ and $S(x)$ are antisymmetric-tensor and scalar densities, respectively, which are of purely hadronic origin. The effective hadronic current (2) is derived by adding the tensor and scalar interactions involving the first-order derivative of $W_\mu(x)$ to the standard weak-interaction Hamiltonian and by performing the Kelly equivalence process[3] up to the g^2 order. The semileptonic interaction Hamiltonian density in second order follows from

$$H_{\text{eff}}(x)=ig^2\int d^4y\,\Delta^W_{\mu\nu}(x-y)T(h^\dagger_\mu(x)l_\nu(y)), \qquad (3)$$

where $\Delta^W_{\mu\nu}(x-y)$ is the propagator of the intermediate vector boson. In the limit $m_W{}^2\to\infty$ with the usual replacement $g^2/m_W{}^2=G/\sqrt{2}$, (3) coincides with the current-current interaction density of the Fermi theory.[4]

In what follows, the predictions for K_{l2} and K_{l3} decays resulting from (3) in such a limit will be compared, in particular, with those of a generalized current-current type of Hamiltonian

$$H_{\text{eff}}=\frac{G}{\sqrt{2}}\sum_i j_i\bar\psi_l\Gamma_i(1+\gamma_5)\psi_{\nu_l}+\text{H.c.}, \qquad (4)$$

where j_i represents the operators caused by the hadronic scalar, pseudoscalar, vector, axial-vector, and tensor interactions, and the Γ_i are the corresponding Dirac matrices. We will see later that the second-class terms in (2) are intrinsically different from the scalar, pseudoscalar, and tensor densities in (4) so that such second-class current effects are much more difficult to verify. We remind the reader that the presence of terms other than $V-A$ in (4) can be measured directly from experiments.[5] Also we will show that there are various ways of explaining the K_{l3} parameters in addition to the one provided by Okubo.

I. K_{l2} DECAY

Only the strange axial-vector part A^K_μ of J_μ and the strange pseudoscalar part P of S contribute to the K_{l2} matrix element. From (3), if we use

$$l_\mu=\bar\psi_l\gamma_\mu(1+\gamma_5)\psi_{\nu_l},$$

we get the matrix element

$$\langle l\bar{\nu}_l|H_{\rm eff}(0)|K^-(p)\rangle$$
$$=\frac{G}{\sqrt{2}}[\langle 0|A_\mu^{K^+}(0)|K^-(p)\rangle - i\langle 0|P(0)|K^-(p)\rangle p_\mu]$$
$$\times \bar{u}_l\gamma_\mu(1+\gamma_5)u_{\nu_l}\sin\theta$$
$$=\frac{G}{\sqrt{2}}\sin\theta[\langle 0|A_\mu^{K^+}(0)|K^-(p)\rangle\bar{u}_l\gamma_\mu(1+\gamma_5)u_{\nu_l}$$
$$+i\langle 0|P(0)|K^-(p)\rangle m_l\bar{u}_l(1+\gamma_5)u_{\nu_l}], \quad (5)$$

where the same Cabibbo angle θ is assumed for both the vector and scalar densities. Use of Dirac's equation is made in the second step of (5). By setting

$$\langle 0|A_\mu^{K^+}(0)|K^-(p)\rangle = +if_K p_\mu, \quad (6a)$$
$$\langle 0|P(0)|K^-(p)\rangle = f_p, \quad (6b)$$

the matrix element (5) reduces to

$$\langle l\bar{\nu}_l|H_{\rm eff}(0)|K^-(p)\rangle$$
$$=\frac{G}{\sqrt{2}}i(f_K+f_p)m_l\bar{u}_l(1+\gamma_5)u_{\nu_l}\sin\theta, \quad (7)$$

so that the ratio of K_{e2} to $K_{\mu 2}$ is unchanged (as Okubo makes it clear in his Letter):

$$\frac{\Gamma(K_{e2})}{\Gamma(K_{\mu 2})}=\frac{m_e^2}{m_\mu^2}\left(\frac{m_K^2-m_e^2}{m_K^2-m_\mu^2}\right)^2 = 2.58\times 10^{-5}.$$

Thus the pseudoscalar admixture introduced by the Okubo second-class current has no measurable effect.

This is to be contrasted with the usual pseudoscalar admixture that has been tested by experiments. Such a pseudoscalar density is conventionally introduced to the K_{l2} matrix element from (4) by[5]

$$\langle l\nu_l|H_{\rm eff}(0)|K^-(p)\rangle$$
$$=\frac{G}{\sqrt{2}}\sin\theta\{\langle 0|A_\mu^{K^+}(0)|K^-(p)\rangle\bar{u}_l\gamma_\mu(1+\gamma_5)u_{\nu_l}$$
$$+im_K\langle 0|P'(0)|K^-(p)\rangle\bar{u}_l(1+\gamma_5)u_\nu\}$$
$$=\frac{G}{\sqrt{2}}i(f_K m_l+f_p' m_K)\bar{u}_l(1+\gamma_5)u_{\nu_l}\sin\theta, \quad (8)$$

where f_p' is defined similarly as in (6b) from the matrix element of P'. It is well known that (8) gives the decay ratio

$$\frac{\Gamma(K_{e2})}{\Gamma(K_{\mu 2})}=\left(\frac{f_K m_e+f_p' m_K}{f_K m_\mu+f_p' m_K}\right)^2\left(\frac{m_K^2-m_e^2}{m_K^2-m_\mu^2}\right)^2, \quad (9)$$

which places an upper limit on the admixture of pseudoscalar current $|f_p'/f_K|<2\times 10^{-3}$ from the observed branching ratio[6] $(1.0\pm 0.6)\times 10^{-5}$. Hence the pseudoscalar current density coming from the Okubo second-class current is different from the conventionally defined pseudoscalar current density and its existence is much more difficult to detect. The same is true in the K_{l3} decay.

II. K_{l3} DECAY

Because of parity considerations, the K_{l3} hadronic matrix element contains only the scalar, vector, and antisymmetric-tensor interaction terms from (4). Thus by considering only such interaction terms in (3), one gets the K_{l3} matrix element

$$\langle \pi^0(k)l\bar{\nu}_l|H_{\rm eff}(0)|K^-(p)\rangle$$
$$=\frac{G}{\sqrt{2}}\sin\theta\langle\pi^0(k)|[V_\mu^{K^+}(0)+2\partial_\nu T_{\mu\nu}(0)$$
$$-\partial_\mu S(0)]|K^-(p)\rangle\bar{u}_l\gamma_\mu(1+\gamma_5)u_{\nu_l}. \quad (10)$$

Then by setting

$$\langle\pi^0(k)|V_\mu^{K^+}(0)|K^-(p)\rangle = \tfrac{1}{2}[(p+k)_\mu f_+(q^2)+q_\mu f_-(q^2)], \quad (11a)$$

$$\langle\pi^0(k)|T_{\mu\nu}(0)|K^-(p)\rangle$$
$$=\tfrac{1}{2}i[(p+k)_\mu q_\nu-(p+k)_\nu q_\mu]G(q^2), \quad (11b)$$
$$\langle\pi^0(k)|S(0)|K^-(p)\rangle = iH(q^2), \quad (11c)$$

with $q=p-k$, (10) becomes

$$\langle\pi^0(k)l\bar{\nu}_l|H_{\rm eff}(0)|K^-(p)\rangle$$
$$=\frac{G}{2\sqrt{2}}\sin\theta[(p+k)_\mu F_+(q^2)+q_\mu F_-(q^2)]$$
$$\times\bar{u}_l\gamma_\mu(1+\gamma_5)u_{\nu_l}, \quad (12)$$

where

$$F_+(q^2)=f_+(q^2)+2q^2G(q^2), \quad (13a)$$
$$F_-(q^2)=f_-(q^2)-2(m_K^2-m_\pi^2)G(q^2)-2H(q^2). \quad (13b)$$

On the other hand, the conventional scalar and tensor interactions are introduced through[5,7]

$$\langle\pi^0(k)l\bar{\nu}_l|H_{\rm eff}(0)|K^-(p)\rangle=\frac{G}{\sqrt{2}}\sin\theta\{i\langle\pi^0(k)|S'(0)|K^-(p)\rangle u_l(1+\gamma_5)u_{\nu_l}$$
$$+\langle\pi^0(k)|V_\mu^{K^+}(0)|K^-(p)\rangle\bar{u}_l\gamma_\mu(1+\gamma_5)u_{\nu_l}+\langle\pi^0(k)|T'_{\mu\nu}(0)|K^-(p)\rangle\bar{u}_l\sigma_{\mu\nu}(1+\gamma_5)u_{\nu_l}\}$$
$$=\frac{G}{2\sqrt{2}}\sin\theta\{-2m_K f_S(q^2)\bar{u}_l(1+\gamma_5)u_{\nu_l}+[f_+(q^2)(p+k)_\mu+f_-(q^2)q_\mu]\bar{u}_l\gamma_\mu(1+\gamma_5)u_{\nu_l}$$
$$+\frac{i}{m_K}[(p+k)_\mu q_\nu-(p+k)_\nu q_\mu]f_T(q^2)\bar{u}_l\sigma_{\mu\nu}(1+\gamma_5)u_{\nu_l}\}, \quad (14)$$

where the form factors $f_S(q^2)$, $f_T(q^2)$, and $f_\pm(q^2)$ are similarly defined as in (11). By making use of Dirac equations, (14) is reduced to the form of (12) in which

$$F_+ = f_+(q^2) + \frac{2m_l}{m_K} f_T(q^2), \tag{15a}$$

$$F_- = f_-(q^2) - \frac{2m_K}{m_l} f_S(q^2) - 2\frac{(p+k)\cdot(2p_l - q)}{m_l m_K} f_T(q^2), \tag{15b}$$

where p_l is the lepton momentum.

Notice that (15) is basically different from (13); while it is not possible to predict the admixture of scalar or tensor currents in (13) from experiments, such admixtures in (15) can be tested from the branching-ratio experiment of $\Gamma(K_{l3})/\Gamma(K_{\mu 3})$, from the polarization, or from the Dalitz-plot analysis because of dependence of (15) on the lepton mass and energies. The latter type of admixture has been investigated, but available data are consistent with pure vector coupling in (14).[8] We note that there have also been some theoretical attempts[9] to understand the K_{l3} parameters within the context of (15) but usually with $f_T = 0$.

To examine various possible ways to confront the experimental situation for K_{l3} decay, we define the K_{l3} parameters from (13) by the relations

$$F_+(q^2) = F_+(0)\left(1 + \Lambda_+ \frac{q^2}{m_\pi^{\ 2}}\right), \tag{16}$$

$$\xi_F = F_-(0)/F_+(0). \tag{17}$$

In addition, we define Λ_0 from the scalar form factor

$$D(q^2) = (m_K^{\ 2} - m_\pi^{\ 2}) F_+(0)\left(1 + \Lambda_0 \frac{q^2}{m_\pi^{\ 2}}\right), \tag{18}$$

which is related to the matrix element for the divergence of the effective hadronic current by

$$\begin{aligned} \langle \pi^0(k) | \partial_\mu h_\mu^{K^+}(0) | K^-(p)\rangle &= -\tfrac{1}{2} i \{(m_K^{\ 2} - m_\pi^{\ 2}) f_+(q^2) + q^2[f_-(q^2) - 2H(q^2)]\} \\ &\equiv -\tfrac{1}{2} i D(q^2). \end{aligned} \tag{19}$$

We will denote the corresponding parameters with $f_\pm(q^2)$ only as λ_+, ξ_f, and λ_0, respectively. These two sets of parameters are related through [10]

$$\Lambda_+ = \lambda_+ + 2G(0) m_\pi^{\ 2}/f_+(0), \tag{20}$$

$$\Lambda_0 = \lambda_0 - 2H(0) m_\pi^{\ 2}/[f_+(0)(m_K^{\ 2} - m_\pi^{\ 2})], \tag{21}$$

$$\begin{aligned} \xi_F &= \xi_f - \frac{2(m_K^{\ 2} - m_\pi^{\ 2})G(0)}{f_+(0)} - \frac{2H(0)}{f_+(0)} \\ &= \frac{m_K^{\ 2} - m_\pi^{\ 2}}{m_\pi^{\ 2}}(\Lambda_0 - \Lambda_+). \end{aligned} \tag{22}$$

Since the parameters Λ_+ and ξ_F (or Λ_+ and Λ_0) can be determined from experiment, speculations about the values of H and G are sensitive not only to the data used for Λ_+ and Λ_0 but also to the theoretical models used for λ_+ and λ_0. In order to estimate the contributions from the second-class currents $G(q^2)$ and $H(q^2)$ in (13) to the measured form factors $F_\pm(q^2)$, however, we must depend on some theoretical models to calculate the pure vector form factors $f_\pm(q^2)$. Thus it is clear that there are a number of possible ways to explain almost any given values of the K_{l3} parameters.

Okubo has in effect focused his attention on (20). By using $\lambda_+ = m_\pi^{\ 2}/m_{K^*}^{\ 2}$ as given by a standard K^*-dominance model (here Okubo has assumed that $\Lambda_0 = \lambda_0 = 0$ in addition to $H = 0$), he concludes that a significant contribution from $G(q^2)$ is needed to give a large Λ_+ of order 0.06–0.08 and a corresponding large negative ξ_F. The experimental situation for Λ_+ is far from conclusive; a recent analysis[7, 11] gives $\Lambda_+ = 0.017 \pm 0.008$ for K^0 decays and $\Lambda_+ = 0.030 \pm 0.007$ for K^+ decays, both of which are consistent with K^* dominance and $G(0) = 0$. Thus there will be no improvement for the corresponding ξ_F value unless $\Lambda_0 = \lambda_0 \neq 0$ or $H \neq 0$.

If Λ_+ turns out to be of order 0.04–0.05, the use of a dipole form for $f_+(q^2)$, in analogy to the electromagnetic form factors of nucleons, would give a good fit without the need for introducing an additional contribution from $G(q^2)$. On the other hand,[12] if $\Lambda_+ = 0.06$–0.08, then use of the Callan-Treiman relation for the on-mass-shell form factors,

$$f_+(m_K^{\ 2}) + f_-(m_K^{\ 2}) = f_K/f_\pi, \tag{23}$$

together with phenomenological result $f_K/[f_\pi f_+(0)] \cong 1.28$, can easily give $\xi_f \cong -0.65$ even if $H(q^2) = G(q^2) = 0$.

In order to compare (21) with experiment, one can either use the Callan-Treiman relation or κ dominance for the pure vector vertex. Such conventional models[13] give a small λ_0 of order 0.02–0.03. If Λ_+ is as large as 0.06–0.08, then from (21) and (22) one can still get a large negative value of ξ_F even if $H = 0$. However, an average of world data[7] gives $\Lambda_0 \cong -0.24 \pm 0.020$, so that the presence of a contribution from $H(q^2)$ is needed to make this compatible with the conventional models for λ_0. Then the corresponding ξ_F value can be made of order -0.6 even if $G = 0$ and Λ_+ is given by K^* dominance.[14]

Finally we remark that the difficulty encountered here of distinguishing the form factors of the second-class currents may generally be true for any strangeness-changing processes,[15] as there are no strict principles like G conjugation or time-reversal invariance that serve to eliminate form factors allowed by Lorentz invariance.

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[8]The experimental upper limits on the scalar and tensor terms are summarized in Ref. 7; $|f_S/f_+| < 0.18$ and $|f_T/f_+| < 0.58$ in K^+_{e3}, and $|f_S/f_+| < 0.15$ and $|f_T/f_+| < 0.22$ in $K^+_{\mu 3}$.

[9]B. G. Kenny, Phys. Rev. Letters 20, 1217 (1968); S. L. Marateck and S. P. Rosen, Phys. Letters 19B, 497 (1969).

[10]Note from (13) that $f_+(0) = F_+(0)$.

[11]C. Rubbia reported also $\Lambda_+ = 0.016$ for neutral K decay at the Austin Meeting of the American Physical Society, Division of Particles and Fields, 1970 (unpublished), in contrast to $\Lambda_+ = 0.08 \pm 0.01$ of C.-Y. Chien *et al.*, Phys. Letters 33B, 627 (1970).

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Comments on Hadronic Helicity Conservation in Inelastic Lepton-Nucleon Reactions*

M. Elitzur†‡ and A. Pais
Rockefeller University, New York, New York 10021
(Received 16 February 1971)

If and where inelastic lepton-nucleon reactions satisfy hadronic helicity conservation in an arbitrary hadronic frame, isotropy is shown to follow with respect to an azimuthal angle, properly defined. For all instances other than s-channel helicity conservation, another dynamical variable must be made to vary in order to observe this azimuthal independence at a given incoming laboratory energy of the leptons. The discrimination between different hadronic frames with regard to possible helicity conservation demands some care, since such conservation can lead to approximately the same isotropy effect in certain specified kinematic regions.

I. INTRODUCTION

Some implications of hadronic helicity conservation have recently been discussed[1] for electroproduction and neutrino-production reactions of the type

$$l+N \to l+N+X, \tag{1}$$

where l = lepton, N = nucleon, and X is a hadronic complex. These reactions are inclusive to the extent that integrations are performed over all relative momentum variables related to X. Thus X is characterized by its over-all four-momentum only. The lepton coupling is assumed to be local. Then s-channel helicity conservation implies that the differential cross section is isotropic with respect to the azimuthal angle ϕ between the leptonic and the hadronic plane, taken in the laboratory system. In these reactions, helicity conservation can therefore be tested without recourse to polarization information of beams, targets, or final products. The same will be true for what follows. The purpose of this comment is twofold.

(a) The isotropy in ϕ is independent of the values of the other dynamical variables which may be taken as follows. (Lepton masses are neglected